M5-branes from gauge theories on S^5

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KIAS Autumn Symposium on String/M theory talk based on:

Hee-Cheol Kim and S.K.,

"M5-branes from gauge theories on the 5-sphere"

arXiv:1206.6339

Other related works:

5d SYM, 6d (2,0) (2010): [Lambert, Papageorgakis] [Lambert, Papageorgakis, Schmidt-Sommerfeld] [Douglas]

DLCQ description of M5's (1997) (2011):

[Aharony, Berkooz, Kachru, Seiberg, Silverstein] [Aharony, Berkooz, Seiberg] [Lambert, Richmond]

Instanton partition function & M5 (2011): [H.-C.Kim, S.K., E.Koh, K.Lee, S.Lee]

SYM's on S^5 and their partition functions (2012):

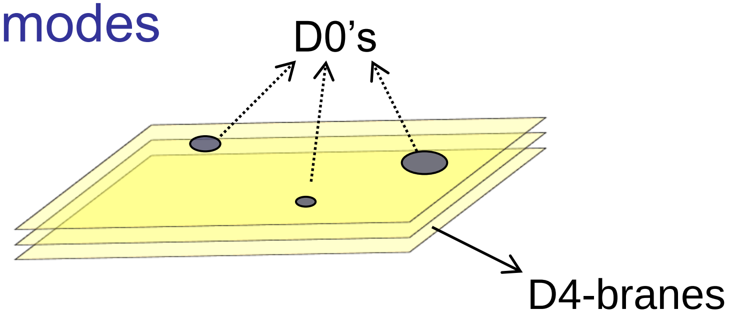
[Hosomichi, Seong, Terashima] [Kallen, Qiu, Zabzine] [Kallen, Minahan, Nedeline, Zabzine] [Imamura]

Motivation

- Most mysterious object of M-theory: M5-branes (much more than M2)
- 6d (2,0) SCFT at low energy (known w/ ADE gauge groups)
- A conclusive microscopic description is yet unknown, N^3 degrees of freedom on N M5…

- Flat space: compactify M5 on a circle: D4-branes, **5d maximal SYM** at low energy. Nonperturbative (or exact) treatment in 5d captures (at least some) 6d physics.
- Instanton "solitons" : D0's on D4's = KK modes

$$F_{\mu\nu} = \star_4 F_{\mu\nu}$$ on R^4



Aspects of 5d SYM

- At least apparently, nonrenormalizable: consistent quantum computation?
- Maximal SYM: UV finite? [Lambert, Papageorgakis, Schmidt-Sommerfeld] [Douglas]

- A more **conservative approach**: "BPS observables" could be reliably calculable in low energy effective descriptions of consistent theories.

1. Instanton partition function on $R^4 \times S^1$ computed & used to study 5d SYM [Nekrasov], and also 6d (2,0) on $R^4 \times S^1 \times S^1$ [H.-C.Kim, S.K., E.Koh, K.Lee, S.Lee]
2. 4d N=2 SUGRA localization & black hole partition function [Dabholkar et.al.]

- Today's talk: 6d (2,0) theory on $S^5 \times S^1$ from (maximal) SYM on S^5
- Similar Euclidean type IIA/M-theory relation: Gopakumar-Vafa

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Indices for 6d (2,0) theories

- Consider 6d (2,0) SCFT on S^5 x R (radial quantization)
- OSp(8|4) symmetry: 16 Poincare SUSY Q & conformal SUSY S
- Bosonic $SO(6,2) \times SO(5)_R$; Cartans E; SO(6) j_1, j_2, j_3; $SO(5)_R$ R_1, R_2

- Index: counts states annihilated by one or more pair(s) of Q,S
- Simplest index that we consider in this talk:

$$\mathrm{tr}[(-1)^F e^{-\beta(E-R_1)}]$$

- 16 SUSY commutes with $E - R_1$: For Q's, E = ½ and R_1 = +1/2
- Path integral for the index preserves 16 SUSY (either in 6d or 5d)

- Caution: This is **NOT** the half-BPS partition function

Maximal SYM on 5-sphere

- Action: 16 SUSY given by Killing spinor eqn. $\nabla_\mu \epsilon = \frac{1}{2r}\gamma_\mu \hat{\gamma}^{45}\epsilon$ (a=1,2,3; i=4,5)

$$S = \frac{1}{g_{YM}^2}\int d^5\sqrt{g}\,\mathrm{tr}\Big[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi^I D^\mu\phi^I + \frac{i}{2}\lambda^\dagger\gamma^\mu D_\mu\lambda - \frac{1}{4}[\phi^I,\phi^J]^2 - \frac{i}{2}\lambda^\dagger\hat{\gamma}^I[\lambda,\phi^I]$$
$$+\frac{4}{2r^2}(\phi^a)^2 + \frac{3}{2r^2}(\phi^i)^2 - \frac{i}{4r}\lambda^\dagger\hat{\gamma}^{45}\lambda - \frac{1}{3r}\epsilon_{abc}\phi^a[\phi^b,\phi^c]\Big]$$

- More motivation (quadratic terms): reduce 6d Abelian (2,0) theory on S^1

6d conformal mass: $m^2 = 4$

$$\frac{4}{2r^2}(\phi^i)^2 + \frac{1}{2r^2}(\phi^i)^2 - \frac{i}{4r}\lambda^\dagger\hat{\gamma}^{45}\lambda$$

Scherk-Schwarz $(\text{mass})^2$ contributions from R_1 chemical potential in 6d to 5d reduction

- Superalgebra: SU(4|2), subgroup of Osp(8|4) for 6d (2,0) commuting with $E - R_1$ in the index
- 5d coupling ~ S^1 radius ~ chemical potential: $\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1} = \frac{2\pi}{r\beta}$

5-sphere partition function

- Localization: SUSY path integrals are exactly computable with off-shell nilpotent symmetry, $Q^2 = 0$ (or some trivial generalization of it):

$$Z(\beta) = \int e^{-S - tQV} \ : \ t \text{ independent}$$

- Off-shell subgroup of SU(4|2): 5d N=1 SYM [Hosomichi, Seong, Terashima]:

1. vector multiplet part is made off-shell with 3 auxiliary fields; D^1, D^2, D^3
2. hypermultiplet part: only one SUSY made "nilpotent" with 2 auxiliary fields.

- Decomposition of our fields:

$$\text{vector}: \ A_\mu \ , \chi, \ \phi \equiv \phi^3$$
$$\text{hyper}: \ q^1 \sim \phi^4 - i\phi^5, \ q^2 \sim \phi^1 + i\phi^2, \ \psi$$

- Q-exact deformation: $\delta\left((\delta\chi)^\dagger\chi\right) + \frac{1}{2}\delta\Big((\delta\psi)^\dagger\psi + \psi^\dagger(\delta\psi^\dagger)^\dagger\Big)$

$$Q^{(\frac{1}{2},\frac{1}{2})}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}} \ : \ \text{BPS bound } E = 2R_1 + 2R_2 + j_1 + j_2 + j_3$$

Some calculation

- Saddle point equations $v^\mu = \epsilon^\dagger \gamma^\mu \epsilon$ is Killing vector for Hopf fiber of S^5

$$F_{\mu\nu} = \frac{1}{2}\sqrt{g}\epsilon_{\mu\nu\alpha\beta\gamma} v^\alpha F^{\beta\gamma} \,, \quad D_\mu \phi = 0 \,, \quad D^3 = \frac{i}{r}\phi$$

all other fields = 0

- Instantons on CP^2 base of S^5 : we discuss their contribution later.
- Perturbative sector: constant scalar matrix $\lambda = r\phi$
- Classical contribution: $e^{-\frac{2\pi^2 \mathrm{tr}(\lambda^2)}{\beta}}$
- Determinant from quadratic fluctuations: 5d N=1 with extra hyper mass

$$\text{vector}: \prod_{\alpha \in \text{root}} \prod_{n=-\infty}^{\infty} (n + i\alpha(\lambda))^{1+\frac{3n}{2}+\frac{n^2}{2}} \sim \prod_\alpha \alpha \prod_{n=1}^{\infty} (n + i\alpha(\lambda))^{n^2+2}$$

$$\text{hyper}: \prod_\alpha \prod_{n=-\infty}^{\infty} \left(n + \frac{3}{2} + m + i\alpha(\lambda)\right)^{-(1+\frac{3n}{2}+\frac{n^2}{2})} = \prod_\alpha \prod_{n=1}^{\infty} \left(n - \frac{1}{2} + m + i\alpha(\lambda)\right)^{-\frac{n^2-n}{2}} \left(n + \frac{1}{2} - m + i\alpha(\lambda)\right)^{-\frac{n^2+n}{2}}$$

- Maximal SUSY point: m = ½ (our case) or – ½ , more cancelation

$$\prod_{\alpha \in \text{root}} \prod_{k=1}^{\infty} (k^2 + r^2 \alpha(\phi)^2) = \prod_{\alpha \in \text{root}} \frac{2\pi \sinh(\pi r \alpha(\phi))}{\pi r \alpha(\phi)}$$

Perturbative partition function

- The matrix integral for perturbative part:

$$Z_{\text{pert}} = \frac{1}{|W|}\int d\lambda\, e^{-\frac{2\pi^2 \text{tr}(\lambda^2)}{\beta}} \prod_{\alpha\in\text{root}} 2\sinh(\pi\alpha(\lambda))$$

- Same matrix integral as pure Chern-Simons partition function on S^3 [Kapustin, Willett, Yaakov]: identify parameters as $\beta = -\frac{2\pi i}{k}$
- U(N) gauge group:

$$Z_{\text{pert}} = \left(\frac{\beta}{2\pi}\right)^{\frac{N}{2}} e^{\beta\frac{N(N^2-1)}{6}} \prod_{n=1}^{N-1}(1-e^{-n\beta})^{N-n}$$

- "Almost" an index, apart from the 1st prefactor (later)
- Degeneracy information: later
- Casimir "energy" factor: other semisimple G : $\frac{N(N^2-1)}{6} \to f^{abc}f^{abc} = \frac{c_2|G|}{6}$

The vacuum Casimir "energy"

- Vacuum expectation value of $E - R_1$ on S^5 x R
- Free theory: sum of oscillator 0-point energies: regularization dependent

$$\epsilon_0 = \lim_{\beta' \to 0} \mathrm{tr} \left[(-1)^F \frac{E}{2} e^{-\beta' E} \right]$$

- Our regularization: should demand regularization to preserve 16 SUSY

$$(\epsilon_0)_{\mathrm{index}} = \lim_{\beta' \to 0} \mathrm{tr} \left[(-1)^F \frac{E - R_1}{2} e^{-\beta' (E - R_1)} \right]$$

- Indeed, the coefficient of our large N "index Casimir energy" differs from the "conventional" one calculated from AdS_7 x S^4 gravity: [Awad, Johnson]

$$(\epsilon_0)_{\mathrm{index}} = -\frac{N^3}{6} \neq (\epsilon_0)_{\mathrm{gravity}} = -\frac{5N^3}{24}$$

- However, this is a robust illustration of N^3 scaling in the 6d (2,0) theory.

Non-perturbative corrections

- We don't have a derivation, but a simple proposal. (derivation in progress)

- Consistency requirement and issue from 5d/6d viewpoints

1. Instanton expansion: Z_{pert} can be corrected only by power series in

$$e^{-\frac{4\pi^2}{g_{YM}^2}\cdot 2\pi r} = e^{-\frac{4\pi^2}{\beta}} \qquad \text{for } \beta \ll 1$$

2. Does the full S^5 partition function take the form of an index?

$$Z_{\text{pert+inst}} = e^{-\beta\epsilon_0}\sum_E \Omega_E e^{-\beta E} \quad \text{with } \Omega_E : \text{integers for } \beta \gg 1$$

- Having both 1. and 2. satisfied is highly nontrivial.

Warming-up: index for the U(1) 6d (2,0)

- Tensor supermultiplet: $H_{MNP} = \star_6 H_{MNP}, \ \Psi^i_\alpha = \Psi^{R_1 R_2}_{j_1 j_2 j_3}, \ \phi^I$
- BPS fields: $\Psi^{+,+}_{-,+,+}, \ \Psi^{+,+}_{+,-,+}, \ \Psi^{+,+}_{+,+,-}, \ \phi^{(+1,0)}, \ \phi^{(0,+1)}$

  holomorphic derivatives: $\partial_{(1,0,0)}, \ \partial_{(0,1,0)}, \ \partial_{(0,0,1)}$

  $Q^{(\frac{1}{2},\frac{1}{2})}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}$: BPS bound $E = 2R_1 + 2R_2 + j_1 + j_2 + j_3$
- BPS constraint: $(\nabla\!\!\!/ \Psi)^{(+,+)}_{+,+,+} = 0$
- Letter index:

$$f(\beta) \equiv \mathrm{tr}_{\mathrm{letter}}\left[(-1)^F e^{-\beta(E-R_1)}\right] = \frac{(e^{-\beta}+e^{-2\beta}) - 3e^{-2\beta} + e^{-3\beta}}{(1-e^{-\beta})^3} = \frac{e^{-\beta}}{1-e^{-\beta}}$$

- Full index:

$$I(\beta) = e^{-\beta\epsilon_0}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f(n\beta)\right] = e^{\frac{\beta}{24}}\prod_{n=1}^{\infty}\frac{1}{1-e^{-n\beta}} = \eta(e^{-\beta})$$

$$= \left(\frac{\beta}{2\pi}\right)^{\frac{1}{2}} e^{\frac{\pi^2}{6\beta}}\prod_{n=1}^{\infty}\frac{1}{1-e^{-\frac{4\pi^2}{\beta}}}$$

- Completely compatible with 5d structure: pert/non-pert decomposition

Non-perturbative correction for U(N)

- Proposal for general N: $Z = Z_{\rm pert} Z_{\rm inst}\ ,\quad Z_{\rm inst}^{U(N)} = (Z_{\rm inst}^{U(1)})^N = \left[\frac{2\pi}{\beta}\eta(e^{-\beta})\right]^N$
- Tests:

1. 5d instanton expansion: OK
2. Index? Prefactors obstructing it cancel out between two contributions

- Final U(N) index: $$Z = e^{\beta\left(\frac{N(N^2-1)}{6}+\frac{N}{24}\right)} \prod_{n=1}^{N} \frac{1}{(1-e^{-n\beta})^n} \prod_{n=N+1}^{\infty} \frac{1}{(1-e^{-n\beta})^N}$$
- Another test at large N: we get MacMahon function

$$\prod_{n=1}^{\infty} \frac{1}{(1-e^{-n\beta})^n}$$

- This is exactly what one gets from supergravity index on $AdS_7 \times S^4$.

Concluding remarks

- "S^5 partition function = 6d index" : constrain 5d theories w/ 6d fixed pts?
- Generalize with more chemical potentials? 6d (1,0) theories from 5d?

- "Perturbative partition function on S^5 ~ pure CS partition function on S^3"
- Any physical reason? Important roles of Wilson loops in 5d?
- Fundamental Wilson loop ~ N : same as M2 Wilson surfaces in AdS_7

- Systematic derivation of instanton corrections?
- Index Casimir energy & other measures of d.o.f? anomaly?
- Other 6-manifolds with S^1 factor: more 5d approach to 6d (2,0)?
- Special role of 16 SUSY? So far, it just provided technical simplifications.