AdS/CMT from Wrapped M5-branes

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Plan

1. Introduction and Summary
2. Consistent truncations
3. Holographic superconductivity
4. $N = 2, D = 4$ supergravity from M5-branes

Based on
J. Gauntlett, A. Donos (Imperial College London), NK (KHU Seoul), O. Varela (AEI Potsdam), 1009.3805 and submitted to JHEP

Principle of Holography

- AdS/CFT correspondence: proposed in 1997, and has been successfully applied to $D = 4$ SYMs, and many other examples.
- A d-dimensional strongly-coupled field theory is equivalent to a weakly coupled (classical) gravity system in $d + 1$-dimensions
    - maybe really nontrivial quantitive checks are possible only for $N = 4, D = 4$ YM, and $N = 6, D = 3$ CS.
- Might be applicable to any strongly coupled quantum field theory?

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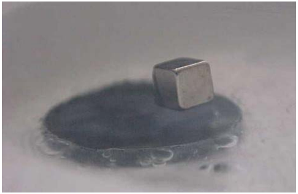
AdS/CMT

- We'd like to apply the holographic principle to condensed matter physics, like superconductivity.
- CMT is, unlike quantum gravity, amenable to (table-top) experiments. Then we might really test the idea of holography and its realization using string theory.

Superconductivity

- First discovered in 1911 by Onnes.
- Resistivity drops to zero at low temperature.
- Meissner effect: Magnetic fields are repeled by superconductor.



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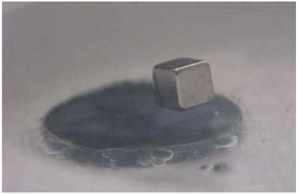

Theory of superconductivity

- Landau, Ginzburg (1950), Bardeen, Cooper, Schrieffer (1957)
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Holographic superconductor?

- Gubser, 0801.2977; Hartnoll, Herzog, Horowitz, 0803.3295, 0810.1563 etc.
- For a phenomenological description, one needs minimally a $D = 4$ classical AdS gravity with a massless gauge field and a charged scalar.
    - On CFT side, global U(1) symmetry and an operator with nonzero charge.
    - One looks for a charged black hole with scalar hair: Holographic version of spontaneous symmetry breaking.

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Arguments for 'scalar hair' in AdS RN

- Usually, charged particles pair-created at horizon either falls into BH, or escapes to infinity.

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SC from M5-branes

- M5-branes are solitonic objects of M-theory (11d unification of string theories)
- Can lead to lower-dim theories through wrapping and/or intersecting.
- We considered a supersymmetric configuration of M5-branes wrapping SLAG 3-cycle (preserves SUSY and guarantees stability) and discovered there is a scalar field leading to holographic superconductivity.
- As a candidate for ground state at zero temperature, we numerically constructed a solution with Lifshitz-like scaling whose critical exponent $z \sim 39$.

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Top-down models of hol. superconductor

- We want a $D = 4$ (bulk) theory which captures exact string/M-theory solutions behind it.
- CONSISTENT TRUNCATION: If a lower-dim supergravity is a consistent truncation of $D = 10/11$ supergravity, we can construct exact higher-dim solutions for any lower-dim model solution.

Consistent truncations: Maximal susy

- For example: Maximal gauged supergravity in $D = 4/5/7$ are shown/believed to be consistent truncations of $D = 11/10(IIB)/11$ supergravity.
- For $AdS_7 \times S^4$, see Nastase, Vaman, van Nieuwenhuizen (1999)
- For $D = 4$ sugra, $N = 8$ gauged supergravity has $35_v + 35_s$ scalar fields.
- Too many fields and we need a further consistent truncations, or some different route to $D = 4$, for practical reasons.

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Consistent truncations involving Sasaki-Einstein spaces

- See J.Gauntlett, S.Kim, O.Varela, D.Waldram, 0901.0676; Cassani, Dall'Agata, Faedo, 1003.4283; Liu, Szepietowski, Zhao, 1003.5374; Gauntlett, Varela, 1003.5642
- $D = 11$ sugra ansatz: around $AdS_4 \times SE_7$ solutions.

$$
\begin{aligned}
ds^2 &= ds_4^2 + e^{2U}(KE_6) + e^{2V}(\eta + A_1)^2 \\
G_4 &= 6e^{-6U-V}(1 + h^2 + |\chi|^2)\mathrm{vol}_4 + H_3 \wedge (\eta + A_1) + H_2 \wedge J \\
&\quad + dh \wedge J \wedge (\eta + A_1) + 2hJ \wedge J \\
&\quad + \sqrt{3}(\chi(\eta + A_1) \wedge \Omega - \tfrac{i}{4} D\chi \wedge \Omega + c.c.)
\end{aligned}
$$

$N = 2$ supergravity from Sasaki-Einstein 7-manifolds

- $D = 4$ fields: metric, massless vector A_1, real scalars U, V, h, one complex scalar χ, p-form field strength H_p for $p = 2, 3$:

  $N = 2$ gravity, a vector and a hypermultiplet.

- Exhibits holographic superconductivity. (J. Gauntlett, J. Sonner, T. Wiseman 2009)
- Universal subsector of any Sasaki-Einstein.

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Crash course on Hol. Superconductors

- See e.g. Denef and Hartnoll, 0901.1160
- Minimally, we need AdS gravity, a Maxwell field and at least one charged field which will spontaneously break $U(1)$.

$$L = \frac{M^2}{2}R + \frac{3M^2}{L^2} - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2$$

- Planck mass M, AdS_4 radius L, gauge coupling g, scalar field ϕ with mass m and charge q.
- In supergravity, we usually have a number of charged fields. Can be tachyonic, but always above the Breitenlohner-Freedman bound around a supersymmetric vacuum.
  - For AdS_{d+1}, $m_{BF}^2 = -d^2/4L^2$

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- Standard AdS/CFT dictionary relates a CFT operator with a bulk scalar field. For AdS_4/CFT_3,

$$\Delta(\Delta-3)=(mL)^2$$

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Bound for Holographic Superconductivity

- From Denef and Hartnoll, 0901.1160
- One solves the equation for charged scalar field in AdS RN background and look for threshold unstable mode.
- With $\gamma^2 = 2g^2(ML)^2$, we have instability (Hol. SC) if

$$\boxed{q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3)}$$

Maximal gauged supergravity in $D = 7$

- M-theory compactified on S^4. The susy vacuum corresponds to $AdS_7 \times S^4$. M5-brane geometry.
- $SL(5, R)$ global symmetry, $SO(5)$ is gauged.
  - Scalar manifold: $SL(5, R)/SO(5)$ and 14 scalars 14
  - $SO(5)$ gauge group, 10 vector fields $10 \times (7 - 2) = 50$
  - Five 3-form fields $5 \times (5 \cdot 4 \cdot 3/3 \cdot 2) = 50$
  - Metric $5 \cdot 6/2 - 1 = 14$
- Known to be a consistent truncation of $D = 11$ supergravity. (Nastase, Vaman, van Nieuwenhuizen 1999)
- SUSY vacuum is AdS_7 with all matter fields turned off.

Dimensional Reduction leading to lower dim AdS

- Unlike ungauged sugra with Minkowski vacuum, it is not trivial to realize spontaneous dimensional reduction. Obviously, the internal space cannot be chosen as T^d.
- One nice way of dim reduction with susy is to identifying spin connection of the internal space with gauge fields.
  - Recall $\delta\psi_\mu \sim (\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + A_\mu^{IJ}\Gamma_{IJ} + \cdots)\epsilon$
  - Correspond to susy cycles in special holonomy manifolds, and usually one obtains $AdS_{7-p} \times \Sigma_p$ vacua.
  - Known as magneto-vac solutions in the past. For $D = 7$ see Pernici and Sezgin (1985).
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SUSY of SLAG cycles

- SUSY cycles are subspace of special holonomy manifolds.
- They do not have special holonomy in general, but when branes are wrapped around them some fraction of susy is preserved.
  - In Math, they are called calibrated, and associated is an invariant tensor called calibration.
  - Roughly speaking, the nontrivial spin connection is cancelled by gauge connection (physically this is from the curvature of transverse space within the special holonomy manifold)
- For SLAG p-cycle,

$$\boxed{\text{Spin connection } SO(p) \leftrightarrow \text{Gauge connection } SO(p)}$$

M5-branes on 3-cycles

- M5-brane has $SO(5)$ global symmetry - from 5d transverse space.
- $SO(3)$ spin-connections should be cancelled by turning on the bulk gauge fields in supergravity
  - SLAG 3-cycle in CY3 (1/4-BPS) : $SO(5) \rightarrow SO(3)$
  - Associative 3-cycle in G_2 (1/8-BPS) : $SO(5) \rightarrow SO(4) \sim SU(2) \times SU(2)$.
- In the near-horizon limit, the $D = 4$ theories have $N = 2$ and $N = 1$ supersymmetry, respectively.

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$D=7$ supergravity

- $SL(5,R)$ matrix T, $SO(5)$ gauge field strength $F^{ij}_{(2)}$, 3-form fields $S^i_{(3)}$.

$$
\begin{aligned}
\mathcal{L}_7 \;=\;& R*\mathbf{1} - \tfrac{1}{4} T^{-1}_{ij} *DT_{jk} \wedge T^{-1}_{k\ell} DT_{\ell i} - \tfrac{1}{4} T^{-1}_{ik} T^{-1}_{j\ell} *F^{ij}_{(2)} \wedge F^{k\ell}_{(2)} \\
& -\tfrac{1}{2} T_{ij} *S^i_{(3)} \wedge S^j_{(3)} + \frac{1}{2g} S^i_{(3)} \wedge DS^i_{(3)} - \frac{1}{8g} \epsilon_{ij_1\cdots j_4} S^i_{(3)} \wedge F^{j_1 j_2}_{(2)} \wedge F^{j_3 j_4}_{(2)} \\
& + \frac{1}{g} \Omega_{(7)} - V*\mathbf{1},
\end{aligned}
$$

- Scalar potential $V = \frac{g^2}{2}(2T_{ij}T_{ij} - (T_{ii})^2)$
- $\delta\Omega_{(7)} = \frac{3}{4}\delta^{klmn}_{ijpq} F^{ij} \wedge F^{kl} \wedge F^{mn} \wedge \delta A^{pq}$

M5 on SLAG3 (or 3 in 6)

- Metric ansatz

$$ds_7^2 = ds_4^2 + ds^2(\Sigma_3)$$

  with $\Sigma_3 = S^3$ or H^3.
- Break $SO(5) \to SO(3)$, and identify the gauge connection with spin connection on Σ_3.
- $a, b = 1, 2, 3,\ \alpha, \beta = 4, 5$

$$A^{ab} = \frac{1}{g}\omega^{ab}, \text{ others vanish}$$

- $T = \mathrm{diag}(e^{-4\lambda}, e^{-4\lambda}, e^{-4\lambda}, e^{6\lambda}, e^{6\lambda})$
- Allows a fixed point for $\Sigma_3 = H^3$: $AdS_4 \times H^3/\Gamma$ solution

Fluctuations and a bigger truncated set

- Allow all fields consistent with $SO(5) \to SO(3)$ breaking.
  - From metric, we have $D=4$ metric and a scalar (breathing mode).

$$ds_7^2 = e^{-6\phi} ds_4^2 + e^{4\phi} ds^2(\Sigma_3)$$

  - Gauge fields: real scalar β, complex scalar θ, graviphoton A_1.

$$A_{(1)}^{ab} = \frac{1}{g}\bar{\omega}^{ab} + \beta \epsilon_{abc}\, \bar{e}^c$$
$$A_{(1)}^{a\alpha} = -A^{\alpha a} = \theta^\alpha \bar{e}^a$$
$$A_{(1)}^{\alpha\beta} = \epsilon^{\alpha\beta} A_1$$

Dimensional reduction cont'd

- 3-form fields: 2-form B_2, 1-form C_1, complex 3-form h_3, complex scalar χ

$$\begin{aligned} S^a_{(3)} &= B_2 \wedge \bar{e}^a + C_1 \wedge \epsilon_{abc} \bar{e}^b \wedge \bar{e}^c \\ S^\alpha_{(3)} &= h^\alpha_3 + \chi^\alpha \mathrm{vol}(\Sigma_3) \end{aligned}$$

- From scalars T, we have a charged scalar $\mathcal{N}$.

$$T_{ab} = e^{-4\lambda}\delta_{ab}\,, \quad T_{a\alpha} = 0\,, \quad T_{\alpha\beta} = e^{6\lambda}\mathcal{N}_{\alpha\beta}$$

Counting of the modes

- Metric, graviphoton A_1 should make sugra multiplet.
- 10 scalars $\phi, \lambda, \mathcal{N}_{\alpha\beta}, \beta, \theta_\alpha, B_2, \chi_\alpha$
- 1 (massive) vector C_1
- h_3 is non-dynamical.
- In total, we have 2 hypers and 1 massive vector multiplet.

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Scalar potential

$$
\begin{aligned}
V \;=\; & -g^2\Big\{3le^{-10\phi} - \tfrac{3}{8}e^{8\lambda-14\phi}(l - 2\beta^2 - 2\theta^T\theta)^2 \\
& + \tfrac{1}{2}e^{-6\phi}\Big[3e^{-8\lambda} + e^{12\lambda}[(\mathrm{Tr}\mathcal{N})^2 - 2\mathrm{Tr}(\mathcal{N}\mathcal{N})] + 6e^{2\lambda}\mathrm{Tr}\mathcal{N}\Big] \\
& - \tfrac{3}{2}e^{-10\phi}\Big[e^{10\lambda}(\theta^T\mathcal{N}\theta) - 2\theta^T\theta + e^{-10\lambda}(\theta^T\mathcal{N}^{-1}\theta)\Big] \\
& - 6e^{-2\lambda-14\phi}\beta^2(\theta^T\mathcal{N}^{-1}\theta) - \tfrac{1}{2g^2}e^{6\lambda-18\phi}(\chi^T\mathcal{N}\chi)\Big\}
\end{aligned}
$$

※ $l = \pm 1$ is the sign of scalar curvature for Σ_3

Vacua

- SUSY vacuum
  - With $l = -1$, $e^{-20\phi} = e^{10\lambda} = 2$.
  - AdS radius $g^2 R^2 = 2$.
  - ϕ, λ with masses $M^2 R^2 = 3 \pm \sqrt{17}$.
  - β with $M^2 R^2 = 2$.
  - χ, θ with $M^2 R^2 = 5,\ 3/2 + \sqrt{17}/2$.
  - $\mathcal{N}$ with $M^2 R^2 = 4$.
- NON-susy vacuum
  - With $l = -1,\ e^{-20\phi} = 486/625,\ e^{10\lambda} = 10$.
  - $g^2 R^2 = 5\sqrt{6}/9$.
  - Can also compute mass spectrum: all above the BF bound (in fact, no tachyons)

Einstein-Maxwell sector and charged BH

- In the action, we have couplings like

$$-\frac{1}{2}e^{-12\lambda+6\phi}F_2 \wedge *F_2 - \frac{3}{2}e^{-4\lambda+2\phi}B_2 \wedge *B_2 - \frac{3gl}{2}B_2 \wedge F_2 + \cdots$$

- At the susy vacuum, if we set other fields to zero the eoms are reduced to

$$L = R + 3\sqrt{2}g^2 - \frac{1}{\sqrt{2}}F \wedge *F$$

  which allows ordinary AdS RN black hole solutions.
- Then, do we have holographic superconductivity?

Unstable mode

- We have charged scalars $\chi, \theta, \mathcal{N}$.
- Would like to use DH bound: For us, $M^2_{DH} = 2$, $g^2_{DH} = 1/\sqrt{2}$, $L^2_{DH} = R^2$, $\gamma^2_{DH} = 2\sqrt{2}R^2$.
- For χ, θ, $q_{DH} = g$ DH bound requires $M^2R^2 \leq 1/2$. They are too massive and do not destabilze the RN BH.
- For $\mathcal{N}$, $q_{DH} = 2g$ and DH bound requires $M^2R^2 \leq 13/2$. With $M^2R^2 = 4$, they lead to spontaneous symmetry breaking and superconductivity!

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Unstable mode

- Consider fluctuations of the gauge field.

$$\delta F_2 \sim vol(AdS_2), \quad \delta B_2 \sim vol(R^2)$$

  and coupled neutral scalars ϕ, λ.
- It turns out there is one more unstable mode.
- We have two competing order parameters, which have their own T_c.

Unstable mode

- Consider fluctuations of the gauge field.

$$\delta F_2 \sim vol(AdS_2), \quad \delta B_2 \sim vol(R^2)$$

  and coupled neutral scalars ϕ, λ.
- It turns out there is one more unstable mode.
- We have two competing order parameters, which have their own T_c.

Numerical analysis and determination of T_c

- In order to determine the critical temperature, we solve numerically the scalar eq or vector eq.
- For instance $\mathcal{N} = \mathrm{diag}(e^{\rho}, e^{-\rho})$ and set $\rho = \rho(r)$.
- Look for a solution which is regular near horizon and normalizable at r infinity. Changing T.
- Results: $\gamma_{DH} T_c/\mu = 0.001$ for complex scalar, and 0.0045 for vector-tensor fluctuation.
- There should be two branches of black holes. The superconducting one has a lower T_c.

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Lifshitz solutions

- Metric $ds^2 = -\frac{r^{2z}}{L^{2z}} dt^2 + \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2}(dx_1^2 + dx_2^2)$.
- Turn on massive vector and others: $A_1 = qr^z/L^z dt, C_1 = -cr^z/L^{z+1} dt, B_2 = br^2/L^3 dx_1 \wedge dx_2$ and also ρ, λ, ϕ should have nontrivial values.
- All the equations - Einstein, Maxwell, scalar - become algebraic.
- 8 unknowns : $q, b, c, z, \rho, \lambda, \phi, zL$.
- We established there's at least one physical solution, with $z = 39.05....$

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Discussion

- Have done...
  - Constructed a new $N = 2, D = 4$ as a consistent trunc of M-theory. Exhibits hol. superconductivity.
  - Identified the gauging, and the scalar manifold as a coset $\frac{SU(1,1)}{U(1)} \times \frac{G_{2(2)}}{SO(4)}$.
  - Two unstable modes and computed their critical temperature.
  - Numerically constructed Lifshitz solution. $z \sim 39$.

  - Comments
  - There were some no-go statements about Lifshitz sol other than $z = 2$ (Li, Nishioka, Takayanagi 0908.0363) Our construction evades those, for instance the metric is not a direct product $Lif(4) \times M_7$.
  - A similar construction using gauged sugra reported by Gregory, Parameswaran, Tasinato, Zavala 1009.3445)

Discussion

- To do...
  - What is the dual field theory exactly, and the order parameter correspond to what?
  - Should construct the BHs with scalar or vector hair.