"Normal generation of nonspecial line bundles on multiple coverings"

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Any line bundle $\mathcal{L}$ on a smooth curve C of genus g with $\deg\mathcal{L} \geq 2g + 1$ is normally generated, i.e., $\varphi_{\mathcal{L}}(C) \subseteq \mathbb{P}^0(C, \mathcal{L})$ is projectively normal. However, it has known that more various line bundles of degree d failing to be normally generated appear on multiple covering curves of genus g as d becomes smaller than $2g + 1$. Thus, investigating the normal generation of line bundles on multiple coverings is a natural approach to the normal generation on algebraic curves.

In this paper, we focus on the normal generation of nonspecial line bundles on multiple coverings. To do this, we introduce a concrete description for a nonspecial line bundle $\mathcal{L}$ on C such as

$$\mathcal{L} \sim \mathcal{K}_C - \phi^* g_t^0 - B + E$$

for some $B \geq 0, E > 0$, where ϕ is its covering morphism. Using this description, we obtain conditions for nonspecial line bundles $\mathcal{L}$ being normally generated in terms of B and E. We also construct nonspecial line bundles possessing the intended normal generation property on multiple coverings.