Mass-deformed $\mathcal{N} = 6$ Chern-Simons-matter theory and its gravity duals

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Based on

"Supersymmetric vacua of mass-deformed M2-brane theory",
H-C. Kim & Seok Kim,
[Nucl. Phys. B839, 96 (2010)]
and
"Gravity duals of $\mathcal{N} = 6$ mass-deformed Chern-Simons-matter theory"
H-C. Kim & Seok Kim
in progress

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Introduction

- Various studies on 3d superconformal Chern-Simons-matter theories.

- One interesting example is ABJM theory.
  $\mathcal{N} = 6$ $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter theory.
  [Aharony-Bergman-Jafferis-Maldacena]

- 4d gauge theories having mass gap show very rich structure.
- gauge/gravity duality [Polchiski-Strassler], [Klebanov-Strassler], ...

- Known non-conformal M2-brane theories are mass-deformation of CFT : have not been explored well.
  - Could be potentially applied to strongly coupled 3d condenced matter system having mass gap.

Motivation

- Gravity solutions of mass-deformed M2 brane theory are known for k=1. [Bena-Warner], [Lin-Lunin-Maldacena]

- The discrepancy of classical SUSY vacua between field theory and gravity has been a puzzle.

  [Gomis, Rodriguez-Gomez, Van Raamsdonk & Verlinde]

  $\Rightarrow$ Quantum effect must be taken into account.

- For general k, the exact SUSY vacua have been unknown in both field theory & gravity.

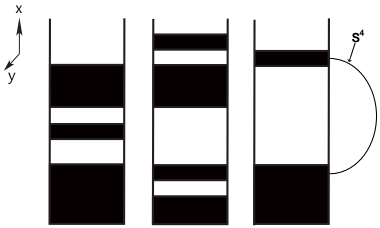
Gravity solutions of mass-deformed M2 branes

- Asymptotically $AdS_4 \times S^7$ geometry deformed by magnetic 4-form flux dual to fermion mass term. [BW], [LLM]
  - 1/2 BPS and smooth.
  - $SO(4) \times SO(4)$ isometry.

(LLM obtained this from type IIB plane wave solution : T-dual & M-lift)

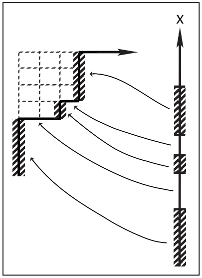
$$
\begin{aligned}
ds_{11}^2 &= e^{\frac{4\Phi}{3}}(-dt^2 + d\omega_1^2 + d\omega_2^2) + e^{-\frac{2\Phi}{3}}\left[h^2(dy^2 + dx^2) + ye^{G}d\Omega_3^2 + ye^{-G}d\tilde{\Omega}_3^2\right] , \\
e^{-2\Phi} &= h^2 - h^{-2}V^2 , \\
G_4 &= -d(e^{2\Phi}h^{-2}V) \wedge dt \wedge d\omega_1 \wedge d\omega_2 + \frac{1}{4}\left[Vd(y^2e^{-2G}) - h^2e^{-3G} *_2 d(y^2e^{2G})\right] \wedge d\tilde{\Omega}_3 \\
&\quad + \frac{1}{4}\left[Vd(y^2e^{2G}) + h^2e^{3G} *_2 d(y^2e^{-2G})\right] \wedge d\Omega_3
\end{aligned}
$$

- M2-branes are polarized into M5-branes wrapping either S^3 or $\tilde{S}^3$.



- 11d solutions are classified by the "droplet" structure on x axis.

- Magnetic flux on 4-cycle is proportional to the length of the strip (the length is quantized).



- Map between the droplets and Young diagrams
  - The size of the Young diagram = The number of M2 branes.

- The number of the solutions with N M2 charge is partitions of N.
  $\Rightarrow$ each solution is dual to a SUSY vacuum in QFT.

ABJM theory

- $\mathcal{N}=6\ \ U(N)_k \times U(N)_{-k}$ Chern-Simons gauge theory with bi-fundamental 4 hypermultiplets, $\{Z_\alpha, \Psi^\alpha\}$. [ABJM]
- N coincident M2-branes at the fixed point of the orbifold $\mathbb{R}^8/\mathbb{Z}_k$. $\rightarrow$ dual to $AdS_4 \times S^7/\mathbb{Z}_k$

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{k}{4\pi}\epsilon^{\mu\nu\lambda} Tr\left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - \tilde{A}_\mu \partial_\nu \tilde{A}_\lambda - \frac{2i}{3}\tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\lambda\right) \\
& - Tr\left(D_\mu \bar{Z}^\alpha D^\mu Z_\alpha + i\bar{\Psi}_\alpha \gamma^\mu D_\mu \Psi^\alpha\right) \\
& + \frac{2\pi i}{k} Tr\Big(\bar{Z}^\alpha Z_\alpha \bar{\Psi}_\beta \Psi^\beta - Z_\alpha \bar{Z}^\alpha \Psi^\beta \Psi_\beta + 2 Z_\alpha \bar{Z}^\beta \Psi^\alpha \bar{\Psi}_\beta - 2\bar{Z}^\alpha Z_\beta \bar{\Psi}_\alpha \Psi^\beta \\
& \quad + \epsilon_{\alpha\beta\gamma\delta} \bar{Z}^\alpha \Psi^\beta \bar{Z}^\gamma \Psi^\delta - \epsilon^{\alpha\beta\gamma\delta} Z_\alpha \bar{\Psi}_\beta Z_\gamma \bar{\Psi}_\delta\Big) \\
& - \frac{4\pi^2}{3k^2} Tr\Big(6 Z_\alpha \bar{Z}^\beta Z_\beta \bar{Z}^\alpha Z_\gamma \bar{Z}^\gamma - 4 Z_\alpha \bar{Z}^\beta Z_\gamma \bar{Z}^\alpha Z_\beta \bar{Z}^\gamma \\
& \quad - Z_\alpha \bar{Z}^\alpha Z_\beta \bar{Z}^\beta Z_\gamma \bar{Z}^\gamma - Z_\alpha \bar{Z}^\beta Z_\beta \bar{Z}^\gamma Z_\gamma \bar{Z}^\alpha\Big).
\end{aligned}
$$

notation of [HLLLP]

- The theory has a manifest $SU(4) \times U(1)_b$ global symmetry that is a subgroup of $SO(8)$ R-symmetry.
- When $k = 1$ or 2, the theory has an enhanced $\mathcal{N} = 8$ supersymmetry.

Mass deformation

- ABJM theory allows a mass deformation preserving $\mathcal{N} = 6$ Poincare supersymmetry.

- Mass deformation with a mass matrix $M = \mu\,\mathrm{diag}(1, 1, -1, -1)$.

$$\mathcal{L}_m = -Tr\left(M^2 \bar{Z} Z + i\bar{\Psi} M \Psi\right) - \frac{4\pi}{k} Tr\left(Z_\alpha \bar{Z}^\alpha Z_\beta \bar{Z}^\gamma M^\beta{}_\gamma - \bar{Z}^\alpha Z_\alpha \bar{Z}^\beta Z_\gamma M^\gamma{}_\beta\right)$$

[HLLLP]

- It breaks the global symmetry, $SU(4)$, down to $SU(2) \times SU(2) \times U(1)$.
  $\Rightarrow Z^I = (Z_\alpha, \bar{W}^{\dot{\alpha}})$

Supersymmetric vacua

- The classical SUSY vacua are given by a direct sum of the two types of blocks,

improved version of [Gomis, et al.]

1-th type : $(n-1)\times n$ rectangular block with $\bar{W}^{\dot{1}} = \bar{W}^{\dot{2}} = 0$

$$Z_1^{(n)} = \mu^{\frac{1}{2}} \begin{pmatrix} \sqrt{n-1} & 0 & & & & \\ & \sqrt{n-2} & 0 & & & \\ & & \ddots & \ddots & & \\ & & & \sqrt{2} & 0 & \\ & & & & 1 & 0 \end{pmatrix}, \quad Z_2^{(n)} = \mu^{\frac{1}{2}} \begin{pmatrix} 0 & 1 & & & & \\ & 0 & \sqrt{2} & & & \\ & & \ddots & \ddots & & \\ & & & 0 & \sqrt{n-2} & \\ & & & & 0 & \sqrt{n-1} \end{pmatrix}$$

2-nd type : $n\times (n-1)$ rectangular block with $Z_1 = Z_2 = 0$

$$\bar{W}^{\dot{1}}_{(n)} = \mu^{\frac{1}{2}} \begin{pmatrix} \sqrt{n-1} & & & & \\ 0 & \sqrt{n-2} & & & \\ & 0 & \ddots & & \\ & & \ddots & \sqrt{2} & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}, \quad \bar{W}^{\dot{2}}_{(n)} = \mu^{\frac{1}{2}} \begin{pmatrix} 0 & & & & \\ 1 & 0 & & & \\ & \sqrt{2} & \ddots & & \\ & & \ddots & 0 & \\ & & & \sqrt{n-2} & 0 \\ & & & & \sqrt{n-1} \end{pmatrix}$$

- The most general solution is

$$Z_a = \begin{pmatrix} Z_a^{(n_1)} & & & & & \\ & \ddots & & & & \\ & & Z_a^{(n_i)} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}, \quad \bar{W}^{\dot{a}} = \begin{pmatrix} 0 & & & & & \\ & \ddots & & & & \\ & & 0 & & & \\ & & & \bar{W}^a_{(n_i+1)} & & \\ & & & & \ddots & \\ & & & & & \bar{W}^a_{(n_f)} \end{pmatrix}$$

- Many discrete classical SUSY vacua.

- Let us define N_n and N'_n, the number of the first and the second type of blocks with n columns/rows, respectively.

- A set of discrete classical SUSY vacua labeled by N_n, N'_n.

- Two constraints

$$\sum_{n=1}^{\infty}\left[nN_n+(n-1)N'_n\right]=N\,,\quad \sum_{n=1}^{\infty}\left[(n-1)N_n+nN'_n\right]=N$$

$$\Rightarrow\ \sum_{n=1}^{\infty}(n-\frac{1}{2})\left(N_n+N'_n\right)=N,\quad \sum_{n=1}^{\infty}N_n=\sum_{n=1}^{\infty}N'_n.$$

- The number of classical vacua with only these constraints is bigger than the expected number, partitions of N. [Gomis, et al.]
(Ex : $\sum_{n=1}^{\infty}(nN_n)=N$, for partitions of N).

- Does quantum effect break supersymmetry?

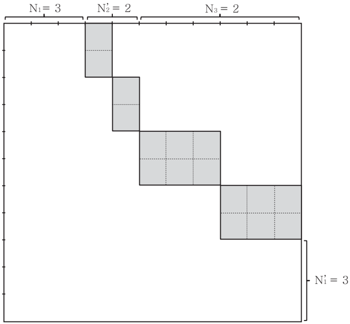
- The unbroken gauge symmetry of the classical vacua

$$\bigotimes_{n=1}^{\infty} \left[U(N_n) \times U(N'_n) \right]$$

- Example

$$N_1 = 3, N_3 = 2 \text{ and}$$
$$N'_1 = 3, N'_2 = 2$$

$\Rightarrow U(3)^2 \times U(2)^2$ unbroken gauge group.



Quantum vacua

- For general k, the theory is strongly coupled and perturbation theory does not work.

- Witten index : (# of bosonic - # of fermionic) SUSY vacua.
  - Invariant under parameter deformation of the theory.
  - It turns out that it counts the number of SUSY vacua for our case.

- We will add $\mathcal{N}=2(or\,3)$ Yang-Mills term for $U(N)\times U(N)$ gauge group and compactify on two-torus with radius R.
  - Theory of interest : $g_{YM}\to\infty$, $R\to\infty$.
  - Calculation : $g_{YM}\to 0$, $R\to 0$.

- Split the three mass scales as $kg_{YM}^2 \ll \frac{\mu}{k} \ll \frac{1}{R}$ and integrate out all higher mass fields.

- In IR, we will encounter the $\mathcal{N}=2(3)$ YM-CS gauge theory corresponding to the unbroken gauge symmetry by the classical vacuum.

- The index for $\mathcal{N}=2(3)$ YM-CS theory at level k with $U(n)$ is

$$\begin{cases} |k| \geq n\,, & \begin{pmatrix} |k| \\ n \end{pmatrix} \equiv \frac{|k|!}{n!(|k|-n)!}\,. \\ |k| < n\,, & 0\,. \end{cases}$$

[Witten], [Acharya-Vafa], [K. Ohta], [Bergman-Hanany -Karch-Kol]

- When $|k| < n$, the index vanishes, which indicates that supersymmetry is dynamically broken.

- For each classical SUSY vacuum parametrized by N_n, N'_n, we get the quantum degeneracy,

$$\prod_{n=1}^{\infty}\left[\begin{pmatrix} |k| \\ N_n \end{pmatrix}\begin{pmatrix} |k| \\ N'_n \end{pmatrix}\right]$$

- Considering the quantum degeneracy, we can compute the generating function of the index for general k,

$$\begin{aligned} I_k(q) &\equiv Tr[(-1)^F q^N] \\ &= \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^{|k|}} \sum_{\substack{n_1,n_2,\cdots,n_{|k|}=-\infty \\ (n_1+n_2+\cdots+n_{|k|}=0)}}^{\infty} q^{\frac{1}{2}\left(n_1^2+n_2^2+\cdots+n_{|k|}^2\right)} \end{aligned}$$

[H-C. Kim, S. Kim]

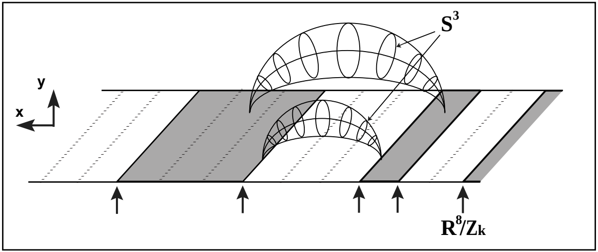
- This result reproduces the partitions of N for $k=1$.

$$I_1(q) = \prod_{n=1}^{\infty} \frac{1}{1-q^n}$$

agrees with LLM solutions for $k=1$!!

Gravity revisited

- Firstly, we can try to perform $\mathbb{Z}_k$ quotient on LLM solution in order to obtain the solutions for $k \neq 1$. $\Rightarrow$ asymptotically $AdS_4 \times S^7/\mathbb{Z}_k$
- We need to check the followings to confirm that we found right solutions.
  1. Preserve at least 12 supersymmetries
  2. The correct number of solutions (compared to field theory result)
  3. Quantization of four-form flux on each 4-sphere
- Torsion 3-cycle $S^3/\mathbb{Z}_k$ appears due to $\mathbb{Z}_k$ quotient.



- Discrete torsion : a discrete holonomy of 3-form field on a torsion 3-cycle in $S^7/\mathbb{Z}_k$, which takes value in $0 \leq l < k$.

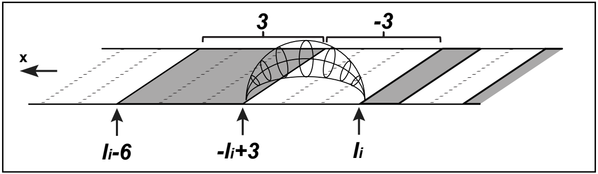
$$\frac{1}{2\pi}\int_{S^3/Z_k} C_3 = \frac{l}{k}$$

  - Sitting at $\mathbb{R}^8/\mathbb{Z}_k$ fixed points.
  - Compensate the fractional 4-form flux on a 4-cycle.

- Total flux on each strip(4-cycle),

$$\left[\int_{S^4/\mathbb{Z}_k} G_4 + \int_{S^3/\mathbb{Z}_k,\mathrm{north}} C_3 + \int_{S^3/\mathbb{Z}_k,\mathrm{south}} C_3\right] \sim \ n : \text{Integer}$$

$\Rightarrow$ Constraints on adjacent torsions.



- Correspond to l fractional M2-branes on $\mathbb{R}^8/\mathbb{Z}_k$ orbifold singularity.
  $\Rightarrow$ $\underline{\mathcal{N} = 3\ U(l)_k \text{ Chern-Simons theory}}$. [Aharony-Bergman-Jafferis]

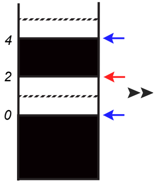
- Extra M2-charge from a discrete torsion.

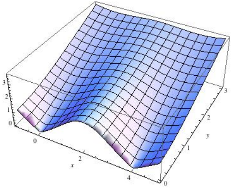
$$kQ_{M2} = \frac{l(k-l)}{2}$$

[Aharony-Hashimoto-Hirano-Ouyang], [Sethi], [Hanany-Kol]

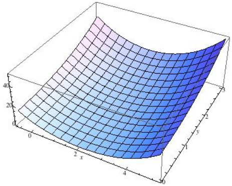
- Careful study on orientation of fractional M2 shows those on "white-to-black" line (read from bottom) are anti-M2
  $\Rightarrow$ exclude them in supersymmetric solutions.

- Exercise : probe regular $M2$ & $\bar{M}2$.





M2



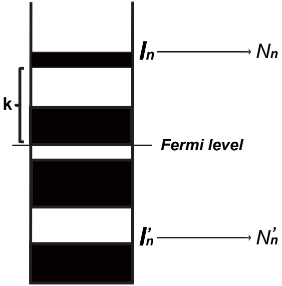
anti-M2

- Each discrete torsion gives the degeneracy associated with $U(l_i)_k$ CS theory (w/ UV completion having many SUSY).

$$\begin{pmatrix} |k| \\ l_i \end{pmatrix} \equiv \frac{|k|!}{l_i!(|k|-l_i)!} \quad \text{for each } l_i$$

- Define l_n/l'_n as torsions at the n-th level above/below the Fermi surface.

  $\Rightarrow \prod_{n=1}^{\infty} U(l_n) \times U(l'_n)$ gauge theory



- We can identify $l_n(l'_n)$ to $N_n(N'_n)$.
  - N_n, N'_n : unbroken gauge symmetry in QFT.
  - To make the map of vacua/gravity solutions look simpler, we assigned 'artificial torsions' $l_n = k$ at some points, which are equivalent to zero.

Counting vacua

- M2-charge of the fermion droplet,

$$kQ_{M2} = \sum_{n=1}^{\infty} \left[\overbrace{\left(k(n-1)(l_n + l'_n) + \frac{l_n^2 + (l'_n)^2}{2} \right)}^{\text{flux contribution(Young diagram size)}} + \overbrace{\left(\frac{l_n(k-l_n)}{2} + \frac{l'_n(k-l'_n)}{2} \right)}^{\text{torsion contribution}} \right]$$

$$= \quad k\sum_{n=1}^{\infty}(n-\frac{1}{2})(l_n + l'_n) = kN \; \Rightarrow \; \textbf{Field theory counting}!!$$

- And the constraint, $\sum_{n=1}^{\infty} l_n = \sum_{n=1}^{\infty} l'_n$.

- Two constraints & the degeneracy,

Perfectly matches!!

Check

1. W-boson mass spectrum for BPS mode.
   - In field theory, the classical W-boson mass linking length n to m first type blocks (when perturbation theory is reliable),
     $\sim \frac{\mu}{k}(n-m)$.
   - In gravity, the energy density of an open membrane (an open string in IIA),
     $\sim \frac{\mu}{k}(n-m) + \frac{\mu}{k^2}(l_n - l_m)$
   - The mass spectrum of BPS mode matches for large k .
     $\Rightarrow$ Quantum correction fo BPS modes?
     (Mass & central charge corrected by loop effect : Kink, Vortices, ...)

Conclusions

- To correctly count SUSY vacua, we need to take into account the dynamical SUSY breaking and quantum degeneracy in field theory.
- In gravity, $\mathbb{Z}_k$ quotient introduces new degrees of freedom, discrete torsion, at the orbifold singularity.
  - These extra degrees of freedom are described by fractional M2-branes.
- Including the extra degeneracy from the discrete torsion, the gravity solutions dual to supersymmetric vacua of mass-deformed CSm theory for general k can be found.

Other Applications

- Vortex solution
- Domain wall
- Non-relativistic limit
- Anyons, FQHE