AGT and S-duality wall

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Outlines

1. Reviews on AGT conjectures
2. Further tests on AGT
3. S-duality walls
4. Computation of partition function of T(SU(2))
5. Conclusions and Future directions

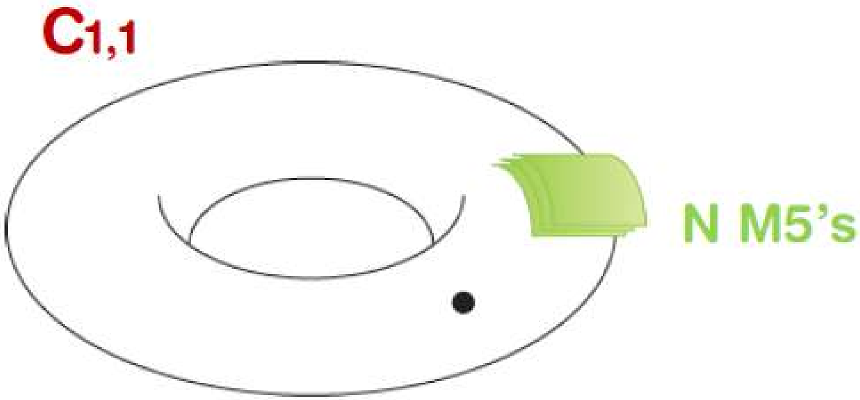
Motivation

- Undestanding AGT conjectures better
- New perspectives on 3d SCFTs?

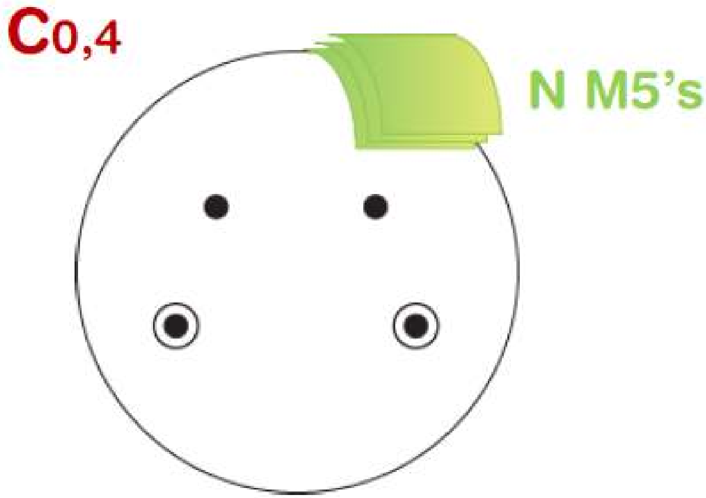
AGT conjectures

- Gaiotto found huge classes of 4d N=2 SCFTs realized as N M5 branes wrapping on a Riemann surface $C_{g,n}$ with genus g and punctures n

(1) N=2* SU(N) SYM :



(2) N=2 SU(N) Nf=2N SQCD :

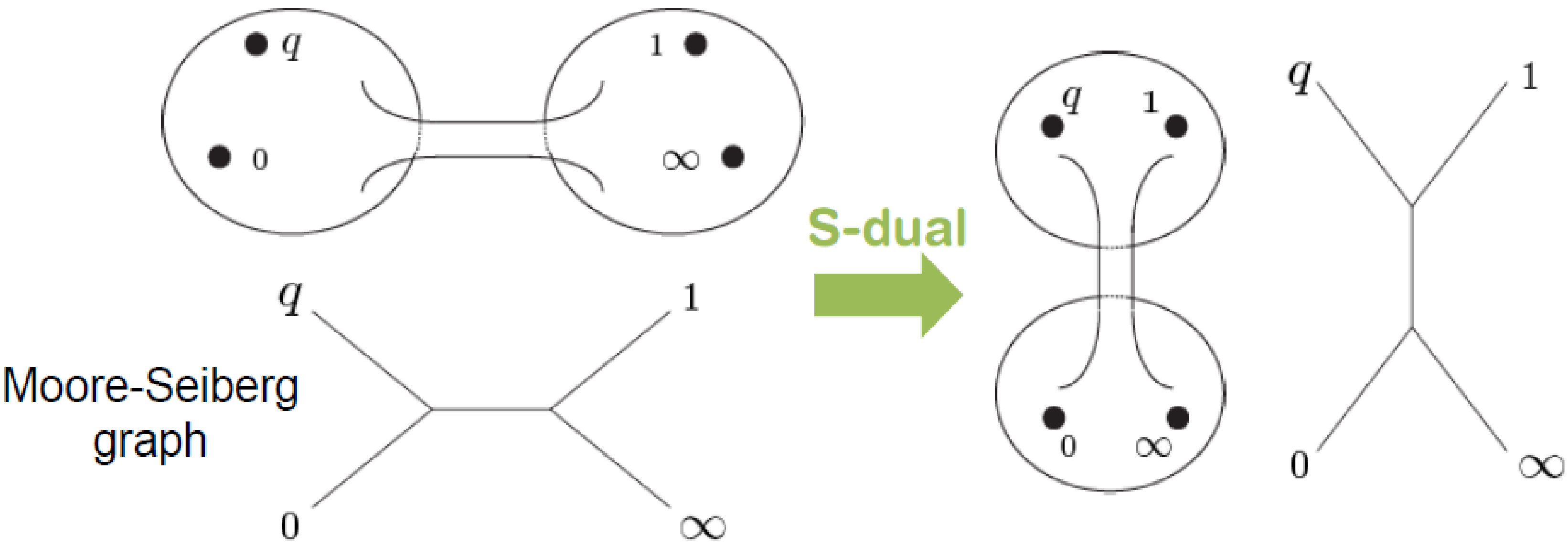


AGT conjectures

- AGT conjectures concern on such theories with N=2
- A correspondence between certain 4d N=2 SCFTs and 2d CFTs
- Specifically 4d SCFTs are given by quiver gauge with $SU(2)$ gauge groups at each node
- Corresponding 2d CFT in this case is a Liouville theory
- For general N, the corresponding 2d CFTs are Toda theory.

- Riemann surface Σ mediates this correspondence.
- In a Liouville theory side, one can consider the correlation functions of primary operators on such Σ.
- For an arbitrary pants decomposition Σ, one can associate Moore-Seiberg graph drawn on Σ.
- In 4d SCFTs, one has a quiver gauge theories realized as 2 M5s wrapping on Σ.
- Specific pants decomposition leads to the different members which are related via S-dualities

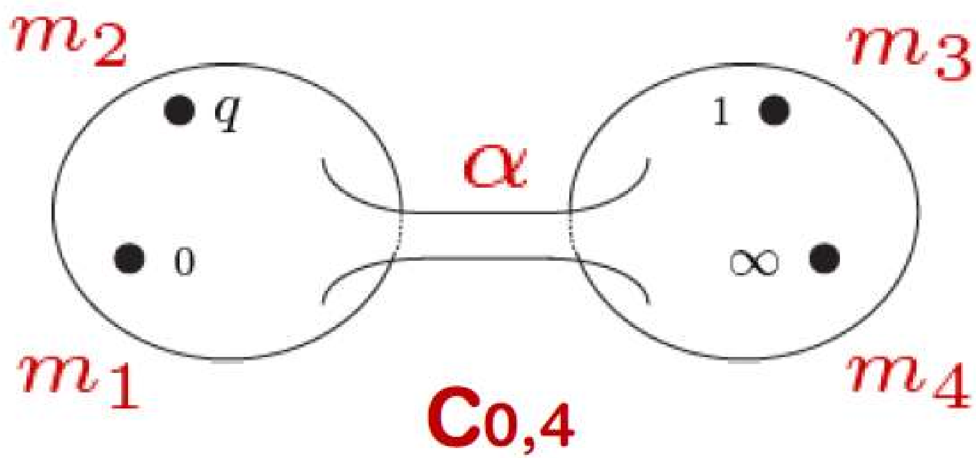
Mapping class group of $\Sigma \rightarrow$ S-duality

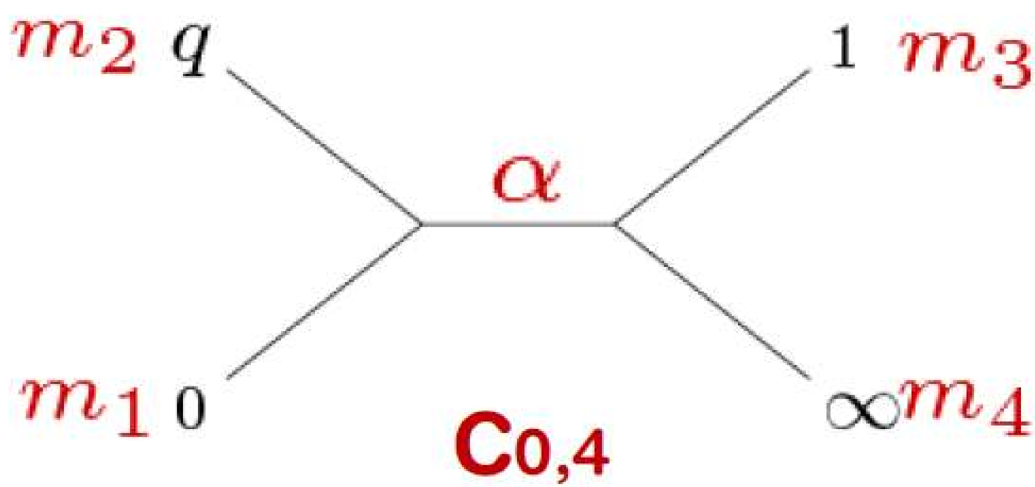


$$q = e^{2\pi i \tau}$$

$$Z = \int d\nu(\alpha)\, \overline{\mathcal{F}}^{(\sigma)}_{\alpha,E}(\tau)\mathcal{F}^{(\sigma)}_{\alpha,E}(\tau)$$

- Partition function of gauge theories on S^4 matches with correlation functions of Liouville theory with central charges $c = 25$ and Liouville coupling $b = 1$
- Liouville momenta $E = E_i$ for external legs (from punctures) and $\alpha = \alpha_a$ denotes internal momenta
- F conformal blocks
- gauge coupling τ determines the complex structure of Σ
- E masses of the matter multiplets α Coulomb vevs
- $F, \bar{F}$ Nekrasov partition function on R^4 with Ω deformation with $\epsilon_1 = \epsilon_2 = 1$


m_2
q
m_3
1
α
0
∞
m_1
m_4
$C_{0,4}$


m_2 q
1 m_3
α
m_1 0
∞ m_4
$C_{0,4}$

- Conformal blocks are defined for specific pants decomposition σ while correlation functions are independent of σ
- Under the change of the decomposition

$$\mathcal{F}^{(\sigma)}_{\alpha,E}(\tau) \;=\; \int d\nu(\alpha')\; g^{(\sigma,\sigma')}_{(\alpha,\alpha',E)} \mathcal{F}^{(\sigma')}_{\alpha',E}(\tau)$$

- S-duality kernel $\quad g^{(\sigma,\sigma')}_{(\alpha,\alpha',E)}$

Further tests of AGT conjectures

Insertion of the nonlocal operators

1. Wilson loop and t'Hooft loop → Verlinde loop operator [Drukker, Gomis, Okuda, Teschner]
2. surface opertor(vortex string) → degenerate field opertor [Alday, Gaiotto, Gukov, Tachikawa, Verlinde]
3. domain wall → defect operator

generalized defect opertor and Janus domain wall

Janus domain wall; 1/2 BPS solution interpolating N=4 SYM with different coupling τ, τ'
The partition function of the Janus configuration is given by [Drukker, Gaiotto, Gomis]

$$\begin{aligned} & \int da \bar{Z}_{Nekrasov}(\tau) Z_{Nekrasov}(\tau') \\ = & \int d\nu(\alpha) \bar{F}_{\alpha,m}(\tau) F_{\alpha,m}(\tau') \end{aligned}$$

S-duality wall

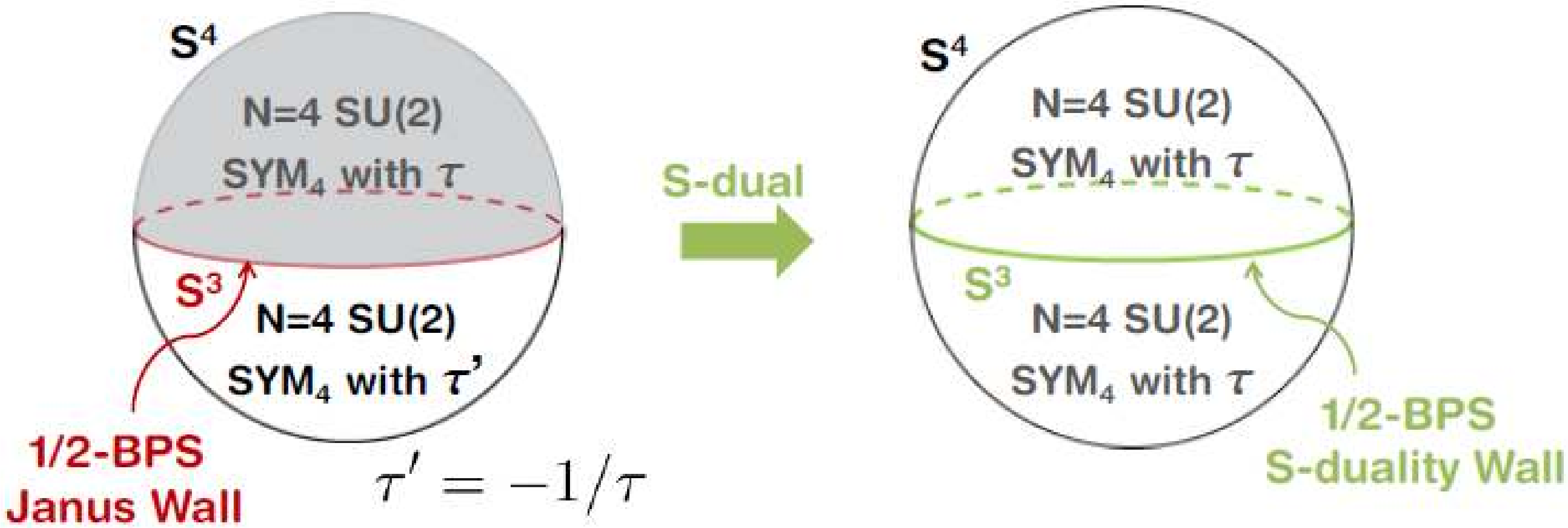
Interesting case is $\tau' = g\tau$ where τ is a mapping class element.

$$F_{\alpha,m}(g\tau) = \int d\nu(\alpha) S_{\alpha,\beta,m} F_{\beta,m}(\tau)$$

(generalized S-duality)
$S_{\alpha,\beta,m}$ is called Moore-Seiberg Kernel

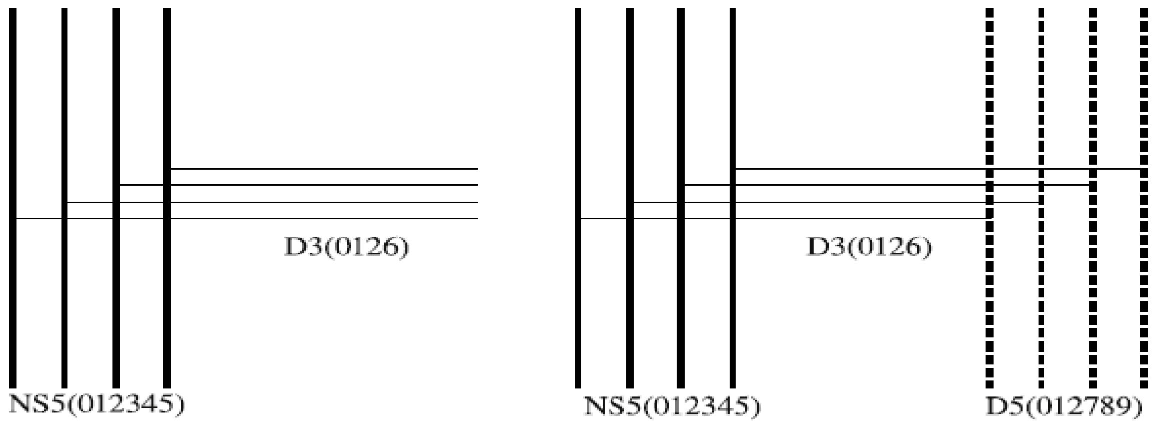
$$\int d\nu(\alpha) \bar{F}_{\alpha,m}(\tau) F_{\alpha,m}(g\tau) = \int d\nu(\alpha) d\nu(\beta) \bar{F}_{\alpha,m}(\tau) S_{\alpha,\beta,m} F_{\beta,m}(\tau)$$

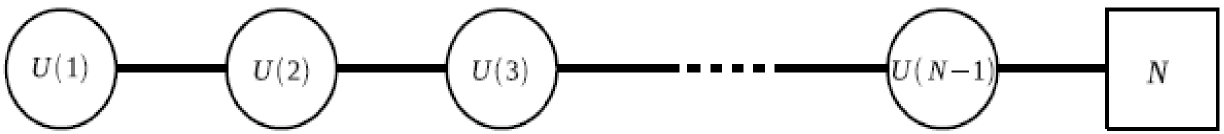
- What does correspond to the Moore-Seiberg Kernel in 4d side?
- Defect theory living on the S-duality wall of N=4 SYM theory is $T(G)$ for gauge group G.



$T(G)$ theory

- $T(G)$ theory is N=4 SCFT in 3-dimension which comes from S-dual of the Dirichlet B.C. of N=4 SYM without Nahm poles.
- it has global symmetry $G \times \tilde{G}$
- 3D mirror of $T(G) = T(\tilde{G})$
- coupled to 4d SYM, Coulomb branch parameter a turns into mass term μ for $T(G)$





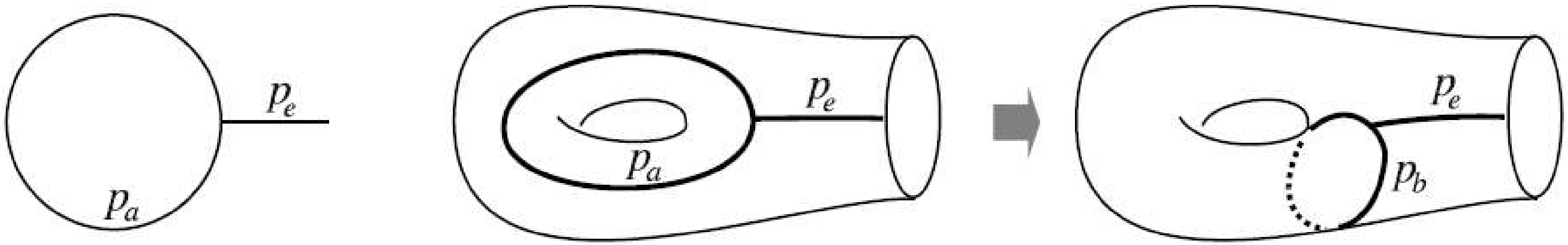
$T(G)$ theory

- When it is coupled to 4d SYM of both sides, Coulomb branch parameter of one theory gives mass term μ while the other gives F.I term η

$T(SU(2))$ theory

- Proposal: Moore-Seiberg Kernel= partition function of $T(SU(2))$ theory
- Moore-Seiberg Kernel is known
- Partition function of $T(SU(2))$ theory on S^3 can be computed by the localization technique. [Kapustin, Willett, Yaakov]
- We find the precise match!

- Moore-Seiberg kernel for one-point conformal block [Teschner]



$$S_{(p_a,p_b,p_e)} = \frac{2^{\frac{3}{2}}}{s_b(p_e)} \int_{\mathbb{R}} dr \frac{s_b(p_b + r + \frac{1}{2}p_e + \frac{iQ}{4})}{s_b(p_b + r - \frac{1}{2}p_e - \frac{iQ}{4})} \frac{s_b(p_b - r + \frac{1}{2}p_e + \frac{iQ}{4})}{s_b(p_b - r - \frac{1}{2}p_e - \frac{iQ}{4})} e^{4\pi i p_a r}$$

$$s_b(x) = \prod_{m,n\in\mathbb{Z}_{\geq 0}} \frac{mb + nb^{-1} + \frac{Q}{2} - ix}{mb + nb^{-1} + \frac{Q}{2} + ix}$$

- When $\frac{Q}{2} + ip_e \to 0$, conformal block reduces to the Virasoro character. S-duality kernel simplifies to

$$S_{(p_a, p_b, p_e)} = \frac{\sqrt{2}\cos(4\pi p_a p_b)}{\sinh(2\pi b p_b)\sinh(2\pi p_b/b)}$$

- 3d $T(SU(2))$ theory is an N=4 SQED with two fundamental hypers $(q_i, \tilde{q}^i)$

$$\mathcal{L}_{\text{mass}} = \int d^4\theta \left[q^{1\dagger} e^{-2V_{\text{mass}}} q_1 + q^{2\dagger} e^{+2V_{\text{mass}}} q_2 + \tilde{q}_1^{\dagger} e^{+2V_{\text{mass}}} \tilde{q}^1 + \tilde{q}_2^{\dagger} e^{-2V_{\text{mass}}} \tilde{q}^2 \right]$$

$$V_{\text{mass}} = -i\theta\bar{\theta}\mu$$

$$\mathcal{L}_{\text{FI}} = -\frac{4}{\pi} \int d^4\theta \, V_{\text{FI}} \Sigma = -\frac{2}{\pi} \zeta D$$

- the contribution from the 4 matter multiplets

$$Z(\sigma + \mu) Z(\sigma - \mu) Z(-\sigma - \mu) Z(-\sigma + \mu)$$

$$Z(\sigma) \equiv \prod_{n=1}^{\infty} \left(\frac{n + \frac{1}{2} + i\sigma}{n - \frac{1}{2} - i\sigma} \right)^n = s_{b=1}\left(\frac{i}{2} - \sigma\right)$$

- The partition function of $T(SU(2))$ theory coupled to N=4 SYM

$$Z_{\rm 3D}^{\mathcal{N}=4} = \int d\sigma \, \frac{s_{b=1}(\mu+\sigma+\frac{i}{2})}{s_{b=1}(\mu+\sigma-\frac{i}{2})} \frac{s_{b=1}(\mu-\sigma+\frac{i}{2})}{s_{b=1}(\mu-\sigma-\frac{i}{2})} e^{4\pi i \zeta \sigma}$$

- matches with S-duality kernel with

$$b=1, \quad r=\sigma, \quad p_a=\zeta, \quad p_b=\mu, \quad p_e=0$$

$$m_e = Q/2 + ip_e = b \neq 0$$
$$b=1$$

$m_e = m_{adj} + b$ [Okuda, Pestun]

$N = 2^*$ SYM and mass-deformed $T(SU(2))$

- N=4 SYM corresponds to Liouville on a one punctured torus with p_e=0
- mass deformation of N=4 SYM induce a mass term of $T(SU(2))$ preserving N=2 SUSY and $SU(2)_N \times SU(2)_R$ global symmetry
- matter multiplet contribution; gauging antidiagonal of $U(1)_N \times U(1)_R$

$$\mathcal{L}_{\rm def} = \int d^4\theta \ \phi^\dagger e^{4V_{\rm def}} \phi + \sum_{i=1}^{2} \left[q^{i\dagger} e^{-2V_{\rm def}} q_i + \tilde{q}_i^\dagger e^{-2V_{\rm def}} \tilde{q}^i \right]$$

$$V_{\rm def} = im\theta\bar{\theta}/2$$

$$Z(\sigma + \mu - \tfrac{m}{2}) Z(\sigma - \mu - \tfrac{m}{2}) Z(-\sigma - \mu - \tfrac{m}{2}) Z(-\sigma + \mu - \tfrac{m}{2})$$

$$Z_{\rm 3D}^{\mathcal{N}=2^*} = \int d\sigma \ \frac{s_{b=1}(\mu + \sigma + \frac{m}{2} + \frac{i}{2})}{s_{b=1}(\mu + \sigma - \frac{m}{2} - \frac{i}{2})} \frac{s_{b=1}(\mu - \sigma + \frac{m}{2} + \frac{i}{2})}{s_{b=1}(\mu - \sigma - \frac{m}{2} - \frac{i}{2})} e^{4\pi i \zeta \sigma}$$

$$m = p_e$$

Self-mirror property

- Partition function of mass deformed $T(SU(2))$ is invariant under the exchange of μ and FI term with $m \to -m$.
- Crucial ingredient is the invariance of $F_{m,b}$ under the Fourier transform

$$F_{m,b}(x) \equiv \frac{s_b(x + \frac{m}{2} + \frac{iQ}{4})}{s_b(x - \frac{m}{2} - \frac{iQ}{4})}$$

$$\int dx e^{-2\pi i p x} F_{m,b}(x) \;=\; F_{-m,b}(p)$$

- For $m = 0$, this implies $\frac{1}{cosh \pi x}$ is invariant under the Fourier transform; Crucial for the proof of the mirror symmetry of N=4 3d theories in [Kapustin, Willett, Yaakov]

Generalization to $SU(N)$

- $N = 2^*$ theories with other gauge group
- S-duality wall is given by $T(G)$ with a real mass deformation
- Mass comes from the gauging a $U(1)$ symmetry commuting with $G \times \tilde{G}$ and flipping sign under the mirror symmetry; anti diagonal combination of $U(1)_N \times U(1)_R$
- One can work out the partition function of the gauge theory
- S-dual kernel of Toda theory to be computed

Conclusions and Future directions

- S-duality kernel and the partition function of 3d theory matches
- The next step is to understand the S-duality wall for $SU(2)$ with 4 flavors (in progress)
- Interesting setting to understand the N=4 3d SCFTs
- M5 branes wrapping on a suitable 3 manifold ?