Holographic QCD in three dimensions

Deog-Ki Hong

Pusan National University

KIAS, 14-17 October, 2010

Based on arXiv:1003.1306 with H.-U Yee

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

- In this talk I discuss the parity anomaly, the LE effective lagrangian and baryons in 3d QCD.

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

- In this talk I discuss the parity anomaly, the LE effective lagrangian and baryons in 3d QCD.

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

- In this talk I discuss the parity anomaly, the LE effective lagrangian and baryons in 3d QCD.

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

- In this talk I discuss the parity anomaly, the LE effective lagrangian and baryons in 3d QCD.

Why 3D QCD?

- High temperature limit of 4D QCD.
- Toy model for QCD
- Universality class with strongly coupled planar systems (e.g. Hubbard model at half filling or High T_c cuprates)

$$U(2n) \mapsto U(n) \times U(n)$$

- In this talk I discuss the parity anomaly, the LE effective lagrangian and baryons in 3d QCD.

Planar fermions:

- In 3D spin-1/2 fermions are described by two-component (Weyl) spinors:

$$\mathcal{L}_{\rm free} = \bar{\psi}\left(i\gamma^{\mu}_{2\times 2}\partial_{\mu} - m\right)\psi\,. \qquad (1)$$

- The 2×2 Dirac matrices are given as:

$$\gamma^0_{2\times 2} = \sigma^3, \quad \gamma^1_{2\times 2} = i\sigma^1, \quad \gamma^2_{2\times 2} = i\sigma^2$$

- There is no γ_5 in odd dimensions and the fermion mass term is therefore real.

Planar fermions:

- In 3D spin-1/2 fermions are described by two-component (Weyl) spinors:

$$\mathcal{L}_{\rm free} = \bar{\psi}\left(i\gamma^{\mu}_{2\times 2}\partial_{\mu} - m\right)\psi\,. \qquad (1)$$

- The 2×2 Dirac matrices are given as:

$$\gamma^0_{2\times 2} = \sigma^3, \quad \gamma^1_{2\times 2} = i\sigma^1, \quad \gamma^2_{2\times 2} = i\sigma^2$$

- There is no γ_5 in odd dimensions and the fermion mass term is therefore real.

Planar fermions:

- In 3D spin-1/2 fermions are described by two-component (Weyl) spinors:

$$\mathcal{L}_{\text{free}} = \bar{\psi}\left(i\gamma^{\mu}_{2\times 2}\partial_{\mu} - m\right)\psi\,. \tag{1}$$

- The 2×2 Dirac matrices are given as:

$$\gamma^0_{2\times 2} = \sigma^3, \quad \gamma^1_{2\times 2} = i\sigma^1, \quad \gamma^2_{2\times 2} = i\sigma^2$$

- There is no γ_5 in odd dimensions and the fermion mass term is therefore real.

Planar fermions:

- Under the parity, P_2, $x = (t, x_1, x_2) \mapsto x' = (t, -x_1, x_2)$, the fermions transform to

$$\psi'(x') = e^{i\delta}\gamma^1_{2\times 2}\psi(x)\,.$$

- The mass term changes its sign:

$$P_2^{-1}\mathcal{L}_{\rm free}P_2 = \bar{\psi}'(x')\left(i\gamma^\mu_{2\times 2}\partial'_\mu + m\right)\psi'(x')\,.$$

- Lagrangian is P_2-invariant, if $m = 0$.

Planar fermions:

- Under the parity, P_2,
  $x = (t, x_1, x_2) \mapsto x' = (t, -x_1, x_2)$, the fermions transform to

$$\psi'(x') = e^{i\delta}\gamma^1_{2\times 2}\psi(x)\,.$$

- The mass term changes its sign:

$$P_2^{-1}\mathcal{L}_{\text{free}}P_2 = \bar{\psi}'(x')\left(i\gamma^{\mu}_{2\times 2}\partial'_{\mu} + m\right)\psi'(x')\,.$$

- Lagrangian is P_2-invariant, if $m = 0$.

Planar fermions:

- Under the parity, P_2, $x = (t, x_1, x_2) \mapsto x' = (t, -x_1, x_2)$, the fermions transform to

$$\psi'(x') = e^{i\delta}\gamma^1_{2\times 2}\psi(x)\,.$$

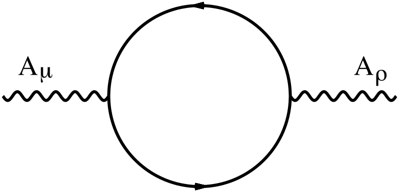
- The mass term changes its sign:

$$P_2^{-1}\mathcal{L}_{\text{free}}P_2 = \bar{\psi}'(x')\left(i\gamma^\mu_{2\times 2}\partial'_\mu + m\right)\psi'(x')\,.$$

- Lagrangian is P_2-invariant, if $m = 0$.

Parity Anomaly:

- However the parity is broken at the quantum level,

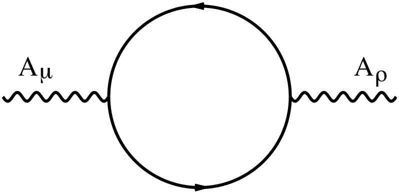


- It contains at low energy a CS term,

$$\mathcal{L}_{\text{CS}} = \frac{e^2}{8\pi} \frac{\Lambda}{|\Lambda|} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

Parity Anomaly:

- However the parity is broken at the quantum level,



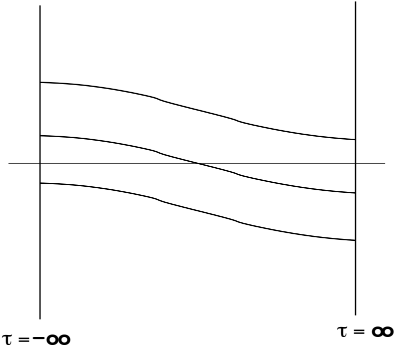
- It contains at low energy a CS term,

$$\mathcal{L}_{\rm CS} = \frac{e^2}{8\pi}\frac{\Lambda}{|\Lambda|}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

Parity Anomaly in QCD_3:

- Effective action is not gauge-invariant:

$$\det\left(\partial\!\!\!/ + A\!\!\!/\right) \longrightarrow (-1)^n \det\left(\partial\!\!\!/ + A\!\!\!/\right)$$



Parity Anomaly in QCD_3:

- To restore the gauge invariance one should add a term which cancels $(-1)^n$,

$$S_{\mathrm{ct}} = \pi \int \omega_3(A) \longrightarrow S_{\mathrm{ct}}(A) + n\,\pi\,.$$

- For even number of flavors, one can define a Dirac spinor, $\Psi = (\psi_L, \psi_R)^T$, which has a parity-invariant Dirac mass. The effective action is hence parity-invariant.

Parity Anomaly in QCD_3:

- To restore the gauge invariance one should add a term which cancels $(-1)^n$,

$$S_{\rm ct} = \pi \int \omega_3(A) \longrightarrow S_{\rm ct}(A) + n\,\pi\,.$$

- For even number of flavors, one can define a Dirac spinor, $\Psi = (\psi_L, \psi_R)^T$, which has a parity-invariant Dirac mass. The effective action is hence parity-invariant.

Parity Anomaly in QCD_3:

- To restore the gauge invariance one should add a term which cancels $(-1)^n$,

$$S_{\rm ct} = \pi \int \omega_3(A) \longrightarrow S_{\rm ct}(A) + n\,\pi\,.$$

- For even number of flavors, one can define a Dirac spinor, $\Psi = (\psi_L, \psi_R)^T$, which has a parity-invariant Dirac mass. The effective action is hence parity-invariant.

Dynamical mass generation

- For an even number $(2n)$ of flavors

$$\Psi_i = \begin{pmatrix} \psi_i \\ \psi_{i+n} \end{pmatrix}, \quad \gamma^\mu = \gamma^\mu_{2\times 2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- We have U(2) 'chiral symmetry', generated by $\mathbf{1}_{4\times 4}$, γ^3, γ^5, $[\gamma^3, \gamma^5]$.

Dynamical mass generation

- For an even number $(2n)$ of flavors

$$\Psi_i = \begin{pmatrix} \psi_i \\ \psi_{i+n} \end{pmatrix}, \quad \gamma^\mu = \gamma^\mu_{2\times 2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

- We have $\mathrm{U}(2)$ ‘chiral symmetry’, generated by $\mathbf{1}_{4\times 4}$, γ^3, γ^5, $[\gamma^3, \gamma^5]$.

Dynamical mass generation

- For $2n$ massless flavors we have $\mathrm{U}(2n)$ ‘chiral’ symmetry and non-anomalous P_4 parity:

$$\Psi_i(x) \underset{P_4}{\longmapsto} \Psi_i'(x') = e^{i\delta} \begin{pmatrix} 0 & \gamma^1_{2\times 2} \\ \gamma^1_{2\times 2} & 0 \end{pmatrix} \Psi_i(x)\,.$$

Symmetry breaking pattern

- QCD_3 is strongly coupled at low energy and confining. (KKN)
- Schwinger-Dyson analysis shows quarks get dynamical mass (Appelquist and Nash '90).
- The chiral symmetry is spontaneously broken and quarks get dynamical mass: $U(2n) \mapsto U(n) \times U(n)$.
- Half of them get mass m_{dyn} and the other half get $-m_{\mathrm{dyn}}$.

Symmetry breaking pattern

- QCD_3 is strongly coupled at low energy and confining. (KKN)
- Schwinger-Dyson analysis shows quarks get dynamical mass (Appelquist and Nash '90).
- The chiral symmetry is spontaneously broken and quarks get dynamical mass: $U(2n) \mapsto U(n) \times U(n)$.
- Half of them get mass m_{dyn} and the other half get $-m_{\text{dyn}}$.

Symmetry breaking pattern

- QCD_3 is strongly coupled at low energy and confining. (KKN)
- Schwinger-Dyson analysis shows quarks get dynamical mass (Appelquist and Nash '90).
- The chiral symmetry is spontaneously broken and quarks get dynamical mass: $U(2n) \mapsto U(n) \times U(n)$.
- Half of them get mass m_{dyn} and the other half get $-m_{\mathrm{dyn}}$.

Vafa-Witten:

- The gauge-invariant regularization gives non-positive measure:

$$\det\left(i\not{D}\right) = \pm\sqrt{\det\left(i\not{D}_4\right)}\,.$$

- But, for $N_f = 2n$ flavors the quark determinant is positive ($m \to 0$):

$$\det\left[(i\not{D}+im)(i\not{D}-im)\right]^{\frac{N_f}{2}} = \det\left[-(\not{D})^2 + m^2\right]^{\frac{N_f}{2}} \geq 0$$

- The vector symmetry $U(n) \times U(n)$ can not be spontaneously broken in P_4 invariant 3D theory. (Vafa-Witten '84)

Vafa-Witten:

- The gauge-invariant regularization gives non-positive measure:

$$\det\left(i\not{D}\right)=\pm\sqrt{\det\left(i\not{D}_4\right)}\,.$$

- But, for $N_f = 2n$ flavors the quark determinant is positive ($m \to 0$):

$$\det\left[(i\not{D}+im)(i\not{D}-im)\right]^{\frac{N_f}{2}}=\det\left[-(\not{D})^2+m^2\right]^{\frac{N_f}{2}}\geq 0$$

- The vector symmetry $U(n)\times U(n)$ can not be spontaneously broken in P_4 invariant 3D theory. (Vafa-Witten '84)

Vafa-Witten:

- The gauge-invariant regularization gives non-positive measure:

$$\det\left(i\not{D}\right) = \pm\sqrt{\det\left(i\not{D}_4\right)}\,.$$

- But, for $N_f = 2n$ flavors the quark determinant is positive ($m \to 0$):

$$\det\left[(i\not{D}+im)(i\not{D}-im)\right]^{\frac{N_f}{2}} = \det\left[-(\not{D})^2 + m^2\right]^{\frac{N_f}{2}} \geq 0$$

- The vector symmetry $U(n) \times U(n)$ can not be spontaneously broken in P_4 invariant 3D theory. (Vafa-Witten '84)

Coleman-Witten:

- Suppose the order parameter is a quark bilinear, $M_i^j = \langle \bar{\psi}_i \psi^j \rangle$, $g \in \mathrm{U}(2n)$:

$$M \longmapsto g^\dagger M g \, ; \; M \underset{P_4}{\longmapsto} P_4^{-1} M P_4 = -I_1 M I_1 \, .$$

- The vacuum energy in the large N_c limit

$$V = N_c \, \mathrm{Tr} \, F(M^2) = N_c \sum_i F(\lambda_i) \, ,$$

- The minimum occurs at $\lambda_i = \kappa^2$ and P_4 invariance requires $\mathrm{Tr} M = 0$. The unbroken symmetry is $U(n) \times U(n)$, if $\kappa \neq 0$.

Coleman-Witten:

- Suppose the order parameter is a quark bilinear, $M_i^j = \langle \bar{\psi}_i \psi^j \rangle$, $g \in \mathrm{U}(2n)$:

$$M \longmapsto g^\dagger M g \, ; \; M \underset{P_4}{\longmapsto} P_4^{-1} M P_4 = -I_1 M I_1 \, .$$

- The vacuum energy in the large N_c limit

$$V = N_c \, \mathrm{Tr} \, F(M^2) = N_c \sum_i F(\lambda_i) \, ,$$

- The minimum occurs at $\lambda_i = \kappa^2$ and P_4 invariance requires $\mathrm{Tr} M = 0$. The unbroken symmetry is $U(n) \times U(n)$, if $\kappa \neq 0$.

Coleman-Witten:

- Suppose the order parameter is a quark bilinear, $M_i^j = \langle \bar{\psi}_i \psi^j \rangle$, $g \in \mathrm{U}(2n)$:

$$M \longmapsto g^\dagger M g \,;\; M \underset{P_4}{\longmapsto} P_4^{-1} M P_4 = -I_1 M I_1 \,.$$

- The vacuum energy in the large N_c limit

$$V = N_c \,\mathrm{Tr}\, F(M^2) = N_c \sum_i F(\lambda_i)\,,$$

- The minimum occurs at $\lambda_i = \kappa^2$ and P_4 invariance requires $\mathrm{Tr} M = 0$. The unbroken symmetry is $U(n) \times U(n)$, if $\kappa \neq 0$.

Low Energy Effective Lagrangian of QCD_3

- Consider composite fields for $g \in U(2n)$

$$\phi(x) = \lim_{y \to x} \frac{|x-y|^{\gamma}}{\kappa} \psi(y)\bar{\psi}(x) \longmapsto g\phi g^{\dagger} \,.$$

- Ground state: $\langle \phi \rangle = I_3 = \mathrm{diag}\left(\mathbf{1}_{n\times n}, -\mathbf{1}_{n\times n}\right)$
- Nambu-Goldstone bosons are described by

$$\mathcal{L}_{\mathrm{B}} = \frac{f_{\pi}^2}{2} \mathrm{Tr}(\partial_{\mu}\phi)^2 = \mathrm{Tr}\left[\left(\partial_{\mu} - i\bar{A}_{\mu}\right) g^{\dagger} \left(\partial_{\mu} + i\bar{A}_{\mu}\right) g\right] \,.$$

- Redundancy of $g(x)$ is removed by gauge sym.:

$$\bar{A}_{\mu} \longmapsto u^{\dagger}\bar{A}_{\mu}u - i\partial_{\mu}u^{\dagger} \,,\; u \in \mathrm{SU}(n)_1 \times \mathrm{SU}(n)_2 \times \mathrm{U}(1)_3$$

Low Energy Effective Lagrangian of QCD_3

- Consider composite fields for $g \in U(2n)$

$$\phi(x) = \lim_{y \to x} \frac{|x-y|^{\gamma}}{\kappa} \psi(y)\bar{\psi}(x) \longmapsto g\phi g^{\dagger} \,.$$

- Ground state: $\langle \phi \rangle = I_3 = \mathrm{diag}\left(\mathbf{1}_{n\times n}, -\mathbf{1}_{n\times n}\right)$
- Nambu-Goldstone bosons are described by

$$\mathcal{L}_{\mathrm{B}} = \frac{f_{\pi}^2}{2} \mathrm{Tr}(\partial_{\mu}\phi)^2 = \mathrm{Tr}\left[\left(\partial_{\mu} - i\bar{A}_{\mu}\right) g^{\dagger} \left(\partial_{\mu} + i\bar{A}_{\mu}\right) g\right] \,.$$

- Redundancy of $g(x)$ is removed by gauge sym.:

$$\bar{A}_{\mu} \longmapsto u^{\dagger}\bar{A}_{\mu}u - i\partial_{\mu}u^{\dagger} \,,\; u \in \mathrm{SU}(n)_1 \times \mathrm{SU}(n)_2 \times \mathrm{U}(1)_3$$

Low Energy Effective Lagrangian of QCD_3

- Consider composite fields for $g \in U(2n)$

$$\phi(x) = \lim_{y \to x} \frac{|x-y|^\gamma}{\kappa} \psi(y)\bar{\psi}(x) \longmapsto g\phi g^\dagger \,.$$

- Ground state: $\langle \phi \rangle = I_3 = \mathrm{diag}\left(\mathbf{1}_{n\times n}, -\mathbf{1}_{n\times n}\right)$
- Nambu-Goldstone bosons are described by

$$\mathcal{L}_{\mathrm{B}} = \frac{f_\pi^2}{2} \mathrm{Tr}(\partial_\mu \phi)^2 = \mathrm{Tr}\left[\left(\partial_\mu - i\bar{A}_\mu\right) g^\dagger \left(\partial_\mu + i\bar{A}_\mu\right) g\right] \,.$$

- Redundancy of $g(x)$ is removed by gauge sym.:

$$\bar{A}_\mu \longmapsto u^\dagger \bar{A}_\mu u - i\partial_\mu u^\dagger \,,\; u \in \mathrm{SU}(n)_1 \times \mathrm{SU}(n)_2 \times \mathrm{U}(1)_3$$

Low Energy Effective Lagrangian of QCD_3

- Consider composite fields for $g \in U(2n)$

$$\phi(x) = \lim_{y \to x} \frac{|x-y|^\gamma}{\kappa} \psi(y)\bar{\psi}(x) \longmapsto g\phi g^\dagger .$$

- Ground state: $\langle \phi \rangle = I_3 = \mathrm{diag}\left(\mathbf{1}_{n\times n}, -\mathbf{1}_{n\times n}\right)$
- Nambu-Goldstone bosons are described by

$$\mathcal{L}_\mathrm{B} = \frac{f_\pi^2}{2} \mathrm{Tr}(\partial_\mu \phi)^2 = \mathrm{Tr}\left[\left(\partial_\mu - i\bar{A}_\mu\right) g^\dagger \left(\partial_\mu + i\bar{A}_\mu\right) g\right] .$$

- Redundancy of $g(x)$ is removed by gauge sym.:

$$\bar{A}_\mu \longmapsto u^\dagger \bar{A}_\mu u - i\partial_\mu u^\dagger , \; u \in \mathrm{SU}(n)_1 \times \mathrm{SU}(n)_2 \times \mathrm{U}(1)_3$$

Effective Largangian

- The effective Lagrangian should match P_2-anomaly. Consider two-point functions of $j_i^\mu = \bar{\psi}_i \gamma^\mu_{2\times 2} \psi_i$ $(i = 1, \cdots, 2n)$:

$$\left\langle j_i^\mu(k)\, j_j^\nu(-k) \right\rangle = \lim_{m\to 0} \frac{m_i}{|m_i|} \delta_{ij} \frac{N_c}{4\pi} \epsilon^{\mu\lambda\nu} k_\lambda \,,$$

- To match the parity anomaly we need to include CS terms such a way that preserves P_4 parity (Rajeev et al '92),

$$\mathcal{L}_{\rm eff} = \mathcal{L}_B + \frac{N_c}{4\pi} \mathcal{L}_{\rm CS}\left(\bar{A}_1\right) - \frac{N_c}{4\pi} \mathcal{L}_{\rm CS}\left(\bar{A}_2\right) + \cdots \,,$$

Effective Largangian

- The effective Lagrangian should match P_2-anomaly. Consider two-point functions of $j_i^\mu = \bar{\psi}_i \gamma^\mu_{2\times 2} \psi_i$ $(i = 1, \cdots, 2n)$:

$$\left\langle j_i^\mu(k)\, j_j^\nu(-k) \right\rangle = \lim_{m\to 0} \frac{m_i}{|m_i|} \delta_{ij} \frac{N_c}{4\pi} \epsilon^{\mu\lambda\nu} k_\lambda \,,$$

- To match the parity anomaly we need to include CS terms such a way that preserves P_4 parity (Rajeev et al '92),

$$\mathcal{L}_{\rm eff} = \mathcal{L}_B + \frac{N_c}{4\pi} \mathcal{L}_{\rm CS}\left(\bar{A}_1\right) - \frac{N_c}{4\pi} \mathcal{L}_{\rm CS}\left(\bar{A}_2\right) + \cdots \,,$$

Baryon as a vortex

- The manifold of Nambu-Goldstone fields of 3d QCD has a nontrivial topology

$$\Pi_2\left(\frac{\mathrm{SU}(2n)}{\mathrm{SU}(n)\times SU(n)\times \mathrm{U}(1)_3}\right)=\Pi_1\left(\mathrm{U}(1)_3\right)=Z\,.$$

- It should allow a vortex (baby Skyrmion),

$$Q=\int \mathrm{d}^2x\, J_0=\frac{1}{2\pi}\int \mathrm{d}^2x\,\epsilon_{0ij}\partial_i\bar{A}_{3j}\,.$$

- $\mathrm{U}(1)_3$ vorticity is the baryon number:

$$\langle J^\mu(k)\,J^\nu_{35}(-k)\rangle=\frac{N_c}{2\pi}\epsilon^{\mu\lambda\nu}k_\lambda+\mathcal{O}(k^2)\,.$$

Baryon as a vortex

- The manifold of Nambu-Goldstone fields of 3d QCD has a nontrivial topology

$$\Pi_2\left(\frac{\mathrm{SU}(2n)}{\mathrm{SU}(n)\times SU(n)\times \mathrm{U}(1)_3}\right)=\Pi_1\left(\mathrm{U}(1)_3\right)=Z\,.$$

- It should allow a vortex (baby Skyrmion),

$$Q=\int \mathrm{d}^2x\, J_0=\frac{1}{2\pi}\int \mathrm{d}^2x\,\epsilon_{0ij}\partial_i\bar{A}_{3j}\,.$$

- $\mathrm{U}(1)_3$ vorticity is the baryon number:

$$\left\langle J^{\mu}\left(k\right)J^{\nu}_{35}\left(-k\right)\right\rangle=\frac{N_c}{2\pi}\epsilon^{\mu\lambda\nu}k_{\lambda}+\mathcal{O}(k^2)\,.$$

Baryon as a vortex

- The manifold of Nambu-Goldstone fields of 3d QCD has a nontrivial topology

$$\Pi_2\left(\frac{\mathrm{SU}(2n)}{\mathrm{SU}(n)\times SU(n)\times \mathrm{U}(1)_3}\right)=\Pi_1\left(\mathrm{U}(1)_3\right)=Z\,.$$

- It should allow a vortex (baby Skyrmion),

$$Q=\int \mathrm{d}^2x\, J_0=\frac{1}{2\pi}\int \mathrm{d}^2x\, \epsilon_{0ij}\partial_i\bar{A}_{3j}\,.$$

- $\mathrm{U}(1)_3$ vorticity is the baryon number:

$$\langle J^\mu\left(k\right)J^\nu_{35}\left(-k\right)\rangle=\frac{N_c}{2\pi}\epsilon^{\mu\lambda\nu}\,k_\lambda+\mathcal{O}(k^2)\,.$$

Baryon as a vortex

- The Lagrangian should have a mutual CS term to match the discrete anomaly,

$$\mathcal{L}_{\text{eff}} \ni \mathcal{L}_{mCS} = \frac{N_c}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu\bar{A}_{3\lambda}$$

- The quark number current becomes

$$\langle J^\mu\rangle = \frac{\delta S_{\text{eff}}(A)}{\delta A_\mu} = \frac{N_c}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu\bar{A}_{3\lambda} + \cdots$$

Baryon as a vortex

- The Lagrangian should have a mutual CS term to match the discrete anomaly,

$$\mathcal{L}_{\text{eff}} \ni \mathcal{L}_{mCS} = \frac{N_c}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu\bar{A}_{3\lambda}$$

- The quark number current becomes

$$\langle J^\mu\rangle = \frac{\delta S_{\text{eff}}(A)}{\delta A_\mu} = \frac{N_c}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu\bar{A}_{3\lambda} + \cdots$$

Brane setup and geometry

- N_c D3 branes wrapping S^1 and N_f probe D7:

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○	×	×	×	×	×	×
D7	○	○	○	×	○	○	○	○	○	×

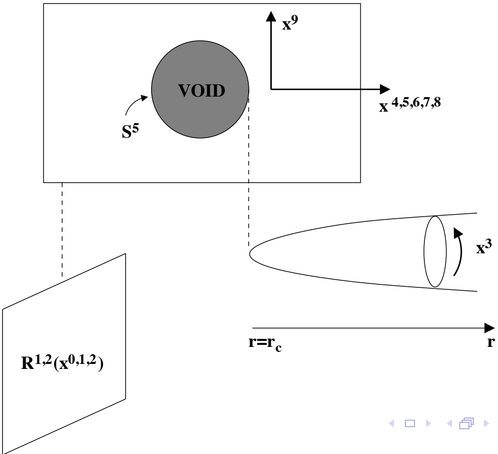
$$ds^2 = \frac{r^2}{L^2}\left(f(r)\left(dx^3\right)^2 + \left(dx^\mu\right)^2\right) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + L^2 d\Omega_5^2$$

$$F_5^{RR} = \frac{(2\pi l_s)^4 N_c}{\mathrm{Vol}(S^5)}\epsilon_5\,, \quad e^\phi = g_s\,.$$

$$(\epsilon_5 = \sin^4\theta\, d\theta \wedge \epsilon_4)$$

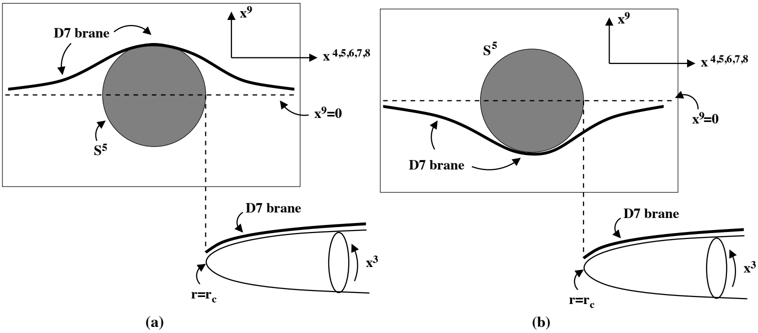
Brane setup and geometry

$$L^4 = 4\pi g_s N_c l_s^4 \;, f(r) = 1 - \frac{M_{KK}^4 L^8}{16 r^4} \;, x^3 \sim x^3 + \frac{2\pi}{M_{KK}} \;.$$



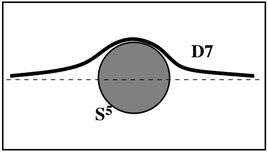
Brane setup and geometry

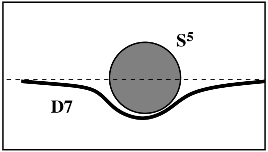
- D7 embedding:



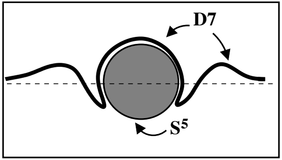
Holographic parity anomaly:

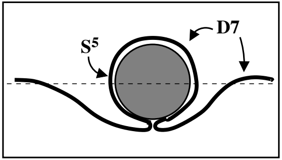
- Holographic parity anomaly:





(a) (b)





(c)

Holographic parity anomaly:

- A D7 wrapping S^5 and sitting at $r = r_c$ introduces one unit of C_0^{RR} monodromy along x_3: D3 worldvolume action contains for k units (uv data)

$$\mu_3 \frac{(2\pi l_s^2)^2}{2!} \int_{D3} C_0^{RR} \wedge F \wedge F = \frac{k}{4\pi} \int_{\mathbb{R}^{1,2}} A \wedge F \quad .$$

- The upper embedding and lower embedding of D7 brane differ by a single unit of CS term in QCD_3: The CS coefficient is therefore quantized.

Holographic parity anomaly:

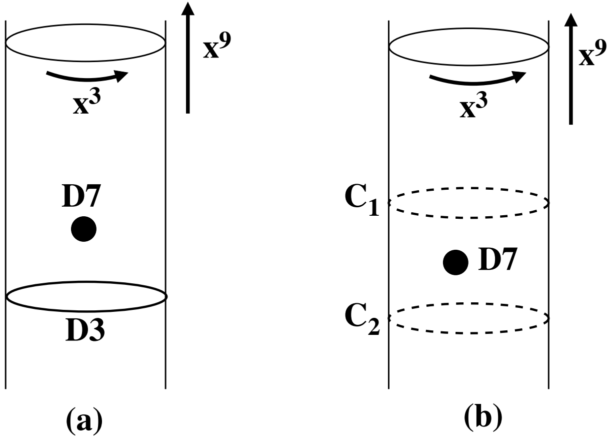
- A D7 wrapping S^5 and sitting at $r = r_c$ introduces one unit of C_0^{RR} monodromy along x_3: D3 worldvolume action contains for k units (uv data)

$$\mu_3 \frac{(2\pi l_s^2)^2}{2!} \int_{D3} C_0^{RR} \wedge F \wedge F = \frac{k}{4\pi} \int_{\mathbb{R}^{1,2}} A \wedge F \quad .$$

- The upper embedding and lower embedding of D7 brane differ by a single unit of CS term in QCD_3: The CS coefficient is therefore quantized.

Weak-coupling D-brane picture

- Another view of our D-brane setup:



Weak-coupling D-brane picture of parity anomaly:

- $C_1 - C_2$ is homological to a circle on (x_3, x_9) around D7:

$$\int_{C_1} F_1^{RR} - \int_{C_2} F_1^{RR} = 1$$

- The D3 world-volume gauge theory contains a piece induced by background C_0^{RR}

$$\frac{1}{4\pi}\int_{D3} F_1^{RR} \wedge A \wedge F = \frac{1}{4\pi}\left[\int_{C_1 \mathrm{or} C_2} F_1^{RR}\right]\int_{\mathbb{R}^{1,2}} A \wedge F$$

Weak-coupling D-brane picture of parity anomaly:

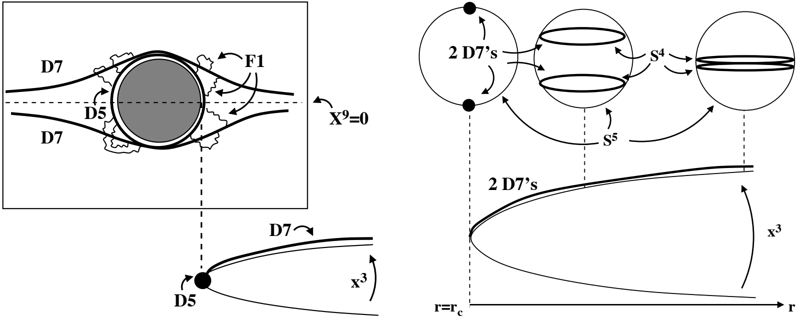
- $C_1 - C_2$ is homological to a circle on (x_3, x_9) around D7:

$$\int_{C_1} F_1^{RR} - \int_{C_2} F_1^{RR} = 1$$

- The D3 world-volume gauge theory contains a piece induced by background C_0^{RR}

$$\frac{1}{4\pi}\int_{D3} F_1^{RR} \wedge A \wedge F = \frac{1}{4\pi}\left[\int_{C_1 \mathrm{or} C_2} F_1^{RR}\right]\int_{\mathbb{R}^{1,2}} A \wedge F$$

Baryons as D5 wrapping S^5



- S^5 wrapped by D5 carries $\int_{S^5} F_5^{RR} = N_c$ fundamental string charges. D7 wraps S^4 inside S^5. ($N_f = 2n$ from now on.)

Baryons as D5 wrapping S^5

- D5 wrapping S^5 at r_c ends on D7 at two intersecting points, as monopoles in D3/D1 (Callan+Maldacena, '97)
- D5, suspended between two sets of D7 branes at $r > r_c$, can be identified as carrying a monopole charge $(+1, -1)$ with respect to $U(n) \times U(n)$ gauge symmetry on the D7 branes world-volume, where the charges sit in the trace part of $U(n)$.
- Since S^4 is common, D5 is a monopole in $(r, x^{0,1,2})$ on D7 after integrating over $S^4 \subset S^5$.

Baryons as D5 wrapping S^5

- D5 wrapping S^5 at r_c ends on D7 at two intersecting points, as monopoles in D3/D1 (Callan+Maldacena, '97)
- D5, suspended between two sets of D7 branes at $r > r_c$, can be identified as carrying a monopole charge $(+1, -1)$ with respect to $U(n) \times U(n)$ gauge symmetry on the D7 branes world-volume, where the charges sit in the trace part of $U(n)$.
- Since S^4 is common, D5 is a monopole in $(r, x^{0,1,2})$ on D7 after integrating over $S^4 \subset S^5$.

Baryons as D5 wrapping S^5

- D5 wrapping S^5 at r_c ends on D7 at two intersecting points, as monopoles in D3/D1 (Callan+Maldacena, '97)
- D5, suspended between two sets of D7 branes at $r > r_c$, can be identified as carrying a monopole charge $(+1, -1)$ with respect to $U(n) \times U(n)$ gauge symmetry on the D7 branes world-volume, where the charges sit in the trace part of $U(n)$.
- Since S^4 is common, D5 is a monopole in $(r, x^{0,1,2})$ on D7 after integrating over $S^4 \subset S^5$.

Baryons as D5 wrapping S^5

- $\theta(r)$ describe D7 emb. $(d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_4^2)$
- Induced metric on D7 is

$$g^* = \frac{r^2}{L^2}(dx^\mu)^2 + \left(\frac{L^2}{r^2 f} + L^2\left(\frac{d\theta}{dr}\right)^2\right) dr^2 + L^2 \sin^2\theta d\Omega_4^2$$

- Worldvolume action on D7 is

$$S = S_{\text{DBI}} + \mu_7 \frac{(2\pi l_s^2)^2}{2!} \int C_4^{RR} \wedge F \wedge F$$

$$C_4^{RR} \sim N_c \left[\left(\theta - \frac{\pi}{2} \pm \frac{\pi}{2}\right) - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta \right] \epsilon_4$$

Baryons as D5 wrapping S^5

- $\theta(r)$ describe D7 emb. $(d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_4^2)$
- Induced metric on D7 is

$$g^* = \frac{r^2}{L^2}(dx^\mu)^2 + \left(\frac{L^2}{r^2 f} + L^2\left(\frac{d\theta}{dr}\right)^2\right) dr^2 + L^2 \sin^2\theta d\Omega_4^2$$

- Worldvolume action on D7 is

$$S = S_{\rm DBI} + \mu_7 \frac{(2\pi l_s^2)^2}{2!} \int C_4^{RR} \wedge F \wedge F$$

$$C_4^{RR} \sim N_c \left[\left(\theta - \frac{\pi}{2} \pm \frac{\pi}{2}\right) - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta \right] \epsilon_4$$

Baryons as D5 wrapping S^5

- $\theta(r)$ describe D7 emb. $(d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_4^2)$
- Induced metric on D7 is

$$g^* = \frac{r^2}{L^2}(dx^\mu)^2 + \left(\frac{L^2}{r^2 f} + L^2\left(\frac{d\theta}{dr}\right)^2\right) dr^2 + L^2 \sin^2\theta d\Omega_4^2$$

- Worldvolume action on D7 is

$$S = S_{\mathrm{DBI}} + \mu_7 \frac{(2\pi l_s^2)^2}{2!} \int C_4^{RR} \wedge F \wedge F$$

$$C_4^{RR} \sim N_c \left[\left(\theta - \frac{\pi}{2} \pm \frac{\pi}{2}\right) - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta\right] \epsilon_4$$

Electric charge of Monopole

- Consider a D7, embedded in the upper hemisphere of S^5 $(x^9 > 0)$. $(0 \leq \theta \leq \frac{\pi}{2})$ with

$$\tilde{\mu}_7 \int_{S^4} C_4^{RR} \wedge F \wedge F = \frac{\Theta(r)}{8\pi^2} F \wedge F$$

$$\Theta(r) = \frac{16}{3} N_c \left(\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta \right)$$

- Integrating Θ over r, it leads to (flavor) CS term of boundary theory:

$$\left. \frac{\Theta(\infty)}{8\pi^2} A \wedge F = \frac{N_c}{8\pi} A \wedge F \right|_{r=\infty}$$

Electric charge of Monopole

- Consider a D7, embedded in the upper hemisphere of S^5 $(x^9 > 0)$. $(0 \leq \theta \leq \frac{\pi}{2})$ with

$$\tilde{\mu}_7 \int_{S^4} C_4^{RR} \wedge F \wedge F = \frac{\Theta(r)}{8\pi^2} F \wedge F$$

$$\Theta(r) = \frac{16}{3} N_c \left(\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta \right)$$

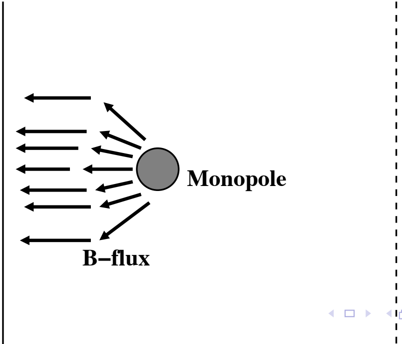
- Integrating Θ over r, it leads to (flavor) CS term of boundary theory:

$$\frac{\Theta(\infty)}{8\pi^2} A \wedge F = \frac{N_c}{8\pi} A \wedge F \Big|_{r=\infty}$$

Electric charge of Monopole

- Electric charge due to Witten effect plus medium (Lee) effect:

$$\rho_e = -\vec{\nabla} \cdot \vec{\Pi} = -\vec{\nabla} \cdot \left(\frac{\vec{E}}{e^2} + \frac{\Theta}{4\pi^2} \vec{B} \right)$$



Electric charge of Monopole

- The magnetic monopole is a dyonic object:

$$\vec{\nabla}\cdot\left(\frac{\vec{E}}{e^2}\right) = -Q_b\delta^3(\vec{x}-\vec{x}_0)\,,\ \vec{\nabla}\cdot\vec{B} = 2\pi\delta^3(\vec{x}-\vec{x}_0)\,.$$

- The physical charge density of the system is

$$\rho_e = -\vec{\nabla}\cdot\vec{\Pi} = -\frac{1}{4\pi^2}(\vec{\nabla}\Theta)\cdot\vec{B} + \left(Q_b - \frac{\Theta(\vec{x}_0)}{2\pi}\right)\delta(\vec{x}-\vec{x}_0)\,.$$

- Electric charge deposited in medium is

$$\Delta Q = -\left[\int d^2x\,\frac{B_r}{4\pi^2}\right]\int_{r_c}^{r} dr'\,\partial_{r'}\Theta(r') = \frac{\Theta(r)}{2\pi},$$

Electric charge of Monopole

- The magnetic monopole is a dyonic object:

$$\vec{\nabla} \cdot \left(\frac{\vec{E}}{e^2} \right) = -Q_b \delta^3(\vec{x} - \vec{x}_0) \, , \; \vec{\nabla} \cdot \vec{B} = 2\pi \delta^3(\vec{x} - \vec{x}_0) \, .$$

- The physical charge density of the system is

$$\rho_e = -\vec{\nabla} \cdot \vec{\Pi} = -\frac{1}{4\pi^2} (\vec{\nabla}\Theta) \cdot \vec{B} + \left(Q_b - \frac{\Theta(\vec{x}_0)}{2\pi} \right) \delta(\vec{x} - \vec{x}_0) \, .$$

- Electric charge deposited in medium is

$$\Delta Q = -\left[\int d^2x \, \frac{B_r}{4\pi^2} \right] \int_{r_c}^{r} dr' \, \partial_{r'} \Theta(r') = \frac{\Theta(r)}{2\pi},$$

Electric charge of Monopole

- The magnetic monopole is a dyonic object:

$$\vec{\nabla} \cdot \left(\frac{\vec{E}}{e^2}\right) = -Q_b \delta^3(\vec{x} - \vec{x}_0)\,, \ \vec{\nabla} \cdot \vec{B} = 2\pi \delta^3(\vec{x} - \vec{x}_0)\,.$$

- The physical charge density of the system is

$$\rho_e = -\vec{\nabla} \cdot \vec{\Pi} = -\frac{1}{4\pi^2}(\vec{\nabla}\Theta) \cdot \vec{B} + \left(Q_b - \frac{\Theta(\vec{x}_0)}{2\pi}\right)\delta(\vec{x} - \vec{x}_0)\,.$$

- Electric charge deposited in medium is

$$\Delta Q = -\left[\int d^2x \, \frac{B_r}{4\pi^2}\right] \int_{r_c}^{r} dr' \, \partial_{r'}\Theta(r') = \frac{\Theta(r)}{2\pi},$$

Electric charge of Monopole

- Applying to our situation of $U(1) \times U(1)$ gauge theory of $(\Theta(r), -\Theta(r))$ angle, a monopole of charge $(+1, -1)$ will have a total charge under the diagonal $U(1)$ given by, since $\Theta(r_c) = 0$,

$$Q = -N_c$$

- Baryons are therefore realized as dyonic monopoles in hQCD_3.
- They are equivalent to (dyonic) 't Hooft-Polyakov monopoles in a different gauge, where X^9 has a nontrival configuration.

Electric charge of Monopole

- Applying to our situation of $U(1) \times U(1)$ gauge theory of $(\Theta(r), -\Theta(r))$ angle, a monopole of charge $(+1, -1)$ will have a total charge under the diagonal $U(1)$ given by, since $\Theta(r_c) = 0$,

$$Q = -N_c$$

- Baryons are therefore realized as dyonic monopoles in hQCD_3.
- They are equivalent to (dyonic) 't Hooft-Polyakov monopoles in a different gauge, where X^9 has a nontrival configuration.

Electric charge of Monopole

- Applying to our situation of $U(1) \times U(1)$ gauge theory of $(\Theta(r), -\Theta(r))$ angle, a monopole of charge $(+1, -1)$ will have a total charge under the diagonal $U(1)$ given by, since $\Theta(r_c) = 0$,

$$Q = -N_c$$

- Baryons are therefore realized as dyonic monopoles in hQCD_3.
- They are equivalent to (dyonic) 't Hooft-Polyakov monopoles in a different gauge, where X^9 has a nontrival configuration.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.

Conclusion

- We have constructed a holographic dual of QCD_3 with D3/D7.
- Brane realization of parity anomaly: CS level number is the number of D7 branes.
- For even flavors quarks get dynamical mass, breaking $U(2n)$ to $U(n) \times U(n)$.
- Bulk D7 action gives the UV complete effective action for QCD_3.
- Baryons are (dyonic) mag. monopole with N_c electric charge by Witten effect.
- As applications, one needs T, μ, B, E and study phase diagram and transport.