Brane Tilings and Smooth Toric Fano's

Amihay Hanany

Introduction

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- What is the gauge theory on its world volume?

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- What is the gauge theory on its world volume?
- A lot of progress has been made
- Many Quiver CS theories

M2 branes and CY4

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- For given CY4 find the CS theory on M2

M2 branes and CY4

- For given CY4 find the CS theory on M2
- Are there any special CY4 we should pay attention to?

In 3+1 dimensions

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- D3 brane probing singular CY3

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- Orbifolds

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In 3+1 dimensions

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- Orbifolds
- toric CY3
- dP's (del Pezzo surfaces)

del Pezzo Surfaces

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- 2d surfaces with positive curvature

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- 2d surfaces with positive curvature
- special attention since there are finitely many of them
- each dP: take a line bundle with negative curvature such that in total CY3

Fano 3-folds

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- A line bundle with negative curvature
- Total space is a CY4
- 3d space is called Fano 3-fold

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- in 1d - classification of Riemann surfaces
- only 1 complex line with positive curvature
- Sphere - P^1
- corresponding CY2: well studied C^2/Z_2

Del Pezzo Surfaces

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- In 2d - 10 smooth surfaces with positive curvature

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Del Pezzo Surfaces

- In 2d - 10 smooth surfaces with positive curvature
- $P^1 x P^1$, P^2, blowups of P^2 by n points
- n=0,1,...,8
- Out of them 5 are toric and 5 are not.

Being Toric

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- The rank of the symmetry group equals the dimension of the CY
- non-toric means the rank is less than the dimension - harder
- Use Toric Geometry

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- Large but still finite..

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- There are the order of 1000 smooth Fano's
- Large but still finite..
- Only 18 are toric!

Study smooth toric
Fano 3-folds

Examples of Smooth Toric Fano's

Examples of Smooth Toric Fano's

- P^3, P^2xP^1, $P^1xP^1xP^1$, dP_nxP^1, etc

	Sym	Toric Data	Geometry	Id of [58]	(b_2, g)
$\mathbb{P}^3$	$U(4)$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix}$	$\mathbb{P}^3$	4	$(1, 33)$
$\mathcal{B}_4$	$[3, 2, 1]$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}$	$\mathbb{P}^2 \times \mathbb{P}^1$	24	$(2, 28)$
$\mathcal{B}_1$	$[3, 1^2]$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & 0 \end{pmatrix}$	$\mathbb{P}(\mathcal{O}_{\mathbb{P}^2} \oplus \mathcal{O}_{\mathbb{P}^2}(2))$	35	$(2, 32)$
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$\mathcal{C}_4$	$[2^2, 1^2]$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 \end{pmatrix}$	$dP_1 \times \mathbb{P}^1$	123	$(3, 25)$
$\mathcal{C}_5$	$[2^2, 1^2]$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 & 0 \end{pmatrix}$	$\mathbb{P}(\mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^1}(1, -1))$	68	$(3, 23)$
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$\mathcal{D}_1$	$[2, 1^3]$	$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 \end{pmatrix}$	$\mathbb{P}^1$-blowup of $\mathcal{B}_2$	131	$(3, 26)$
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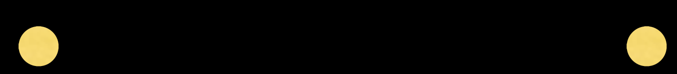

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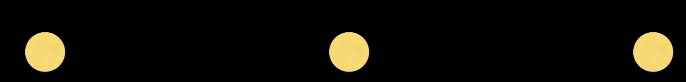

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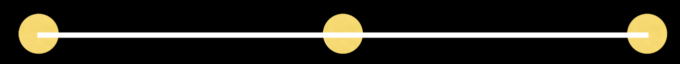

del Pezzo's
Fano 2-folds

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- 2d polygon with vertices on a 2d lattice

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- 1 internal point

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- 2d polygon with vertices on a 2d lattice
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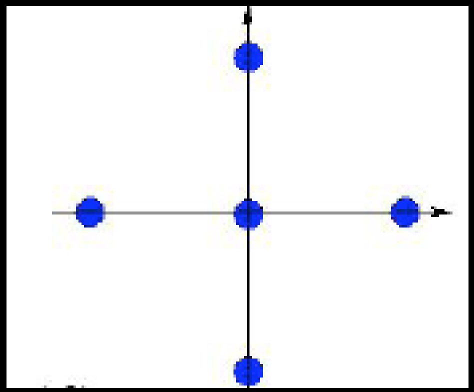
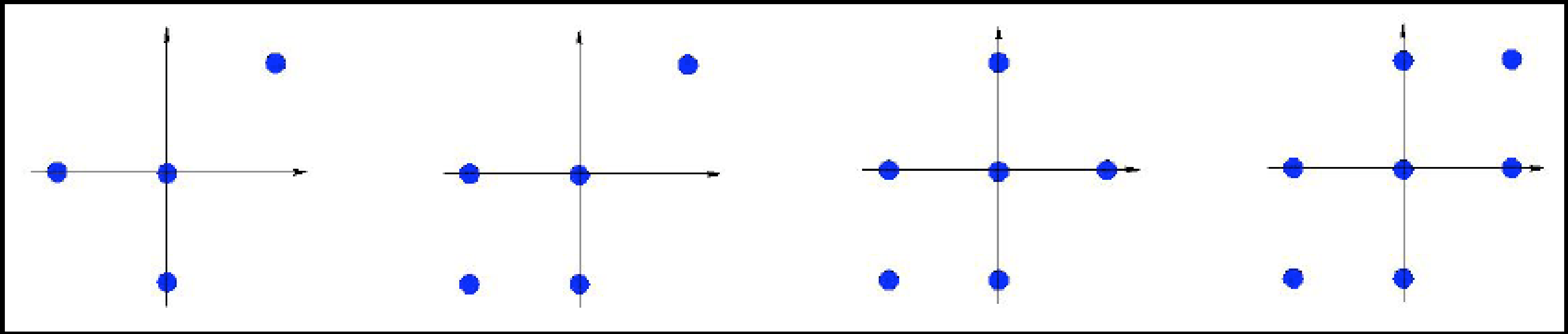
- Precisely 5 such polygons

5 diagrams
1 internal point





Toric Diagrams
smooth Fano 3-folds

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smooth Fano 3-folds

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Toric Diagrams smooth Fano 3-folds

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- no points on edges

Toric Diagrams smooth Fano 3-folds

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- 1 internal point
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Toric Diagrams smooth Fano 3-folds

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Construct the gauge theory dual to Fano's

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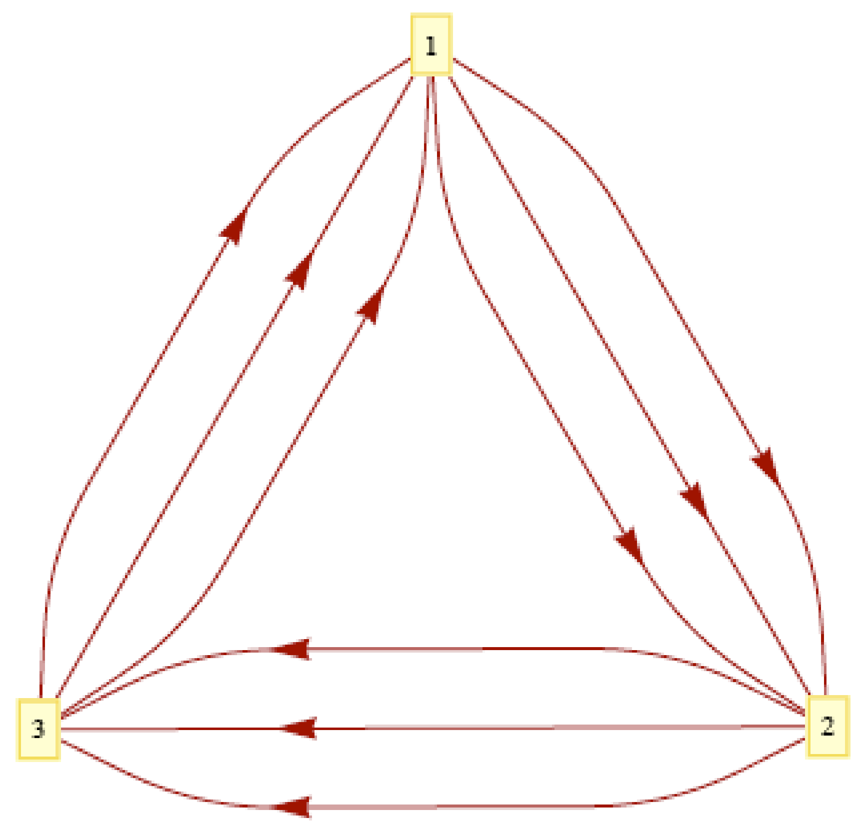
- Given by a Brane Tiling with CS levels on edges

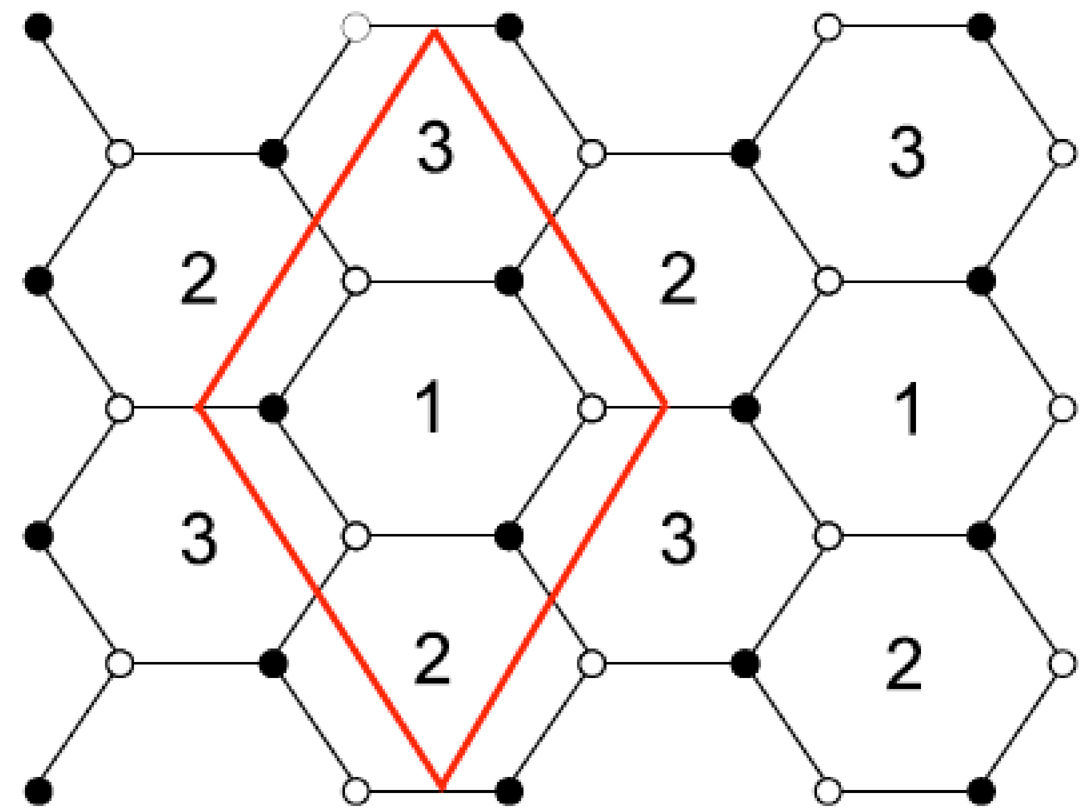
Construct the gauge theory dual to Fano's

- Given by a Brane Tiling with CS levels on edges
- A combinatorial formalism to compute the toric data

Example

$\mathcal{B}_4$ **(Toric Fano 24):** $\mathbb{P}^2 \times \mathbb{P}^1$ **(The** $M^{1,1,1}$ **Theory)**





$$W = \mathrm{Tr}\left(\epsilon_{ijk} X^i_{12} X^j_{23} X^k_{31}\right)$$

$$\vec{k} = (1, -2, 1)$$

Combinatorial data

$$k_a = \sum_i d_{ai} n_i$$

$$\begin{aligned} k_1 &= n^1_{12} + n^2_{12} + n^3_{12} - n^1_{31} - n^2_{31} - n^3_{31} \\ k_2 &= n^1_{23} + n^2_{23} + n^3_{23} - n^1_{12} - n^2_{12} - n^3_{12} \\ k_3 &= n^1_{31} + n^2_{31} + n^3_{31} - n^1_{23} - n^2_{23} - n^3_{23} \end{aligned}$$

$$n^1_{12} = -n^1_{23} = 1, \qquad n^i_{jk} = 0 \text{ otherwise}$$

$$K = \left(\begin{array}{c|ccc} & w_1 & w_2 & w_3 \\ \hline b_1 & z^{n^1_{31}} & z^{n^3_{12}} & y z^{n^2_{23}} \\ b_2 & \frac{1}{x} z^{n^3_{23}} & z^{n^2_{31}} & z^{n^1_{12}} \\ b_3 & z^{n^2_{12}} & \frac{x}{y} z^{n^1_{23}} & z^{n^3_{31}} \end{array} \right)$$

$$\begin{aligned} \mathrm{perm}(K) = &\; x y^{-1} z^{(n^1_{12} + n^1_{23} + n^1_{31})} + y z^{(n^2_{12} + n^2_{23} + n^2_{31})} + x^{-1} z^{(n^3_{12} + n^3_{23} + n^3_{31})} \\ &+ z^{(n^1_{12} + n^2_{12} + n^3_{12})} + z^{(n^1_{23} + n^2_{23} + n^3_{23})} + z^{(n^1_{31} + n^2_{31} + n^3_{31})} \\ = &\; x y^{-1} + y + x^{-1} + z + z^{-1} + 1 \end{aligned}$$

Toric Diagram

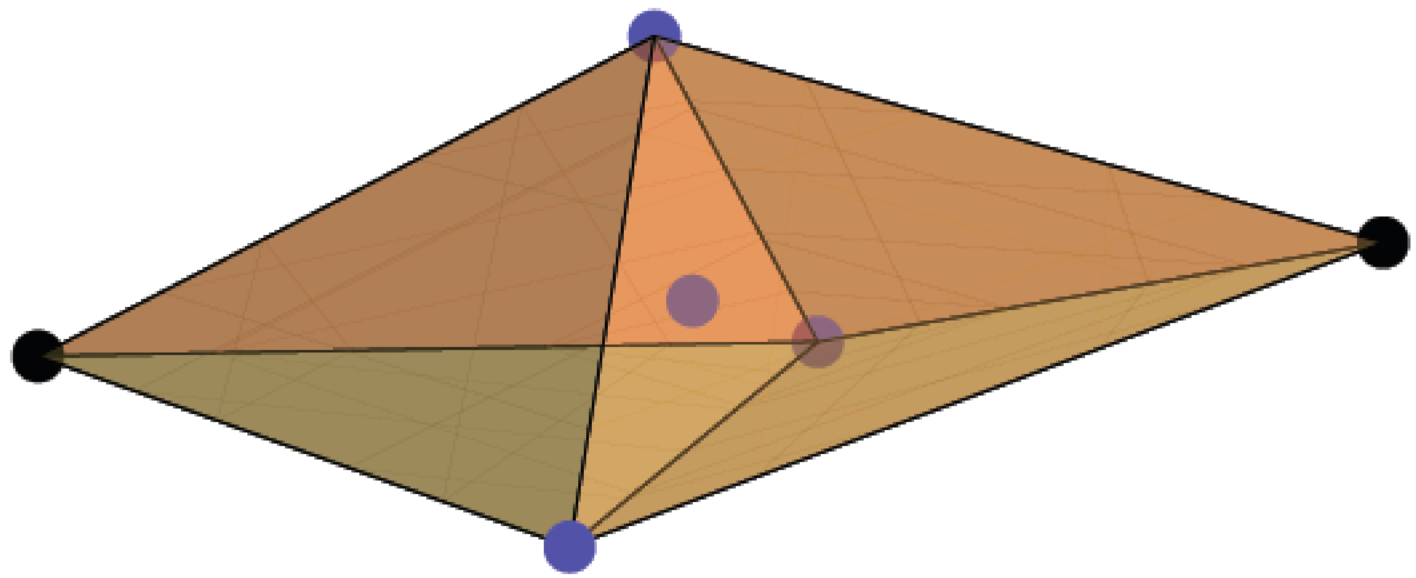


Figure 2: The toric diagram of the $M^{1,1,1}$ theory.

Hilbert Series

$$g^{\mathrm{mes}}(t;\mathcal{B}_4) = \frac{1+26t^{18}+26t^{36}+t^{54}}{(1-t^{18})^4}$$

$$= \sum_{n=0}^{\infty} [3n,0;2n]\, t^{18n}$$

Lattice of generators

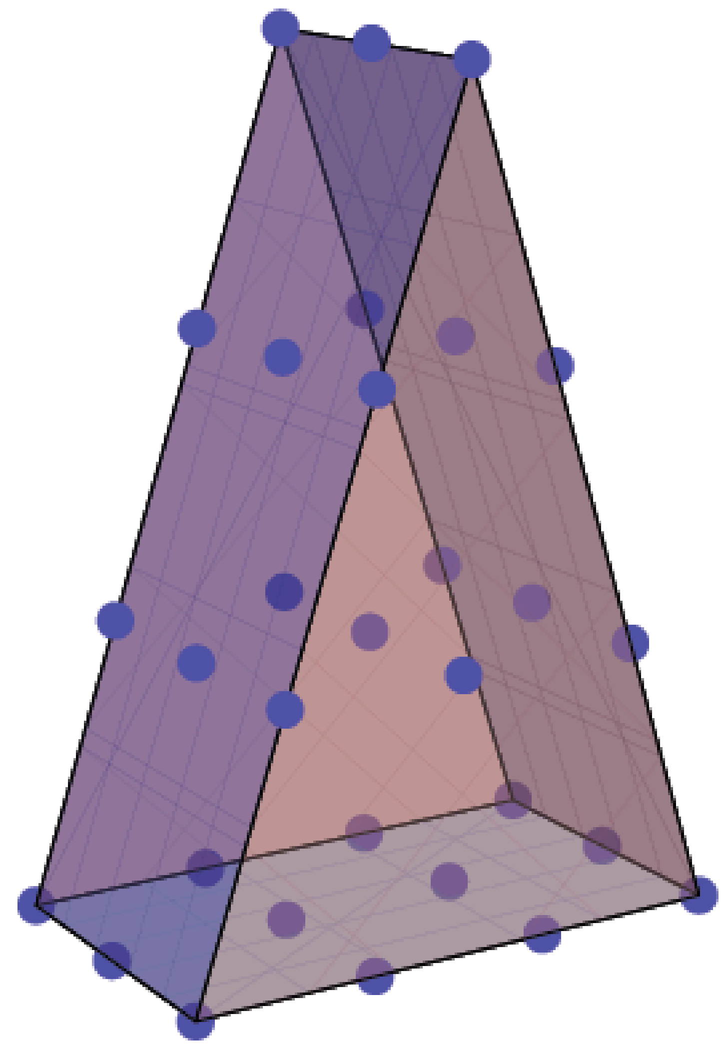


Figure 3: The lattice of generators of the $M^{1,1,1}$ theory.

Table of R charges

Quiver fields	R-charge
X^i_{12}	$7/9$
X^i_{23}	$7/9$
X^i_{31}	$4/9$

Table 4: R-charges of the quiver fields for the $M^{1,1,1}$ theory.

Fano 3-folds

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- translate toric data to brane tilings
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- $\mathbb{P}^3$, $\mathbb{P}^2$ x $\mathbb{P}^1$, $\mathbb{P}^1$ x $\mathbb{P}^1$ x $\mathbb{P}^1$, dP_1 x $\mathbb{P}^1$

Summary

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- Have introduced Fano 3-folds into M theory backgrounds

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- Compute various properties

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Thank you