Abstract

Speaker: Fabrizio Catanese(Bayreuth, KIAS)

Title: Moduli spaces of automorphism marked varieties: curves, surfaces and the absolute Galois group

Abstract:

For several reasons it is interesting to consider moduli spaces of triples (X,G,a) where X is a projective variety, G is a finite group, and a is an effective action of G on X. If X is the canonical model of a variety of general type, then G is acting linearly on some pluricanonical model, and we have a moduli space which is a finite covering of a closed subspace M^G of the moduli space.

In the case of curves this investigation is related to the description of the singular locus of the moduli space M_g, for instance of its irreducible components (due by Cornalba), and of its compactification ₩bar{M_g} (done by the author).

In the case of surfaces there is another occurrence of Murphy's law, as shown in my joint work with Ingrid Bauer: the deformation equivalence for minimal models S and for canonical models differs drastically (nodal Burniat surfaces being the easiest example).

In the case of curves, there are interesting relations with topology. Moduli spaces of curves with a group G of automorphisms of a fixed topological type have a description by Teichmueller theory, which naturally leads to conjecture genus stabilization for rational homology groups.

I will then describe two equivalent description of its irreducible components, surveying known irreducibility results for some special groups. A new fine homological invariant was introduced in my joint work with Loenne and Perroni: it allows to prove genus stabilization in the ramified case, extending a theorem of Dunfield and Thurston in the easier unramified case.

An important application is the following.

In the 60's J. P. Serre showed that there exists a field automorphism s in the

absolute Galois group Gal, and a variety X defined over a number field, such that X and the Galois conjugate variety X^s have non isomorphic fundamental groups, in particular they are not homeomorphic.

In a joint paper with I. Bayer and F. Grunewald we prove a strong sharpening of this phenomenon discovered by Serre. Theorem. If s is not in the conjugacy class of the complex conjugation then there exists a surface (isogenous to a product) X such that X and the Galois conjugate variety X^s have non isomorphic fundamental groups.

Moreover, we give some faithful actions of Gal, related among them, in particular, we show that Gal acts faithfully on the set of connected components of the (coarse) moduli spaces of surfaces of general type.