Couplings between M2-branes and Bulk Form Fields

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Outline

- Effective actions of multiple M2-branes
- Deformation of BLG and ABJM theories
- Gauge invariant WZ-type coupling
- Consistency check for single M2-brane
- Reduction to type IIA string theory
- SUSY preserving mass deformation
- Discussion

Based on arXiv:0905.4840 JHEP 09,
arXiv:1009.5209

Effective actions of multiple M2-branes

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Effective action of D2-branes : N=8 SYM → Effective action of M2-branes

Strong coupling limit (IR limit)

$$\frac{1}{g_{\mathrm{YM}}^2} = \frac{l_p^3}{R_{11}^2}$$

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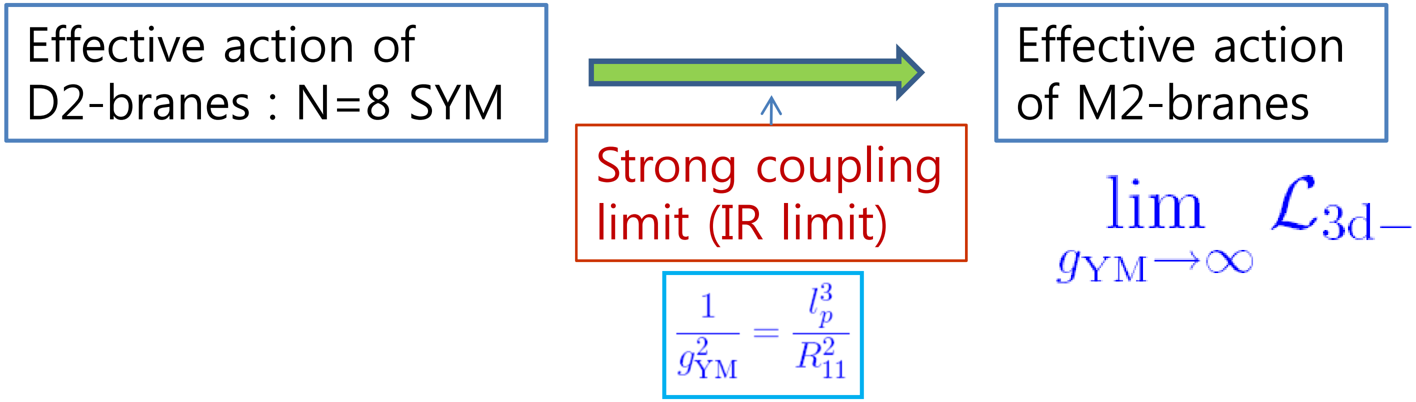

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➔ **superconformal Chern-Simons matter theory**

[Schwarz 04]

- **Two superconformal Chern-Simons matter theories with higher number of supersymmetry :**

  N=8 3-algebra based BLG theory (gauge group: SO(4))

  [Bagger-Lambert, Gustavsson 07]

  N=6 ABJM theory with U(N)XU(N) gauge group

  [Aharony-Bergman-Jafferis-Maldacena 08]

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  - Supersymmetry enhancement for k=1,2

Deformation of BLG and ABJM theories

- **Supersymmetry preserving mass deformation:**

  BLG theory : [Gomis-Salim-Passerini; Hosomichi-Lee-Lee 0804],

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- **Higher derivative corrections:**

  BLG theory : [Ezhuthachan, Mukhi, Papageorgakis 0903],

  ABJM theory (U(1)XU(1)): [Sasaki 0912]

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- Question : [Kim-OK-Nakajima-Tolla 1009]

  How can we construct a WZ-type coupling for the multiple M2-branes with general setting?

1

Construction of WZ-type coupling

1

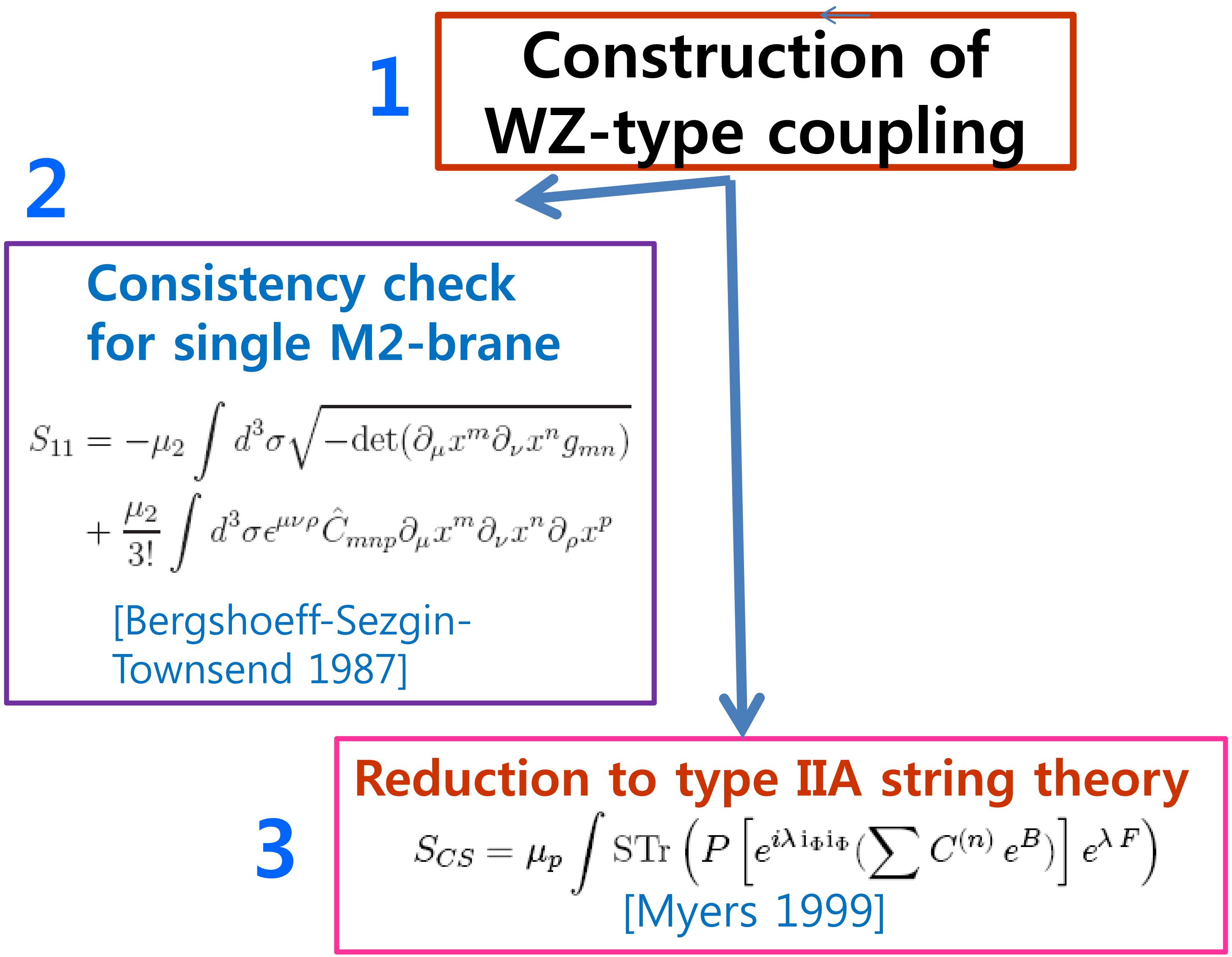
Construction of WZ-type coupling

2

Consistency check for single M2-brane

$$S_{11} = -\mu_2 \int d^3\sigma \sqrt{-\det(\partial_\mu x^m \partial_\nu x^n g_{mn})}$$
$$+ \frac{\mu_2}{3!} \int d^3\sigma \epsilon^{\mu\nu\rho} \hat{C}_{mnp} \partial_\mu x^m \partial_\nu x^n \partial_\rho x^p$$

[Bergshoeff-Sezgin-Townsend 1987]



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[Bergshoeff-Sezgin-Townsend 1987]

3

Reduction to type IIA string theory

$$S_{CS} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda \mathrm{i}_\Phi \mathrm{i}_\Phi}(\sum C^{(n)}\, e^B)\right] e^{\lambda F}\right)$$

[Myers 1999]

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4

Supersymmetry preserving mass deformation:

$$S_\mu = \frac{4\pi\mu}{k} \int d^3x \, \mathrm{Tr}\big(M_B{}^C Y^A Y_A^\dagger Y^B Y_C^\dagger \cdot - M^B_C Y_A^\dagger Y^A Y_B^\dagger Y^C \big)$$

[Hosomichi-Lee^3-Park; Gomis-Gomes- Raamsdonk-Virlinde 0807]

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Gauge invariant WZ-type coupling

- **ABJM action:**

$$\boxed{S=\int d^3x\,\left(\mathcal{L}_0+\mathcal{L}_{\mathrm{CS}}-V_{\mathrm{ferm}}-V_{\mathrm{bos}}\right)}$$

$$\begin{aligned}
\mathcal{L}_0 &= \mathrm{tr}\left(-D_\mu Y_A^\dagger D^\mu Y^A + i\psi^{\dagger A}\gamma^\mu D_\mu \psi_A\right)\,,\\
\mathcal{L}_{\mathrm{CS}} &= \frac{k}{4\pi}\,\epsilon^{\mu\nu\rho}\,\mathrm{tr}\left(A_\mu\partial_\nu A_\rho+\frac{2i}{3}A_\mu A_\nu A_\rho-\hat{A}_\mu\partial_\nu\hat{A}_\rho-\frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right)\,,\\
V_{\mathrm{ferm}} &= \frac{2\pi i}{k}\mathrm{tr}\Big(Y_A^\dagger Y^A\psi^{\dagger B}\psi_B-Y^AY_A^\dagger\psi_B\psi^{\dagger B}+2Y^AY_B^\dagger\psi_A\psi^{\dagger B}-2Y_A^\dagger Y^B\psi^{\dagger A}\psi_B\\
&\qquad\qquad+\epsilon^{ABCD}Y_A^\dagger\psi_BY_C^\dagger\psi_D-\epsilon_{ABCD}Y^A\psi^{\dagger B}Y^C\psi^{\dagger D}\Big),\\
V_{\mathrm{bos}} &= -\frac{4\pi^2}{3k^2}\mathrm{tr}\Big(Y_A^\dagger Y^AY_B^\dagger Y^BY_C^\dagger Y^C+Y^AY_A^\dagger Y^BY_B^\dagger Y^CY_C^\dagger+4Y_A^\dagger Y^BY_C^\dagger Y^AY_B^\dagger Y^C\\
&\qquad\qquad-6Y^AY_B^\dagger Y^BY_A^\dagger Y^CY_C^\dagger\Big).
\end{aligned}$$

- **Field contents of ABJM theory :**

$(Y^A)^a{}_{\hat{a}}$: $\mathbf{4}$ of SU(4), bifundamental of $U(N)_L \times U(N)_R$

$(\psi_A)^a{}_{\hat{a}}$: $\bar{\mathbf{4}}$ of SU(4), bifundamental of $U(N)_L \times U(N)_R$

$(A_\mu)^a{}_b$: Level k $U(N)_L$ gauge field

$(\hat{A}_\mu)^{\hat{a}}{}_{\hat{b}}$: Level $-k$ $U(N)_R$ gauge field

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- **Bosonic potential can be written by**

$$\boxed{V_{\rm bos} = \frac{32\pi^2}{3k^2}\left|\beta^{BC}_A + \delta^{[B}_A \beta^{C]D}_D\right|^2} \qquad |\mathcal{O}|^2 \equiv {\rm tr}\mathcal{O}^\dagger\mathcal{O}.$$

$$\beta^{AB}_C \equiv \tfrac{1}{2}(Y^A Y^\dagger_C Y^B - Y^B Y^\dagger_C Y^A) \quad \Longleftrightarrow \quad [X, Y; Z]$$

$$X = X_a T^a \quad [T^a, T^b; T^c] = f^{ab}{}_{cd} T^d$$

[Bagger-Lambert 0807]

- **Gauge covariant building blocks of ABJM theory:**

$$D_\mu Y^A, \quad \beta^{AB}_{C}, \quad F_{\mu\nu}, \quad \hat{F}_{\mu\nu}, \quad (\mathrm{c.c})$$

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$$C^{(3)}, \quad C^{(6)}, \quad D_\mu Y^A, \quad \beta^{AB}_{C}, \quad F_{\mu\nu}, \quad \hat{F}_{\mu\nu}, \quad (\mathrm{c.c})$$

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Form fields depend on transverse scalars

Gauge invariant WZ-type coupling

- **3-form field coupling:** **linearized 3-form field and**

$$\begin{aligned} S_C^{(3)} = \mu_2 \int d^3x\, \frac{1}{3!}\epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[& C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y_B^\dagger\big) \\ & + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y_C^\dagger) + (\mathrm{c.c.}) \Big] \end{aligned}$$

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$$\lambda = 2\pi l_{\mathrm{P}}^{3/2}$$

- **Form fields have multiple gauge indices:**

  - each term has same number of fields in bifundamental and antibifundamental representations

  - all gauge indices are contracted to guarantee gauge invariance

  → invariant under the orbifold transformation

$$\begin{aligned}
\{\mathrm{Tr}\}\big(C_{\mu\nu A} D_\rho Y^A\big) &= (C_{\mu\nu A})^{\hat a}_{a} (D_\rho Y^A)^a{}_{\hat a} \\
\{\mathrm{Tr}\}\big(C_{\mu AB} D_\nu Y^A D_\rho Y^B\big) &= (C_{\mu AB})^{\hat a\hat b}_{ab} (D_\nu Y^A)^a{}_{\hat a} (D_\rho Y^B)^b{}_{\hat b} \\
\{\mathrm{Tr}\}\big(C_{\mu A\bar B} D_\nu Y^A D_\rho Y^\dagger_B\big) &= (C_{\mu A\bar B})^{\hat a b}_{a\hat b} (D_\nu Y^A)^a{}_{\hat a} (D_\rho Y^\dagger_B)^{\hat b}{}_b \\
\{\mathrm{Tr}\}\big(C_{AB\bar C} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C\big) &= (C_{AB\bar C})^{\hat a\hat b c}_{ab\hat c} (D_\mu Y^A)^a{}_{\hat a} (D_\nu Y^B)^b{}_{\hat b} (D_\rho Y^\dagger_C)^{\hat c}{}_c \\
\{\mathrm{Tr}\}\big(C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C\big) &= (C_{ABC})^{\hat a\hat b\hat c}_{abc} (D_\mu Y^A)^a{}_{\hat a} (D_\nu Y^B)^b{}_{\hat b} (D_\rho Y^C)^c{}_{\hat c}
\end{aligned}$$

$$\{\mathrm{Tr}\}\left(C_{\mu\nu A}D_\rho Y^A\right) = (C_{\mu\nu A})^{\hat{a}}_{a}(D_\rho Y^A)^{a}{}_{\hat{a}}$$

$$\{\mathrm{Tr}\}\left(C_{\mu AB}D_\nu Y^A D_\rho Y^B\right) = (C_{\mu AB})^{\hat{a}\hat{b}}_{ab}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^B)^{b}{}_{\hat{b}}$$

$$\boxed{\{\mathrm{Tr}\}\left(C_{\mu A\bar{B}}D_\nu Y^A D_\rho Y^\dagger_B\right) = (C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^\dagger_B)^{\hat{b}}{}_{b}}$$

$$\{\mathrm{Tr}\}\left(C_{AB\bar{C}}D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C\right) = (C_{AB\bar{C}})^{\hat{a}\hat{b}c}_{ab\hat{c}}(D_\mu Y^A)^{a}{}_{\hat{a}}(D_\nu Y^B)^{b}{}_{\hat{b}}(D_\rho Y^\dagger_C)^{\hat{c}}{}_{c}$$

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$$\boxed{(C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}} = \left(F^{\hat{a}}_{a}G^{b}_{\hat{b}} + H^{\hat{a}}_{\hat{b}}I^{b}_{a}\right)_{\mu A\bar{B}}}$$

$$(C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^\dagger_B)^{\hat{b}}{}_{b} = \left(\mathrm{Tr}(FD_\nu Y^A)\mathrm{Tr}(GD_\rho Y^\dagger_B)\right)_{\mu A\bar{B}} + \left(\mathrm{Tr}(HD_\nu Y^A ID_\rho Y^\dagger_B)\right)_{\mu A\bar{B}}$$

$$\{\mathrm{Tr}\}\left(C_{\mu\nu A} D_\rho Y^A\right) = (C_{\mu\nu A})^{\hat{a}}_{a}(D_\rho Y^A)^{a}{}_{\hat{a}}$$

$$\{\mathrm{Tr}\}\left(C_{\mu AB} D_\nu Y^A D_\rho Y^B\right) = (C_{\mu AB})^{\hat{a}\hat{b}}_{ab}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^B)^{b}{}_{\hat{b}}$$

$$\boxed{\{\mathrm{Tr}\}\left(C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B\right) = (C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^\dagger_B)^{\hat{b}}{}_{b}}$$

$$\{\mathrm{Tr}\}\left(C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C\right) = (C_{AB\bar{C}})^{\hat{a}\hat{b}c}_{ab\hat{c}}(D_\mu Y^A)^{a}{}_{\hat{a}}(D_\nu Y^B)^{b}{}_{\hat{b}}(D_\rho Y^\dagger_C)^{\hat{c}}{}_{c}$$

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$$(C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}}(D_\nu Y^A)^{a}{}_{\hat{a}}(D_\rho Y^\dagger_B)^{\hat{b}}{}_{b} = \cancel{\left(\mathrm{Tr}(F D_\nu Y^A)\mathrm{Tr}(G D_\rho Y^\dagger_B)\right)_{\mu A\bar{B}}} + \left(\mathrm{Tr}(H D_\nu Y^A I D_\rho Y^\dagger_B)\right)_{\mu A\bar{B}}$$

- **Allow single trace terms only:**
  no multi-trace terms in IIA string theory

- **6-form field coupling:**

$$
\begin{aligned}
S_C^{(6)} =& \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{\ C} + 3\lambda \big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{\ \ D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y^\dagger_C \beta^{AB}_{\ \ D} \big) \\
&+ 3\lambda^2 \big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{\ \ E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y^\dagger_D \beta^{BC}_{\ \ E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y^\dagger_C D_\rho Y^\dagger_D \beta^{AB}_{\ \ E} \big) \\
&+ \lambda^3 \big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{\ \ F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_E \beta^{CD}_{\ \ F} \\
&+ C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{BC}_{\ \ F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}_{\ \ F} \big) + (\mathrm{c.c.}) \Big)
\end{aligned}
$$

$\mu_2' = \tau \lambda \mu_2$

τ is a dimensionless parameter which will be fixed after reduction to type IIA string theory

- **6-form field coupling:**

$$\boxed{\begin{aligned} S_C^{(6)} =&\mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{C} + 3\lambda (C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y^\dagger_C \beta^{AB}_{D}) \\ & + 3\lambda^2 (C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y^\dagger_D \beta^{BC}_{E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y^\dagger_C D_\rho Y^\dagger_D \beta^{AB}_{E}) \\ & + \lambda^3 (C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_E \beta^{CD}_{F} \\ & + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{BC}_{F} + \boxed{C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}_{F})} + (\mathrm{c.c.}) \Big) \end{aligned}}$$

$\mu_2' = \tau\lambda\mu_2$ τ is a dimensionless parameter which will be fixed after reduction to type IIA string theory

$$\begin{aligned} & C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}_{F} \\ & \qquad = (C_{AB\bar{C}\bar{D}\bar{E}\bar{F}})^{abc\hat{d}}_{\hat{a}\hat{b}\hat{c}d} (D_\mu Y^\dagger_C)^{\hat{a}}_{a} (D_\nu Y^\dagger_D)^{\hat{b}}_{b} (D_\rho Y^\dagger_E)^{\hat{c}}_{c} (\beta^{AB}_{F})^{d}_{\hat{d}} \end{aligned}$$

➔ **allow single trace terms**

Consistency check for single M2-brane

- **Comparison with the well known effective action of a single M2-brane with WZ-type coupling:**

$$S_{11} = -\mu_2 \int d^3\sigma \sqrt{-\det(\partial_\mu x^m \partial_\nu x^n g_{mn})} + \frac{\mu_2}{3!} \int d^3\sigma \epsilon^{\mu\nu\rho} \hat{C}_{mnp} \partial_\mu x^m \partial_\nu x^n \partial_\rho x^p$$

[Bergshoeff-Sezgin-Townsend 1987]

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⇓ static gauge

$$\boxed{\frac{\mu_2}{3!} \int d^3x \epsilon^{\mu\nu\rho} \left(\hat{C}_{\mu\nu\rho} + 3\lambda \hat{C}_{\mu\nu I} \partial_\rho X^I + 3\lambda^2 \hat{C}_{\mu IJ} \partial_\nu X^I \partial_\rho X^J + \lambda^3 \hat{C}_{IJK} \partial_\mu X^I \partial_\nu X^J \partial_\rho X^k \right)}$$

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$$\boxed{\frac{\mu_2}{3!} \int d^3x \epsilon^{\mu\nu\rho} \left(\hat{C}_{\mu\nu\rho} + 3\lambda \hat{C}_{\mu\nu I} \partial_\rho X^I + 3\lambda^2 \hat{C}_{\mu IJ} \partial_\nu X^I \partial_\rho X^J + \lambda^3 \hat{C}_{IJK} \partial_\mu X^I \partial_\nu X^J \partial_\rho X^k \right)}$$

⇕ ?

$$\boxed{\begin{aligned} S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[& C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 (C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y_B^\dagger) \\ & + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y_C^\dagger) + (\mathrm{c.c.}) \Big] \end{aligned}}$$

- **U(1)XU(1) ABJM theory: no bosonic potential**

$$\boxed{S_{\rm CS} = \int d^3x \frac{k}{4\pi} \epsilon^{\mu\nu\rho} (A_\mu \partial_\nu A_\rho - \hat{A}_\mu \partial_\nu \hat{A}_\rho)}$$

$$\big\downarrow$$

$$\boxed{S_{\rm CS} = \int d^3x \frac{k}{2\pi} \epsilon^{\mu\nu\rho} A^+_\mu F^-_{\nu\rho}}$$

$$A^\pm_\mu = \frac{1}{2}(A_\mu \pm \hat{A}_\mu)$$

$$F^-_{\mu\nu} = \partial_\mu A^-_\nu - \partial_\nu A^-_\mu$$

$$D_\mu Y^A = \partial_\mu Y^A + 2iA^-_\mu Y^A$$

- A^+_μ **is an auxiliary field ➔ integrate out:**

$$F^-_{\mu\nu} = 0$$

- ## $A_\mu^- = 0$ **gauge**

$$C_{\mu\nu\rho} = \hat{C}_{\mu\nu\rho} \qquad C_{\mu\nu A} = \frac{1}{2}\left(\hat{C}_{\mu\nu A} - i\hat{C}_{\mu\nu A+4}\right)$$

$$C_{\mu AB} = \frac{1}{4}\left(\hat{C}_{\mu AB} - i\hat{C}_{\mu A+4B} - i\hat{C}_{\mu AB+4} - \hat{C}_{\mu A+4B+4}\right)$$

$$C_{\mu A\bar{B}} = \frac{1}{4}\left(\hat{C}_{\mu AB} - i\hat{C}_{\mu A+4B} + i\hat{C}_{\mu AB+4} + \hat{C}_{\mu A+4B+4}\right)$$

$$\begin{aligned} C_{ABC} = \frac{1}{8}\Big(&\hat{C}_{ABC} - i\hat{C}_{A+4BC} - i\hat{C}_{AB+4C} - i\hat{C}_{ABC+4} \\ &- \hat{C}_{AB+4C+4} - \hat{C}_{A+4BC+4} - \hat{C}_{A+4B+4C} + i\hat{C}_{A+4B+4C+4}\Big) \end{aligned}$$

$$\begin{aligned} C_{AB\bar{C}} = \frac{1}{8}\Big(&\hat{C}_{ABC} - i\hat{C}_{A+4BC} - i\hat{C}_{AB+4C} + i\hat{C}_{ABC+4} \\ &+ \hat{C}_{AB+4C+4} + \hat{C}_{A+4BC+4} - \hat{C}_{A+4B+4C} - i\hat{C}_{A+4B+4C+4}\Big) \end{aligned}$$

Reduction to type IIA string theory

- **Mukhi-Papageorgakis (MP) Higgsing procedure:**

  \- nonvanishing vacuum expectation value (vev)

  ➔ U(N)XU(N) gauge group is broken to U(N)

Reduction to type IIA string theory

- **Mukhi-Papageorgakis (MP) Higgsing procedure:**
  - nonvanishing vacuum expectation value (vev)

    ➔ U(N)XU(N) gauge group is broken to U(N)

- **After the breakdown of the gauge group:**

$$Y^A = Y_0^A T^0 + iY_\alpha^A T^\alpha$$

SU(N) generator

U(1) generator

$$= \tilde{X}^A + i\tilde{X}^{A+4}$$

$Y_0^A,\ Y_\alpha^A$: complex numbers

$\tilde{X}^A,\ \tilde{X}^{A+4}$: Hermitians

$$Y^a = \tilde{X}^a + i\tilde{X}^{a+4}, \quad (a = 1, 2, 3)$$

$$Y^4 = \frac{v}{2}\mathbb{I} + \tilde{X}^4 + i\tilde{X}^8$$

$$\boxed{v \longrightarrow \infty, \qquad k \longrightarrow \infty, \qquad \frac{v}{k} \longrightarrow \text{fixed}}$$

$$D_\mu Y^a = \tilde{D}_\mu \tilde{X}^a + i\tilde{D}_\mu \tilde{X}^{a+4}$$
$$D_\mu Y^4 = \tilde{D}_\mu \tilde{X}^4 + i\tilde{D}_\mu \tilde{X}^8 + ivA^-_\mu$$

$$\beta^{a4}_4 = \frac{v}{2}([\tilde{X}^a,\, \tilde{X}^4] + i[\tilde{X}^{a+4},\, \tilde{X}^4]),$$
$$\beta^{ab}_4 = \frac{v}{4}([\tilde{X}^a,\, \tilde{X}^b] + i[\tilde{X}^a,\, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4},\, \tilde{X}^b] - [\tilde{X}^{a+4},\, \tilde{X}^{b+4}])$$
$$\beta^{a4}_b = \frac{v}{4}([\tilde{X}^a,\, \tilde{X}^b] - i[\tilde{X}^a,\, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4},\, \tilde{X}^b] + [\tilde{X}^{a+4},\, \tilde{X}^{b+4}])$$

$$\boxed{v \longrightarrow \infty, \qquad k \longrightarrow \infty, \qquad \frac{v}{k} \longrightarrow \text{fixed}}$$

$$D_\mu Y^a = \tilde{D}_\mu \tilde{X}^a + i\tilde{D}_\mu \tilde{X}^{a+4}$$
$$D_\mu Y^4 = \tilde{D}_\mu \tilde{X}^4 + \boxed{i\tilde{D}_\mu \tilde{X}^8 + ivA_\mu^-} \longrightarrow ivA_\mu^-$$

$$\beta^{a4}_{\ 4} = \frac{v}{2}([\tilde{X}^a, \tilde{X}^4] + i[\tilde{X}^{a+4}, \tilde{X}^4]),$$
$$\beta^{ab}_{\ 4} = \frac{v}{4}([\tilde{X}^a, \tilde{X}^b] + i[\tilde{X}^a, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4}, \tilde{X}^b] - [\tilde{X}^{a+4}, \tilde{X}^{b+4}])$$
$$\beta^{a4}_{\ b} = \frac{v}{4}([\tilde{X}^a, \tilde{X}^b] - i[\tilde{X}^a, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4}, \tilde{X}^b] + [\tilde{X}^{a+4}, \tilde{X}^{b+4}])$$

➔ integrate out the auxiliary field A_μ^-

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➔ integrate out the auxiliary field A^-_μ

➔ **ABJM theory** with U(N)XU(N) gauge group is reduced to **Yang-Mills theory** with U(N) gauge group

$$\boxed{v \longrightarrow \infty, \qquad k \longrightarrow \infty, \qquad \frac{v}{k} \longrightarrow \text{fixed}}$$

$$\begin{aligned} D_\mu Y^a &= \tilde{D}_\mu \tilde{X}^a + i\tilde{D}_\mu \tilde{X}^{a+4} \\ D_\mu Y^4 &= \tilde{D}_\mu \tilde{X}^4 + \boxed{i\tilde{D}_\mu \tilde{X}^8 + ivA_\mu^-} \longrightarrow ivA_\mu^- \end{aligned}$$

$$\begin{aligned} \beta^{a4}_{\ \ 4} &= \frac{v}{2}([\tilde{X}^a, \tilde{X}^4] + i[\tilde{X}^{a+4}, \tilde{X}^4]), \\ \beta^{ab}_{\ \ 4} &= \frac{v}{4}([\tilde{X}^a, \tilde{X}^b] + i[\tilde{X}^a, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4}, \tilde{X}^b] - [\tilde{X}^{a+4}, \tilde{X}^{b+4}]) \\ \beta^{a4}_{\ \ b} &= \frac{v}{4}([\tilde{X}^a, \tilde{X}^b] - i[\tilde{X}^a, \tilde{X}^{b+4}] + i[\tilde{X}^{a+4}, \tilde{X}^b] + [\tilde{X}^{a+4}, \tilde{X}^{b+4}]) \end{aligned}$$

➔ integrate out the auxiliary field A_μ^-

➔ **ABJM theory** with U(N)XU(N) gauge group is reduced to **Yang-Mills theory** with U(N) gauge group

- **Reduction of WZ-type coupling?**

- **WZ-type coupling in type IIA string theory:**

$$S_{CS} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda \mathrm{i}_\Phi \mathrm{i}_\Phi} \left(\sum C^{(n)} \, e^{B} \right) \right] e^{\lambda \, F} \right)$$

[Myers 1999]

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[Myers 1999]

- **P=2 case:**

$$\begin{aligned} S_{SC} = \mu_2 \int \mathrm{STr}\Big[&P[C^{(3)}] + P[C^{(1)} \wedge \mathcal{F}] \\ &+ i\lambda i_\Phi i_\Phi \big[P[C^{(5)}] + P[C^{(3)} \wedge \mathcal{F}] + \frac{1}{2} P[C^{(1)} \wedge \mathcal{F}^2]\big] + \cdots \Big] \end{aligned}$$

$$\mathcal{F} = B + \lambda F$$

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$$\begin{aligned} S_{SC} = \mu_2 \int \mathrm{STr}\, & \boxed{P[C^{(3)}]} + P[C^{(1)} \wedge \mathcal{F}] \\ & + \boxed{i\lambda i_\Phi i_\Phi \big[P[C^{(5)}]} + P[C^{(3)} \wedge \mathcal{F}] + \frac{1}{2} P[C^{(1)} \wedge \mathcal{F}^2]] + \cdots \Big] \end{aligned}$$

Linear terms in form fields

$\mathcal{F} = B + \lambda F$

$$
\begin{aligned}
S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \, \{\mathrm{Tr}\} \Big[& C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y_B^\dagger \big) \\
& + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y_C^\dagger) + (\mathrm{c.c.}) \Big]
\end{aligned}
$$

$$
\begin{aligned}
S_C^{(6)} = & \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \, \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{\ C} + 3\lambda \big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{\ \ D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y_C^\dagger \beta^{AB}_{\ \ D} \big) \\
& + 3\lambda^2 \big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{\ \ E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y_D^\dagger \beta^{BC}_{\ \ E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y_C^\dagger D_\rho Y_D^\dagger \beta^{AB}_{\ \ E} \big) \\
& + \lambda^3 \big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{\ \ F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y_E^\dagger \beta^{CD}_{\ \ F} \\
& + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta^{BC}_{\ \ F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y_C^\dagger D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta^{AB}_{\ \ F} \big) + (\mathrm{c.c.}) \Big)
\end{aligned}
$$

$$
\begin{aligned}
S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[& C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y_B^\dagger \big) \\
& + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y_C^\dagger) + (\mathrm{c.c.}) \Big]
\end{aligned}
$$

$$
\begin{aligned}
S_C^{(6)} = & \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{\ \ \ C} + 3\lambda \big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{\ \ \ D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y_C^\dagger \beta^{AB}_{\ \ \ D} \big) \\
& + 3\lambda^2 \big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{\ \ \ E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y_D^\dagger \beta^{BC}_{\ \ \ E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y_C^\dagger D_\rho Y_D^\dagger \beta^{AB}_{\ \ \ E} \big) \\
& + \lambda^3 \big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{\ \ \ F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y_E^\dagger \beta^{CD}_{\ \ \ F} \\
& + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta^{BC}_{\ \ \ F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y_C^\dagger D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta^{AB}_{\ \ \ F} \big) + (\mathrm{c.c.}) \Big)
\end{aligned}
$$

1. **No restriction except for the single traceness**

$$S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B \big) \\ + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C) + (\mathrm{c.c.}) \Big]$$

$$S_C^{(6)} = \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}{}_{C} + 3\lambda \big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}{}_{D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y^\dagger_C \beta^{AB}{}_{D} \big) \\ + 3\lambda^2 \big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}{}_{E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y^\dagger_D \beta^{BC}{}_{E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y^\dagger_C D_\rho Y^\dagger_D \beta^{AB}{}_{E} \big) \\ + \lambda^3 \big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}{}_{F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_E \beta^{CD}{}_{F} \\ + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{BC}{}_{F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}{}_{F} \big) + (\mathrm{c.c.}) \Big)$$

MP Higgs mechanism

1. **No restriction except for the single traceness**
2. **Can we reproduce the WZ-type coupling in IIA theory?**

$$S_{SC} = \mu_2 \int \mathrm{STr} \Big[P[C^{(3)}] + i\lambda i_\Phi i_\Phi \big[P[C^{(5)}] \big] \Big]$$

$$S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B \big) \\ + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C) + (\mathrm{c.c.}) \Big]$$

$$S_C^{(6)} = \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{\ \ C} + 3\lambda \big(C_{\mu\nu ABC\bar{D}} D_\rho Y^A \beta^{BC}_{\ \ D} + C_{\mu\nu AB\bar{C}\bar{D}} D_\rho Y^\dagger_C \beta^{AB}_{\ \ D} \big) \\ + 3\lambda^2 \big(C_{\mu ABCD\bar{E}} D_\nu Y^A D_\rho Y^B \beta^{CD}_{\ \ E} + C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y^\dagger_D \beta^{BC}_{\ \ E} + C_{\mu AB\bar{C}\bar{D}\bar{E}} D_\nu Y^\dagger_C D_\rho Y^\dagger_D \beta^{AB}_{\ \ E} \big) \\ + \lambda^3 \big(C_{ABCDE\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^C \beta^{DE}_{\ \ F} + C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_E \beta^{CD}_{\ \ F} \\ + C_{ABC\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{BC}_{\ \ F} + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^\dagger_C D_\nu Y^\dagger_D D_\rho Y^\dagger_E \beta^{AB}_{\ \ F} \big) + (\mathrm{c.c.}) \Big)$$

MP Higgs mechanism

1. No restriction except for the single traceness
2. Can we reproduce the WZ-type coupling in IIA theory?
3. Relations among form fields in string/M-theory

$$S_{SC} = \mu_2 \int \mathrm{STr} \Big[P[C^{(3)}] + i\lambda i_\Phi i_\Phi \big[P[C^{(5)}] \big] \Big]$$

- **3-form fields:**

$$
\begin{aligned}
S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!}\epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[& C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + 3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B \big) \\
& + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C \big) + (\mathrm{c.c.}) \Big]
\end{aligned}
$$

- **3-form fields:**

$$S_C^{(3)} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \{\mathrm{Tr}\} \Big[C_{\mu\nu\rho} + 3\lambda C_{\mu\nu A} D_\rho Y^A + \boxed{3\lambda^2 \big(C_{\mu AB} D_\nu Y^A D_\rho Y^B + C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B \big)} \\ + \lambda^3 (C_{ABC} D_\mu Y^A D_\nu Y^B D_\rho Y^C + C_{AB\bar{C}} D_\mu Y^A D_\nu Y^B D_\rho Y^\dagger_C) + (\mathrm{c.c.}) \Big]$$

$$\{\mathrm{Tr}\}(C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B) = (C_{\mu A\bar{B}})^{\hat{a}b}_{a\hat{b}} (D_\nu Y^A)^a_{\hat{a}} (D_\rho Y^\dagger_B)^{\hat{b}}_b$$

After the symmetry breaking scalar fields are all in the adjoint representation of the unbroken U(N) ➔ hatted and unhatted gauge indices are indistinguishable.

$$\{\mathrm{Tr}\}(C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B) = (C_{\mu A\bar{B}})^{cd}_{ab} (D_\nu Y^A)^a_c (D_\rho Y^\dagger_B)^b_d$$

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$$= \mathrm{Tr}(C_{\mu A\bar{B}}^{(1)} D_\nu Y^A D_\rho Y_B^\dagger) + \mathrm{Tr}(C_{\mu A\bar{B}}^{(2)} D_\rho Y_B^\dagger D_\nu Y^A)$$

$$\boxed{Y^A \to \tfrac{v}{2}\delta^{A4} + Y^A}$$

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$$= \boxed{\mathrm{Tr}(C_{\mu A\bar{B}}^{(1)} D_\nu Y^A D_\rho Y_B^\dagger) + \mathrm{Tr}(C_{\mu A\bar{B}}^{(2)} D_\rho Y_B^\dagger D_\nu Y^A)}$$

$$\boxed{Y^A \to \tfrac{v}{2}\delta^{A4} + Y^A}$$

$$\epsilon^{\mu\nu\rho}\{\mathrm{Tr}\}(C_{\mu AB} D_\nu Y^A D_\rho Y^B) = \boxed{\epsilon^{\mu\nu\rho}\mathrm{Tr}(C_{\mu AB}^{(3)} D_\nu Y^A D_\rho Y^B)}$$

$$\begin{aligned}&\epsilon^{\mu\nu\rho}\{\mathrm{Tr}\}\big(C_{\mu A\bar{B}}D_\nu Y^A D_\rho Y^\dagger_B + C_{\mu AB}D_\nu Y^A D_\rho Y^B\big) + (\mathrm{c.c.})\\ = \quad &\epsilon^{\mu\nu\rho}\mathrm{Tr}\big(\tilde{C}_{\mu ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j + v\tilde{B}_{\mu i}\langle\!\langle A^-_\nu \tilde{D}_\rho \tilde{X}^i\rangle\!\rangle\big)\end{aligned}$$

- **Restrictions for form fields in M-theory:**

$$\boxed{C^{(1)}_{\mu A\bar{B}} - C^{(1)\dagger}_{\mu B\bar{A}} = C^{(2)}_{\mu A\bar{B}} - C^{(2)\dagger}_{\mu B\bar{A}}, \quad C^{(3)}_{\mu AB} = C^{(3)}_{\mu[AB]}, \qquad (A, B = 1, 2, 3, 4)}$$

- **Relations form fields in string theory and M-theory:**

$$\boxed{\begin{aligned}
&\tilde{B}_{\mu 4} = 4i\big(C^{(1)}_{\mu 4\bar{4}} - C^{(1)\dagger}_{\mu 4\bar{4}}\big)\\
&\tilde{B}_{\mu a} = 2i\big(C^{(1)}_{\mu 4\bar{a}} - C^{(1)\dagger}_{\mu 4\bar{a}} + C^{(1)}_{\mu a\bar{4}} - C^{(1)\dagger}_{\mu a\bar{4}} + C^{(3)}_{\mu 4a} - C^{(3)\dagger}_{\mu 4a}\big)\\
&\tilde{B}_{\mu a+4} = 2\big(C^{(1)}_{\mu 4\bar{a}} + C^{(1)\dagger}_{\mu 4\bar{a}} - C^{(1)}_{\mu a\bar{4}} - C^{(1)\dagger}_{\mu a\bar{4}} - C^{(3)}_{\mu 4a} - C^{(3)\dagger}_{\mu 4a}\big)\\
&\tilde{C}_{\mu 4a} = C^{(1)}_{\mu 4\bar{a}} + C^{(1)\dagger}_{\mu 4\bar{a}} - C^{(1)}_{\mu a\bar{4}} - C^{(1)\dagger}_{\mu a\bar{4}} + C^{(3)}_{\mu 4a} + C^{(3)\dagger}_{\mu 4a}\\
&\tilde{C}_{\mu 4a+4} = -i\big(C^{(1)}_{\mu 4\bar{a}} - C^{(1)\dagger}_{\mu 4\bar{a}} + C^{(1)}_{\mu a\bar{4}} - C^{(1)\dagger}_{\mu a\bar{4}} - C^{(3)}_{\mu 4a} + C^{(3)\dagger}_{\mu 4a}\big)\\
&\tilde{C}_{\mu ab} = C^{(1)\dagger}_{\mu a\bar{b}} - C^{(1)}_{\mu b\bar{a}} + C^{(2)}_{\mu a\bar{b}} - C^{(2)\dagger}_{\mu b\bar{a}} - C^{(3)\dagger}_{\mu ab} + C^{(3)}_{\mu ab}\\
&\tilde{C}_{\mu ab+4} = i\big(C^{(1)\dagger}_{\mu a\bar{b}} - C^{(1)}_{\mu b\bar{a}} - C^{(2)}_{\mu a\bar{b}} + C^{(2)\dagger}_{b\bar{a}} + C^{(3)\dagger}_{\mu ab} - C^{(3)}_{\mu ba}\big)\\
&\tilde{C}_{\mu a+4b+4} = C^{(1)\dagger}_{\mu a\bar{b}} - C^{(1)}_{\mu b\bar{a}} + C^{(2)}_{\mu a\bar{b}} - C^{(2)\dagger}_{\mu b\bar{a}} + C^{(3)\dagger}_{\mu ab} - C^{(3)}_{\mu ab}
\end{aligned}}$$

- **Up to leading order in the vev:**

$$
\begin{aligned}
S^{(3)}_{\tilde{C}} = \mu_2 \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \Big(& \tilde{C}_{\mu\nu\rho} + 3\lambda (\tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}^i + v \tilde{B}_{\mu\nu} A^-_\rho) + 3\lambda^2 (\tilde{C}_{\mu i j} \tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j \\
& + v \tilde{B}_{\mu i} \langle\!\langle A^-_\nu \tilde{D}_\rho \tilde{X}^i \rangle\!\rangle) + \lambda^3 (\tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k + v \tilde{B}_{ij} \langle\!\langle A^-_\mu \tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j \rangle\!\rangle) \Big)
\end{aligned}
$$

- **Up to leading order in the vev:**

$$S^{(3)}_{\tilde{C}} = \mu_2 \int d^3x \, \frac{1}{3!}\epsilon^{\mu\nu\rho}\Big(\tilde{C}_{\mu\nu\rho} + 3\lambda(\tilde{C}_{\mu\nu i}\tilde{D}_\rho \tilde{X}^i + v\tilde{B}_{\mu\nu}A^-_\rho) + 3\lambda^2(\tilde{C}_{\mu ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j$$

$$+ v\tilde{B}_{\mu i}\langle\langle A^-_\nu \tilde{D}_\rho \tilde{X}^i\rangle\rangle) + \lambda^3(\tilde{C}_{ijk}\tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k + vB_{ij}\langle\langle A^-_\mu \tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j\rangle\rangle)\Big)$$

+

$$\int d^3x \, \mathrm{Tr}\Big(- \tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - v^2 A^-_\mu A^{-\mu} + \frac{k}{2\pi}\, \epsilon^{\mu\nu\rho} A^-_\mu \tilde{F}_{\nu\rho} - V_{\mathrm{bos}}\Big) + \mathcal{O}(\frac{1}{v})$$

- **Solving EOM for** A^-_μ

$$A^-_\mu = \frac{k}{4\pi v^2}\epsilon_\mu{}^{\nu\rho}\Big(\tilde{F}_{\nu\rho} + \mu_2 v\lambda \frac{2\pi}{k} P[\tilde{B}_{\nu\rho}]\Big) = \frac{1}{2g_{\mathrm{YM}}v}\,\epsilon_\mu{}^{\nu\rho}\Big(\tilde{F}_{\nu\rho} + \frac{1}{\tilde{\lambda}} P[\tilde{B}_{\nu\rho}]\Big)$$

$$P[\tilde{B}_{\mu\nu}] = \frac{1}{2}\Big(\tilde{B}_{\mu\nu} + \lambda\langle\langle \tilde{B}_{\mu i}\tilde{D}_\nu \tilde{X}^i\rangle\rangle + \frac{\lambda^2}{3}\langle\langle \tilde{B}_{ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j\rangle\rangle\Big)$$

$$\tilde{F}_{\mu\nu} = \partial_\mu A^+_\nu - \partial_\nu A^+_\mu + i[A^+_\mu,\, A^+_\nu]$$

- **Linearized non-Abelian DBI action for D2-branes:**

$$\int d^3x \Bigg\{ \frac{1}{g_{\rm YM}^2}\Big[-\tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - \frac{1}{2}\Big(\tilde{F}_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[\tilde{B}_{\mu\nu}]\Big)^2 + \frac{1}{8}[\tilde{X}^i,\, \tilde{X}^j]^2\Big]$$

$$+ \frac{\mu_2}{3!}\epsilon^{\mu\nu\rho}\big(\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda}\tilde{C}_{\mu\nu i}\tilde{D}_\rho \tilde{X}^i + 3\tilde{\lambda}^2 \tilde{C}_{\mu ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j + \tilde{\lambda}^3 \tilde{C}_{ijk}\tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k\big)\Bigg\}$$

- **Linearized non-Abelian DBI action for D2-branes:**

$$\int d^3x \left\{ \frac{1}{g_{\rm YM}^2}\left[-\tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - \frac{1}{2}\left(\boxed{\tilde{F}_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[\tilde{B}_{\mu\nu}]} \right)^2 + \frac{1}{8}[\tilde{X}^i, \tilde{X}^j]^2 \right] \right.$$

$$\left. + \frac{\mu_2}{3!}\epsilon^{\mu\nu\rho}\left(\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda}\tilde{C}_{\mu\nu i}\tilde{D}_\rho \tilde{X}^i + 3\tilde{\lambda}^2 \tilde{C}_{\mu ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j + \tilde{\lambda}^3 \tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k \right) \right\}$$

- In addition to the natural couplings of the D2-brane to the R-R 3-form fields in type IIA theory, the gauge invariant combination $\tilde{F}_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[\tilde{B}_{\mu\nu}]$ appeared.

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$$+ \frac{\mu_2}{3!}\epsilon^{\mu\nu\rho}\big(\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda}\tilde{C}_{\mu\nu i}\tilde{D}_\rho \tilde{X}^i + 3\tilde{\lambda}^2 \tilde{C}_{\mu ij}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j + \tilde{\lambda}^3 \tilde{C}_{ijk}\tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k\big) \Bigg\}$$

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- **WZ-type coupling for R-R 5-form fields:**

$$S_{\tilde{C}}^{(5)} = -\frac{\mu_2 \tilde{\lambda}}{2}\int d^3x \epsilon^{\mu\nu\rho}\frac{1}{3!}\Big(i\tilde{C}_{\mu\nu\rho ij}[\tilde{X}^i, \tilde{X}^j] + 3i\tilde{\lambda}\tilde{C}_{\mu\nu ijk}\langle\!\langle [\tilde{X}^i, \tilde{X}^j]\tilde{D}_\rho \tilde{X}^k \rangle\!\rangle$$

$$+ 3i\tilde{\lambda}^2 \tilde{C}_{\mu ijkl}\langle\!\langle [\tilde{X}^i, \tilde{X}^j]\tilde{D}_\nu \tilde{X}^k \tilde{D}_\rho \tilde{X}^l \rangle\!\rangle + i\tilde{\lambda}^3 \tilde{C}_{ijklm}\langle\!\langle [\tilde{X}^i, \tilde{X}^j]\tilde{D}_\mu \tilde{X}^k \tilde{D}_\nu \tilde{X}^l \tilde{D}_\rho \tilde{X}^m \rangle\!\rangle \Big)$$

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$$\int d^3x \Big\{ \frac{1}{g_{\mathrm{YM}}^2}\Big[-\tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - \frac{1}{2}\Big(\boxed{\tilde{F}_{\mu\nu} + \frac{1}{\tilde{\lambda}}P[\tilde{B}_{\mu\nu}]}\Big)^2 + \frac{1}{8}[\tilde{X}^i,\, \tilde{X}^j]^2\Big]$$
$$+ \frac{\mu_2}{3!}\epsilon^{\mu\nu\rho}\big(\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda}\tilde{C}_{\mu\nu i}\tilde{D}_\rho \tilde{X}^i + 3\tilde{\lambda}^2 \tilde{C}_{\mu i j}\tilde{D}_\nu \tilde{X}^i \tilde{D}_\rho \tilde{X}^j + \tilde{\lambda}^3 \tilde{C}_{ijk}\tilde{D}_\mu \tilde{X}^i \tilde{D}_\nu \tilde{X}^j \tilde{D}_\rho \tilde{X}^k\big)\Big\}$$

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$$+ 3i\tilde{\lambda}^2 \tilde{C}_{\mu ijkl}\langle\!\langle[\tilde{X}^i, \tilde{X}^j]\tilde{D}_\nu \tilde{X}^k \tilde{D}_\rho \tilde{X}^l\rangle\!\rangle + \boxed{i\tilde{\lambda}^3 \tilde{C}_{ijklm}\langle\!\langle[\tilde{X}^i, \tilde{X}^j]\tilde{D}_\mu \tilde{X}^k \tilde{D}_\nu \tilde{X}^l \tilde{D}_\rho \tilde{X}^m\rangle\!\rangle\Big)}$$

SUSY preserving mass deformation

- **SUSY preserving mass-deformation for the ABJM theory is possible** [Hosomichi-Lee-Lee-Lee-Park 08] [Gomis-Gomez-Raamsdonk-Verlinde 08]
- SU(4) R-symmetry in the original ABJM

  ➔ **SU(2)XSU(2)XU(1)**

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  ➔ **SU(2)XSU(2)XU(1)**
- Several different methods to obtain the mass-deformed ABJM theory: **N=1 superfield formalism, F-term deformation, D-term deformation.**

  ➔ **All these deformations are equivalent**

  [Kim-Kim-O.K.-Nakajima 09]

- **Classical vacua (V=0) were known.**
  [Gomis-Gomez-Raamsdonk-Verlinde 08]
  ➔ too many solutions
  ➔ Supersymmetric vacua for k=1[Kim-Kim 2010]

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One-to-one correspondence

Dual gravity: Lin-Lunin-Maldacena (04) geometry

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Dual gravity: Lin-Lunin-Maldacena (04) geometry

- **Mass parameter in ABJM theory**
  ➔ **turning on 4-form flux** [Lambert-Richmond 0908]

- **SUSY preserving mass deformation & WZ-type coupling:**

$$S_\mu = \mu^2 \int d^3x \, \mathrm{Tr}\big(Y^A Y_A^\dagger\big) + \frac{4\pi\mu}{k} \int d^3x \, \mathrm{Tr}\big(M_B{}^C Y^A Y_A^\dagger Y^B Y_C^\dagger - M^B{}_C Y_A^\dagger Y^A Y_B^\dagger Y^C\big)$$

$$= \mu^2 \int d^3x \, \mathrm{Tr}\big(Y^A Y_A^\dagger\big) - \frac{2\pi\mu}{k} \int d^3x \, \mathrm{Tr}\big(T_{AB\bar{C}\bar{D}} Y_D^\dagger \beta^{AB}_{\ \ C}\big) + (\mathrm{c.c.})$$

$$M_A{}^B = \mathrm{diag}(1,1,-1,-1) \qquad T_{AB\bar{C}\bar{D}} = M_A{}^D \delta_B^C - M_B{}^D \delta_A{}^C$$

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$$\boxed{S_\mu^{(6)} = \mu_2' \int d^3x\, \frac{1}{3!}\epsilon^{\mu\nu\rho}\mathrm{Tr}\big(C_{\mu\nu\rho AB\bar{C}}\beta^{AB}_{C}\big) + (\mathrm{c.c.})}$$

$$C_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2}\epsilon_{\mu\nu\rho}T_{AB\bar{C}\bar{D}}Y_D^\dagger, \quad C^\dagger_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2}\epsilon_{\mu\nu\rho}T^\dagger_{AB\bar{C}\bar{D}}Y^D$$

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$$\boxed{S_\mu^{(6)} = \mu_2' \int d^3x \, \frac{1}{3!} \epsilon^{\mu\nu\rho} \mathrm{Tr}\big(C_{\mu\nu\rho AB\bar{C}} \beta^{AB}_{\ \ C}\big) + (\mathrm{c.c.})}$$

$$C_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2} \epsilon_{\mu\nu\rho} T_{AB\bar{C}\bar{D}} Y_D^\dagger, \quad C^\dagger_{\mu\nu\rho AB\bar{C}} = -\frac{2\mu}{\lambda\mu_2} \epsilon_{\mu\nu\rho} T^\dagger_{AB\bar{C}\bar{D}} Y^D$$

$$\Longleftrightarrow \; F_{\mu\nu\rho AB\bar{C}\bar{D}} = F^\dagger_{\mu\nu\rho AB\bar{C}\bar{D}} = -\frac{2\mu}{\lambda\mu_2} \epsilon_{\mu\nu\rho} T_{AB\bar{C}\bar{D}} \; \Longleftrightarrow \; F_{AB\bar{C}\bar{D}} = -\frac{2\mu}{\lambda\mu_2} T_{AB\bar{C}\bar{D}}$$

Discussion

- Gauge invariant WZ-type coupling:
  - form fields depend on the scalar fields
  - single trace terms
- General setting ➔ restriction on the form fields

  ➔ explicit WZ-type coupling?

- Extension: nonlinear couplings,

  monopole operator for k=1,2: $\tilde{Y}^{A\hat{a}}{}_{a} = T^{\hat{a}\hat{b}}_{ab} Y^{Ab}{}_{\hat{b}}$

- Myers effect in M-theory, supersymmetry, relations with string theory

Single trace terms

$$\boxed{\{\mathrm{Tr}\}(C_{\mu A\bar{B}} D_\nu Y^A D_\rho Y^\dagger_B)}$$

single trace terms

$$\boxed{C^{\hat{a}b}_{a\hat{b}} = H^{\hat{a}}_{\hat{b}} I^b_a}$$

$$\{\mathrm{Tr}\}(C\,DY\,DY^\dagger) = C^{\hat{a}b}_{a\hat{b}}(DY)^a_{\hat{a}}(DY^\dagger)^{\hat{b}}_b = \mathrm{Tr}(H\,DY^A I\,DY^\dagger)$$

$$\boxed{Y^A \to \tfrac{v}{2}\delta^{A4} + Y^A}$$

h_i and i_i Depend on v, k, l_p

$$H^a{}_b = h_0\delta^a_b + h_1\tilde{Y}^a_b + h_2(\tilde{Y}\tilde{Y})^a{}_b + \cdots\,,$$

$$I^a{}_b = i_0\delta^a_b + i_1\tilde{Y}^a_b + i_2(\tilde{Y}\tilde{Y})^a{}_b + \cdots\,,$$

$$\boxed{\mathrm{Tr}(H\,DY^A I\,DY^\dagger) = \mathrm{Tr}\big(C^{(1)}DY\,DY^\dagger + C^{(2)}DY^\dagger DY + \mathcal{H}DY\mathcal{I}DY^\dagger\big)}$$

$$C^{(1)} = (h_0 + h_1\tilde{Y} + h_2\tilde{Y}\tilde{Y} + \cdots)i_0$$

$$C^{(2)} = (i_0 + i_1\tilde{Y} + i_2\tilde{Y}\tilde{Y} + \cdots)h_0,$$

$$\mathcal{H} = h_1\tilde{Y} + h_2\tilde{Y}\tilde{Y} + \cdots\,,$$

$$\mathcal{I} = i_1\tilde{Y} + i_2\tilde{Y}\tilde{Y} + \cdots\,.$$