On Large N Solution of Gaiotto-Tomasiello Theory

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Introduction

Question: What happens in AdS/CFT correspondence in the presence of (fundamental) flavors ?

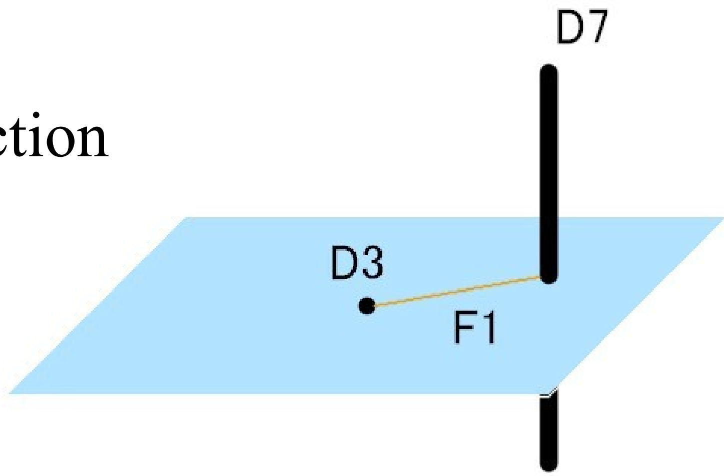
E.g. Flavors added to N=4 SYM_4 [Karch, Katz]

$M \ll N$ (# of flavors) (rank) ⇨ Flavors are introduced by M probe D7-branes.

Dual geometry is still $\mathrm{AdS}_5 \times S^5$.

For a large M, i.e. M=(N), the back-reaction of D7 to geometry becomes relevant.

⇨ What happens in such cases ?



Conformal SQCD

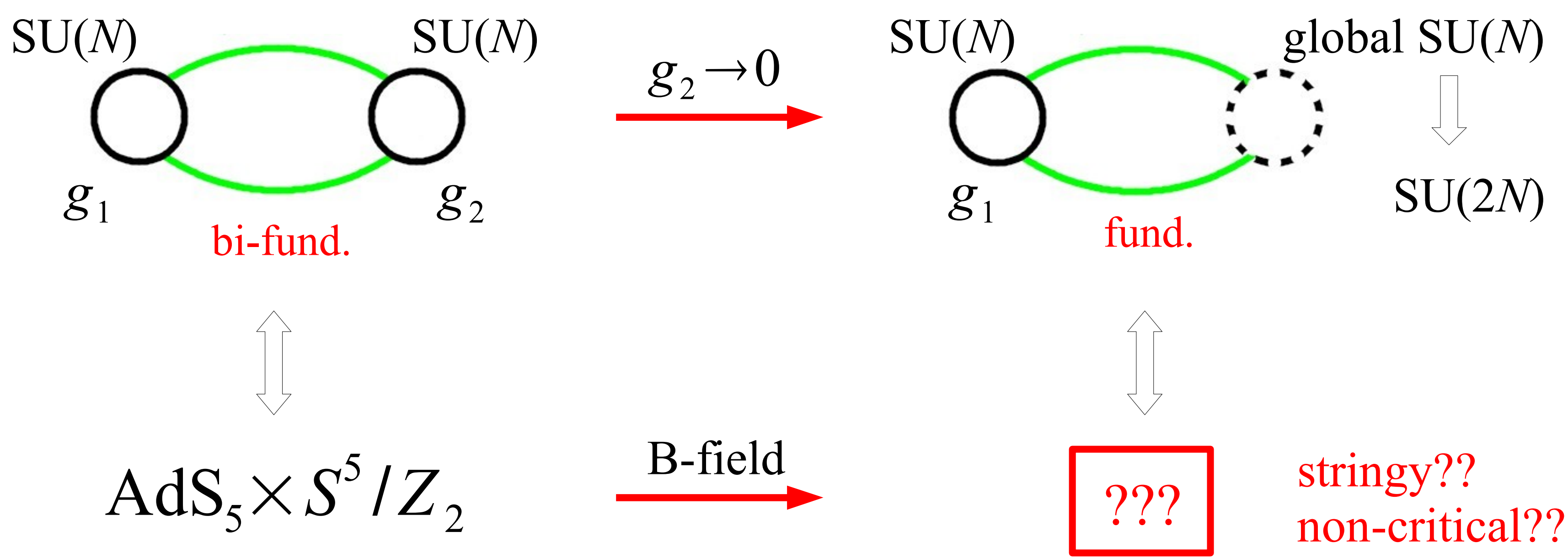
[Gadde,Pomoni,Rastelli]

N=2 SYM with flavors with $M=O(N)$ is generically non-conformal even in large N limit.

Conformal for $M=2N$ ⇔ (???) Another AdS_5 solution

A useful viewpoint:

Conformal SQCD is obtained from N=2 quiver theory in a limit.



Another approach to gravity dual

[Gaiotto, Maldacena]

Conformal SQCD can be realized in terms of M5-branes wrapping on a Riemann surface.

⇨ Near-horizon geometry is schematically

$$\mathrm{AdS}_5 \times \Sigma_2 \times S^4.$$

(not a direct product)

Riemann surface

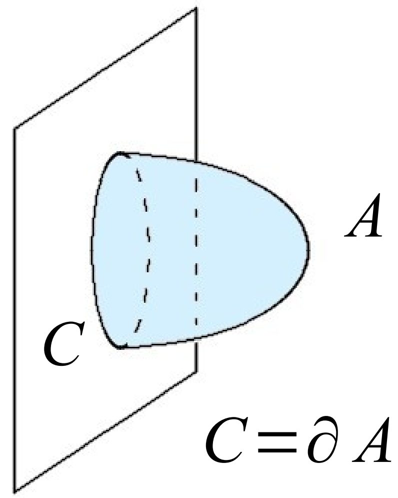
It is observed that the dual geometry has

- $A_{2N\text{-}1}$ singularity in the bulk,
- Type IIA reduction with stringy curvature.

Wilson loops in AdS/CFT

[Maldacena]
[Rey, Yee]

Wilson loops are nice observables to probe dual geometry.



$$\langle W[C]\rangle \sim e^{-S_{NG}[A]}, \quad S_{NG} \propto \mathrm{Area}[A]$$

Typically, $\mathrm{Area}[A] \propto \sqrt{\lambda}. \quad (\lambda = g_{YM}^2 N)$

$\Longrightarrow$ $$\langle W[C]\rangle \sim e^{c\sqrt{\lambda}}$$ in large N, large λ.

``AdS behavior''

In fact, $\langle W[C]\rangle$ can be calculated exactly for

- N=4 SYM_4 [Pestun]
- ABJM [Marino, Putrov]

$\Longrightarrow$ SUGRA results including c are reproduced.

Wilson loop in conformal SQCD

[Rey, TS]

Localization formulae for Wilson loops are available for all N=2 (Lagrangian) gauge theories with matters.

[Pestun]

$$\langle W_R[C] \rangle = \frac{1}{Z} \int dM \; e^{-\frac{1}{g^2}\mathrm{tr}\, M^2} Z_{1-loop}(M) Z_{inst}(M)\, \mathrm{tr}_R\, e^M$$

This can be evaluated by saddle-point approximation.

(exact in large N limit)

Schematic form of saddle-point equation for conformal SQCD:

$$\frac{1}{\lambda}\phi_i + \underline{V_F(\phi_i)} = V_{ad}(\phi_i).$$

external potential induced by matters independent of λ

??? $\Rightarrow$ Eigenvalues are confined even in large λ.

$$\langle W[C] \rangle \overset{??}{\sim} O(1)$$

non-AdS??

Our observation : AdS behavior of Wilson loop seems to disappear in the presence of many flavors.

⇕

Possibly related to singular natures of the dual geometries proposed so far.

However, our analysis is not precise enough to conclude.

⇒ Find an analogous setup where a more precise calculations can be performed.

In the following, we consider Gaiotto-Tomasiello theory with the help of localization technique.

Gaiotto-Tomasiello theory [Gaiotto, Tomasiello]

ABJ(M) theory is specified by integers (N_1, N_2, k).

⇨ Generalization in which two Chern-Simons level may differ, i.e. parametrized by (N_1, N_2, k_1, k_2), $k_1+k_2\neq 0$.

SUSY is reduced to $\mathrm{N}\leq 3$.

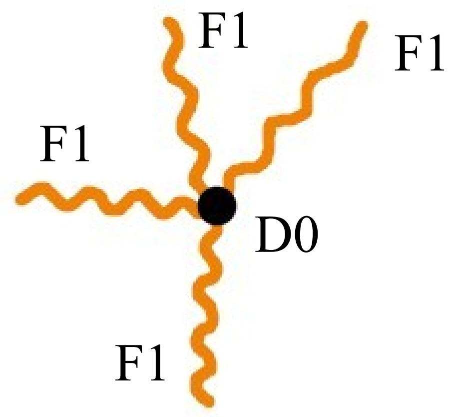
We focus on an N=3 Chern-Simons-matter theory (GT theory) whose field content coincides with ABJM.

- conformal quantum mechanically.
- couplings : $\frac{N_1}{k_1}, \frac{N_2}{k_2}$ and $\frac{k_1}{k2}$ in planar limit.
- There might exist a choice of parameters for which the theory would not exist (like ABJ).

⇨ $N_1=N_2, \quad k_1\leq |k_2|$ should exist.

Gravity dual : AdS$_4$ solution in massive Type IIA. [GT]

To see a piece of evidence, consider a D0-brane.



$\exists$ CS-coupling $\int_{D0} F_{10} A$

⇨ IIA strings must be attached.
Opposite endpoints form a state with a non-zero charge.

⇕

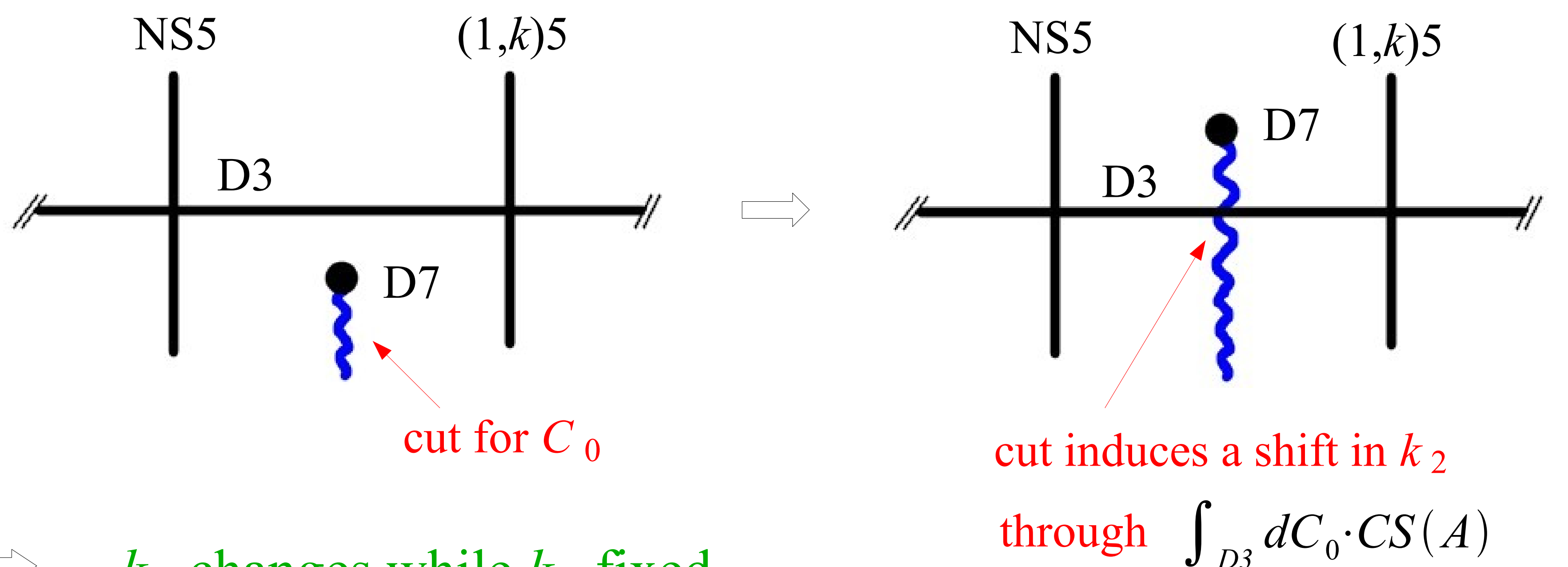
A local operator : monopole operator (string = Wilson line)

In $U(1)_{k_1} \times U(1)_{k_2}$ theory, monopole op. with charge (1,1) is equivalent to Wilson line with charge (k_1, k_2).

⇨ $$k_1 + k_2 = F_{10}$$

A brane construction of (N=0) CSM with F_{10} [Bergman, Lifschytz]

In Type IIB picture,



⇒ k_2 changes while k_1 fixed.

Take a limit $|k_2| \to \infty$ ⇒ $\frac{N_2}{k_2} \to 0.$

One set of gauge fields decouple, bi-fund. become fund.

⇒ Analogy to conformal SQCD !!

Localization in CSM

[Kapustin, Willett, Yaakov]

Localization formula for GT theory :

$$Z_{GT} = \int \prod_{i=1}^{N_1} d\phi_i \prod_{a=1}^{N_2} d\tilde{\phi}_a \, e^{i\pi(k_1 \sum_i \phi_i + k_2 \sum_a \tilde{\phi}_a)} \times \frac{\prod_{i<j} \sinh^2 \pi(\phi_i - \phi_j) \prod_{a<b} \sinh^2 \pi(\tilde{\phi}_a - \tilde{\phi}_b)}{\prod_{i,a} \cosh^2 \pi(\phi_i - \tilde{\phi}_a)}$$

Difference from ABJ : (k_1, k_2) instead of $(k, -k)$

No dependence on superpotential

Wilson loop (1/3-BPS) ⟺ insertion of $\sum_i e^{2\pi\phi_i}$

Note: The decoupling limit sets $\tilde{\phi}_a = 0$ by the stationary phase.

⟹ N=3 Chern-Simons with flavors

Large N Solution of GT theory [TS]

Take a limit : (N_1, N_2, k_1, k_2) all proportional to k, $k \to \infty$.

⇨ saddle-point approx. is exact.

Saddle-point eqs.

$$\frac{k_1}{2\pi} u_i = \sum_{j \neq i}^{N_1} \coth \frac{u_i - u_j}{2} + \sum_{a=1}^{N_2} \tanh \frac{u_i - v_a}{2},$$

$$\frac{k_2}{2\pi} v_a = \sum_{b \neq a}^{N_2} \coth \frac{v_a - v_b}{2} + \sum_{i=1}^{N_1} \tanh \frac{v_a - u_i}{2}.$$

(after rescaling and analytic cont.)

Introducing new variables $z_i = e^{u_i}$, $w_a = e^{v_a}$, these become

$$\kappa_1 \log z_i = t_1 + t_2 + \frac{2t_1}{N_1} \sum_{j \neq i}^{N_1} \frac{z_j}{z_i - z_j} - \frac{2t_2}{N_2} \sum_{a=1}^{N_2} \frac{w_a}{z_i + w_a},$$

$$\kappa_2 \log w_a = t_1 + t_2 + \frac{2t_2}{N_2} \sum_{b \neq a}^{N_2} \frac{w_b}{w_a - w_b} - \frac{2t_1}{N_1} \sum_{i=1}^{N_1} \frac{z_i}{w_a + z_i}.$$

$$\kappa_{1,2} = \frac{k_{1,2}}{k} \qquad t_{1,2} = \frac{N_{1,2}}{k}$$

Resolvent : $v(z)=t_1\int dx\,\rho(x)\frac{x}{z-x}-t_2\int dx\,\tilde{\rho}(x)\frac{x}{z+x}.$

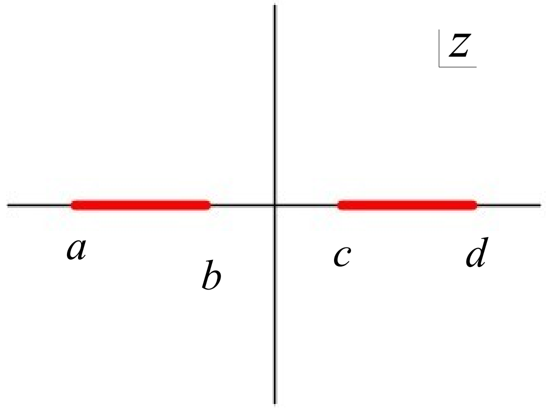
e.v. density for z 's — e.v. density for w 's

$\Longrightarrow$

$$\kappa_1\log y-t = v(y+i0)+v(y-i0),\quad (c<y<d)$$
$$\kappa_2\log(-y)-t = v(y+i0)+v(y-i0),\quad (a<y<b)$$

$(t=t_1+t_2)$

Here we assumed the eigenvalue distribution like this.



Boundary conditions :

$$v(z)\ \rightarrow\ \begin{cases} O(z^{-1}), & (z\rightarrow\infty) \\ -t. & (z=0)\end{cases}$$

∃ An analytic function satisfying the saddle-point equations and the boundary conditions !!!

$$\frac{v(z)}{\sqrt{(z-a)(z-b)(z-c)(z-d)}} = \frac{\kappa_1}{2\pi}\int_c^d dx \frac{f_+(x, t/\kappa_1)}{z-x} + \frac{\kappa_2}{2\pi}\int_a^b dx \frac{f_-(x, t/\kappa_2)}{z-x}$$

where $$f_\pm(x,s) = \frac{\pm\log(\pm e^{-s} x)}{\sqrt{|(x-a)(x-b)(x-c)(x-d)|}}.$$

It can be shown that $ab=1$, $cd=1$ are consistent, and then

$$\oint_{C_{1,2}} \frac{dz}{2\pi i}\frac{v(z)}{z} = t_{1,2}$$

determine the remaining parameters.

Note : The solution is reduced to ABJM solution when $k_1 + k_2 = 0$.

Large $|k_2|$ limit

If $N_1 = N_2$ and $k_1 + k_2 = 0$, then Wilson loop is known to have AdS behavior. What happens in the limit $|k_2| \to \infty$?

Easy way to see : Take the limit at the level of partition function.

$$\Longrightarrow \qquad u_i + t_2 \tanh\frac{u_i}{2} = \frac{t_1}{N_1}\sum_{j\neq i}^{N} \coth\frac{u_i - u_j}{2}.$$

Define the resolvent : $$v(z) = t_1 \int dx\, \rho(x) \frac{x}{z-x}.$$

The solution is

$$v(z) = \frac{1}{2}\log\left[\frac{e^{-t_1}}{2+a+a^{-1}}\left(z+1-\sqrt{(z-a)(z-a^{-1})}\right)^2\right]$$
$$+\frac{t_2}{2}\frac{z-1}{z+1} - \frac{t_2}{\sqrt{2+a+a^{-1}}}\frac{\sqrt{(z-a)(z-a^{-1})}}{z+1}.$$

Wilson loop is obtained from the resolvent as

$$\langle \sum_i e^{u_i} \rangle = \frac{1}{t_1} \lim_{z\to\infty} z\cdot v(z)$$

$$= \frac{1}{t_1} \sinh^2 u + \cosh u - 1, \qquad (a = e^{2u})$$

where $2\cosh u \cdot \log\cosh u = t_1$.

Therefore, we find

$$\langle \sum_i e^{u_i} \rangle = o(t_1).$$

power-like !
non-ADS !

The non-exponential behavior of Wilson loop discussed in 4-dim. is shown to exist rigorously in a 3-dim. analogue.

Summary

- AdS/CFT correspondence in the presence of many flavors are discussed, focusing on Wilson loops.
- In 4-dim. it was suggested that Wilson loop would not exhibit the AdS behavior.
- Gaiotto-Tomasiello theory is discussed as 3-dim. analogue. Planar resolvent is obtained.
- In the decoupling limit, AdS behavior is shown to disappear.

What do they mean ??

Possible interpretations :

- No AdS dual ?
- Stringy AdS factor ?
- Classical AdS with stringy cycles in internal space ?
- etc. ?