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Black Hole Formations
as
Holographic Quantum Quenches

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based on

arXiv:1005.3348 (JHEP 1007:071,2010)

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arXiv:1008.3439 with Tomoki Ugajin (IPMU)

arXiv:1010.???? with Wei Li (IPMU)

① Introduction

Clearly, string theory has lead to tremendous revolutions on our understandings of black holes (BHs), especially for BPS BHs.

This has been enabled essentially by using holography or AdS/CFT.

However, this direction of study is still far from complete.

➡ Non-SUSY situations ? Dynamical evolutions ?

For example, the following problems are still open:

(i) Entropy of Schwarzschild BH in flat spacetime ?

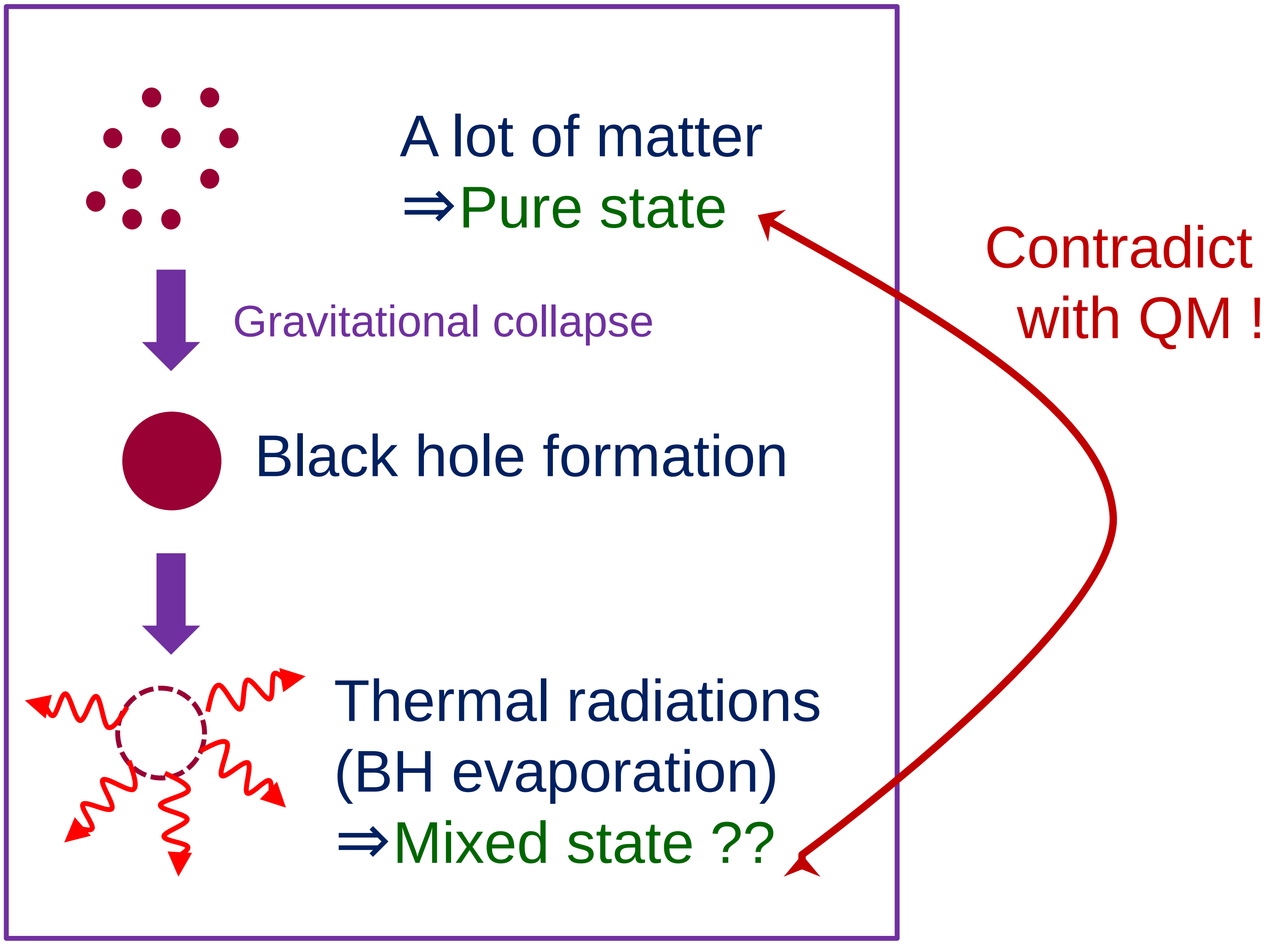
→ Is there any holography in flat spacetime ?

(ii) Complete understanding on how the BH information problem is avoided in string theory ?

[For static BH, there have been considerable developments:
e.g. Maldacena 01', Festuccia-Liu 07', Hayden-Preskill 07',
Sekino-Susskind 08', Iizuka-Polchinski 08' …]

→ How to describe evaporations of BHs in holography ?

A Quick Sketch of BH Information Problem



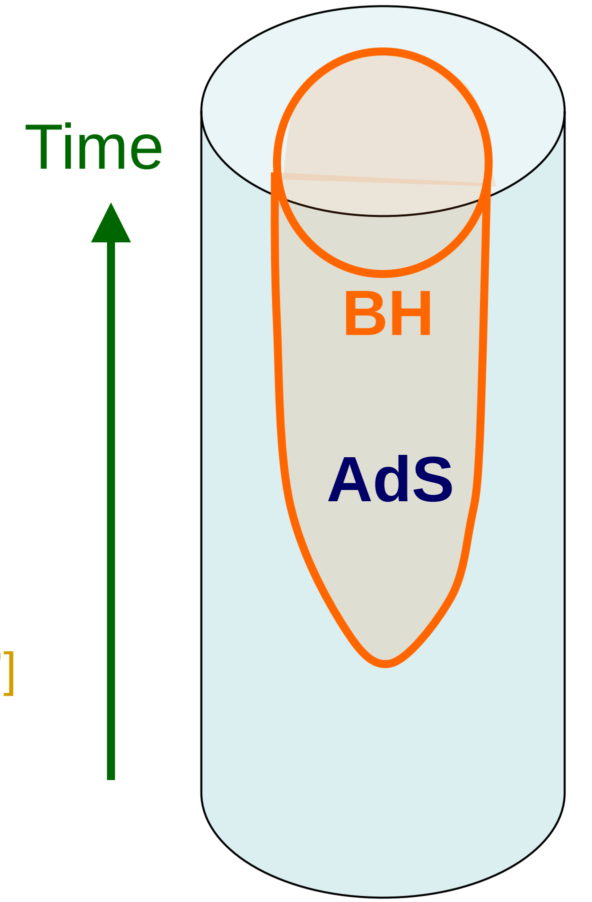
BH formation in AdS/CFT

Thermalization in CFT

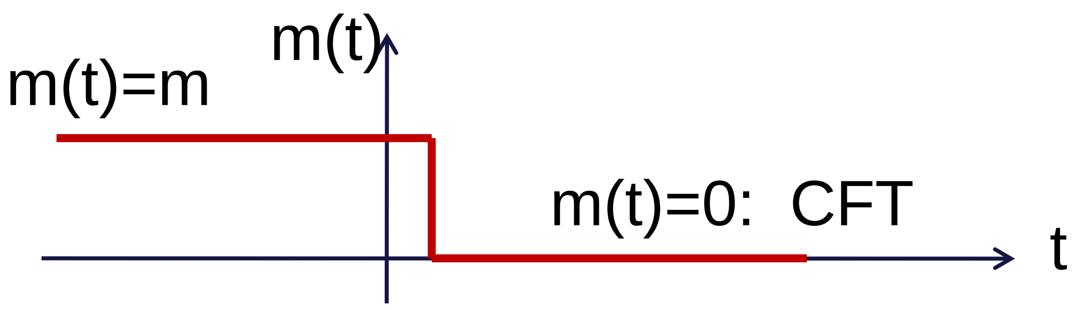
⟺ BH formation in AdS

In particular, instantaneous excitations of CFTs are called **quantum quenches**.

[e.g. Calabrese-Cardy 05’-10’]



e.g. Mass quench



Entropy Puzzle in AdS/CFT

(i) In the CFT side, the von-Neumann entropy remains vanishing under a unitary evolutions of a pure state.

$$\rho_{tot}(t) = U(t,t_0)\left|\Psi_0\right\rangle\left\langle\Psi_0\right|U(t,t_0)^{-1}$$

$$\Rightarrow \quad S(t) = -Tr\,\rho_{tot}(t)\,\log\,\rho_{tot}(t) = S(t_0).$$

(ii) In the gravity dual, its holographic dual inevitably includes a black hole at late time and thus the entropy looks non-vanishing !

Thus (i) and (ii) contradict !

We will resolve this issue using *entanglement entropy* and study *quantum quenches* as CFT duals of BH creations and evaporations.

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② Quantum Quenches and Emergent Horizons

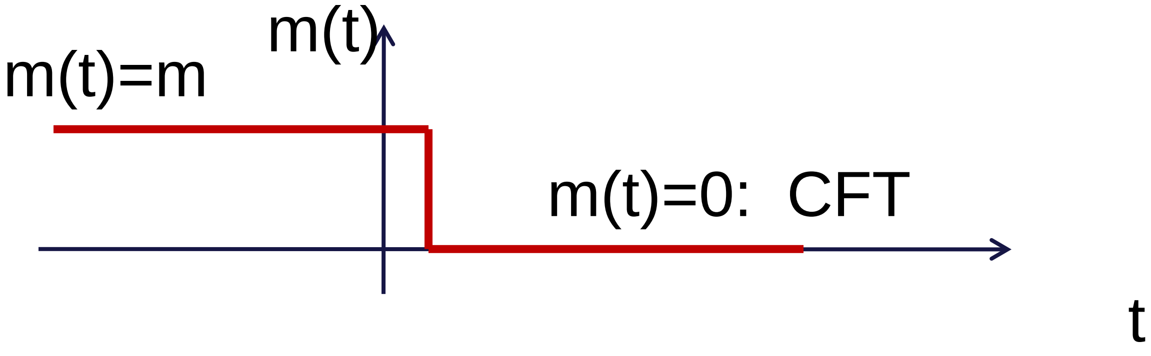
(2-1) What are quantum quenches ?

A quantum quench is a time-dependent process when we suddenly change parameters of a quantum manybody system at T=0

→ Cold atom experiments

This typically leads to an *effective thermalization,* though the system always remains to be a *pure state*.

Ex. A mass quench for a scalar field



Brief review of mass quench in free scalar

[Calabrese-Cardy 05', Sotiriadis-Calabrese-Cardy 08']

Consider a harmonic oscillator whose frequency is shifted from ω_0 to ω at t=0. (The initial temperature is $T_0 = \beta_0^{-1}$)

$$H_0 = \omega_0 \left(a_0 a_0^+ + \frac{1}{2} \right) \quad \Rightarrow \quad H = \omega \left(a a^+ + \frac{1}{2} \right).$$

The creation operators are related by Bogoliubov transformation:

$$a^+ = \cosh\varsigma \, a_0^+ + \sinh\varsigma \, a_0 ,$$

$$a = \sinh\varsigma \, a_0^+ + \cosh\varsigma \, a_0 ,$$

$$e^{\varsigma} \equiv \sqrt{\frac{\omega}{\omega_0}} \quad .$$

The T-product correlation function reads

$$T\left\langle x(t_1)x(t_2)\right\rangle = T\ \mathrm{Tr}[e^{-\beta_0 H_0}\hat{x}(t_1)\hat{x}(t_2)]$$

$$= \frac{1}{2\omega}e^{-i\omega|t_2-t_1|} + \left[\frac{\omega_0}{4}\left(\frac{1}{\omega^2}+\frac{1}{\omega_0^2}\right)\coth\frac{\beta_0\omega_0}{2} - \frac{1}{2\omega}\right]\cos\omega(t_1-t_2)$$

$$-\frac{\omega_0}{4}\left(\frac{1}{\omega^2}-\frac{1}{\omega_0^2}\right)\coth\frac{\beta_0\omega_0}{2}\cos\omega(t_1+t_2) \quad .$$

Time translation invariance breaking term

On the other had, the thermal correlation function at T=1/β is

$$T\left\langle x(t_1)x(t_2)\right\rangle_{th} = T\ \mathrm{Tr}[e^{-\beta H}\hat{x}(t_1)\hat{x}(t_2)]$$

$$= \frac{1}{2\omega}e^{-i\omega|t_2-t_1|} + \frac{\cos\omega(t_1-t_2)}{\omega(e^{\beta\omega}-1)} \quad .$$

To analyze the free scalar field, we just need to consider the momentum dependent frequency:

$$\omega_0 \to \omega_0(k) = \sqrt{m_0^2 + k^2} \ , \qquad \omega \to \omega(k) = \sqrt{m^2 + k^2} \quad .$$

Importantly, after the Fourier transformation, we can neglect the time translation invariant breaking term in the real space correlation function at late time.

Finally, it approaches the thermal 2pt. function at the eff. temp.

$$\beta_{eff}(k) = \frac{1}{\omega(k)} \log \frac{(\omega(k) - \omega_0(k))^2 + e^{\beta_0 \omega_0(k)} (\omega(k) + \omega_0(k))^2}{(\omega(k) + \omega_0(k))^2 + e^{\beta_0 \omega_0(k)} (\omega(k) - \omega_0(k))^2} \quad .$$

Note: The effective temperature is momentum dependent.

The deviation from the grand-canonical ensemble is expected when the system is integrable.

[Cold atom exp. : T. Kinoshita, T. Wenger, D. S. Weiss, Nature (London) 440, 900 (2006)

Generalized Gibbs ensemble: Rigol –Dunjko-Yurovsky-Olshanii 06’]

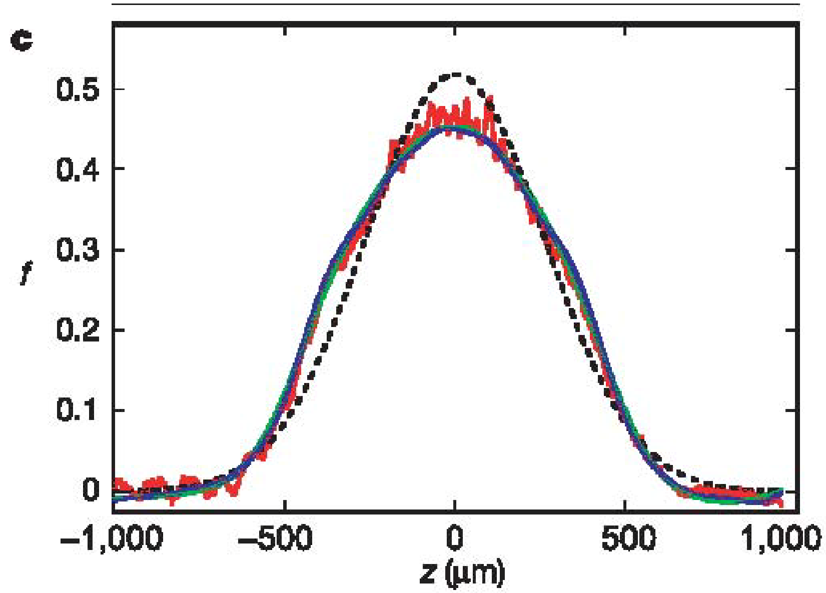


Figure 4 | Projected versus actual $f(p_{ex})$ for various γ_d, the dephased average peak coupling strength. The blue and green curves are $f(p_{ex})$ for $t_d = 15\tau$, rescaled to account for loss and convolved with the known heating

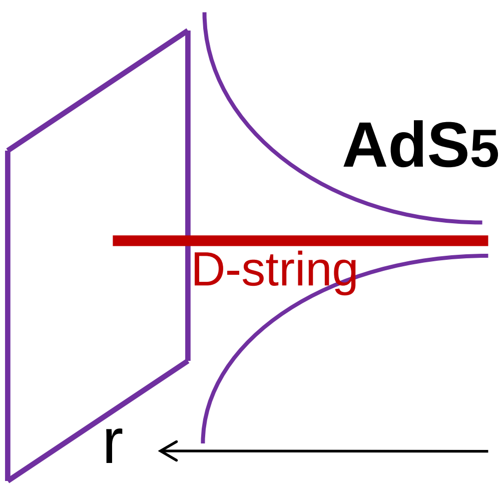
(2-2) Emergent Static BH from Probe D-branes

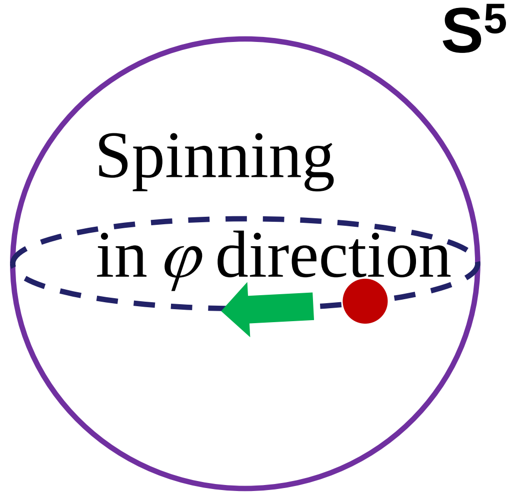
[Das-Nishioka-TT 10']

As an exercise, consider a D-string extending in the radial direction and rotating in the φ direction. Its action looks like

$$S_{DBI} = -T_{D1}\int dtdr\sqrt{-G} = -T_{D1}\int dtdr\sqrt{1+r^2(\varphi')^2-(\dot{\varphi})^2/r^2} \quad .$$

Assume the following profile: $\varphi(t,r) = \omega t + g(r), \qquad \theta = \frac{\pi}{2}.$





The equation of motion leads to

$$\frac{\partial}{\partial r}\left(\frac{r^2\varphi'}{\sqrt{1+r^2\varphi'^2-\dot{\varphi}^2/r^2}}\right)=\frac{\partial}{\partial t}\left(\frac{\dot{\varphi}/r^2}{\sqrt{1+r^2\varphi'^2-\dot{\varphi}^2/r^2}}\right),$$

and thus we find

$$g'(r)=\sqrt{\frac{1-\omega^2/r^2}{A^2r^4-r^2}},$$

$$S_{DBI}=-T_{D1}\int dtdr\sqrt{\frac{A(r^2-\omega^2)}{Ar^2-1}}\quad .$$

To avoid a singularity at r=ω, we need to simply require

$$A=\frac{1}{\omega^2}\quad .$$

This leads to the D-string solution

$$\varphi(t,r) = \omega t - \frac{\omega}{r} + \varphi_0 , \qquad \theta = \frac{\pi}{2}.$$

Its induced metric looks like

$$ds^2 = -(r^2 - \omega^2)dt^2 + \frac{2\omega^2}{r^2} dtdr + \left(\frac{1}{r^2} + \frac{\omega^2}{r^4} \right) dr^2.$$

By redefining the time coordinate

$$\tau = t - \frac{1}{r} - \frac{1}{2\omega} \log \frac{r - \omega}{r + \omega} \quad .$$

This leads to the `2d BTZ' blackhole (or AdS2) metric

$$ds^2 = -(r^2 - \omega^2)d\tau^2 + \frac{dr^2}{r^2 - \omega^2} \quad .$$

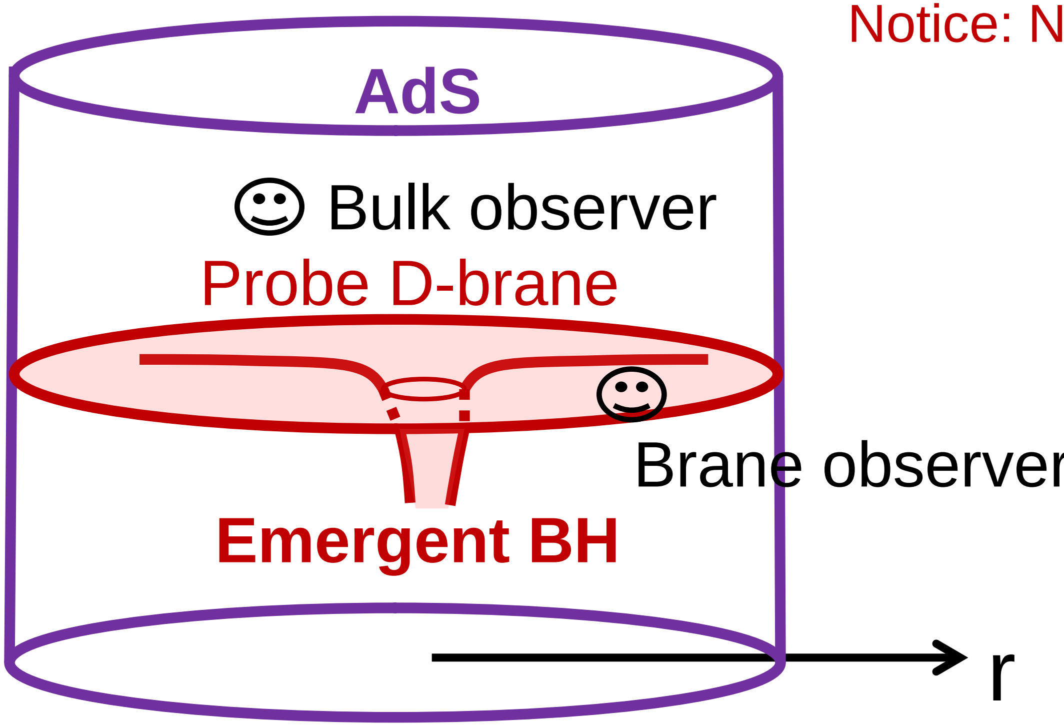
Note: We can generalize such a solution to Dp-branes.

Claim

Thermalization in a probe D-brane

⇔ Horizon formation in its induced metric on the D-brane

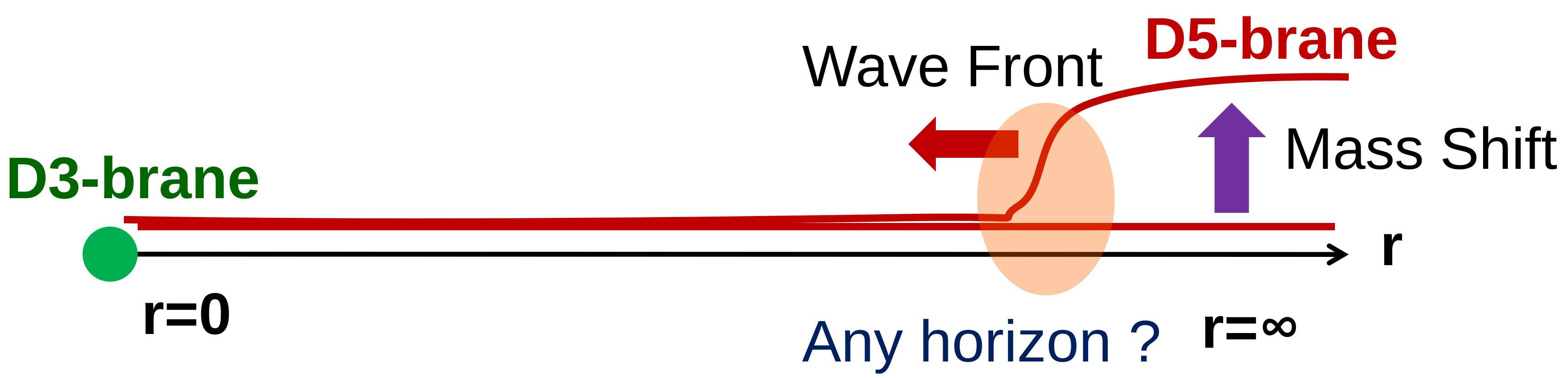
Notice: No horizon in the bulk !



(2-3) Holographic Mass Quench

Consider a probe D5-brane in $AdS_5 \times S^5$ dual to a defect CFT. The mass of hypermultiplets from D3-D5 open strings is given by the separation between N D3-branes and the D5-brane.

Thus , the mass quantum quench is triggered by suddenly shifting the position of D5-brane at the AdS boundary.



D5-brane Solution

Express $AdS_5 \times S^5$ by

$$ds^2_{IIB} = -r^2 dt^2 + r^2 \sum_{i=1}^{3} dx_i^2 + \frac{dr^2}{r^2} + (d\theta^2 + \cos^2\theta d\Omega_2^2 + \sin^2\theta d\Omega_2'^2).$$

⇒ The D5 is wrapped on (t,x1,x2,r) and Ω_2 .

Define $\eta = r\sin\theta, \quad z = \dfrac{1}{r\cos\theta}$.

$\eta(t,z)$ ⇒ the profile of D5-brane .

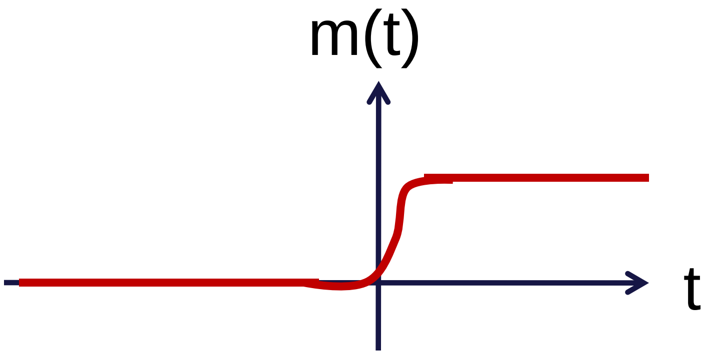
We numerically solve PDEs for DBI action to find $\eta(t,z)$.

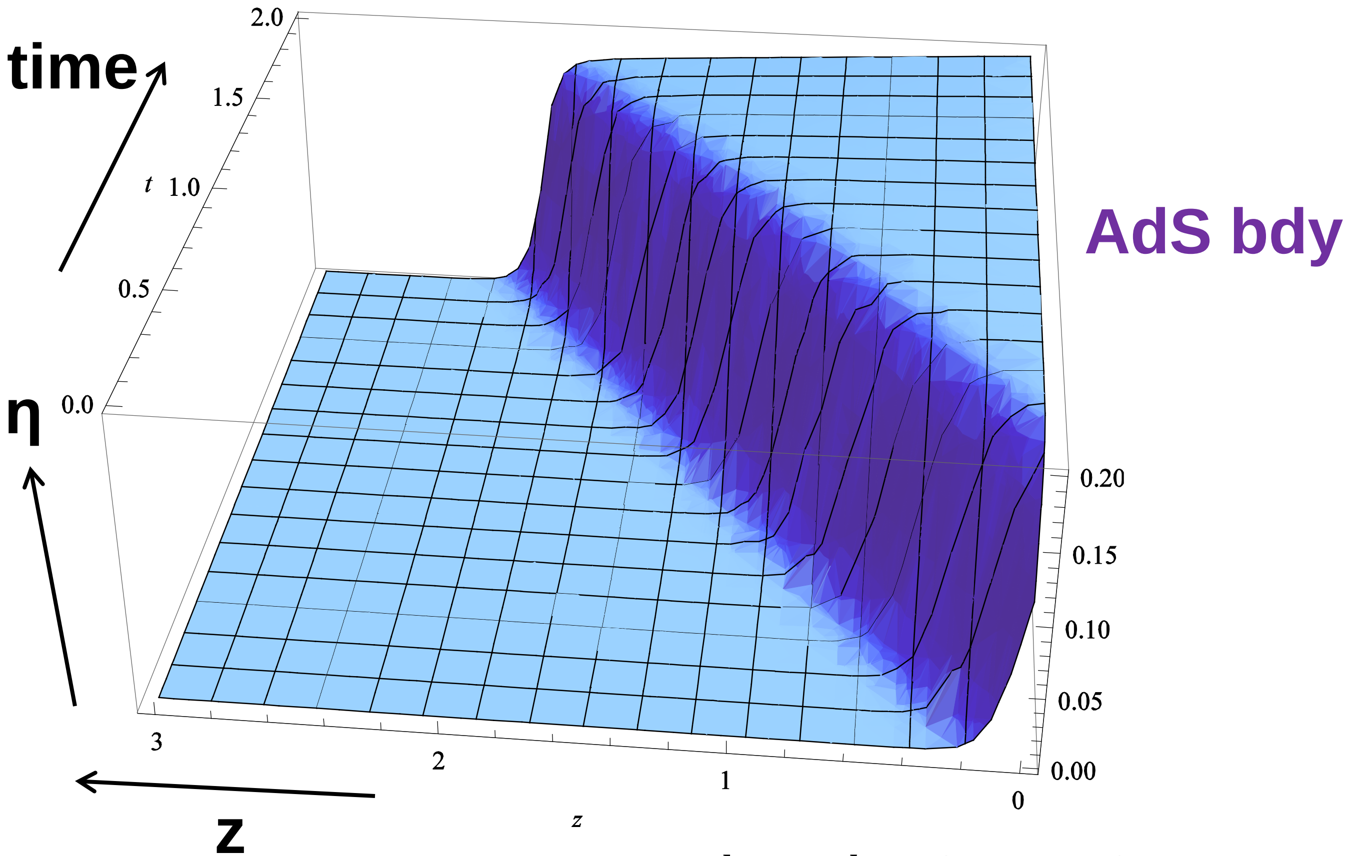
→ Compute the induced metric of this D5-brane !

Numerical D5-brane Solution

For the time-dependent mass as

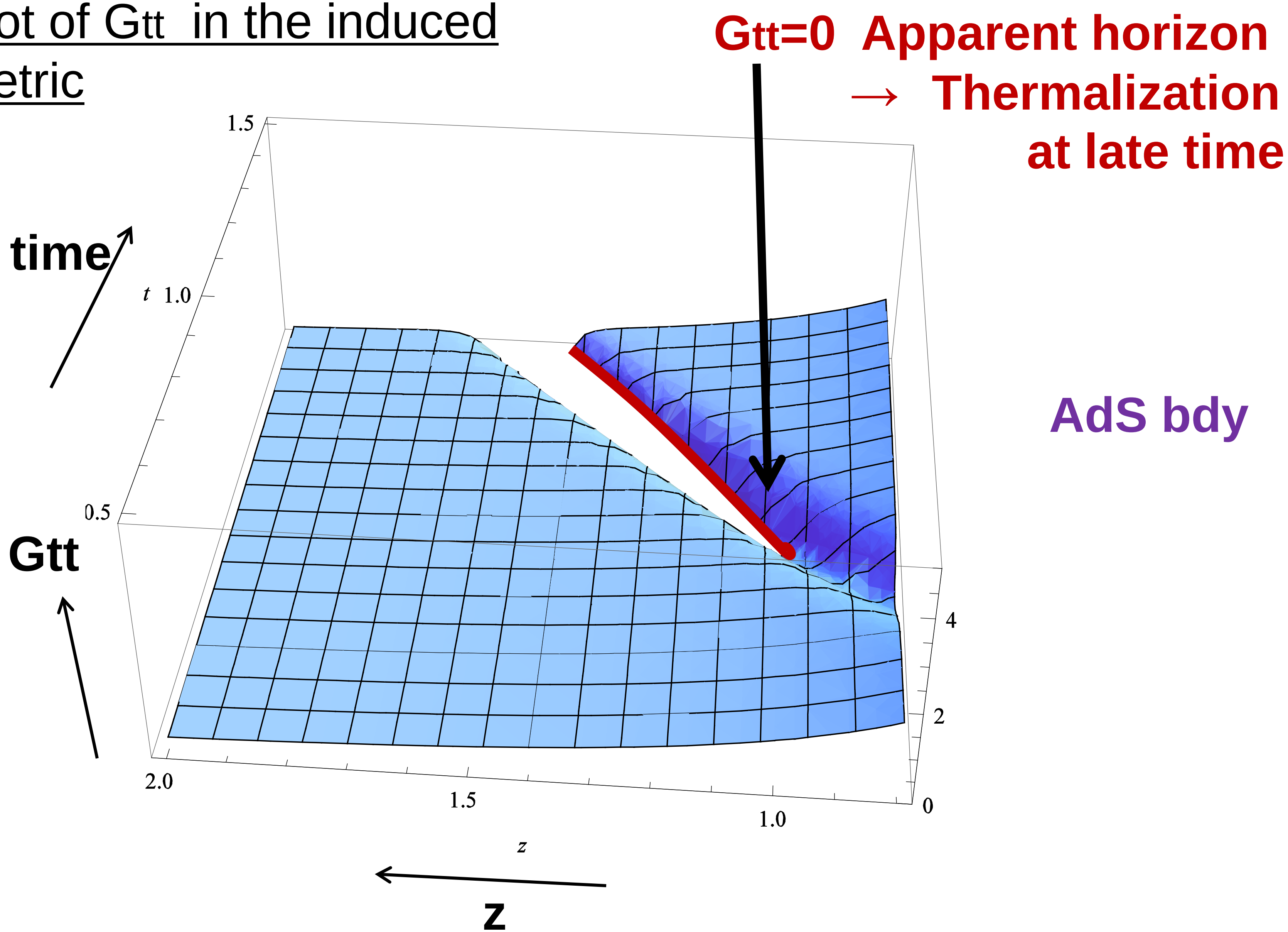
$$m(t) = m_0 (1 + \tanh kt) \equiv \eta(t,0).$$





We choose $k = 10, \quad m_0 = 0.1.$

<u>Plot of Gtt in the induced metric</u>



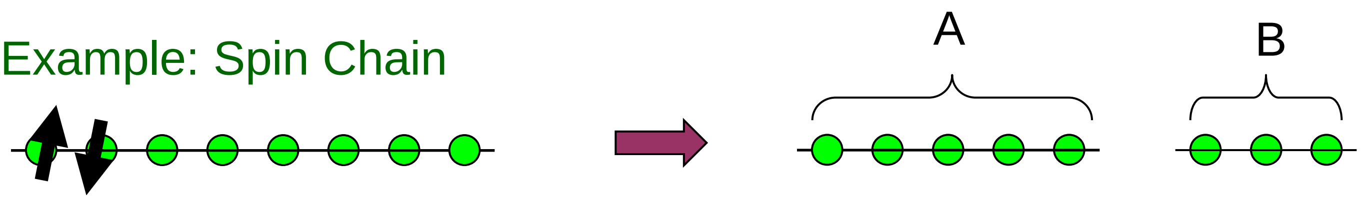
③ BH formations as Pure States in AdS/CFT

(3-1) Holographic Entanglement Entropy

Divide a given quantum system into two parts A and B.
Then the total Hilbert space becomes factorized

$$H_{tot} = H_A \otimes H_B \quad .$$

Example: Spin Chain



We define the reduced density matrix ρ_A for A by

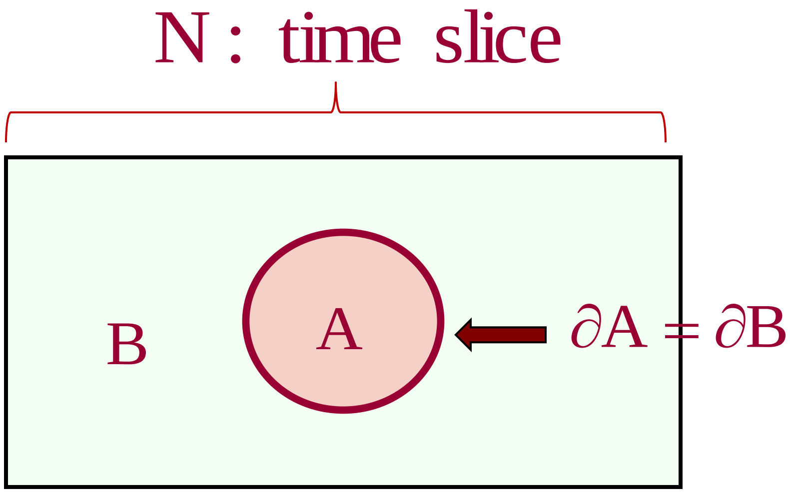
$$\rho_A = \mathrm{Tr}_B \rho_{tot} \quad ,$$

taking trace over the Hilbert space of B .

Now the entanglement entropy S_A is defined by the von-Neumann entropy

$$S_A = -\mathrm{Tr}_A \, \rho_A \log \rho_A \quad .$$

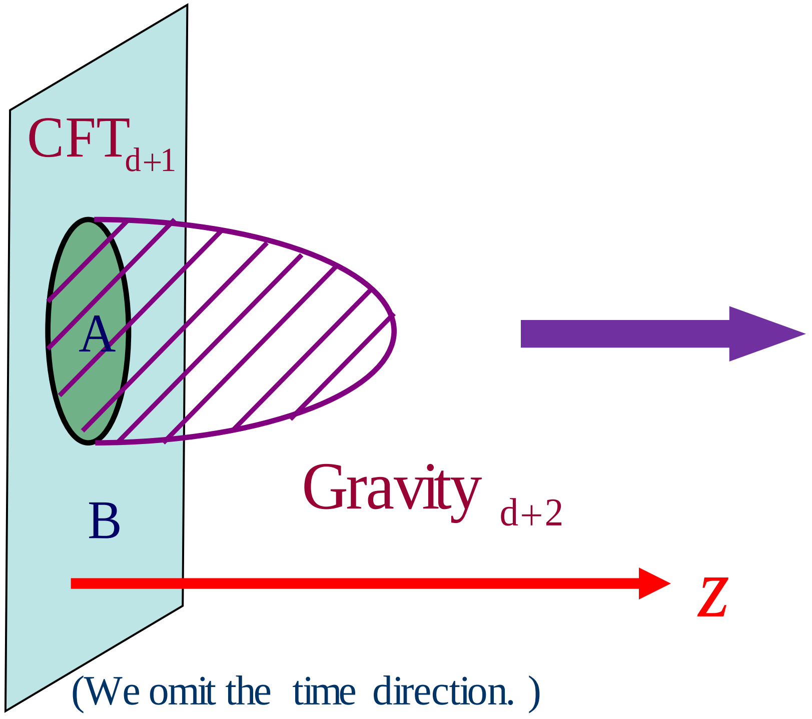
In QFTs, it is defined geometrically (called geometric entropy).



Holographic Formula

The holographic entanglement entropy S_A is given by the area of minimal surface whose boundary coincides with ∂A.

[Ryu-TT, 06']



$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

(`Bekenstein-Hawking formula' when γ_A is the horizon.)

Comments

- A heuristic derivation from bulk to boundary relation is in [Fursaev, 06']. Many evidences and no counter examples for 4 years, in spite of the absence of complete proof.

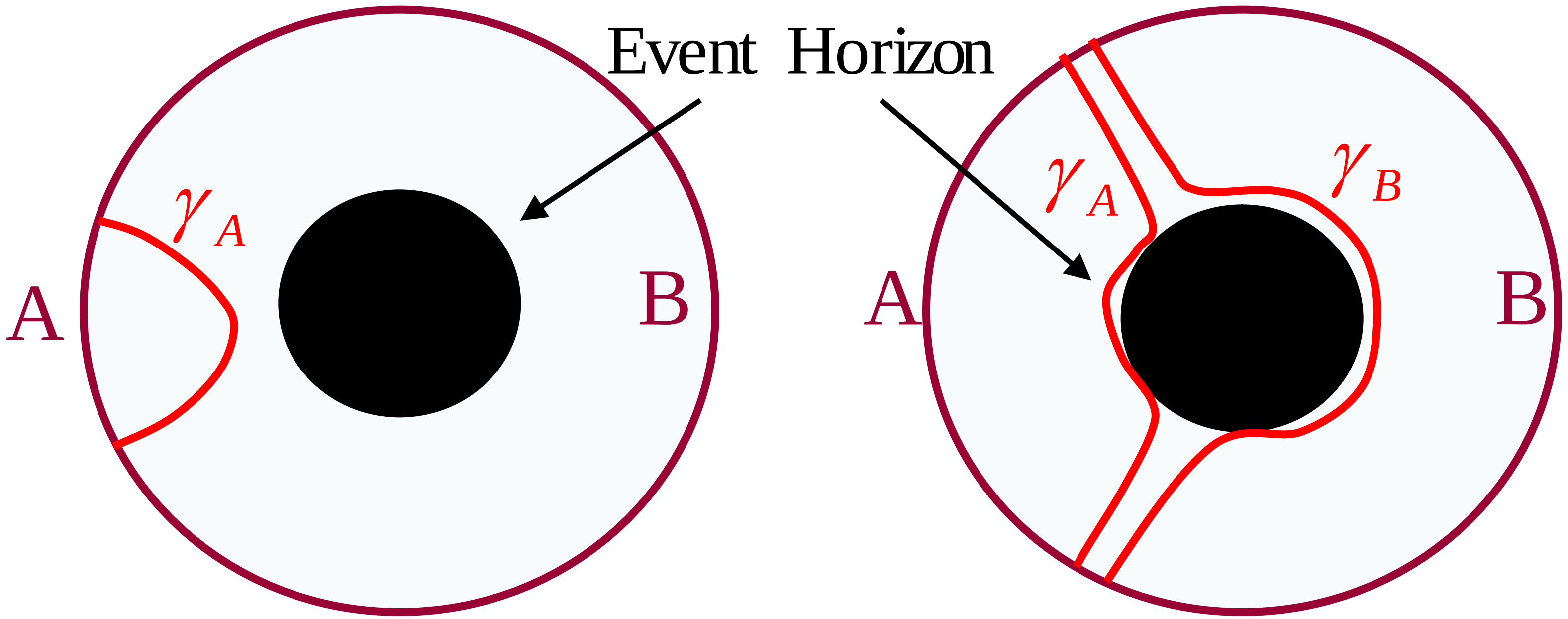
- We need to employ extremal surfaces in the time-dependent spacetime. [Hubeny-Rangamani-TT, 0705.0016]

- In the presence of a black hole horizon, the minimal surfaces typically wraps the horizon.

  $\Rightarrow$ Reduced to the Bekenstein-Hawking entropy, consistently.

EE from AdS BH

(i) Small A (ii) Large A



$S_A \neq S_B$ *if* ρ_{tot} is not pure.

(3-2) Resolution of the Puzzle via Entanglement Entropy

Our Claim: The non-vanishing entropy appears only after coase-graining. The von-Neumann entropy itself is vanishing even in the presence of black holes in AdS.

First, notice that the (thermal) entropy for the total system can be found from the entanglement entropy via the formula

$$S_{tot} = \lim_{|B|\to 0}(S_A - S_B).$$

This is indeed vanishing if we assume the pure state relation $S_A=S_B$.

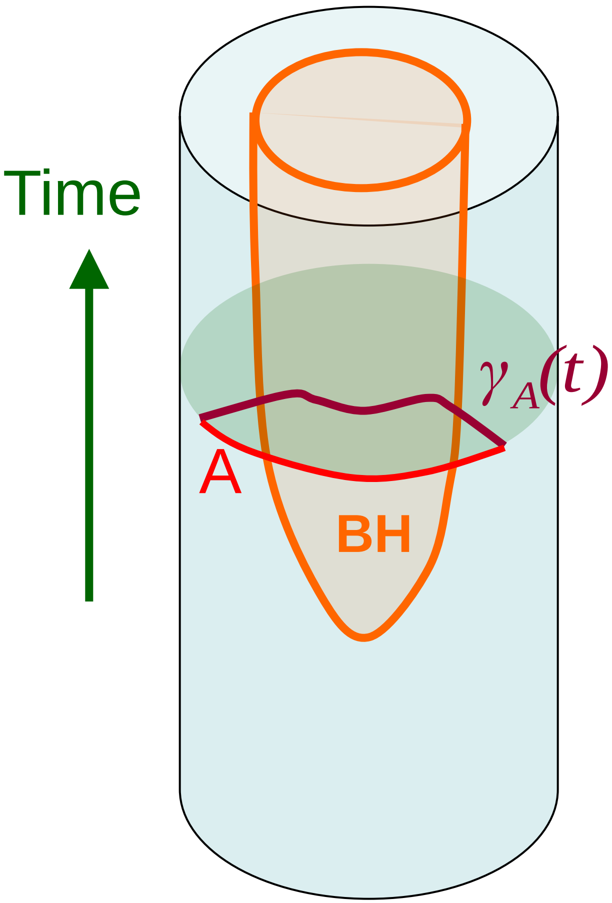
Indeed, we can holographically show this as follows:

$$S_A(t) = \min\left[\frac{\mathrm{Area}(\gamma_A(t))}{4G_N}\right],$$

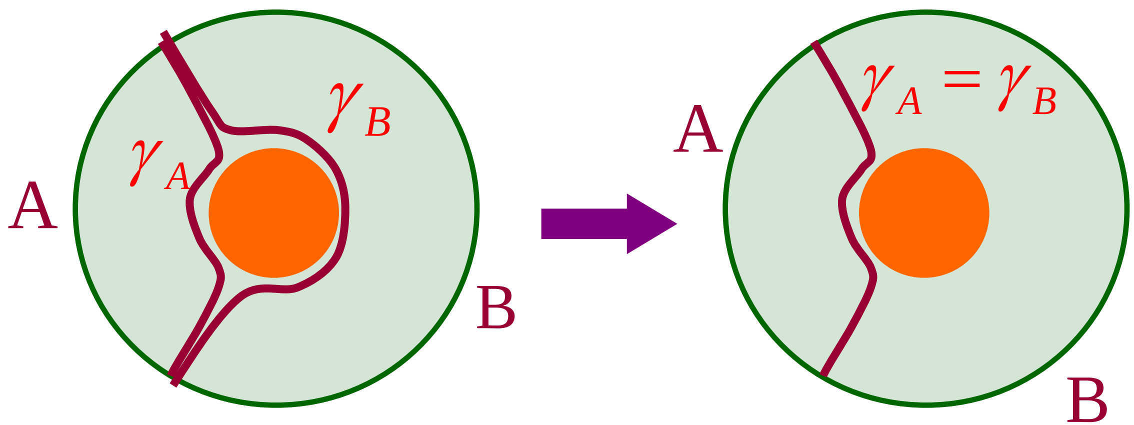
$\gamma_A(t) =$ extremal surfaces homotopic to $A(t)$

such that $\partial\gamma_A(t) = \partial A(t)$.

[Hubeny-Rangamani-TT 07']



Black hole formation in global AdS_{d+2}



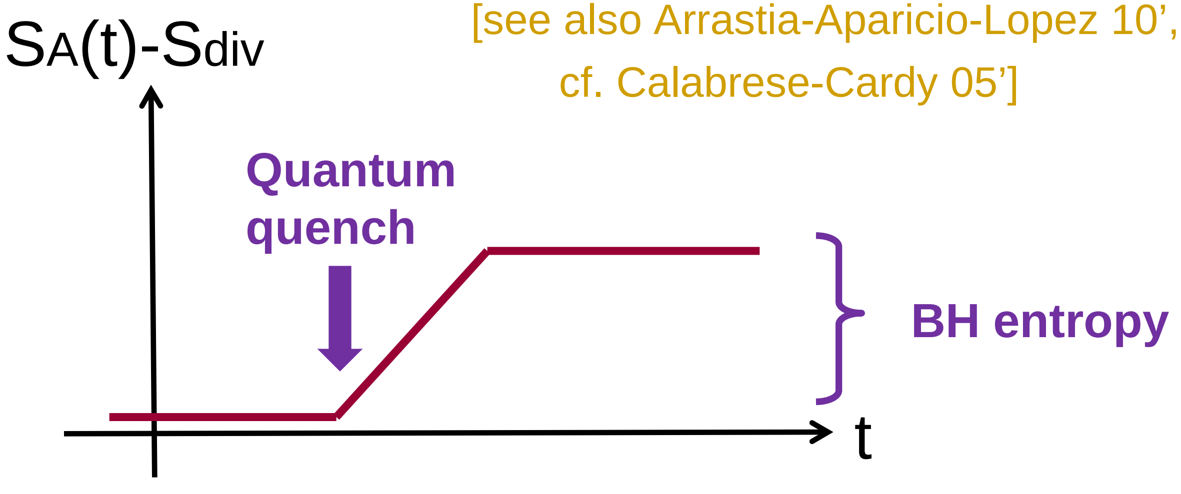
Continuous deformation leads to $S_A = S_B$

Therefore, if the initial state does not include BHs, then always we have $S_A=S_B$ and thus $S_{tot}=0$.

In such a pure state system, the total entropy is not useful to detect the BH formation.

Instead, the entanglement entropy S_A can be used to probe the BH formation as a **coarse-grained entropy**.

[see also Arrastia-Aparicio-Lopez 10', cf. Calabrese-Cardy 05']



④ Entanglement Entropy as Coarse-grained Entropy

[Ugajin-TT 10']

(4-1) Evolution of Entanglement Entropy and BH formation

As an explicit example , consider the 2D free Dirac fermion on a circle.

AdS/CFT: free CFT ⟺ quantum gravity
with a lot of quantum corrections !

Assuming that the initial wave function $|\Psi_0\rangle$ flows into a boundary fixed point as argued in [Calabrese-Cardy 05'], we can approximate by

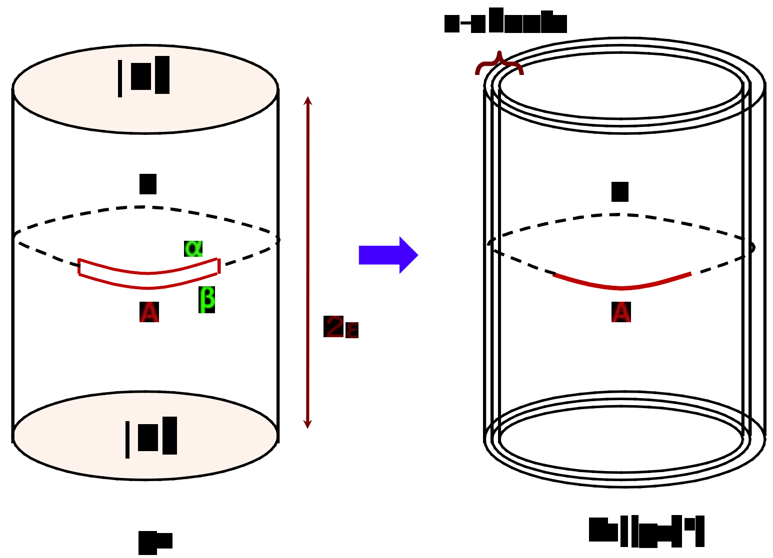
$$|\Psi_0\rangle = e^{-\varepsilon H}|B\rangle \quad ,$$

where $|B\rangle$ is the boundary state. The constant ε is a regularization parameter and measures the strength of the quantum quench:

$$\Delta m \sim \varepsilon^{-1}$$

.

Calculations of EE



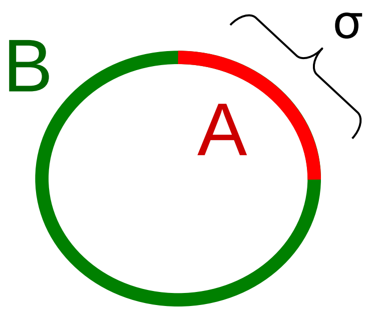
$$S_A = -\frac{\partial}{\partial N} \mathrm{Tr}[(\rho_A)^N]\Big|_{N=1} \quad ,$$

$$\mathrm{Tr}[(\rho_A)^N] = \prod_{a=-\frac{N-1}{2}}^{\frac{N-1}{2}} \frac{\langle B | e^{-2\varepsilon H} \cdot \sigma^{(a)}(y_1, \bar{y}_1) \cdot \sigma^{(-a)}(y_2, \bar{y}_2) | B \rangle}{\langle B | e^{-2\varepsilon H} | B \rangle} \quad .$$

The final result of entanglement entropy is given by

$$S_A(t,\sigma)=\frac{1}{3}\log\frac{2\varepsilon}{\pi a}+\frac{1}{6}\log\frac{\left|\theta_1\left(\frac{i\sigma}{4\varepsilon}\middle|\frac{\pi i}{2\varepsilon}\right)\right|^2\cdot\left|\theta_1\left(\frac{\varepsilon+it}{2\varepsilon}\middle|\frac{\pi i}{2\varepsilon}\right)\right|^2}{\eta\left(\frac{\pi i}{2\varepsilon}\right)^6\cdot\left|\theta_1\left(\frac{2\varepsilon+2it+i\sigma}{4\varepsilon}\middle|\frac{\pi i}{2\varepsilon}\right)\right|\cdot\left|\theta_1\left(\frac{2\varepsilon+2it-i\sigma}{4\varepsilon}\middle|\frac{\pi i}{2\varepsilon}\right)\right|}\quad,$$

where $a=$ UV cut off and $0\leq\sigma<2\pi$.



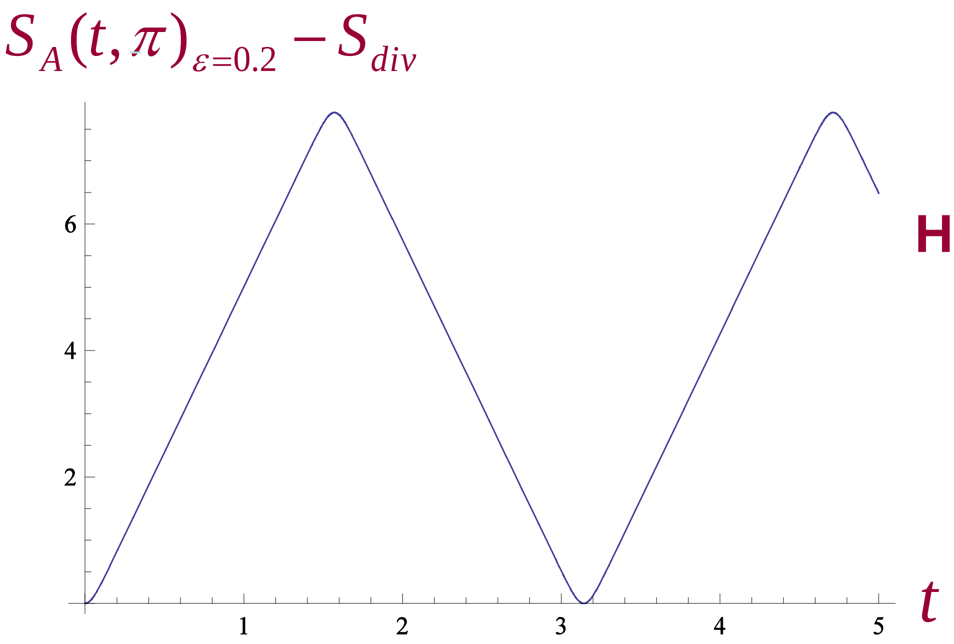
This satisfies

$S_A(t,\sigma)=S_A(t,2\pi-\sigma)\equiv S_B(t,\sigma).\quad\Rightarrow$ Pure State

$S_A(t+\pi,\sigma)=S_A(t,\sigma)\quad\Rightarrow$ Recurrence special to the free field theory

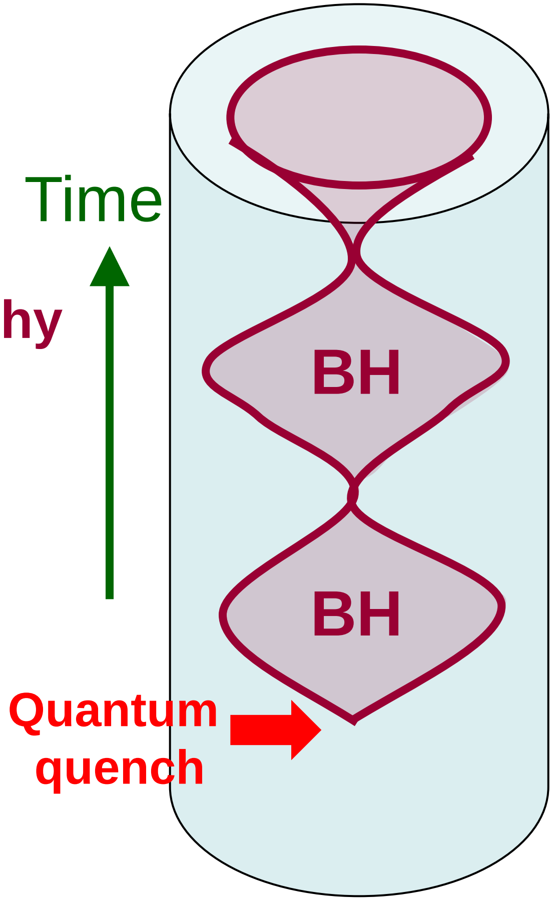
(much shorter than the Poincare recurrence)

Time evolution of entanglement entropy



Quantum quench in free CFT

Holography



BH formation and evaporation
in extremely quantum gravity

No information paradox at all in either side !

(4-2) More comments

First of all, we can confirm that the scalar field X
(= bosonization of the Dirac fermion) is thermally excited:

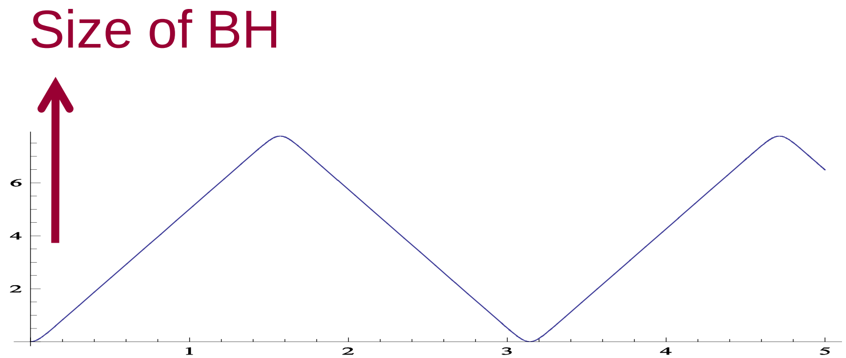
$$\text{Oscillators}: \quad X(\tau,\sigma) \Rightarrow (\alpha_n , \tilde{\alpha}_n).$$

$$E_n = \langle \alpha_{-n}\alpha_n \rangle = \frac{n}{2\pi(e^{4\varepsilon n}-1)} \quad \Rightarrow \quad T_{eff} = \frac{1}{4\varepsilon} \quad .$$

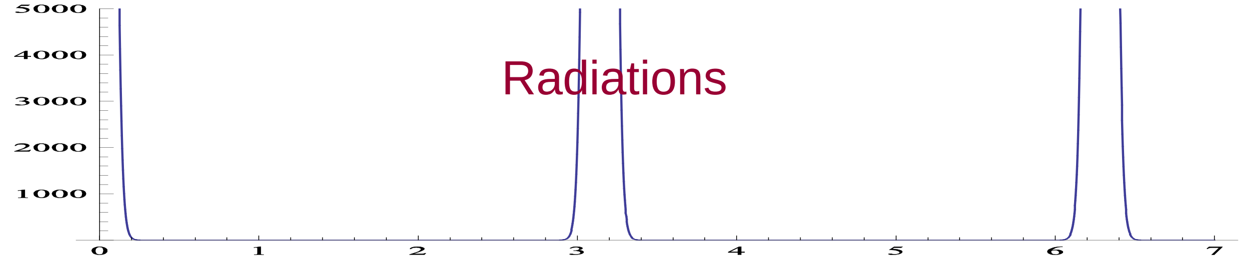
$$\rho_{\text{left}} \equiv \text{Tr}_{\text{right}}\, e^{-2\varepsilon H} |B\rangle\langle B| = \prod_{m=0}^{\infty}\sum_{n=0}^{\infty} e^{-4\varepsilon mn} \frac{(\alpha_{-m})^n}{\sqrt{n!}} |0\rangle\langle 0| \frac{(\alpha_m)^n}{\sqrt{n!}} \quad .$$

Correlation functions

$$S_{eff}(t)$$

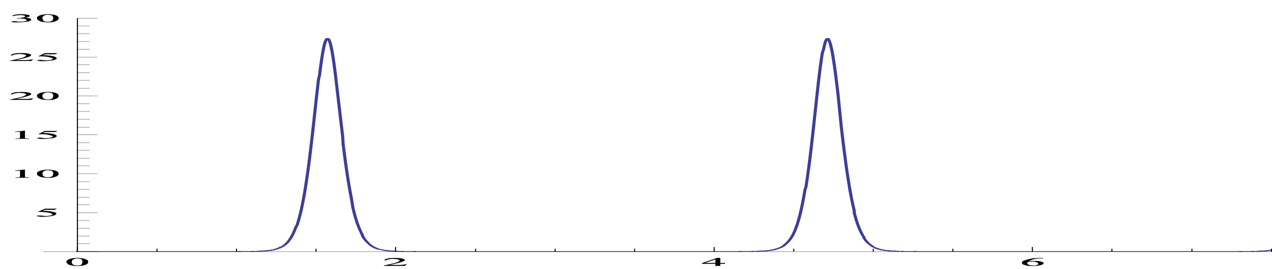


$$\left\langle e^{ikX(t,0)} \right\rangle$$



$$\left\langle e^{ikX(t,0)} e^{-ikX(0,0)} \right\rangle$$

$$\left\langle e^{ikX(t,\pi)} e^{-ikX(0,0)} \right\rangle$$



⑤ Holography and Entanglement in Flat Space

[Li-TT to appear]

The entanglement entropy is a rather general observable in general setup of holography as in our BH example.

This is because EE can be defined in any quantum many body systems.

Motivated by this, finally we would like to discuss what a holography for flat spacetime looks like.

So, simply consider $$R^{d+1}: \quad ds^2 = d\rho^2 + \rho^2 d\Omega_d^2 \quad .$$

$$\text{UV cut off} \Rightarrow \ \rho < \rho_\infty$$

Correlation functions

We can compute the holographic n-point functions following the bulk-boundary relation just like in AdS/CFT.

Assuming a scalar in R^{d+1} like the dilaton:

$$S = \frac{1}{4\pi G_N}\int dx^{d+1}\sqrt{g}\left[f(\phi)\partial_\mu\phi\partial^\mu\phi\right].$$

Then we find that all n-point functions scale simply:

$$\langle O_1(y_1)O_2(y_2)\cdots O_n(y_n)\rangle = \frac{(\rho_\infty)^{d-1}}{G_N}g(y_1, y_2, \cdots, y_n).$$

$\Rightarrow$ Only divergent terms appear !

Adding the (non-local) boundary counter term, all correlation functions become trivial !

Holographic Entanglement Entropy (HEE)

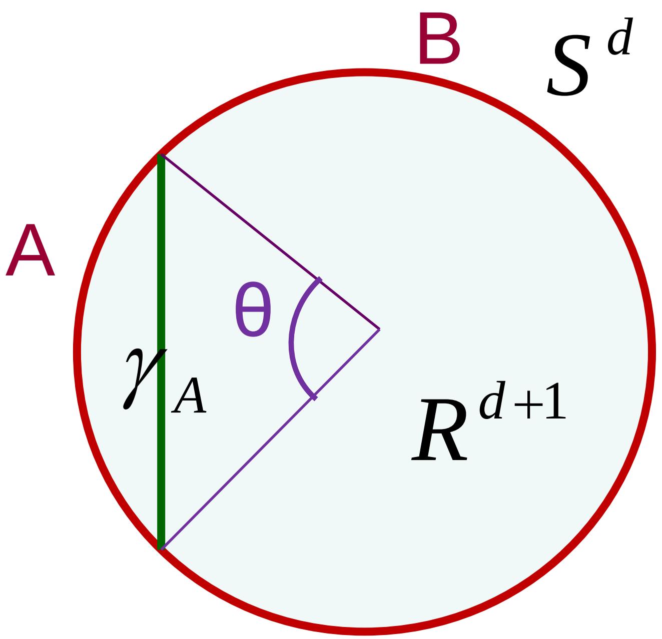
Though this result seems at first confusing, actually it is consistent with the holographic entanglement entropy.

It is easy to confirm that the HEE follows the volume law rather than the standard area law !

$$S_A \sim \frac{(\rho_\infty)^{d-1} \cdot \left(\mathrm{Sin}\frac{\theta}{2}\right)^{d-1}}{4G_N}.$$

$$\mathrm{Area}(\gamma_A) \to \mathrm{Vol}(A)$$

$$\text{in the limit } \theta \to 0 \;!$$



This unusual volume law implies that the subsystem A gets *maximally entangled* with B when A is infinitesimally small.

$$\rho_A = \sum_{M1,M2,..,Mn} \left| M1, M2, \ldots, Mn \right\rangle \left\langle M1, M2, \ldots, Mn \right| = \otimes_i \rho_i$$

Therefore, we have the trivial correlation functions:

$$\left\langle O_1 O_2 \cdots O_n \right\rangle = \mathrm{Tr}[\rho_A O_1 O_2 \cdots O_n] = 0.$$

At the same time, the volume law argues that the holographic dual of flat space is given by a highly non-local theory.

For example, a similar volume law is obtained for

$$S = \int dx^d \sqrt{g}\ [\phi \cdot e^{\sqrt{\nabla^2}} \cdot \phi] \quad .$$

Note: A bit similar to the open string field theory.

Possible Relation to Schwarzschild BH Entropy

Unruh Temp. at the boundary : $T_U = \frac{1}{2\pi\rho_\infty}$.

$$S_A \sim \frac{(\rho_\infty)^{d-1}}{G_N} \sim \frac{1}{G_N} \cdot \frac{1}{(T_U)^{d-1}} .$$

This can be comparable to the Schwarzschild BH entropy:

$$S_{BH} \sim \frac{1}{G_N} \cdot \frac{1}{(T_{BH})^{d-1}}.$$

This suggests that the BH entropy may be interpreted as the entanglement entropy……

⑥ Conclusions

- The quantum quench is a simple and useful process to study dynamical aspects of quantum many body systems. We identified the holographic dual of quantum quenches in CFT with horizon formations in the induced metric of a probe D-brane (`emergent black holes').

- We resolve an entropy puzzle on the thermalization in AdS/CFT by showing the total von-Neumann entropy is always vanishing in spite that the AdS includes BHs at late time.

- We present a toy holographic dual of BH formations and evaporations using quantum quenches.

- We discussed a possible holography for flat space and argued that the dual theory is non-local and is highly entangled.