# The M5-Brane From Below

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NL, C. Papageorgakis arXiv:1007.2982

NL, CP and M. Schmidt-Sommerfeld, arXiv:1012.2882

P. Richmond arXiv:1109.6454

NL, CP, MSS in progress


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5
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# Outline

- Introduction
- A non-Abelian (2,0) algebra
- The view from 5D: SYM as M5's on space-like $S^1$
- The view from 1D: Instanton QM as M5's on light-like $S^1$
- Enter Deconstruction
- Comments

# Introduction

We are gathered here today[1] to talk about the M5-brane.

Low-energy M5-brane dynamics governed by a 6D theory with:

- $(2, 0)$ supersymmetry
- conformal invariance
- $\mathrm{SO}(5)$ R-symmetry

Multiplet contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions

Very rich and novel 6D CFT dual to $AdS_7 \times S^4$

[1] Maybe just the first 100 minutes

Various conjectures on how to define/describe/cope with/compute with the $(2,0)$

- DLCQ of QM on instanton moduli space [Aharony, Berkooz, Kachru, Seiberg, Silverstein]
- Deconstruction from D=4 SCFT [Arkani-Hamed,Cohen,Karch,Motl]
- Strong coupling limit of 5D SYM [Douglas],[NL,CP,MSS]

all based on lower dimensional theories. Here we will discuss how these are interconnected.

More recently there are six-dimensional proposals [Chu],[Ho, Huang, Matsuo].

# A non-Abelian (2,0) algebra

At linearized level the free susy variations are

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma^{\mu\nu\lambda}H_{\mu\nu\lambda}\epsilon \\ \delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}\partial_{\lambda]}\Psi \ , \end{aligned}$$

and the equations of motion are those of free fields with $dH = 0$ (and hence $dH = d\star H = 0$).

Reduction to the D4-brane theory sets $\partial_5 = 0$ and

$$F_{\mu\nu} = H_{\mu\nu 5}$$

We wish to generalise this algebra to nonabelian fields with

$$D_\mu X^I_A = \partial_\mu X^I_A - \tilde{A}_\mu{}^B{}_A X^I_B$$

Upon reduction we expect Yang-Mills susy:

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \Gamma^\alpha\Gamma^I D_\alpha X^I\epsilon + \frac{1}{2}\Gamma^{\alpha\beta}\Gamma^5 F_{\alpha\beta}\epsilon - \frac{i}{2}[X^I, X^J]\Gamma^{IJ}\Gamma^5\epsilon \\ \delta A_\alpha &= i\bar{\epsilon}\Gamma_\alpha\Gamma_5\Psi\ , \end{aligned}$$

Thus we need a term in $\delta\Psi$ that is quadratic in $X^I$ and which has a single $\Gamma_\mu$:

- Invent a field $C^\mu_A$

After starting with a suitably general anstaz we find closure of the susy algebra implies

$$
\begin{aligned}
\delta X^I_A &= i\bar\epsilon\Gamma^I\Psi_A \\
\delta\Psi_A &= \Gamma^\mu\Gamma^I D_\mu X^I_A\epsilon + \frac{1}{12}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_A\epsilon - \frac{1}{2}\Gamma_\lambda\Gamma^{IJ}C^\lambda_B X^I_C X^J_D f^{CDB}{}_A \\
\delta H_{\mu\nu\lambda\ A} &= 3i\bar\epsilon\Gamma_{[\mu\nu}D_{\lambda]}\Psi_A + i\bar\epsilon\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa_B X^I_C\Psi_D f^{CDB}{}_A \\
\delta\tilde A_\mu{}^B{}_A &= i\bar\epsilon\Gamma_{\mu\lambda}C^\lambda_C\Psi_D f^{CDB}{}_A \\
\delta C^\mu_A &= 0
\end{aligned}
$$

where $f^{ABC}{}_D$ are totally anti-symmetric structure constants of the $N=8$ 3-algebra (possibly Lorentzian). Can also have $C^\mu$ and $f^{ABC}{}_D \to f^{AB}{}_C$.

Has $(2,0)$ supersymmetry, SO(5) R-symmetry and scale symmetry ($C^\mu_A$ has dimensions of length)

The algebra closes with the on-shell conditions

$$
\begin{aligned}
0 &= D^2 X^I_A - C^\nu_B C_{\nu G} X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A + fermions \\
0 &= D_{[\mu} H_{\nu\lambda\rho]\ A} + \frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma_B X^I_C D^\tau X^I_D f^{CDB}{}_A + fermions \\
0 &= \Gamma^\mu D_\mu \Psi_A + X^I_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A \\
0 &= \tilde{F}_{\mu\nu}{}^B{}_A - C^\lambda_C H_{\mu\nu\lambda\ D} f^{CDB}{}_A \\
0 &= D_\mu C^\nu_A = C^\mu_C C^\nu_D f^{BCD}{}_A \\
0 &= C^\rho_C D_\rho X^I_D f^{CDB}{}_A = C^\rho_C D_\rho \Psi_D f^{CDB}{}_A = C^\rho_C D_\rho H_{\mu\nu\lambda\ A} f^{CDB}{}_A
\end{aligned}
$$

Can be rephrased within the language of Lie-Crossed-Modules for Gerbes [Palmer, Seamann]

Thus $C^\mu_A$ picks out a fixed direction in space and in the 3-algebra and $C^\mu_A D_\mu = 0$. So apparently we are simply pushed back to 5D.

But no so quick, we can look at the Conserved currents, *e.g.*

$$\begin{aligned} T_{\mu\nu} &= D_\mu X^I_A D_\nu X^{IA} - \frac{1}{2}\eta_{\mu\nu} D_\lambda X^I_A D^\lambda X^{IA} \\ &+\frac{1}{4}\eta_{\mu\nu} C^\lambda_B X^I_A X^J_C C_{\lambda G} X^I_F X^J_E f^{CDBA} f^{EFG}{}_D + \frac{1}{4} H_{\mu\lambda\rho\ A} H_\nu{}^{\lambda\rho\ A} \\ &+fermions \end{aligned}$$

And we also obtain six-dimensional expressions for the supercurrent and central charges.

Here we see that the system is 6-dimensional

$$C^\mu P_\mu = \int d^5 x C^\mu T_{0\mu} \sim \mathrm{Tr} \int F \wedge H \in \mathbb{Z}$$

but with a compact direction (i.e. $\mathbb{R}^5 \times S^1$).

# Spacelike Reduction and DLCQ

For $C_A^\mu = g_{YM}^2 \delta_5^\mu \delta_A^0$ the previous system reduces to 5D SYM

$$P_5 = -\frac{1}{8g_{YM}^2} \int d^4x \, \mathrm{tr}(F_{ij} F_{kl} \varepsilon_{ijkl}) = \frac{k}{R_5}$$

First recall some facts about 5D SYM:

- ◇ Power-counting non-renormalisable, $g_{YM}^2 \sim$ length
  $\Rightarrow$ naively new d.o.f. should appear at some scale
- ◇ M-theory says a UV (strong coupling) fixed point exists and is 6-dimensional: the M5-brane CFT
- ◇ Agrees with Nahm's classification of SCFT's: indeed UV-fixed point theory cannot be 5D

From String Theory the relation between D4- and M5-brane theories given by compactification on $S^1$.

5D SYM also has particle states that carry instanton charge $k$ with mass

$$M \propto \frac{k}{g_{YM}^2} \propto \frac{k}{R_5}$$

Simplest such states just D0's in D4 worldvolume.

Interpretation as momentum on $S^1$ of compactified 6D theory. [Rozali, Berkooz-Rozali-Seiberg]

$\Rightarrow$ Even in Yang-Mills limit this tower of states knows something about M-theory direction.

We argued that, at least for $< X^6 >= v$ 5D SYM contains a complete spectrum of KK modes in the soliton spectrum

- KK tower of W-Bosons given by Dyonic instantons [NL,Tong]
- KK tower of strings given by Monopole strings
- KK tower of photons given by quantum-sized instantons $< \rho >\sim g_{YM}^2/v$

All smooth finite energy states in the correct representation of (2,0) supersymmetry.

So:

- ◇ No more room for any additional UV states.
- ◇ Natural conclusion: 5D SYM is the (2,0) CFT compactified on $S^1$ (see also [Douglas])
- ◇ ... and hence well-defined non-perturbatively (some how). (see also [Douglas])
  - Details of how this works out not clear since 6D CFT contains momentum states which are non-perturbative from the point of view of 5D theory

You might hope that 5DSYM is finite (but apparently it's not at 6 loops [Bern,Douglas,..] to appear)

But this is also naive since one normally says that solitons are suppressed by factors of

$$e^{-1/g_{YM}^2}$$

But $g_{YM}^2$ has dimensions of length so we in fact must have

$$e^{-d/g_{YM}^2}$$

where $d$ is a length-scale (e.g. instanton size, instanton/anti-instanton separation)

So no decoupling from perturbative physics if $d \leq \mathcal{O}(g_{YM}^2)$ (e.g. photon KK tower).

# Light-Like Reduction and DLCQ

We could also consider a null reduction, $x^\mu = (x^+, x^-, x^i)$:

$$C^\mu_A = g^2_{YM}\delta^\mu_+\delta^0_A$$

$$\begin{aligned}
0 &= D^2 X^I_a - \frac{ig}{2}\bar{\Psi}_c\Gamma_+\Gamma^I\Psi_d f^{cd}{}_a \\
0 &= \Gamma^\mu D_\mu\Psi_a + g^2_{YM}X^I_c\Gamma_+\Gamma^I\Psi_d f^{cd}{}_a \\
0 &= D_{[\mu}H_{\nu\lambda\rho]\ a} - \frac{g^2_{YM}}{4}\epsilon_{\mu\nu\lambda\rho\tau+}X^I_c D^\tau X^I_d f^{cd}{}_a + fermions \\
0 &= \tilde{F}_{\mu\nu}{}^b{}_a - g^2_{YM}H_{\mu\nu+\ d}f^{db}{}_a
\end{aligned}$$

where $f^{ab}{}_c = f^{0ab}{}_c$. Curious system with 16 supersymmetries and an $SO(5)$ R-symmetry but $D_+ = 0$

We wish to view $x^-$ as time, solve the non-dynamical equations, and then quantizing using the hamiltonian

$$\mathcal{H} = \mathcal{P}_- = \int d^4x T_{--}$$

Setting the fermions to zero the non-trivial equations are:

$$\begin{aligned} 0 &= D_i D^i X^I \\ 0 &= D^i F_{i-} \\ 0 &= D_- F_{i-} - D^j G_{ij} - ig^4 [X^I, D_i X^I] \end{aligned}$$

with $F_{i-} = -g^2 H_{i-+}$, $F_{ij} = -g^2 H_{ij+}$, $G_{ij} = -g^2 H_{ij-}$ and hence

$$F = - \star F \qquad G = \star G$$

It follows that $A_i$ is determined by the AHDM construction

- introduces moduli $A_i = A_i(m^\alpha)$
- Natural moduli space metric

$$g_{\alpha\beta} = \mathrm{Tr} \int d^4x \delta_\alpha A_i \delta_\beta A_i$$

Furthermore $X^I$ can also be solved for explicitly in terms of $m^\alpha$ and their vev's:

$$X^I = v^I + \mathcal{O}\left(\frac{1}{x^2}\right)$$

Next we note that

$$\partial_- A_i = \frac{\partial A_i}{\partial m^\alpha}\partial_- m^\alpha + D_i\omega$$

where $\omega$ is a gauge transformation that we choose to ensure that $D^i\partial_- A_i = 0$.

- $\Rightarrow D_i D^i A_- = 0 \qquad \Rightarrow \qquad A_- = w + \mathcal{O}\left(\frac{1}{x^2}\right)$

All fields are reduced to functions of the vev's $v^I, w$ and instanton moduli $m^\alpha$

'Time' dependence arises by letting these be functions of $x^-$.

Now

$$\mathcal{P}_+ = \int d^4x T_{-+} = -\frac{2}{g^2}\mathrm{Tr}\int F\wedge F \in \frac{4\pi^2}{g^2}\mathbb{Z}$$

gives the instanton number and

$$\mathcal{P}_i = \int d^4x T_{-i} = \mathrm{Tr}\int F_{ij}F_-{}^j \propto \text{total instanton momentum}$$

The hamiltonian $\mathcal{P}_-$ is:

$$\mathcal{P}_- = \frac{1}{2g^2} g_{\alpha\beta}(\partial_- m^\alpha - L^\alpha)(\partial_- m^\beta - L^\beta) + V$$

where

$$L_\alpha \partial_- m^\alpha = \mathrm{Tr}\oint \partial_- A_r A_- \qquad V = \frac{g^2}{2}\mathrm{Tr}\oint X^I D_r X^I \ .$$

The Superalgebra for this theory is

$$\{\mathcal{Q}_-, \mathcal{Q}_-\} = -\,2\mathcal{P}_-(\Gamma^- C^{-1}) + \mathcal{Z}^I_+(\Gamma^-\Gamma^I C^{-1}) + \mathcal{Z}^{IJ}_{ij+}(\Gamma^{ij}\Gamma^-\Gamma^{IJ} C^{-1})$$
$$\{\mathcal{Q}_+, \mathcal{Q}_+\} = -\,2\mathcal{P}_+(\Gamma^+ C^{-1})$$
$$\{\mathcal{Q}_-, \mathcal{Q}_+\} = -\,2\mathcal{P}_i(\Gamma^i C^{-1}) + \mathcal{Z}^I_i(\Gamma^i\Gamma^I C^{-1})\,,$$

where the central charges are

$$\mathcal{Z}^I_+ = -\,2\mathrm{Tr}\int d^4x\; F_{-i}D^iX^I$$
$$\mathcal{Z}^I_i = -\,\mathrm{Tr}\int d^4x\; G_{ij}D^jX^I$$
$$\mathcal{Z}^{IJ}_{ij+} = -\,g^2\mathrm{Tr}\int d^4x\; D_{[i}X^ID_{j]}X^J\ .$$

Thus we obtained the [Aharony, Berkooz, Kachru, Seiberg, Silverstein] proposal of $(2,0)$ theory along with explicit expressions for $\mathcal{Z}$, $\mathcal{P}_\mu$ and generalized to include $L^\alpha$, $V$.

Arises naturally from an infinite boost of 5D SYM or D4-branes using the (2,0) theory we constructed above.

So it follows from the conjecture that (2,0) on $S^1$ is 5D SYM in a [Seiberg] limit that $C^\mu$ becomes null:

$$F_{\mu\nu} \to F^-_{\mu\nu} \oplus F^+_{\mu\nu}$$

and $F^+_{\mu\nu}$ decouples as $C^\mu C_\mu \to 0$.

# Enter Deconstruction

To summarize, the $(2,0)$ system that we constructed leads to the following:

- ◇ Proposal (2,0) theory on $S^1$ is exactly 5D SYM for any value of the coupling (see also [Douglas])
- ◇ Rederived and generalized the $(2,0)$ DLCQ theory as the quantum mechanics of instantons [Aharony, Berkooz, Kachru, Seiberg, Silverstein].

In some sense, since quantum 5D SYM isn't defined (unless one can make non-perturbative sense of the Lagrangian), the conjecture that it is $(2,0)$ on $S^1$ is just a definition.

Can we define 5D SYM another way? Deconstruction comes to mind [NL,CP,MSS] in progress:

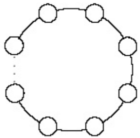

FIG. 1: Theory space for an $SU(k)^N$ model.

Start with $\mathcal{N}=2$, $N_f=2K$ $SU(K)$ SCFT with $<\Phi>=v$ breaking $SU(K)^N \to SU(K)$.

This theory deconstructs 5D SYM on $S^1$ with when $N \to \infty$:

$$2\pi R_5 = \frac{N}{g_4 v}$$

But actually, as noted in [Arkani-Hamed,Cohen,Karch,Motl], because of $SL(2,\mathbb{Z})$ this deconstructs a 6D theory on $S^1 \times S^1$

Extra circle (KK modes come from monopoles = wrapped instantons)

$$2\pi R_6 = \frac{g_4}{v} = \frac{g_{YM}^2}{2\pi}$$

So the conjecture that (2,0) on $S^1$ is 5DSYM comes out as limit of deconstruction conjecture.

# Comments

Other work with [NL,Nastase,CP] derives M5-branes from blow-up M2's in a flux background.

- Momentum as instanton number arises from monopole operators in ABJM

Or with [Jeon, NL, PR] using cubic T-duality of M2's

- Leads to interesting reformulation of 3D and 5D SYM in terms of a local monopole operator

Subsequent work has greatly added to and enhanced the general picture:

- S-duality for 5D SYM on $S^1$ [Tachikawa]
- Bound states in instanton QM [Kim,Kim,Koh,Lee,Lee],
- String Junctions in 5D SYM [Bolognesi, Lee]
- $N^3$ scaling of 5D SYM partition function [Kim,Kim],[Kallen,Minahan,Nedelin,Zabzine]

Other recent work:

- $(2,0)$ amplitudes [Czech, Huang, Rozali]
- More general circle reductions [Gustavsson],[Linander,Ohlsson]

Altogether these paint a consistent, interconnected picture of the M5-brane in terms of lower dimensional theories

- Alternatively, what exactly is the difference between the $(2,0)$ CFT on $S^1$ and 5D SYM?

Some important technical points remain

- Is 5D SYM well-defined non-perturbatively
- In DLCQ picture instanton moduli space has singularities (but these are mild orbifold singularities)
- How much does deconstruction tell us about the full higher dimensional theory