Monopole operators in Chern-Simon-matter theories

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talk based on:

S.K. Nucl. Phys. B821, 241 (2009), **arXiv:0903.4172**

S.K. and **K. Madhu** JHEP 0912, 018, **arXiv:0906.4751**

Hee-Cheol Kim and **S.K.** **arXiv:1007.4560**

S. Cheon, D. Gang and **S.K.** to appear, **arXiv:1011.nnnn…?**

some related works:

M.K. Benna, I.R. Klebanov and T. Klose, arXiv:0906.3008

D. Berenstein and J. Park, arXiv:0906.3817

D. Bashkirov and A. Kapustin, arXiv:1007.4861

Introduction

- A class of Chern-Simons-matter theories describe low energy dynamics of M2's [Bagger-Lambert] [Gustavsson] [Aharony-Bergman-Jafferis-Maldadena] , …
- Simplest: N=6 CSm with $U(N)_k \times U(N)_{-k}$ for N M2-branes on R^8/Z_k
- Many examples with 5,4,3,2,1(,0 ?) SUSY known [ABJ] [Hosomichi-Lee-Lee-Lee-Park] [Imamura et.al.] [Gaiotto-Tomasiello] [Gaiotto-Jafferis] [Hikida-Li-Takayanagi] [Hanany et.al.] [Klebanov et.al.] [Martelli-Sparks], etc.

- Sometimes, expect extra symmetries at strong coupling
- Enhanced SUSY: e.g. N=6 to 8 (at k=1,2) ; N=5 to 6 (at k=1) **(more later)**
- Flavor symmetries: SU(2) x U(1) to SU(3) for N^{010} [Gaiotto-Jafferis]
- How? Delicate roles of **monopole operators**

Monopole operators in 3 dimensional QFT

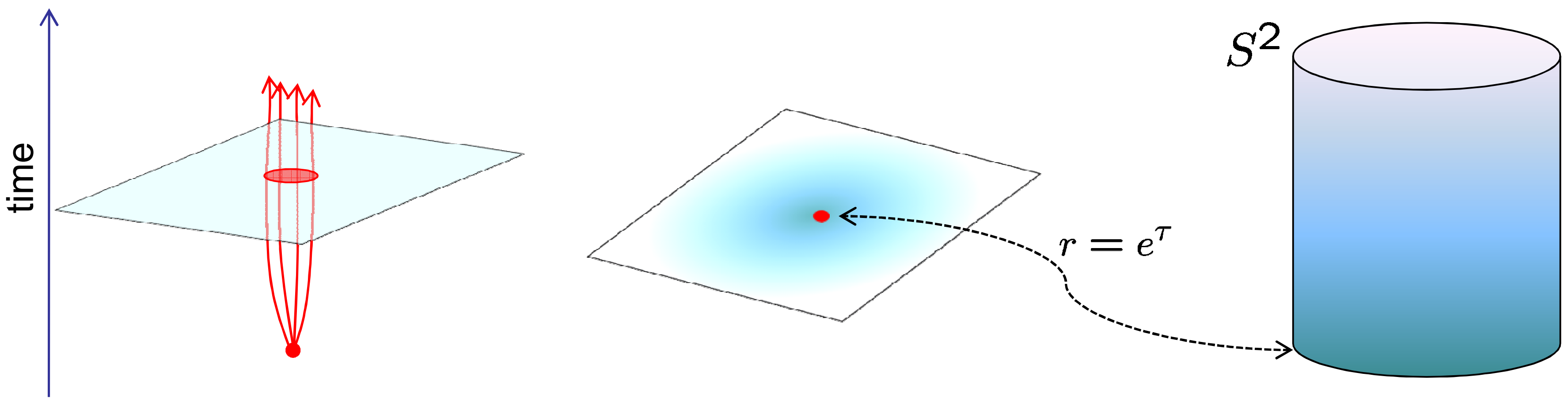
- Change b.c. around the insertion point (in path integral rep. of any observables)
- (singular) b.c. with quantized flux on S^2 surrounding the point

$$\int_{S^2} F = 2\pi H \qquad H = \mathrm{diag}(n_1, n_2, \cdots, n_N)$$ for U(N) gauge group

- The magnetic field near the point may be (but NOT necessarily) uniform

$$F \sim \star_3 d\left(\frac{1}{|x|}\right) H$$ or… more complicated profiles (details later)

- Vortex-creating operators: vortices in theories with mass gap, or (delocalized) fluxes on S^2 of radially quantized CFT



Monopole operators in CSm & AdS/CFT

- Gauss' law: extra excitation of matter fields required.

$$\frac{k}{2\pi} \star_3 F_\mu \sim j_\mu$$

- Extra gauge invariant operators: gauge non-invariance screened by monopoles

$$D_{\text{operator}} \sim k(\text{flux})$$

- Can provide extra currents for enhanced symmetry (at low enough k)
- Extra local operators provide KK states: IIA to M-theory (or D0 branes)

N=6 CSm with $U(N)_k \times U(N)_{-k}$:

$$J_\mu = \frac{k}{4\pi}\epsilon_{\mu\nu\rho}\text{tr}F^{\nu\rho} = \frac{k}{4\pi}\epsilon_{\mu\nu\rho}\text{tr}\tilde{F}^{\nu\rho} \ , \quad \int_{S^2} J_0 = p_{11}$$

$$AdS_4 \times CP^3 \longrightarrow AdS_4 \times S^7 / Z_k$$

- Generally nonperturbative, more difficulty in studying them at low k

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Strong coupling calculations in CSm

- Partition function on $S^2 \times S^1$: more precisely, index (periodic along S1)
- Interpretation as "superconformal index" for local operators

$$I(\beta, \gamma^i) = \mathrm{Tr}\left[(-1)^F e^{-\beta'\{Q,Q^\dagger\}} e^{-\beta(D+j)} e^{-\gamma^i q_i}\right]$$

regulator

dilatation + angular momentum

extra global symmetry commuting with Q

- Counts "states" on $S^2 \times R$: "thermal" replace R by S^1 with radius $\frac{\beta+\beta'}{2\pi}$
- Can use localization to calculate it at arbitrary coupling k [SK]

- Other quantities (where SUSY helps): partition ftn on S^3 [Kapustin-Willett-Yaakov]
- Information on monopoles absent (but more on it tomorrow [Drukker][Suyama]…)
- SC index: contains more information / more complicated to study

Superconformal index

- localization: formal saddle point analysis, consisting of classical & 1 loop
- Saddle points: monopoles on S^2 & commuting holonomies on S^1

$$A = \frac{H}{2}(\pm 1 - \cos\theta)\, d\phi + \frac{\alpha}{\beta} dt \qquad \tilde{A} = \frac{\tilde{H}}{2}(\pm 1 - \cos\theta)\, d\phi + \frac{\tilde{\alpha}}{\beta} dt$$

$$H = \mathrm{diag}(n_1, \cdots, n_N) \quad \alpha = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$
$$\tilde{H} = \mathrm{diag}(\tilde{n}_1, \cdots, \tilde{n}_N) \quad \tilde{\alpha} = \mathrm{diag}(\tilde{\alpha}_1, \cdots, \tilde{\alpha}_N)$$

- Unitary integral expression (whose eigenvalues are given by holonomies): [SK]

$$I(x, y_1, y_2) = x^{\epsilon_0} \int \frac{1}{(\mathrm{symmetry})} \left[\frac{d\alpha_i d\tilde{\alpha}_i}{(2\pi)^2}\right] \prod_{\substack{i<j;\\ n_i = n_j}} \left[2 \sin\left(\frac{\alpha_i - \alpha_j}{2}\right)\right]^2 \prod_{\substack{i<j;\\ \tilde{n}_i = \tilde{n}_j}} \left[2 \sin\left(\frac{\tilde{\alpha}_i - \tilde{\alpha}_j}{2}\right)\right]^2$$
$$\times e^{ik\sum_{i=1}^N (n_i\alpha_i - \tilde{n}_i\tilde{\alpha}_i)} \prod_{i,j=1}^N \exp\left[\sum_{n=1}^\infty \frac{1}{n}\left(f_{ij}^+(x^n, y_1^n, y_2^n) e^{in(\tilde{\alpha}_j - \alpha_i)} + f_{ij}^-(x^n, y_1^n, y_2^n) e^{in(\alpha_i - \tilde{\alpha}_j)}\right)\right]$$
$$\times \prod_{i,j=1}^N \exp\left[\sum_{n=1}^\infty \frac{1}{n}\left(f_{ij}^{\mathrm{adj}}(x^n) e^{-in(\alpha_i - \alpha_j)} + \tilde{f}_{ij}^{\mathrm{adj}}(x^n) e^{-in(\tilde{\alpha}_i - \tilde{\alpha}_j)}\right)\right]$$

$$f_{ij}^+(x, y_1, y_2) = x^{|n_i - \tilde{n}_j|}\left[\frac{x^{1/2}}{1-x^2}\left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}}\right) - \frac{x^{3/2}}{1-x^2}\left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}}\right)\right]$$

$$f_{ij}^{\mathrm{adj}}(x) = -(1 - \delta_{n_i n_j}) x^{|n_i - n_j|} = \begin{cases} 0 & \text{if } n_i = n_j \\ -x^{|n_i - n_j|} & \text{if } n_i \neq n_j \end{cases}$$

$$f_{ij}^-(x, y_1, y_2) = x^{|n_i - \tilde{n}_j|}\left[\frac{x^{1/2}}{1-x^2}\left(\sqrt{y_1 y_2} + \frac{1}{\sqrt{y_1 y_2}}\right) - \frac{x^{3/2}}{1-x^2}\left(\sqrt{\frac{y_1}{y_2}} + \sqrt{\frac{y_2}{y_1}}\right)\right]$$

$$\tilde{f}_{ij}^{\mathrm{adj}}(x) = -(1 - \delta_{\tilde{n}_i \tilde{n}_j}) x^{|\tilde{n}_i - \tilde{n}_j|} = \begin{cases} 0 & \text{if } \tilde{n}_i = \tilde{n}_j \\ -x^{|\tilde{n}_i - \tilde{n}_j|} & \text{if } \tilde{n}_i \neq \tilde{n}_j \end{cases}$$

Possible monopole operators

- Monopoles with $H = \tilde{H}$: have a good semi-classical picture. [ABJM] [Berenstein-Trancanelli] [Berenstein-Park] [SK-Madhu] , etc.

- "diagonal" elements of scalars are neutral under this magnetic field

$$(H\phi - \phi\tilde{H})_{ij} = (n_i - \tilde{n}_j)\phi_{ij} = (n_i - n_j)\phi_{ij}$$

excite ϕ_{ii}'s only

- Gauss' law & e.o.m on S^2 x R: ground states in s-waves (neutral)

$$\frac{k}{2\pi} \star F_t = -i\left(\phi^\dagger \partial_t \phi - \partial_t \phi^\dagger \phi\right)$$

$$\partial_t^2 \phi = -\frac{1}{4}\phi$$

comes from the conformal mass

$$\longrightarrow \quad \phi = v e^{-\frac{it}{2}}$$

$$v^\dagger v = \frac{k}{4\pi} H$$

- Monopoles with $H \neq \tilde{H}$: what are they…?

- As will be explained more later, they come with matters with spin

Comparison to gravity spectrum

- $H \neq \tilde{H}$ essential for precision comparison of AdS$_4$/CFT$_3$ spectra
- For instance, at k=1:

$I_{(3)(3)} =$	$x^{\frac{3}{2}}[$	4	$+4x$	$+0x^2$	$+8x^3$	$-4x^4$	$+8x^5$	$+2x^6$	$+4x^7$	$+0x^8$	$+\mathcal{O}(x^9)$	$]$
$I_{(2,1)(2,1)} =$	$x^{\frac{3}{2}}[$	6	$+20x$	$+24x^2$	$+28x^3$	$+64x^4$	$+34x^5$	$+34x^6$	$+166x^7$	$-32x^8$	$+\mathcal{O}(x^9)$	$]$
$I_{(1,1,1)(1,1,1)} =$	$x^{\frac{3}{2}}[$	4	$+12x$	$+30x^2$	$+52x^3$	$+52x^4$	$+98x^5$	$+170x^6$	$+130x^7$	$+106x^8$	$+\mathcal{O}(x^9)$	$]$
$I_{(2,1)(1,1,1)} + I_{(1,1,1)(2,1)} =$	$x^{\frac{3}{2}}[$					$0x^4$	$+12x^5$	$-20x^6$	$-44x^7$	$+176x^8$	$+\mathcal{O}(x^9)$	$]$
$I_{(3)(2,1)} + I_{(2,1)(3)} =$	$x^{\frac{3}{2}}[$							$+4x^6$	$-16x^7$	$+32x^8$	$+\mathcal{O}(x^9)$	$]$
$I_{(3)(1,1,1)} + I_{(1,1,1)(3)} =$	$x^{\frac{3}{2}}[$										$+\mathcal{O}(x^{12})$	$]$
$I_3(x) =$	$x^{\frac{3}{2}}[$	4	$+4x$	$+2x^2$	$+4x^3$	$+2x^4$	$+4x^5$	$+2x^6$	$+4x^7$	$+2x^8$	$+\mathcal{O}(x^9)$	$]$
$I_1(x)I_2(x) =$	$x^{\frac{3}{2}}[$	6	$+20x$	$+26x^2$	$+36x^3$	$+46x^4$	$+52x^5$	$+66x^6$	$+68x^7$	$+86x^8$	$+\mathcal{O}(x^9)$	$]$
$\frac{1}{3}I_1(x^3)+\frac{1}{2}I_1(x)I_1(x^2)+\frac{1}{6}I_1(x)^3 =$	$x^{\frac{3}{2}}[$	4	$+12x$	$+26x^2$	$+48x^3$	$+64x^4$	$+96x^5$	$+122x^6$	$+168x^7$	$+194x^8$	$+\mathcal{O}(x^9)$	$]$

$$
\begin{aligned}
&I_{(3)(3)} + I_{(2,1)(2,1)} + I_{(1,1,1)(1,1,1)} + 2I_{(2,1)(1,1,1)} + 2I_{(3)(2,1)} + 2I_{(3)(1,1,1)} \\
&= I_3(x) + I_1(x)I_2(x) + \frac{1}{3}I_1(x^3) + \frac{1}{2}I_1(x)I_1(x^2) + \frac{1}{6}I_1(x)^3 + \mathcal{O}(x^{\frac{3}{2}+9})
\end{aligned}
$$

- In physical CSm theories, monopole harmonics back-react to the flux.
- What are the physical picture of these monopoles with $H \neq \tilde{H}$?

Ground states of monopoles

- Ground states for $H = \tilde{H}$: chiral rings ~ diagonal scalars in s-waves
- Degeneracy obtained by "quantizing moduli space" [ABJM] [Hanany et.al.]
- Ground states for $H \neq \tilde{H}$: geometric meaning unclear
- Quantum numbers for U(2) x U(2): for $n_1 \geq \tilde{n}_1 \geq \tilde{n}_2 \geq n_2 (\geq 0)$

$$R = \frac{k}{2}(n_1 + n_2) = \frac{k}{2}(\tilde{n}_1 + \tilde{n}_2) \qquad j = k\tilde{n}_1(\tilde{n}_2 - n_2) \qquad D = R + j$$

- Degeneracy for U(2) x U(2): $\chi_{2j}(r) = \frac{r^{2j+1} - r^{-2j-1}}{r - r^{-1}} = r^{2j} + r^{2j-2} + \cdot + r^{-2j}$

$$\begin{cases} \chi_{kn_2}(r)\chi_{k(2\tilde{n}-n_1)}(r) & \text{for } n_1 \geq \tilde{n}_1 > \tilde{n}_2 \geq n_2 \\ \frac{1}{2}\chi_{kn}(r^2) + \frac{1}{2}\chi_{kn}(r)^2 & \text{for } n_1 = n_2 = \geq \tilde{n}_1 = \tilde{n}_2 \equiv n \\ \chi_{kn_2}(r)^2 + \chi_{kn_2-2}(r)^2 + \cdots + \chi_{0 \text{ or } 1}(r)^2 & \text{for } n_1 > \tilde{n}_1 = \tilde{n}_2 > n_2 \end{cases}$$

- Degeneracy for U(N)xU(N): patterns unclear yet (integral expressions only)
- Comparison to SC index 4d: milder cancelation for non s-waves

Semi-classical studies of monopole operators

- Monopole operators at general k are difficult to study directly.
- In CFT, use "operator-state" map: states on S^2 x R
- For large k, can study them by quantizing 'soliton' solutions
- Often provides intuitions for strong coupling monopoles

- Analysis for $H = \tilde{H}$: matter excitations in s-waves, back-reaction simple

- $H \neq \tilde{H}$: should excite matters charged under the monopole: not uniform on S^2 (intuition: monopole harmonics carry angular momenta)
- As matters back-react, magnetic fields are also non-uniform.

Monopoles with unequal magnetic fluxes

- U(2) x U(2) example: $H=(n_1+n_2,0),\tilde{H}=(n_1,n_2)$ (fluxes positive)

$$A_\mu = \begin{pmatrix} A^1_\mu & 0 \\ 0 & 0 \end{pmatrix} \qquad \tilde{A}_\mu = \begin{pmatrix} \tilde{A}^1_\mu & 0 \\ 0 & \tilde{A}^2_\mu \end{pmatrix}$$

& azimuthal symmetry: all functions independent of θ & $A_\theta=\tilde{A}_\theta=0$

- matter ansatz fixed by consistency of BPS eqns. & Gauss' law

$$\phi_1 = \begin{pmatrix} \psi(\theta) & 0 \\ 0 & 0 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} 0 & 0 \\ \chi(\theta) & 0 \end{pmatrix}$$

(up to global $SU(2)_R$ rotating 2 scalars)

- other two scalars zero with a definite SUSY projector (they are anti-BPS)
- SUSY $(D_1-iD_2)\phi^\dagger_a=0,\ D_3\phi^\dagger_a=\frac{2\pi}{k}(\phi^\dagger_b\phi_b\phi^\dagger_a-\phi^\dagger_a\phi_b\phi^\dagger_b)$ (on R^3) & Gauss' law:

$$x=\cos\theta\ ,\quad {}'=\frac{d}{dx}$$

$$f_1\equiv\frac{2\pi}{k}|\psi|^2,\ f_2\equiv\frac{2\pi}{k}|\chi|^2$$

$$g_{1,2}\equiv A^1_t-\tilde{A}^{1,2}_t,\ h_{1,2}\equiv A^1_\varphi-\tilde{A}^{1,2}_\varphi$$

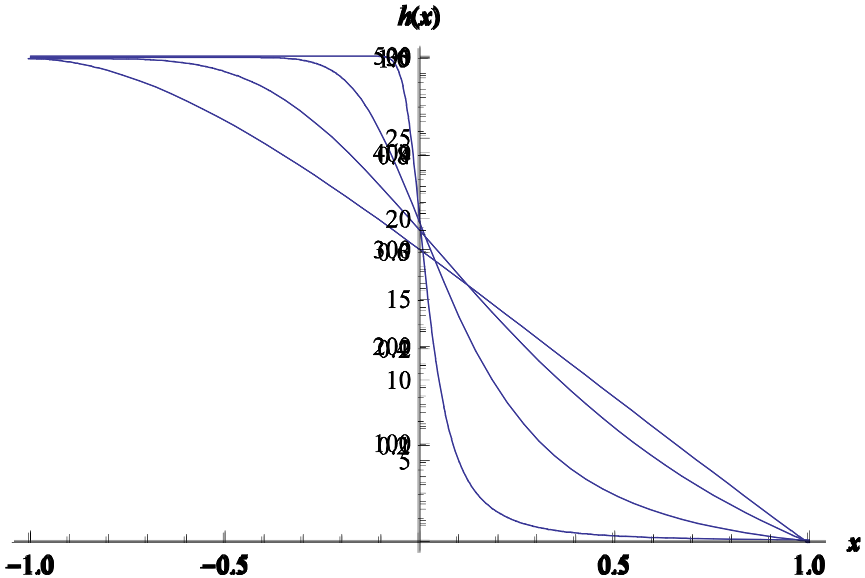
$$\begin{cases} h'_1=2g_2f_2,\ h'_2=2g_1f_1 \\ (1-x^2)g'_1=2h_2f_2,\ (1-x^2)g'_2=2h_1f_1 \\ xf_1=-\left(g_2+h_2+\frac{1}{2}\right) \\ xf_2=-\left(g_1+h_1+\frac{1}{2}\right) \end{cases}$$

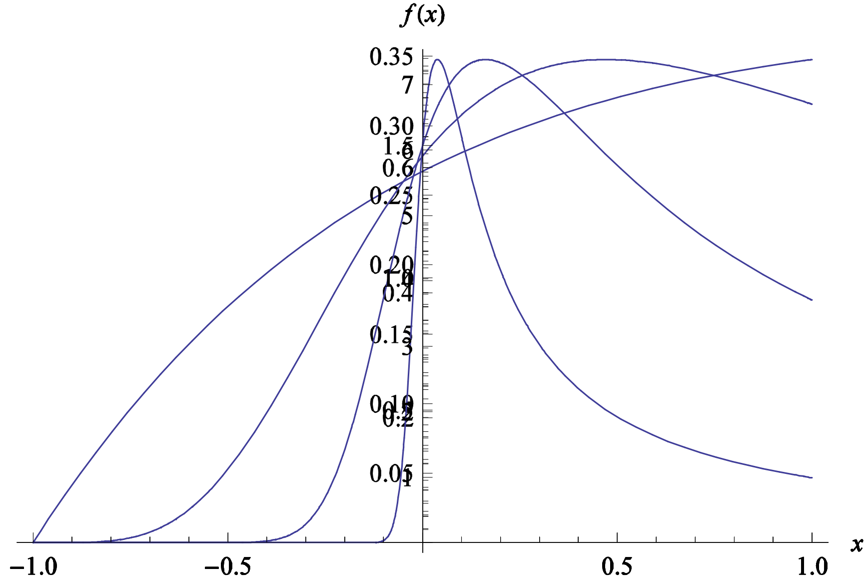
Monopoles with unequal magnetic flux: solutions

- Numerical solutions for the ODE's: e.g. $H = (2n, 0), \tilde{H} = (n, n)$
- magnetic potential & matters: compare to uniform flux $A_\varphi = \frac{n}{2}(1 - \cos\theta) = \frac{n}{2}(1 - x)$

$$h_1 = h_2 \equiv h(x) \qquad\qquad f_1 = f_2 \equiv f(x) = \frac{2\pi}{k}|\psi|^2$$

n=500





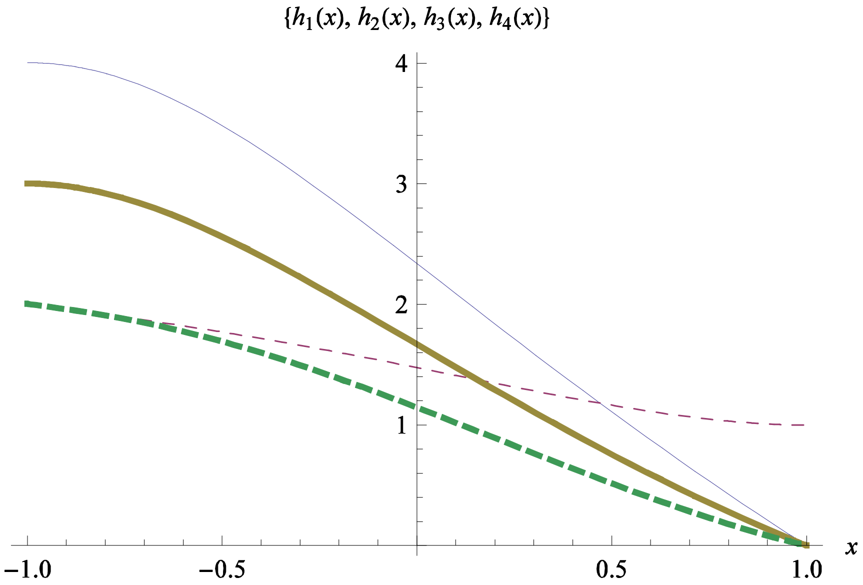
- All quantum numbers (R-charge, spin) predicted from index reproduced.

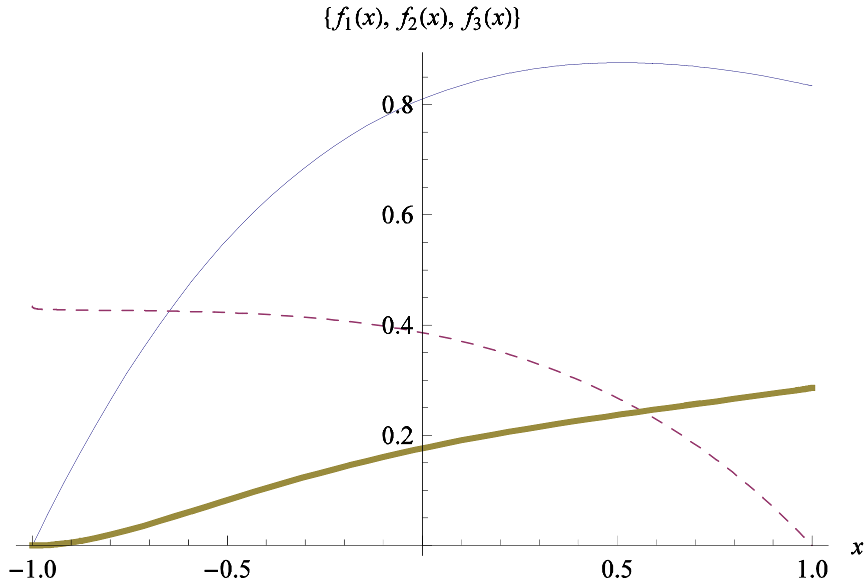
More monopoles with unequal fluxes: U(2)xU(2)

- Quite surprisingly, adding one more flux as $H=(n_1,n_2),\tilde{H}=(\tilde{n}_1,\tilde{n}_2)$ is far more complicated than the previous solutions… (but doable)
- More functions, trickier numerics, consistent (& most general) ansatz

$$A_\mu = \begin{pmatrix} A_\mu^1 & 0 \\ 0 & A_\mu^2 \end{pmatrix} \quad \tilde{A}_\mu = \begin{pmatrix} \tilde{A}_\mu^1 & 0 \\ 0 & \tilde{A}_\mu^2 \end{pmatrix} \qquad \phi_1 = \begin{pmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{pmatrix} \phi_2 = \begin{pmatrix} 0 & \chi_2 \\ \chi_1 & 0 \end{pmatrix}$$

- Solutions with fluxes $H=(4,1),\tilde{H}=(3,2)$





Quantization: ground state degeneracy

- Moduli: SU(2) global symmetry acting on scalars, **2-sphere**
- Symplectic 2-form on solution space ~ Fubini-Study 2-form on CP^1

$$\omega = ik(2\tilde{n}_1 - n_1 + n_2)\omega_{FS}$$

- Degeneracy: SU(2) character of irrep. with total spin $j = \frac{k(2\tilde{n}_1 - n_1 + n_2)}{2}$

$$\chi_{2j}(r) = \frac{r^{2j+1} - r^{-2j-1}}{r - r^{-1}} = r^{2j} + r^{2j-2} + \cdot + r^{-2j}$$

- Agrees with the index degeneracy when $H = (n_1, 0), \tilde{H} = (\tilde{n}_1, \tilde{n}_2)$
- Smaller than index generally: probably our ansatz is not most general

$$\chi_{k(2\tilde{n}_1 - n_1 + n_2)}(r) < \chi_{kn_2}(r)\chi_{k(2\tilde{n}_1 - n_1)}$$

$$= \chi_{k(2\tilde{n}_1 - n_1 + n_2)}(r) + \chi_{k(2\tilde{n}_1 - n_1 + n_2) - 2} + \cdots + \chi_{k(2\tilde{n}_1 - n_1 - n_2)}$$

- Lessons: (1) existence, (2) ground states could be all bosonic, (3) index shows less cancelation with in 3d than 4d

Monopoles in CSm with reduced SUSY

- Strong coupling symmetry enhancements are not so uncommon.

- "N=5 SUSY CSm with O(2N+l) x Sp(2N) at k=1" is same as
  "N=6 SUSY CSm with U(N+l) x U(N) at k=4" [ABJ]

- Previous evidence: same moduli space R^8/Z_4 (~ chiral rings)

- Indices agree between the two theories [S.Cheon-D.Gang-SK] (to appear soon)

1. analytic check of the agreement in the large N limit
2. finite N numerical studies: refines previous proposal (i.e. mapped discrete torsions)

$$U(1+\ell) \times U(1) \sim O(2+\ell) \times Sp(2) \qquad \text{where } \ell = 0, 1, 2, 3$$

Concluding remarks

- Calculation of topological quantities (e.g. index) provides very useful information on strongly interacting QFT, M2, M-theory…
- Semi-classical studies at large k: QFT duals of D0 branes, direct studies

- “N=6 to 8” enhancement studied by calculating spectrum of stress-energy supermultiplet (after certain deformation of the theory) [Bashkirov-Kapustin]
- Similar study of the enhanced supermultiplet for “N=5 to 6” enhancement?

- $N^{3/2}$ for vacuum free energy on 3-sphere [Drukker-Marino-Putrov]
- Same factor from thermal partition function (or more feasibly, index)?

Concluding remarks *(continued)*

- In 4d SCFT, index exhibits too much cancelation of bosonic/fermionic operators: does not show N^2 states at high "temperature" phase
- Can't be used to study SUSY AdS_5 black holes [H.S. Reall et.al.] [Cvetic et.al.]

- Compared to the 4d, monopole indices show less cancelations in 3d.
- Better chance to observe $N^{3/2}$ at high temperature…?

- Supersymmetric correlation functions at strong coupling via localization
- The strong coupling results proportional to $(N^{3/2})^{1-n/2}$ for n-point functions (even for n mutually BPS operators)