Glueballs and Conifolds

(Green's Functions and Non-Singlet Glueballs on Deformed Conifolds)

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... based on work with I. Klebanov, J. Lin, S. Pufu, hep-th/1009.2763

Autumn Symposium on String/M-theory @ KIAS

Plan for this Talk

- General motivation:
  - Wanted: Description of strongly coupled QCD
  - Strategy: AdS/CFT correspondence
  - D-branes at conical singularities to reduce SUSY

1. What is the Conifold?
2. Review AdS/CFT on the Conifold
3. What are Glueballs?
4. Glueball spectrum on Conifolds
   - Deformed $AdS_5 \times T^{1,1}$
   - Deformed $AdS_4 \times V_{5,2}$

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What is the Conifold ?

Undeformed Conifold – Deformed Conifold – Generalized Conifold.

The Conifold

[Candelas, Green, Hübsch, NPB330(1990)]

- **"A" Conifold (CF):** Manifold with isolated conical singularities.
- **"The" CF:** $(2d-2)_R$ dimensional complex curve in $\mathbb{C}^d$ defined by

$$\boxed{z_1^2 + z_2^2 + \ldots + z_d^2 = 0} \qquad z_i \in \mathbb{C}$$

- **Topology:** The CF is a cone :-)

$$z_i \to t z_i \qquad t \in \mathbb{R}^+$$

- **Symmetry:** SO(d) x U(1)

$$z_i = R_{ij} z_j \qquad z_i = e^{i\alpha} z_i$$

- **Geometry:** The CF is a non-compact Calabi-Yau manifold

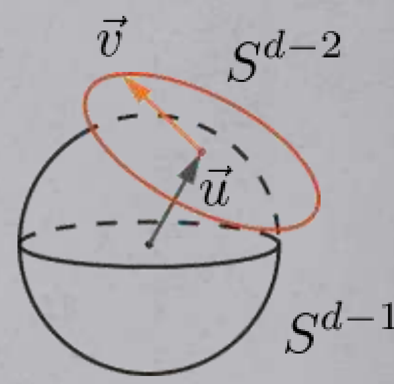
$$ds^2_{\mathrm{CF}} = \partial_i \bar{\partial}_j \mathcal{F}(z, \bar{z})\, dz^i\, d\bar{z}^j \qquad R_{i\bar{j}} = 0$$

Slices of the Conifold: $V_{d,2}$

[Candela, de la Ossa, NPB342(1990)]

- **Slice Σ_r**: Intersect the CF with a sphere of radius r

$$|z_1|^2 + |z_2|^2 + \ldots + |z_d|^2 = r^2$$



- **Stiefel manifold:**

Write $\vec{z} = (z_1, \ldots, z_d) = \vec{u} + i\vec{v}$

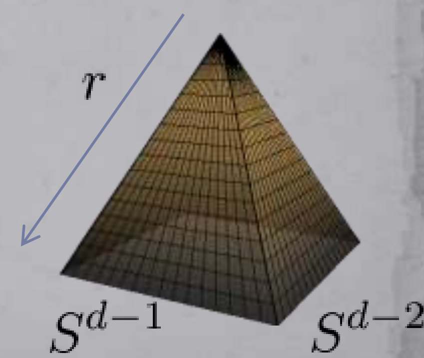
then $\vec{u} \cdot \vec{v} = 0\,, \quad \vec{u}^{\,2} = \vec{v}^{\,2} = \frac{1}{2} r^2$

Σ_r is the "set of all orthonormal 2-frames in d-dimensions $V_{d,2}$"

- **Coset:** Σ_r is also the "coset SO(d)/SO(d-2)"

Rotate $\vec{z}_0 = \frac{r}{\sqrt{2}}(1, i, 0 \ldots, 0)$ to any point in Σ_r

- **Radius and "angles":** $ds^2_{\rm CF} = dr^2 + r^2\, ds^2_{\Sigma_r}$



$$\Sigma_r \;\sim\; V_{d,2} \;\sim\; \mathrm{SO}(d)/\mathrm{SO}(d-2) \;\sim\; S^{d-2} \star S^{d-1}$$

The Deformed Conifold

- "The DCF": $(2d-2)_\mathbb{R}$ dimensional complex curve in $\mathbb{C}^d$ defined by

$$z_1^2 + z_2^2 + \ldots + z_d^2 = \epsilon^2 \qquad \epsilon \in \mathbb{R}^+$$

- **Deformation:**
  - The DCF is a not a cone :-(
  - U(1) is broken to Z_2
  - Tip is blown up to (d-1)-sphere of radius ~ε

$$\vec{u} \cdot \vec{v} = 0\,, \quad \vec{u}^{\,2} = \tfrac{1}{2}(r^2 + \epsilon^2)\,, \quad \vec{v}^{\,2} = \tfrac{1}{2}(r^2 - \epsilon^2)$$

- **Parametrization:** "Radius" τ and "angles" y_i $\quad \mathrm{DCF} = \mathbb{R}_\tau \times \Sigma_\tau$

$$z_i = \frac{\epsilon}{\sqrt{2}} \left(e^{\tau/2} y_i + e^{-\tau/2} \bar{y}_i \right)$$

[PKKL 1009.2763]

with $\sum_i y_i^2 = 0\,, \quad \sum_i |y_i|^2 = 1\,, \quad \sum_i |z_i|^2 = \epsilon^2 \cosh\tau$

Each slice Σ_τ of the DCF looks like a slice Σ_r of the undeformed CF!

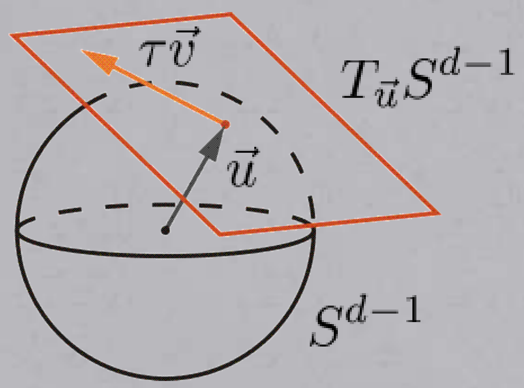
The Deformed Conifold

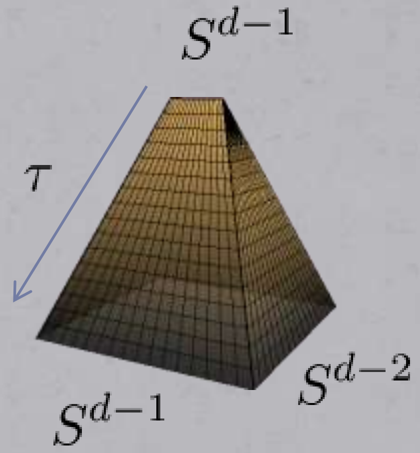
If the DCF is not a cone – what is it ?

Write again as real and imaginary parts $y_i = \frac{1}{\sqrt{2}}(u_i + i v_i)$

$$z_i = \frac{\epsilon}{\sqrt{2}}\left(e^{\tau/2} y_i + e^{-\tau/2}\bar{y}_i\right) = \epsilon\left(u_i \cosh\frac{\tau}{2} + i v_i \sinh\frac{\tau}{2}\right)$$

and deform this smoothly into $z_i \,\dot{\sim}\, u_i + i\,\tau\, v_i$





The DCF is homeomorphic to the "tangent bundle to a d-sphere"

→ Stenzel space

[Stenzel, Manu.Math, 80(1993)]

The Deformed Conifold

Slices: $\Sigma_{\tau=0} \sim V_{d,1} \sim \mathrm{SO}(d)/\mathrm{SO}(d-1) \sim S^{d-1}$

$$\Sigma_{\tau>0} \sim V_{d,2} \sim \mathrm{SO}(d)/\mathrm{SO}(d-2) \sim S^{d-2} \star S^{d-1}$$

Ricci-flat metric: $\mathcal{F}'(\tau) = \epsilon^2 \left[\frac{d-2}{\epsilon^2} \int_0^\tau (\sinh \xi)^{d-2}\, d\xi \right]^{\frac{1}{d-1}}$ [Cvetic, Gibbons, Lü, Pope, CMP232(2003)]

$$ds^2_{\mathrm{DCF}} = \tfrac{1}{4}\mathcal{F}'' d\tau^2 + \mathcal{F}' \coth\tau\, dy_i d\bar{y}_i + \tfrac{1}{2}\mathcal{F}' \operatorname{csch}\tau \left(dy_i dy_i + d\bar{y}_i d\bar{y}_i \right) + \left(\mathcal{F}'' - \mathcal{F}' \coth\tau \right) y_i d\bar{y}_i \bar{y}_j dy_j$$

Applications:

d=3	2d-2 = 4	"ordinary" gravity	[Eguchi, Hanson PLB74(1978)]
d=4	2d-2 = 6	D3 branes	[Candela, de la Ossa NPB342(1990)]
d=5	2d-2 = 8	M2 branes	[Cvetic, Gibbons, Lu, Pope CMP232(2003)]

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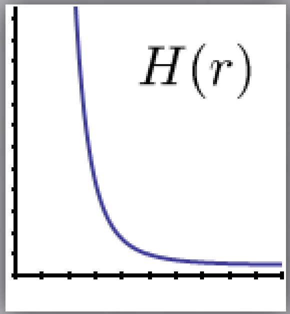
Review of AdS/CFT on Conifold

Focus on d=4 – Undeformed Conifold – Klebanov-Witten Theory – Add fractional branes – Backreaction – Deformed Conifold – Klebanov-Strassler Theory – Cascading Gauge Theory.

D-branes on Conifolds – Supergravity Solutions

- N D3 branes on CF:



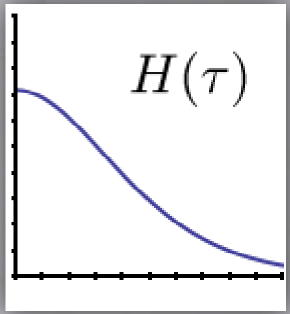
$$ds^2_{10} = H^{-\frac{1}{2}}(r)ds^2_4 + H^{\frac{1}{2}}(r)ds^2_{\mathrm{CF}}$$

with fluxes $\int_{T^{1,1}} F_5 = N$ $\int_{S^3} F_3 = 0$

→ Add M D5 wrapped over 2-Cycle → Collapse to Tip, Backreaction

[Klebanov, Nekrasov NPB574(2000)] [Klebanov, Tseytlin NPB578(2000)] [Klebanov, Strassler JHEP0008:052(2000)]

- N integer D3 & M fractional D3 on DCF:



$$ds^2_{10} = H^{-\frac{1}{2}}(\tau)ds^2_4 + H^{\frac{1}{2}}(\tau)ds^2_{\mathrm{DCF}}$$

with fluxes $\int_{T^{1,1}} F_5 = N_{\mathrm{eff}}(\tau)$ $\int_{S^3} F_3 = M$

Dual Gauge Theories

- **CF ↔ Klebanov-Witten Theory:**

  [Klebanov, Witten, NPB556(1999)]

  $\mathcal{N}$=1 superconformal SU(N) x SU(N) gauge theory in 4 dimension

Chiral matter	$SU(N)^2_{\rm gauge}$	$SU(2)^2_{\rm flavor}$	$U(1)_R$	$U(1)_B$	$\Delta_{\rm UV}$	$\Delta_{\rm IR}$
$(A_i)^a{}_{\hat{b}}$	$(\mathbf{N},\bar{\mathbf{N}})$	$(\mathbf{2},\mathbf{1})$	$\frac{1}{2}$	$+1$	1	$\frac{3}{4}$
$(B_j)^{\hat{a}}{}_b$	$(\bar{\mathbf{N}},\mathbf{N})$	$(\mathbf{1},\mathbf{2})$	$\frac{1}{2}$	-1	1	$\frac{3}{4}$

Superpotential: $h\int d^4x\, d^2\theta\, \epsilon^{ij}\epsilon^{kl}\, {\rm tr}\, A_i B_k A_j B_l$ (unrenormalizable)

- **DCF ↔ Klebanov-Strassler Theory:**

  [Klebanov, Strassler, JHEP0008:052(2000)]

  $\mathcal{N}$=1 susy, non-conformal SU(N+M) x SU(N) gauge theory in 4d,

  Confinement, chiral symmetry breaking, cascading RG flow

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What are Glueballs?

Glueballs in QCD – Glueballs in Klebanov-Strassler Theory – Glueball Masses from Supergravity.

What are Glueballs ?

Bound states of gluons

- Created by $\mathrm{tr}\, F^{\mu\nu} F^{\rho\sigma}$, $\mathrm{tr}\, F^{\mu\nu} D^{\kappa} F^{\rho\sigma}$, $\mathrm{tr}\, F^{\mu\nu} [F^{\rho\sigma}, F^{\kappa\tau}]$ ...
- Very non-perturbative: Large dynamically generated mass
- Quantum numbers:

	Lorentz rep. (s_1, s_2)	J^{PC} (Spin, Parity, Charge conj.)
$S = \mathrm{tr}\, F_{\mu\nu} F^{\mu\nu}$	$(0,0)$	0^{++}
$P = \mathrm{tr}\, \tilde{F}_{\mu\nu} F^{\mu\nu}$	$(0,0)$	0^{-+}
$T_{\alpha\beta} = \mathrm{tr}\, F_{\alpha\mu} F^{\mu}{}_{\beta} - \frac{1}{4} g_{\alpha\beta} S$	$(1,1)$	$0^{++}, 1^{-+}, 2^{++}$

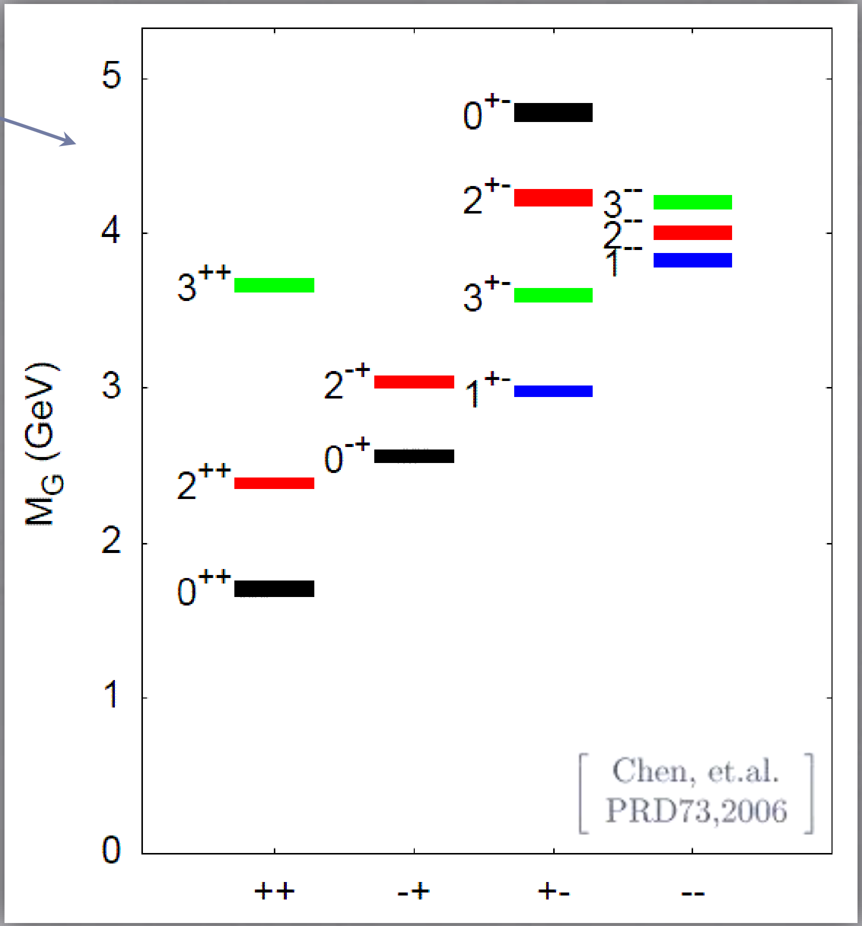
- Hard to idenfity: Mix with mesons $\bar{q}\Gamma q$ and hybrids $\bar{q}\Gamma F^{\mu\nu} q$.

Approaches to the Glueball Mass Spectrum

- Lattice QCD
- Bag model
- Potential model
- Instanton gas model
- QCD sum rules
- Duality OZI models

  ⋮

- Gauge/Gravity duality



Glueball masses from Supergravity

- Gauge theory side:

The bound state masses m_i can be read off from the poles of 2pt ftns

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim \sum_i \frac{c_i}{k^2+m_i^2} + \text{less singular terms}$$

- String theory side:

Such poles corresponds to normalizable solution to the linearized SUGRA e.o.m. for the bulk field Φ dual to the operator $\mathcal{O}$

Simplest example: $\Box_{10}\Phi(x,\tau,y)=0$

Ansatz $\Phi(x,\tau,y)=e^{ik\cdot x}\phi(\tau,y)$

$$\Delta_6\phi(\tau,y) = -m^2 H(\tau)\phi(\tau,y) \qquad m^2=-k_\mu k^\mu$$

Glueballs on the Conifold

- What had been done?
  - Scalar, vector, tensor glueballs
  - Only SO(4)-flavor-singlets
  - Decoupling of sugra equations

[Krasnitz hep-th/0011179]
[Gubser, Herzog, Klebanov JHEP0409:036,2004]
[Berg, Haack, Mück NPB736,2006 NPB789,2008]
[Dymarsky, Melnikov JHEP0805:035,2008]
[Benna, Dymarsky, Klebanov, Solovyov JHEP0806:070,2008]
[Dymarsky, Melnikov, Solovyov JHEP0905:105,2009]
[Gordeli Melnikov 0912.5517]

- What have we done?
  - Minimally coupled scalar: traceless part of metric
  - Spin-2 glueballs (also: Green's functions)
  - Non-trivial SO(4)-flavor quantum numbers
  - in 4d (10d sugra, $V_{4,2} = T^{1,1}$) and 3d (11d sugra, $V_{5,2}$)

$$\mathrm{tr}\big(T_{\alpha\beta} A_1 B_1 B_2^\dagger \ldots\ldots\ldots\big)$$

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Glueball Spectrum on Conifolds

Our Computation – Coordinates and Laplacian – Prediagonalization using Group Theory – Example – Results.

Laplacian on the Generalized Deformed Conifold

$$\Delta_{2d-2} = \mathcal{T} + g_{\mathcal{C}}(\tau)\mathcal{C} + g_{\mathcal{R}}(\tau)\mathcal{R} + g_{\mathcal{L}}(\tau)\mathcal{L}$$

[PKKL 1009.2763]

with

$$\mathcal{T} = \frac{4}{\mathcal{F}''\mathcal{F}'^{d-2}}\partial_\tau\left(\mathcal{F}'^{d-2}\partial_\tau\right)$$

$$\mathcal{C} = y_i y_j \frac{\partial^2}{\partial y_i \partial y_j} + (\bar{y}_i y_j - \delta_{ij} y_k \bar{y}_k)\frac{\partial^2}{\partial y_i \partial \bar{y}_j} + (d-1) y_i \frac{\partial}{\partial y_i} + \text{c.c.}$$

$$\mathcal{R} = \left(y_i \frac{\partial}{\partial y_i} - \bar{y}_i \frac{\partial}{\partial \bar{y}_i}\right)\left(y_j \frac{\partial}{\partial y_j} - \bar{y}_j \frac{\partial}{\partial \bar{y}_j}\right)$$

$$\mathcal{L} = \frac{1}{2}(\bar{y}_i y_j + y_i \bar{y}_j - \delta_{ij} y_k \bar{y}_k)\frac{\partial^2}{\partial y_i \partial y_j} + \frac{d-2}{2}\bar{y}_i \frac{\partial}{\partial y_i} + \text{c.c.}$$

and

$$g_{\mathcal{C}}(\tau) = -\frac{2\coth\tau}{\mathcal{F}'} \qquad g_{\mathcal{R}}(\tau) = -\frac{1}{\mathcal{F}''} + \frac{2\coth\tau}{\mathcal{F}'} \qquad g_{\mathcal{L}}(\tau) = \frac{4\,\mathrm{csch}\,\tau}{\mathcal{F}'}$$

Basis of Functions on the gen. DCF

SO(d) acts on each slice $\Sigma_\tau \simeq V_{d,2}$ – without mixing different slices.

Expand wave function as $$\phi(\tau, y_i, \bar{y}_i) = \sum_\alpha f_\alpha(\tau) F_\alpha(y_i, \bar{y}_i)$$

where $F_\alpha(y_i, \bar{y}_i) \in L^2(\Sigma_\tau)$ is a square integrable function on Σ_τ

$L^2(\Sigma_\tau \cong \frac{\mathrm{SO}(d)}{\mathrm{SO}(d-2)})$ decomposes into irreps of SO(d), but...

1) Which SO(d) representations occur ?

2) How many times does a given SO(d) irrep occur ?

(1 and 2: Range of α)

3) How do the basis functions in those irreps look ?

4) How do the operators in the Laplacian act onto these functions ?

1) Which SO(d) representations occur ?

Build SO(d) reps from tensor products of n y's and $\bar{n}$ $\bar{y}$'s:

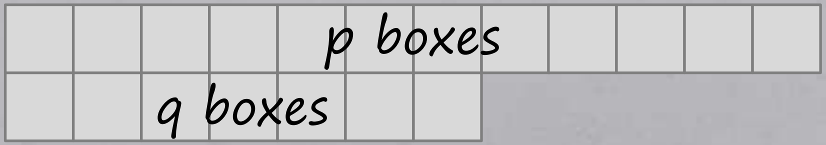
$$F(y_i, \bar{y}_i) = M^{j_1 j_2 \cdots j_{\bar{n}}}_{i_1 i_2 \cdots i_n}\, y_{i_1} y_{i_2} \cdots y_{i_n}\, \bar{y}_{j_1} \bar{y}_{j_2} \cdots \bar{y}_{j_{\bar{n}}}$$

The matrix $M^{j_1 \cdots}_{i_1 \cdots}$ is a representation of SO(d) (in gen. reducible)

Don't overcount, note: $\sum_i y_i^2 = 0$ $\quad \sum_i \bar{y}_i^2 = 0$ $\quad \sum_i |y_i|^2 = 1$

$M^{j_1 \cdots}_{i_1 \cdots}$ is symmetric in **i**'s, symmetric in **j**'s, traceless in ANY pair

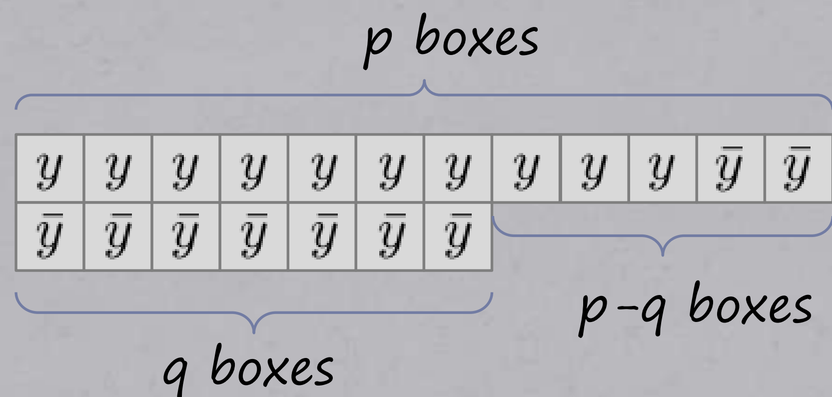
→ At degree n+$\bar{n}$, the only possible representations are (p,q):



p+q = n+$\bar{n}$

2) How many times does (p,q) occur ?

Fill the Young tableau with y's and $\bar{y}$'s



Possible number of y's in the p−q extra boxes: 0, 1, ..., p−q.

→ The representation (p,q) occurs p−q+1 times.

Examples: 2x (1,0) y_i , $\bar{y}_i$

3x (2,0) $y_i y_j$, $y_i \bar{y}_j - \delta_{ij}$, $\bar{y}_i \bar{y}_j$

1x (1,1) $y_i \bar{y}_j - y_j \bar{y}_i$

3) What do the basis functions look like ?

The p-q+1 highest weight states in the irreps (p,q) are

$$F_{p,q,\tilde{m}} = \sqrt{\binom{p-q}{\frac{p-q}{2}-\tilde{m}}}\,(y_2+iy_3)^{\frac{p-q}{2}+\tilde{m}}\,(\bar{y}_2+i\bar{y}_3)^{\frac{p-q}{2}-\tilde{m}}$$

$$\times\left[(y_2+iy_3)(\bar{y}_1+i\bar{y}_4)-(y_1+iy_4)(\bar{y}_2+i\bar{y}_3)\right]^q$$

$$\tilde{m} = -\frac{p-q}{2},\ -\frac{p-q}{2}+1,\ \ldots,\ +\frac{p-q}{2}$$

The possible mixing is reduced to

$$\phi(\tau, y_i, \bar{y}_i) = \sum_{\tilde{m}} f_{\tilde{m}}(\tau) F_{p,q,\tilde{m}}(y_i, \bar{y}_i)$$

4) How does the Laplacian act ?

Introduce some operators: [Gelbart, TAMS192(1974)][Strichartz, CJM27(1975)]

Measure $\tilde{m}$ $\quad \tilde{J}_3 = \frac{1}{2}\left(y_i\partial_i - \bar{y}_i\bar{\partial}_i\right)$

Define ladder operators $\quad \tilde{J}_+ = y_i\bar{\partial}_i \qquad \tilde{J}_- = \bar{y}_i\partial_i$

(these three: $\widetilde{\mathrm{SU}(2)}$)

Measure degree $n+\bar{n}$ $\quad N = y_i\partial_i + \bar{y}_i\bar{\partial}_i$ $\quad \mathrm{U}(1)_N$

Obs: These generators do NOT correspond to isometries of the DCF !

But it turns out that the bits in the Laplacian can be written as

$$\mathcal{C} = p(p+d-2) + q(q+d-4)$$

$$\mathcal{R} = 4\tilde{J}_3^2$$

$$\mathcal{L} = \tilde{J}_+\left(\tfrac{1}{2}N - \tilde{J}_3\right) + \tilde{J}_-\left(\tfrac{1}{2}N + \tilde{J}_3\right)$$

Glueball Example

Take (p,q) = (2,0) then $\tilde{m}$ = -1, 0, +1

$$\phi(\tau, y_i, \bar{y}_i) = f_1(\tau)\,(y_1 + iy_2)^2 + f_0(\tau)\,(y_1 + iy_2)(\bar{y}_1 + i\bar{y}_2) + f_{-1}(\tau)\,(\bar{y}_1 + i\bar{y}_2)^2$$

Because $\mathbb{Z}_2 : y_i \leftrightarrow \bar{y}_i$ is a symmetry, there is a further decoupling.

Even: $f_1^+ = f_1 + f_{-1}\,,\ f_0^+ = f_0$

$$\mathcal{T}\begin{pmatrix} f_1^+ \\ f_0^+ \end{pmatrix} + \begin{pmatrix} 8g_{\mathcal{C}} + 4g_{\mathcal{R}} + m^2 H & 2\sqrt{2}g_{\mathcal{L}} \\ 2\sqrt{2}g_{\mathcal{L}} & 8g_{\mathcal{C}} + m^2 H \end{pmatrix}\begin{pmatrix} f_1^+ \\ f_0^+ \end{pmatrix} = 0$$

Odd: $f_1^- = f_1 - f_{-1}$

$$\mathcal{T} f_1^- + \left(8g_{\mathcal{C}} + 4g_{\mathcal{R}} + m^2 H\right) f_1^- = 0$$

d = 4

Glueball Example

Solve the (system of) ordinary differential equations by shooting method.

Even: Normalizable solutions for

m² = 3.87, 6.08, 6.34, 8.94, 9.3, ...

Large τ asymptotics

$$F_1^+(y,\bar{y}) \to \mathrm{tr}\, T_{\alpha\beta}\big[(A_1B_1)^2 + (B_2^\dagger A_2^\dagger)^2\big] \qquad f_1^+(\tau) \sim e^{-7\tau/3}$$

$$F_0^+(y,\bar{y}) \to \mathrm{tr}\, T_{\alpha\beta} A_1 B_1 B_2^\dagger A_2^\dagger \qquad f_0^+(\tau) \sim e^{-2(1+\sqrt{7})\tau/3}$$

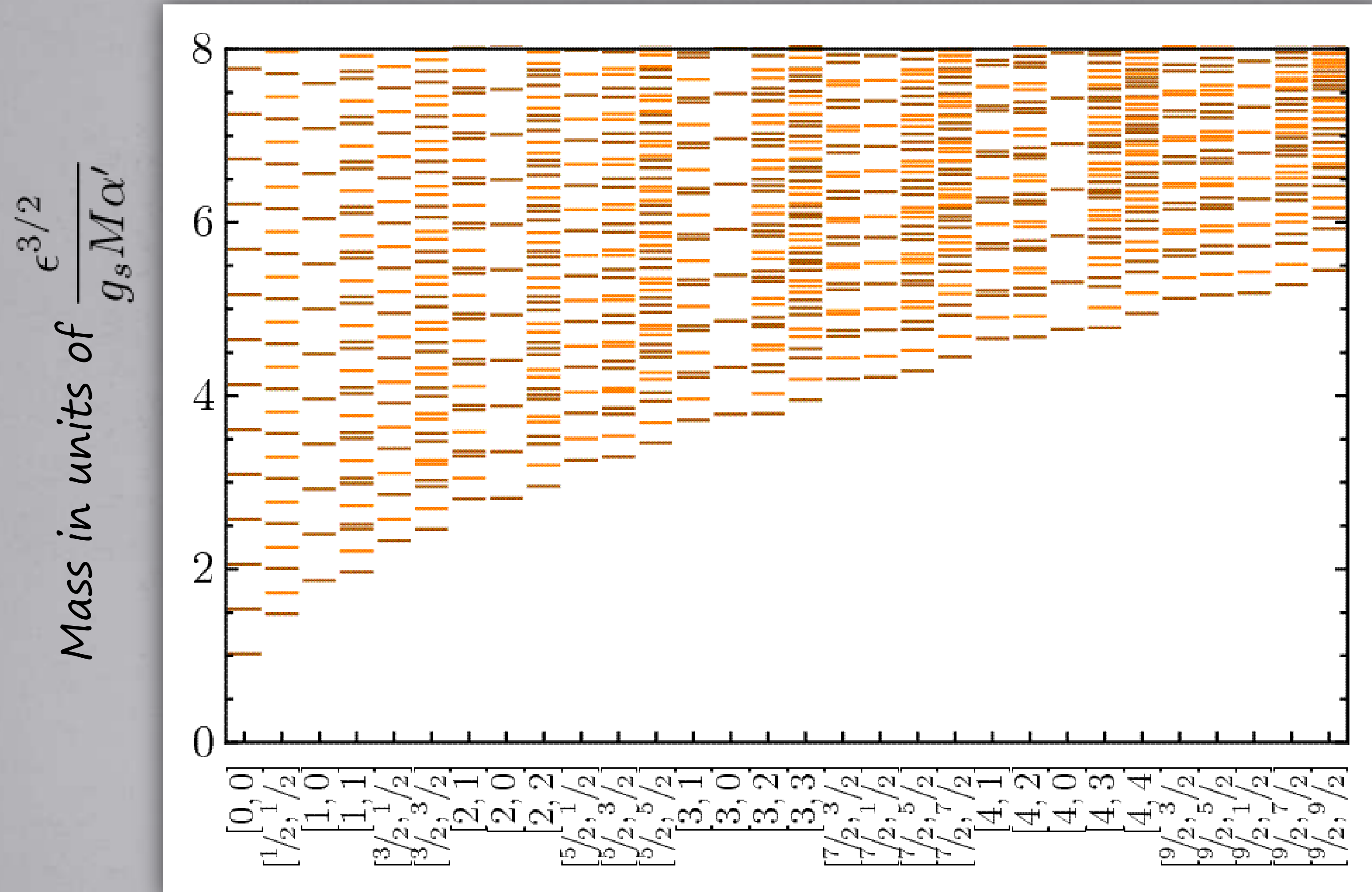
Odd: Normalizable solutions for

m² = 4.88, 7.47, 10.58, 14.23, 18.41, ...

Large τ asymptotics

$$F_{-1}^+(y,\bar{y}) \to \mathrm{tr}\, T_{\alpha\beta}\big[(A_1B_1)^2 - (B_2^\dagger A_2^\dagger)^2\big] \qquad f_{-1}^+(\tau) \sim e^{-7\tau/3}$$

Glueball Mass Spectrum on $T^{1,1}$



$SO(4) = SU(2)_L \times SU(2)_R$ representations $[j_L, j_R]$

Glueball Mass Spectrum on $V_{5,2}$

- **N M2 branes on DCF:** $ds_{11}^2 = H^{-\frac{2}{3}}(\tau) ds_4^2 + H^{\frac{1}{3}}(\tau) ds_{\mathrm{DCF}}^2$

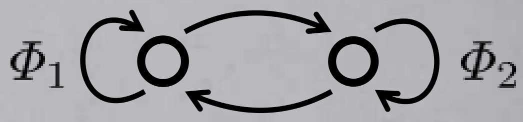
$$\int_{\Sigma_\tau} *G_4 = N(\tau) \qquad \int_{S^4} G_4 = 0$$ (No fractional M-branes !)

[Cvetic, Gibbons, Lu, Pope, CMP232(2003)]

- **Chern-Simons:** $\mathcal{N}$=2 superconformal $U(N)_1 \times U(N)_{-1}$

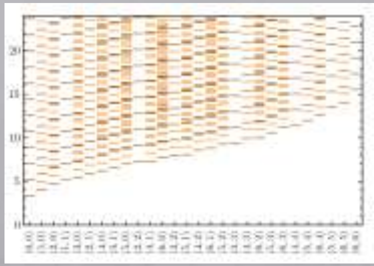
[Martelli, Sparks, JHEP12(2009)]

$$W = \Phi_1(B_1A_1 + B_2A_2) + \Phi_2(A_1B_1 + A_2B_2)$$
$$+\mu\, \mathrm{tr}\left(\Phi_1^2 - \Phi_2^2\right) \; + s\left(\Phi_1^3 + \Phi_2^3\right)$$



- **Glueballs:** $\epsilon^2 \sim \mu^2/s$

$$\Delta_8 \phi(\tau, y) = -m^2 H(\tau) \phi(\tau, y)$$



Summary

- Laplacian on generalized DCF in SO(d) covariant variables
- Computation of mass spectrum for glueballs with flavor charges
- Involves solving coupled ODE's
- Mixing is due to the absence of $U(1)_R$ symmetry on DCF
- For generic glueballs, many dual operators acquire a VEV
- No time to cover:
  - Green's functions: Backreation when mobile D3's are added
- Future:
  - Glueballs with different Lorentz spin; not only minimal scalar