BPS Conditions of Wrapped Branes

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Supersymmetric branes

- D-branes/M-branes are central objects in String duality.
- Turns out they are very useful tool in the study of QFT duality too.
- One can construct a rich set of interesting QFTs by wrapping branes on SUSY cycle Σ (in special holonomy manifold $\mathcal{M}$).
- The lower-dimnensional QFT depends on topology of Σ.
- Through Gauge/Gravity (or AdS/CFT) correspondence, strongly-coupled QFT on branes is described by 10d/11d Supergravity solutions.

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Plan and Refs

- We will review the general properties of susy wrapped brane geometry, and how they can be used to obtain more explicit conditions in specific setups.

Key References:

- A.Fayyazuddin and D.J.Smith, hep-th/9902210
- J.Maldacena and C.Nunez, hep-th/0007018
- B.Brinne, A.Fayyazuddin, T.Z. Husain, D.J.Smith, hep-th/0012194
- J.P.Gauntlett,O.A.Mac Conamhna, T.Mateos, D.Waldram, hep-th/0605146
- M.T.Anderson, C.Beem, N.Bobev, L.Rastelli, arXiv:1109.3724

Plan

1. Supersymmetry - Killing spinor equation and Special Holonomy
2. Killing spinorial geometry technique
3. AdS solutions from D3/M2
4. BPS conditions of Wrapped Branes

Killing spinors

- Supersymmetric (or BPS) solutions allow nontrivial solutions to Killing spinor equations.
- Killing spinor equations are obtained simply by setting $\delta_\epsilon \psi = 0$ in supersymmetric theories (for us, supergravity)
- For supergravity, Killing spinors ϵ are nontrivial functions of spacetime coordinates and subject to a certain set of Gamma matrix projection rules.
- For purely geometric configurations, special holonomy manifolds (e.g. Calabi-Yau) are supersymmetric.
- Basic branes (D-branes, M2 and M5) are 1/2-BPS solutions.

Special holonomy manifolds

- For pure geometry, Killing spinor eq is simply

$$\nabla_m \epsilon = 0$$

- PDE for ϵ and it's useful to consider integrability conditions

$$[\nabla_m, \nabla_n] = \frac{1}{4} R_{mnpq} \gamma^{pq} \epsilon = 0$$

- Obviously Riemann curvature shouldn't be too general (restricted or special holonomy), and ϵ should satisfy certain projection rules for γ matrix.

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Calabi-Yau

- CY2: A 4d manifold with SU(2) holonomy, instead of SO(4)
- Spinor in 4d is $(2,1) \oplus (1,2)$ of $SO(4) = SU(2)_L \times SU(2)_R$. If one selects one specific spinor, it breaks $SO(4) \to SU(2)$.
- Or, when Riemann curvature takes values in $SU(2)_R \in SO(4)$ integrability condition is satisfied by a chiral spinor.

Berger's classification (1955)

Holonomy	Dim	Type	SUSY
$SO(n)$	n	Orientable manifold	NO
$U(n)$	2n	Kahler manifold	NO
$SU(n)$	2n	Calabi-Yau manifold	$1/2^{n-1}$
$Sp(n) \times Sp(1)$	4n	Quaternion-Kahler manifold	NO
$Sp(n)$	4n	Hyperkahler manifold	$3/16$ for $n = 2$
G_2	7	G_2 manifold	$1/8$
$Spin(7)$	8	$Spin(7)$ manifold	$1/16$

Calibration and Calibrated manifolds

- Special holonomy manifolds are associated with certain closed forms (ϕ) called calibration, and calibrated manifolds with respect to ϕ.
- In string theory, they define supersymmetric cycles
- For instance, for CY-n manifolds we have $(1,1)$ form J (Kahler form), holomorphic n-form Ω.
- A calibrated manifold has minimal volume within their homology class.

$$\int_{\Sigma} \mathrm{vol}_{\Sigma} = \int_{\Sigma} \phi \leq \int_{\Sigma'} \mathrm{vol}_{\Sigma'}$$

Supersymmetric Cycles

For CY manifolds:

- Kahler cycles calibrated by J, $J \wedge J$, ...
- SLAG (Special Lagrangian) cycles calibrated by Ω.

In fact requiring special holonomy is equivalent to requiring existence of calibrations:

For CY for instance,

$$SU(n) \text{ holonomy} \leftrightarrow \nabla_m \epsilon = 0 \leftrightarrow dJ = 0, d\Omega = 0$$

Calibrations from Killing spinor

- Calibrations are constructed as spinor bilinears:
  For CY3,
  $$J_{mn} = \epsilon^\dagger \gamma_{mn} \epsilon, \quad \Omega_{mnp} = \epsilon^\dagger \gamma_{mnp} \epsilon$$
  Then $\nabla \epsilon = 0$ implies J, Ω are covariantly constant.

AdS_3 solutions from D3 branes (NK, 2005)

- General ansatz for D3 wrapped on Kahler 2-cycle:

$$ds^2 = e^{2A}(AdS_3) + ds^2(7d)$$

$$F^{(5)} = (1 + *)vol(AdS_3) \wedge F$$

- Then the 10d Killing spinor equation is reduced to 7d one

$$\nabla_a \eta - \frac{e^{-3A}}{4} \not{F} \gamma_a \eta = 0$$

$$\left(\not{\partial} A + \frac{e^{-3A}}{2} \not{F} - ie^{-A} \right) \eta = 0$$

- Then we can study the algebraic and differential properties of spinor bilinears like, $C = \eta^\dagger\eta, K_a = \eta^\dagger\gamma_a\eta$ etc.
- Using KSE, for instance

$$\nabla_a(\eta^\dagger\eta) = (\nabla_a\eta^\dagger)\eta + \eta^\dagger(\nabla_a\eta) = -e^{-3A}F_{ab}\eta^\dagger\gamma^b\eta = \partial_a A\eta^\dagger\eta$$

$$\rightarrow \eta^\dagger\eta = e^A$$

- This can be repeated for higher-rank tensors.

- The result is summarised as

$$ds_7^2 = e^{2A}(d\psi + B)^2 + e^{-2A} ds^2_{Kahler}$$

$$F^{(5)} = (1 + *) vol(AdS_3) \wedge \left(\frac{1}{2} J - \frac{1}{4} d(e^{4A}(d\psi + B) \right)$$

- With $dB = 2\mathcal{R}$ (Ricci 2-form) , $R = 8e^{-4A}$ and

$$\Box R - \frac{1}{2} R^2 + R_{ij} R^{ij} = 0$$

AdS_2 from M2 (Kim, Park 2006)

11d metric

$$ds^2 = e^{2A}(AdS_2) + ds^2(9d)$$

$$F^{(4)} = Vol(AdS_2) \wedge F$$

9d Killing spinor equation

$$ie^{-A}\eta + \partial\!\!\!/ A\eta - \frac{1}{6}e^{-2A}F\!\!\!/\eta = 0$$

$$\nabla_a\eta + \frac{1}{24}e^{-2A}\left(\gamma_a{}^{bc}F_{bc} - 4F_{ab}\gamma^b\right)\eta = 0$$

And you repeat the same procedure ...

The result of Killing spinor analysis is

$$ds^2(9d) = e^{2A}(d\psi + B)^2 + e^{-A} ds_8^2(Kahler)$$

We also have $R = 2e^{-3A}$ and $dB = \mathcal{R}$, and

$$\Box R - \frac{1}{2}R^2 + R_{ij}R^{ij} = 0$$

Generalization to higher dimensions (JPG, NK 2007)

- Encouraged by the appearance of the same equation, we tried to generalize to arbitrary higher dimensions $d = 7, 9, \cdots$.
- Action

$$L_{2n+1} = e^{(1-n)B}\left[R + \frac{n(2n-3)}{2}(\nabla B)^2 + \frac{1}{4}e^{2B}F^2 - \frac{2n}{(n-2)^2}\right]$$

- With Killing spinor equations

$$\left[\nabla\!\!\!/ B + i\frac{2(n-1)}{n-2} + \frac{1}{2}e^{B}F\!\!\!/\right]\eta = 0$$

$$\left[\nabla_c B + \frac{i}{2}\Gamma_c + \frac{1}{8}e^{B}F_{ab}\gamma_c{}^{ab}\right]\eta = 0$$

Then for any $n \geq 3$, the existence of Killing spinor and imposition of Bianchi identity and gauge field eom ($dF = 0, d(e^{(3-n)B} * F = 0)$), the Killing spinor analysis leads to the same equation

$$\boxed{\Box R - \frac{1}{2}R^2 + R_{ij}R^{ij} = 0}$$

- Checked many known solutions satisfy this equation.
- 1/2-BPS solutions of Lin, Lunin, Maldacena are special cases too.
- Some new solutions using simple ansatz.

Maldacena-Nunez solution of Wrapped Branes (2000)

- MN constructed D3 and M5 solutions wrapped on Kahler 2-cycles (H_2) using gauged sugra.
- In 7d gauged supergravity, one identifies $SO(2) \in SO(5)$ gauge connection with spin connection of H^2.

$$ds^2 = e^{2f} \left[ds^2(R^{3,1}) + dr^2 \right] + e^{2g}(dx^2 + dy^2)/y^2$$

$$A^{12} = dx/y, \quad \text{other } A^{IJ} = 0$$

One calar field λ

- Killing spinor equation gives coupled nonlinear ODE for $f(r), g(r), \lambda(r)$.
- AdS fixed point for round H_2 as above. ($e^f \sim 1/r, g = const$)

MN in 11d

- MN prescription was generalized to other supersymmetric cycles (3, 4, and 5-cycles in various special holonomy manifolds)
- It turns out some of D3 and M2 brane wrapped brane solutions can be decribed in a uniform way, using our $\Box R ... = 0$.
- More specifically, we consider D3 wrapping 2-cycle in CY3, and M2 wrapping 2-cycle in CY4.

AdS from Wrapped branes

Metric ansatz

$$ds^2_{2n+4} = \Delta_1 ds^2(H_2) + \Delta_2 d\theta^2 + \frac{sin^2 2\theta}{\Delta_2} D\psi^2 + \cos^2\theta ds^2(KE^+_{2n})$$

Obvious choice for Kahler form is

$$J = \Delta_1 J_2 + \sin 2\theta d\theta \wedge D\psi + \cos^2\theta J_{2n}$$

Check the Kahler condition $dJ = 0$ and find

$$d(D\psi) = J_2 - J_{2n}, \quad \Delta_1 = c + \sin^2\theta$$

Wrapped brane AdS solutions generalized (NK 1206.1535)

It turns out that Ricci is complicated in terms of Δ_2.

$$\mathcal{R} = -d\left\{\left[1 + \frac{1}{\cos^n\theta}\sqrt{\Delta_1\Delta_2}\frac{d}{d\theta}\left(\sqrt{\frac{\Delta_1}{\Delta_2}}\sin 2\theta\cos^n\theta\right)\right]D\psi\right\}$$

But using Gauged Sugra solution as a hint, we find solutions for all n

$$\Delta_2 = 2n\Delta_1 = 2(n\sin^2\theta + 1), \quad R = 4(1+n)/(1+n\sin^2\theta)$$

BPS condition and FS eq

One can find BPS conditions for branes wrapping a particular SUSY cycle:

- Remaining Poincare symmetry + Killing spinors with projection rules
- The result is summarized in the backreacted G-structure manifold
- M5-brane wrapping Kahler 2-cycle in CY2:

$$ds^2 = L^{-1} ds^2(R^{3,1}) + ds^2(M(SU(2)) + L^2(d\vec{y}_3^2)$$

$$d(L^{-1/2}(J_2 + iJ_3)) = 0$$

$$vol_3(Y) \wedge d(LJ_1) = 0$$

$$*_7 F = L^2 d(L^{-2} J_1)$$

- Fayyazuddin-Smith equation (1999), and extended and generalized by Brinne, Fayazzudin, Husain, Smith (2000) and in Gauntlett, Mac Conamhna, Mateos, Waldram (2006)

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Illustration: Flat M5-brane

Supergravity solution for M5-branes:

$$ds^2 = f^{-1/3} dx_{||}^2 (R^{5,1}) + f^{2/3} (dr^2 + r^2 d\Omega_4^2)$$

$$f = 1 + \frac{\pi N l_P^3}{r^3}$$

M5-brane sugra solution from FS-eq

- From BPS condition of wrapped branes, we set

$$ds^2_{11} = e^{2A} ds^2(R^{5,1}) + e^{2B} d\vec{y}_5^2$$

$$\to L^{-1} ds^2(R^{3,1}) + L^{-1}(dx_4^2 + dx_5^2) + L^2(dy_4^2 + dy_5^2) + L^2 d\vec{y}_3^2$$

with $B = -2A = \ln L$ and $L = L(x_4, x_5, y_i)$

$$J_1 = L^{-1} dx_4 \wedge dx_5 + L^2 dy_4 \wedge dy_5$$

$$J_2 + iJ_3 = L^{1/2}(dx_4 + idx_5) \wedge (dy_4 + idy_5)$$

- $d(L^{-1/2}(J_2 + iJ_3)) = 0$ is trivial, $vol(Y) \wedge d(LJ) = 0$ says $\partial_{x_4} L = \partial_{x_5} L = 0$
- $*_7 F = L^2 d(L^{-2} J_1) \Rightarrow *F = \partial_{y_i} L^{-3} vol(R^{5,1}) \wedge dy_i, \quad d * F = 0$ is trivial.
- $F = L^6 \partial_{y_i} L^{-3} \epsilon_{ijklm} dy_j \wedge dy_k \wedge dy_l \wedge dy_m \Rightarrow$ Bianchi requires L^3 be harmonic in 5d

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- From BPS condition of wrapped branes, we set

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MN from BPS conditions

- In 2000, Maldacena and Nunez constructed D3 and M5 solutions wrapped on Kahler 2-cycles (H_2) using gauged sugra.
- In 7d gauged supergravity, one identifies $SO(2) \in SO(5)$ gauge connection with spin connection of H^2.

$$ds^2 = e^{2f}\left[ds^2(R^{3,1}) + dr^2\right] + e^{2g}(dx^2 + dy^2)/y^2$$

$$A^{12} = dx/y, \quad \text{other } A^{IJ} = 0$$

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- Killing spinor equation gives coupled nonlinear ODE for $f(r), g(r), \lambda(r)$.
- AdS fixed point for round H_2 as above. ($e^f \sim 1/r, g = const$)

MN in 11d

- One can uplift the 7d solution to 11d, using the uplifting formula (Cvetic et. al., hep-th/9903214)

$$
\begin{aligned}
ds_{11}^2 &= \Delta^{1/3} ds_7^2 + \tfrac{1}{4}\Delta^{-2/3} ds_4^2 \\
&= \Delta^{1/3} e^{2f} ds^2(R^{3,1}) + \Delta^{1/3} e^{2g} ds^2(H_2) + \Delta^{1/3}(dr^2 + \tfrac{e^{\lambda}}{4} d\theta^2) \\
&\quad + \frac{\Delta^{2/3}}{4} e^{3\lambda}\cos^2\theta d\Omega_2 + \frac{\Delta^{-2/3}}{4} e^{3\lambda}\cos^2\theta (d\phi + \frac{dx}{y})^2
\end{aligned}
$$

- One can identify Fayyazuddin-Smith SU(2)-structure here, i.e. $L^{-1} = \Delta^{1/3} e^{2f}$ etc.
- But we'd like to study more general conditions for M5 wrapping H_2.

MN from FS-eq

- Ansatz for SU(2) structure space:

$$ds_4^2 = \frac{u}{L} ds^2(H_2) + \frac{L^2}{u}(d\rho^2 + \rho^2 D\psi^2), \quad L = \left(\frac{uv}{\rho}\right)^{1/3}$$

$$J = \frac{u}{L} vol(H_2) + \frac{L^2}{u} d\rho \wedge \rho D\phi$$

$$J_2 + iJ_3 = e^{in\phi} L^{1/2} \Omega_{H_2} \wedge (d\rho + i\rho D\phi)$$

- BPS condition $d(L^{-1/2}(J_2 + iJ_3)) = 0$ requires $n = 1, \;\; d(D\phi) = -vol(H_2) = -d\Omega_{H_2}$

BPS condition for M5 on H_2

- Then $dy \wedge d(LJ_1) = 0$ requires

$$v + \partial_\rho u = 0$$

- Finally $dF = 0$ requires

$$\nabla_y^2 u - \rho\partial_\rho \left(\frac{uv}{\rho}\right) = 0$$

  Reduces to a single non-linear PDE [Mac Conamhna 0706.1795]

$$\nabla_y^2 u + \frac{1}{2}\rho\partial_\rho \left(\frac{1}{\rho}\partial_\rho u^2\right) = 0$$

- Cf. AdS solution : $\rho^2 u + 2\vec{y}^2 u^2 = Const$

Uniformization from D=11 supergravity

- We'd like to consider more general set of BPS conditions where the 2-cycle is NOT round H_2 or S_2.
- Allow (x, y) dependence for u, v etc.
- Turns out we need to twist $d\rho$ as well,

$$d\rho \to D\rho = d\rho + B_x dx + B_y dy$$

$$D\phi = d\phi + (A_x - 1/y)dx + A_y dy$$

- $d(L^{-1/2}\Omega) = 0$ requires $B_x = \rho A_y, B_y = -\rho A_x$.

General BPS condition

From BPS conditions:

$$\begin{aligned} \partial_x v - \partial_\rho(\rho v A_y) &= 0 \\ \partial_y v + \partial_\rho(\rho v A_x) &= 0 \\ \partial_x(vA_x) + \partial_y(vA_y) &= 0 \\ v + \partial_\rho u + \frac{1}{2} v\rho\partial_\rho \vec{A}^2 - vF_{xy} &= 0 \end{aligned}$$

From Bianchi:

$$\begin{aligned} \nabla^2(vA_x) - \partial_y\partial_\rho\left(\frac{uv}{\rho}\right) &= 0 \\ \nabla^2(vA_y) + \partial_x\partial_\rho\left(\frac{uv}{\rho}\right) &= 0 \\ \nabla^2 u + (\Delta - \rho\partial_\rho)\left(\frac{uv}{\rho}\right) + \rho^2\nabla^2(v\vec{A}^2) &= 0 \end{aligned}$$

Motivation and things to do

- In Gaiotto's construction of $N = 2$ theories, it is implicitly assumed that the superconformal theory of $N = 2, D = 4$ SCFT from wrapped M5 brane only depends on the topology of 2-cycle (number of holes and punctures) and not the explicit metric.
- In supergravity, it should mean that there cannot be AdS solution with (x, y) dependence.
- Anderson, Beem, Bobev, Rastelli (2011) considered such BPS conditions in gauged sugra, and proved non-existence of new AdS points.

$$(\partial_x^2 + \partial_y^2)\Phi + \partial_r^2 e^{\Phi} = m^2 e^{\Phi}$$

- Prove no-go result in 10d/11d.
- Generalize to 3-cycles, 4-cycles etc.

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