A Proposal for the Worldvolume Action of Multiple M5-Branes

Chong-Sun Chu

National Tsing-Hua University, Taiwan & Durham University, UK

KIAS Autumn Symposium
19/9/2012

based on

- A Theory of Non-Abelian Tensor Gauge Field with Non-Abelian Gauge Symmetry G x G, Chong-Sun Chu, arXiv:1108.5131.
- Non-abelian Action for Multiple Five-Branes with Self-Dual Tensors, Chong-Sun Chu, Sheng-Lan Ko, arXiv:1203.4224.
- Non-Abelian Self-Dual String Solutions, Chong-Sun Chu, Sheng-Lan Ko, Pichet Vanichchapongjaroen, arXiv:1207.1095.

Outline

Outline

Mysteries of M5-branes

- The low energy worldvolume dynamics is expected to be given by a 6d (2,0) SCFT with $SO(5)$ R-symmetry.

  (Strominger, Witten)

  The (2,0) tensor multiples contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions.

  (Gibbons, Townsend; Strominger; Kaplan, Michelson)

However, other than the global symmetries and the spectrum, it is mysterious!

- Gauge symmetry for multiple M5-branes ?
- Interacting self-dual dynamics on M5-branes worldvolume?

Also,

- Quantized geometry for M5-brane in a large constant 3-form C-field?
  Bergshoeff, Berman, Van der Shaar, Sundell; Kawamoto, Sasakura; some recent progress in Chu, Douglas; Chu, Sembi
- Entropy counting N^3 for N number of M5-branes?

  (Klebnov, Tseytlin).

  recent progress in Bolognesi, Lee; Maxfield, Sethi; ...

Self-dual dynamics for multiple M5-branes (?)

- Generally, it is well known to be difficult to write down a Lorentz invariant action for self-dual dynamics.

  (Siegel 84; Floreanini, Jackiw 87)

- For a single M5 case, problem solved by Perry-Schwarz (also Henneaux-Teitelboim) by sacrificing manifest 6d Lorentz symmetry.

  (Perry-Schwarz 97; Henneaux-Teitelboim 88)

  The action was later generalized to include kappa symmetry

  (Aganagic, Park, Popescu, Schwarz, 97)

  Covariant construction given later by PST

  (Pasti-Sorokin-Tonin)

- Not clear how to do this for $N > 1$ due to the other problem that an appropriate generalization of the tensor gauge symmetry was not known.

Enhanced gauge symmetry of multiple M5-branes (?)

- For multiple D-branes, symmetry is enhanced from $U(1)$ to $U(N)$:

$$\delta A^a_\mu = \partial_\mu \Lambda^a + [A_\mu, \Lambda]^a, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + [A_\mu, A_\nu]^a.$$

- For multiple M5-branes, it is not known how to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B^a_{\mu\nu} = \partial_\mu \Lambda^a_\nu - \partial_\nu \Lambda^a_\mu + (?), \quad H^a_{\mu\nu\lambda} = \partial_\mu B^a_{\nu\lambda} + \partial_\nu B^a_{\lambda\mu} + \partial_\lambda B^a_{\mu\nu} + (?).$$

  to have nontrivial self interaction.

- Moreover, exists no-go theorems: there is no nontrivial deformation of the Abelian 2-form gauge theory if locality of the action and the transformation laws are assumed.

  (Henneaux; Bekaert; Sevrin; Nepomechie)

- These no-go theorems, however, suggest an important direction of given up locality.

Outline

Perry-Schwarz formulation

- In the Perry-Schwarz formulation:
  1. x_5 is singled out. i.e. Manifest Lorentz symmetry lost.
  2. The self-dual tensor gauge field is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. i.e. $B_{\mu 5}$ never appear.
- Denote the 5d and 6d coordinates by x^μ and $x^M = (x^\mu, x^5)$. $\eta^{MN} = (-+++++), \epsilon^{01234} = -\epsilon_{01234} = 1, \quad \epsilon^{012345} = -\epsilon_{012345} = 1$.
  The self-duality equation reads

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu} (= H_{5\mu\nu})$$

  where

$$\tilde{H}^{\mu\nu} := \frac{1}{6}\epsilon^{\mu\nu\rho\lambda\sigma} H_{\rho\lambda\sigma}, \qquad H^{\mu\nu\rho} = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda\sigma}\tilde{H}_{\lambda\sigma}.$$

- This can be obtained with the Perry-Schwarz action:

$$S_0(B) = \frac{1}{2}\int d^6x \left(-\tilde{H}^{\mu\nu}\tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu}\partial_5 B_{\mu\nu}\right)$$

- EOM:

$$\epsilon^{\mu\nu\rho\lambda\sigma}\partial_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu, \quad \text{for arbitrary } \alpha_\mu.$$

- The action is invariant under the gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \quad \text{for arbitrary } \varphi_\mu.$$

This allows one to reduce the general solution to the EOM to the first order form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

This is the self-duality equation in this theory.

Modified Lorentz symmetry

- The action has manifest 5d Lorentz invariance and a non-manifest Lorentz symmetry mixing μ with the 5 direction.

Lorentz transformation (active view)

Standard Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot L) B_{\mu\nu} + \delta_{\rm spin} B_{\mu\nu}$$

has a orbital part:

$$\Lambda \cdot L = (\Lambda \cdot x)\partial_5 - x_5(\Lambda \cdot \partial)$$

and a spin part:

$$\delta_{\rm spin} B_{\mu\nu} = \Lambda_\nu B_{\mu 5} - \Lambda_\mu B_{\nu 5}.$$

- PS proposed the modified Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5(\Lambda \cdot \partial) B_{\mu\nu},$$

where $\Lambda_\mu = \Lambda_{5\mu}$ denote the corresponding infinitesimal transformation parameters.

1. On shell, it is equal to the standard Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu} = (\Lambda \cdot x)\partial_5 B_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu}$$

2. Commutator:

$$[\delta_{\Lambda_1}, \delta_{\Lambda_2}]B_{\mu\nu} = \delta^{(5d)}_{\Lambda_{\alpha\beta}} B_{\mu\nu} + \text{EOM} + \text{gauge symmetry}$$

   where

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \qquad \varphi_\nu = x^\alpha \Lambda_{\alpha\lambda} B_\nu{}^\lambda$$

   is a gauge symmetry of the PS theory.
   That it is possible to support 6d Lorentz symmetry without introducing the components $B_{\mu 5}$ is entirely due to the existence of the gauge symmetry in the theory.

3. Modified Lorentz symmetry is typical of action of self-dual dynamics (Siegel 84)

Non-abelian action

Ideas: try to generalize Perry-Schwarz approach

1. Give up manifest 6d Lorentz symmetry
2. Represent the self-dual tensor gauge field by a 5×5 antisymmetric field $B^a_{\mu\nu}$ in the adjoint.
3. Introduce a set of 1-form gauge fields for a gauge group G as suggested by $G \times G$ construction of tensor gauge symmetry
   - Multiple M2-branes (ABJM) was used to probe the M5-branes system. The gauge non-invariance of the boundary Chern-Simons action was shown to imply the existence of a Kac-Moody current algebra on the worldsheet of multiple self-dual strings
   - This was argued to induces a $U(N) \times U(N)$ gauge symmetry in the theory of N coincident M5-branes. (Chu, Smith; Chu)

Still to consider:

- supersymmetry
- PST like covariantization

The Action

- Our proposed action $S = S_0 + S_E$ consists of two pieces:

$$S_0 = \frac{1}{2}\int d^6x \,\mathrm{tr}\left(-\tilde{H}^{\mu\nu}\tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu}\partial_5 B_{\mu\nu}\right),$$

  is the non-abelian generalization of the Perry-Schwarz action, where

$$H_{\mu\nu\lambda} = D_{[\mu}B_{\nu\lambda]}, \quad D_\mu = \partial_\mu + A_\mu.$$

  No $B_{\mu 5}$ and A_5. A_μ lives in 5-dimensions

- $$S_E = \int d^5x \,\mathrm{tr}\left((F_{\mu\nu} - c\int dx_5 \,\tilde{H}_{\mu\nu})E^{\mu\nu}\right).$$

  where $E_{\mu\nu}$ is a 5d auxiliary field implementing the constraint

$$F_{\mu\nu} = c\int dx_5 \,\tilde{H}_{\mu\nu}$$

The action is invariant under:

1. Yang-Mills gauge symmetry

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \delta B_{\mu\nu} = [B_{\mu\nu}, \Lambda], \quad \delta E_{\mu\nu} = [E_{\mu\nu}, \Lambda].$$

2. Tensor gauge symmetry:

$$\delta_T A_\mu = 0, \quad \delta_T B_{\mu\nu} = D_{[\mu}\Lambda_{\nu]}, \quad \delta_T E_{\mu\nu} = 0,$$

for arbitrary $\Lambda_\mu(x^M)$ such that $[F_{[\mu\nu}, \Lambda_{\lambda]}] = 0$.

3. Moreover there is a gauge symmetry

$$\delta E_{\mu\nu} = \alpha_{\mu\nu}$$

for arbitrary $\alpha(x^\lambda)$ such that $D_{[\mu}\alpha_{\nu\lambda]} = 0, \quad D^\mu \alpha_{\mu\lambda} = 0.$

Property 1: Self-Duality

- EOM of $E_{\mu\nu}$ give

$$F_{\mu\nu} = c \int dx_5 \; \tilde{H}_{\mu\nu}.$$

- EOM of $B_{\mu\nu}$

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

  has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \Phi_{\lambda\sigma},$$

  where $D_{[\lambda}\Phi_{\mu\nu]} = 0$.

- Again with an appropriate fixing of the tensor gauge symmetry, one can always reduce the second order EOM to the self-duality equation

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

Property 2: Degrees of freedom

- After eliminating the aux field $E_{\mu\nu}$ and substituting the $F = H$ constraint, the resulting action is highly nonlinear. To count the degrees of freedom, we use the linearized theory.
  - At the quadratic level, the non-abelian action is simply given by $\dim G$ copies of the Perry-Schwarz action, plus the action S_E. We obtain $3 \times \dim G$ degrees of freedom in $B_{\mu\nu}$.
- Our theory contains $3 \times \dim G$ degrees of freedom as required by (2,0) supersymmetry

Property 3: Lorentz Symmetry

- The action is invariant under the 5-μ Lorentz transformation:

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} + x_5 \Lambda^\kappa H_{\kappa\mu\nu} + \Lambda^\kappa \phi_{\mu\nu\kappa},$$

  where

$$\phi^a_{\mu\nu\kappa} = \int dy\ G^{ab\mu'\nu'}_{\mu\nu}(x,y) J^b_{\mu'\nu'\kappa}(y)$$

  $J_{\mu\nu\kappa}$ is some expression linear in H
  and $G^{ab}_{\mu\nu,\mu'\nu'}(x,y)$ is the Green function satisfying

$$\partial_5 G^{ab\mu'\nu'}_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta\gamma}(D^{(y)}_\alpha)^a{}_c G^{cb\mu'\nu'}_{\beta\gamma} = \delta^{\mu'\nu'}_{\mu\nu}\delta^{ab}\delta^{(6)}(x-y)$$

  with the BC: $G^{ab\mu'\nu'}_{\mu\nu}(x,y) = 0, \quad |x_5| \to \infty.$
- As before, the commutator closes to the standard 5d Lorentz transformation plus terms vanishing onshell, plus a gauge transformation.

- Note that our proposed action and Lorentz symmetry are nonlocal.

  We are working in a formulation without $B_{\mu 5}$., it is possible that these nonlocalities are due to the fact that we are working in a gauge fixed version of a covariant formulation (exactly like QED in Coulomb gauge or string in lightcone gauge).

Property 4: Reduction to D4-branes

- Consider a compactification of x_5 on a circle of radius R. The dimensional reduced action reads

$$S = \frac{2\pi R}{2} \int d^5x \, \mathrm{tr} \left(-\tilde{H}_{\mu\nu}^2 + (F_{\mu\nu} - 2\pi Rc\tilde{H}_{\mu\nu}) E^{\mu\nu} \right)$$

- Integrate out $E_{\mu\nu}$, we obtain

$$F_{\mu\nu} = 2\pi Rc\tilde{H}_{\mu\nu}.$$

  and eliminate $\tilde{H}_{\mu\nu}$, we obtain the 5d Yang-Mills action

$$S_{YM} = -\frac{1}{4\pi Rc^2} \int d^5x \; \mathrm{tr} \; F_{\mu\nu}^2.$$

- The action S_{YM} corresponds to the expected form of the YM coupling

$$g_{YM}^2 = Rc^2$$

  and the gauge group in our construction is to be

$$G = U(N)$$

  for a system of N M5-branes.

- But EOM gives $D^\mu F_{\mu\nu} = 0$ instead of

$$D_\mu F^{\mu\nu} = -\frac{\pi R}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]?$$

- Need to be more careful with the implementation of Delta function:

$$\int [DA][DB][DE]e^{-S} = \int [DA][DB]e^{-S_{YM}}\delta(F_{\mu\nu} - 2\pi R\tilde{H}_{\mu\nu}) = \int [DA]e^{-S_{YM}-S'},$$

where consistency requires that

$$\frac{\delta S'}{\delta A_\nu} = \frac{1}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]$$

- The 5d theory is thus given by the action $S_{5d} = S_{YM} + S'$.
  S' describes a high derivative correction term to the Yang-Mills theory since $[F, B] \sim DDB$ and B is of the order of F.
  Q: It might be possible that S' captures the non-abelian DBI action of D4-branes?

Outline

Self-dual string on M5-brane

- M2-branes can end on M5-brane. The endpoint gives strings living on the M5-brane.
- These self-dual strings appear as soliton of the M5-brane theory.
- In the Abelian case, the self-dual string solition has been obtained by Perry-Schwarz and also by Howe-Lambert-West.

Self-Dual String Soliton in the Perry-Schwarz Theory

- The Perry-Schwarz non-linear field equation is given by

$$\tilde{H}_{\mu\nu} = \frac{(1-y_1)H_{\mu\nu 5} + H_{\mu\rho 5}H^{\rho\sigma 5}H_{\sigma\nu 5}}{\sqrt{1 - y_1 + \frac{1}{2}y_1^2 - y_2}},$$

where

$$y_1 := -\frac{1}{2}H_{\mu\nu 5}H^{\mu\nu 5}, \qquad y_2 := \frac{1}{4}H_{\mu\nu 5}H^{\nu\rho 5}H_{\rho\sigma 5}H^{\sigma\mu 5}.$$

- A solution aligning in the x^5 direction is solved by the ansatz:

$$B = \alpha(\rho)dtdx^5 + \frac{\beta}{8}(\pm 1 - \cos\tilde{\theta})d\tilde{\phi}d\tilde{\psi},$$

where the 6d metric is

$$ds^2 = -dt^2 + (dx^5)^2 + d\rho^2 + \rho^2 d\Omega_3^2,$$

with the three-sphere given in Euler coordinates

$$d\Omega_3^2 = \frac{1}{4}[(d\tilde{\psi} + \cos\tilde{\theta}d\tilde{\phi})^2 + (d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\phi}^2)],$$

- Note that $B_{\tilde{\phi}\tilde{\psi}} = \beta(\pm 1 - \cos\tilde{\theta})$ is the potential of a Dirac monopole!

- For this ansatz, $y_1 = \alpha'^{\,2}$, $y_2 = \alpha'^{\,4}/2$ and the EOM reads

$$\alpha'(\rho) = \frac{\beta}{\sqrt{\beta^2 + \rho^6}}.$$

- The solution is regular everywhere:

$$\begin{aligned} \alpha \sim \rho \quad &\text{as} \quad \rho \to 0, \\ \alpha \sim -\frac{\beta}{2\rho^2} + \mathrm{const.} \quad &\text{as} \quad \rho \to \infty. \end{aligned}$$

- The magnetic charge P and electric charge Q (per unit length) of the string:

$$P = \int_{S^3} H, \qquad Q = \int_{S^3} *H,$$

gives

$$P = Q = 2\pi^2 \beta$$

hence the string is self-dual.

- charge quantization condition for self-dual string in 6d

$$PQ' + QP' \in 2\pi \mathbf{Z}$$

(Deser, Gomberoff, Henneaux, Teitelboim)

for the self-dual string gives

$$\beta = \pm\sqrt{\frac{n}{4\pi^3}},$$

i.e.

$$P = Q = \pm\sqrt{n\pi}.$$

- For the non-abelian theory, the equations of motion to be solved are:

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

$$F_{\mu\nu} = c \int dx_5 \, \tilde{H}_{\mu\nu}$$

Need to do:

1. supersymmetrize: Perry-Schwarz self-dual string solution is non-BPS as there is no other matter field turned on to cancel the tensor field force.
2. construct a string aligns in a different direction, say x^4.
   The gauge field is auxiliary:

$$F_{\mu\nu} = c \int dx_5 \, \tilde{H}_{\mu\nu} = c(B_{\mu\nu}(x_5 = \infty) - B_{\mu\nu}(x_5 = -\infty))$$

   Therefore if the non-Abelian solution is translationally invariant along x^5, then $F_{\mu\nu} = 0$ is trivial.

Self-dual string in x^4 direction

- To get a non-trivial solution, try to construct a string in x^4 diection.
- Let us consider a rotation $(x_4, x_5) \to (-x_5, x_4)$, the PS field strength reads

$$H = \frac{\alpha'}{\rho} dtdw\left(xdx + ydy + zdz + x^5 dx^5\right) + \frac{\beta}{\rho^4}\left(x^5 dxdydz - xdydzdx^5 + zdydxdx^5 - ydzdxdx^5\right),$$

This solves the self-duality equation.

- To get B in the PS form, can integrate directly the self-duality equation

$$H_{\mu\nu 5} = \partial_5 B_{\mu\nu}$$

and get

$$B_{ij} = -\frac{1}{2}\frac{\beta\epsilon_{ijk}x_k}{r^3}\left(\frac{x^5 r}{\rho^2} + \tan^{-1}(x^5/r)\right), \qquad B_{tw} = -\frac{\beta}{2\rho^2},$$

$i, j = 1, 2, 3$, $w = x^4$.

- Although the auxillary field does not appear in the PS construction, it is amazing that

$$F_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijk}x_k}{r^3}, \qquad F_{tw} = 0$$

i.e. a Dirac monopole in the (x, y, z) subspace if $c\beta = -2/\pi$!

- The presence of a Dirac monopole was already apparent in the original solution of PS. Here, we reveal that the monopole configuration also appears (more directly) in the auxiliary gauge field.

- It turns out the use of an non-abelian monopole in place of the Dirac monopole is precisely what is needed to construct the non-abelian self-dual string solution.
- Here we have two candidates of non-abelian monopole: the Wu-Yang monopole and the 't Hooft-Polyakov monopole.

Non-abelian Wu-Yang and 't Hooft-Polyakov monopole

Wu-Yang

- Consider $SU(2)$ gauge group

$$[T^a, T^b] = i\epsilon^{abc} T^c, \quad a, b, c = 1, 2, 3.$$

- The non-abelian Wu-Yang monopole is given by

$$A_i^a = -\epsilon_{aik} \frac{x_k}{r^2}, \qquad F_{ij}^a = \epsilon_{ijm} \frac{x_m x_a}{r^4}, \qquad i, j = 1, 2, 3.$$

- Note that the field strength for the Wu-Yang solution is related to the field strength of the Dirac monopole by a simple relation:

$$F_{ij}^a = F_{ij}^{(\mathrm{Dirac})} \frac{x^a}{r}.$$

- Note that the color magnetic charge vanishes

$$\int_{S^2} F^a = 0$$

and the Wu-Yang solution is actually not a monopole. Neverthless it plays a key role in the construction of non-abelian monopole of 't Hooft-Polyakov.

't Hooft-Polyakov

- In the BPS limit, the 't Hooft-Polyakov monopole satisfies

$$\frac{1}{2}\epsilon_{ijk}F_{ij} = D_k\phi, \quad D_k^2\phi = 0$$

where ϕ is an adjoint Higgs scalar field.

- The solution is given by

$$A_i^a = -\epsilon_{aik}\frac{x^k}{r^2}(1-k_v(r)), \qquad \phi^a = \frac{vx^a}{r}h_v(r),$$

where

$$k_v(r) := \frac{vr}{\sinh(vr)}, \qquad h_v(r) := \coth(vr) - \frac{1}{vr}.$$

- Asymptotically $r \to \infty$, we have

$$A_i^a \to -\epsilon_{aik}\frac{x^k}{r^2}, \qquad \phi^a \to \frac{|v|x^a}{r} := \phi_\infty,$$

i.e. precisely the Wu-Yang monopole at infinity.

- Unbroken $U(1)$ gauge symmetry at infinity may be identified as the electromagnetic gauge group and the electromagnetic field strength can be obtained as a projection:

$$\mathcal{F}_{ij} = F^a_{ij} \frac{\phi^a}{|v|} = \epsilon_{ijk} \frac{x^k}{r^3}, \qquad \text{for large } r.$$

The magnetic charge is given by $p = \int_{S^2} \mathcal{F} = 4\pi$, which corresponds to a magnetic monopole of unit charge.

- Note that at the core $r \to 0$, we have

$$A_i \to 0, \qquad \phi \to 0$$

and hence the $SU(2)$ symmetry is restored at the monopole core.

Non-abelian self-dual string solution

- Inspired by the relation of Dirac monopole to the Wu-Yang solution, try the ansatz

$$H^a_{\mu\nu\lambda} = H^{(\mathrm{PS})}_{\mu\nu\lambda}\frac{x^a}{r}$$

Here $r = \sqrt{x^2 + y^2 + z^2}$ and $H^{(\mathrm{PS})}_{\mu\nu\lambda}$ is the field strength for the linearized Perry-Schwarz solution aligning in the x^4 direction. Self-duality is automatically satisfied!

- B can be obtained by integrating $H_{\mu\nu 5} = \partial_5 B_{\mu\nu}$ and we obtain

$$B^a_{\mu\nu} = B^{(\mathrm{PS})}_{\mu\nu}\frac{x^a}{r},$$

- It is amusing that the auxillary field configuration is given by

$$F^a_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijm}x_m x_a}{r^4}, \qquad F^a_{tw} = 0.$$

This is the Wu-Yang monopole if we take $c\beta = -\frac{2}{\pi}$.

- Like the Wu-Yang monopole, the color magnetic charge of our Wu-Yang string solution vanishes.
- Not a problem as we should include scalar fields (which are natural from the point of view of (2,0) supersymmetry).
- Although we do not have the full (2,0) supersymmetric theory, one can argue (a simple dimensional analysis) that the self-duality equation of motion is not modified by the presence of the scalar fields.
- As for the scalar field, the self-interacting potential vanishes if there is only one scalar field turned on (R-symmetry). As a result, the equation of motion of the scalar field is

$$D_M^2 \phi = 0.$$

- A reasonable form of the BPS equation is the non-abelian generalization of the BPS equation of Howe-Lambert-West:

$$H_{ijk} = \epsilon_{ijk}\partial_5\phi, \qquad H_{ij5} = -\epsilon_{ijk}D_k\phi.$$

- This follows immediately from the supersymmetry transformation

$$\delta\psi = (\Gamma^M\Gamma^I D_M\phi^I + \frac{1}{3!2}\Gamma^{MNP}H_{MNP})\epsilon$$

and the 1/2 BPS condition

$$\Gamma^{046}\epsilon = -\epsilon.$$

Note: this is the most natural non-abelian generalization of the abelian (2,0) supersymmetry transformation.

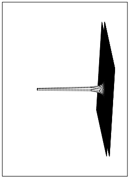
- The BPS equation can be solved with $\phi = B_{tw}$:

$$\phi^a = -(u + \frac{\beta}{2\rho^2})\frac{x^a}{r},$$

- The transverse distance $|\phi|$ defined by $|\phi|^2 = \phi^a\phi^a$ gives

$$|\phi| = |u + \frac{\beta}{2\rho^2}|.$$

- This describes a system of M5-branes with a spike at $\rho = 0$ and level off to u as $\rho \to \infty$. Hence the physical interpretation of our self-dual string is that two M5-branes are separating by a distance u and with an M2-brane ending on them.



- Asymptotic $U(1)$ B-field is $\mathcal{B}_{\mu\nu} \equiv \hat{\phi}^a_\infty B^a_{\mu\nu} = \pm B^{(\mathrm{PS})}_{\mu\nu}$ and we obtain

$$P = Q = -\frac{4\pi}{|c|}.$$

Charge quantization yields

$$c = \pm 4\sqrt{\frac{\pi}{n}}$$

- One may also consider the compactified case with x^5 compactified on a circle.
- Fourier mode expansion

$$H_{MNP} = \sum_n e^{inx^5/R} H^{(n)}_{MNP}(r).$$

Integrating $H_{\mu\nu 5} = \partial_5 B_{\mu\nu}$, we get

$$B_{\mu\nu} = \frac{x^5}{2\pi Rc} F_{\mu\nu}(r) + \sum_{n=-\infty}^{\infty} e^{inx^5/R} B^{(n)}_{\mu\nu}(r),$$

where

$$H^{(n\neq 0)}_{\mu\nu 5}(r) = \frac{in}{R} B^{(n\neq 0)}_{\mu\nu}(r).$$

Note the presence of winding modes.

- Consider an ansatz with the only nonzero components B_{tw} and B_{ij}, the self-duality condition reads

$$D_{[i}B^{(0)}_{jk]} = \epsilon_{ijk}\frac{F_{tw}}{2\pi Rc}, \qquad D_k B^{(0)}_{tw} = -\frac{f_k}{2\pi Rc}$$

$$D_k b^{(n)}_k = \frac{in}{R}B^{(n)}_{tw}, \qquad D_k B^{(n)}_{tw} = -b^{(n)}_k\frac{in}{R}, \qquad n \neq 0,$$

where

$$f_k(r) := \frac{1}{2}\epsilon_{ijk}F_{ij} \qquad \text{and} \qquad b^{(n)}_k(r) := \frac{1}{2}\epsilon_{ijk}B^{(n)}_{ij} \quad \text{for } n \neq 0.$$

- Note the red equation takes exactly the same form as the BPS equation for the HP monopole if we identify $-2\pi RcB^{(0)}_{tw} := \phi$ as the scalar field there.
- Hence it can be solved with A^a_i taken to be that of the HP monopole

$$A^a_i = -\epsilon_{aik}\frac{x^k}{r^2}(1 - k_v(r)).$$

This implies $F_{tw} = 0$.

- The whole system of equations can be solved exactly and we obtain

$$B^a_{tw} = -\frac{h_v(r)}{2\pi Rc}\frac{vx^a}{r} + \sum_{n\neq 0}\alpha_n e^{inx^5/R}\frac{e^{-|n|r/R}}{vr}\left(1+\frac{vR}{|n|}\coth(vr)\right)\frac{vx^a}{r},$$

$$B^a_{ij} = \frac{x^5}{2\pi Rc}F^a_{ij}(r) + c_0 F^a_{ij}(r) + \sum_{n\neq 0} e^{inx^5/R} B^{a\,(n)}_{ij}(r),$$

where

$$B^{a\,(n)}_{ij} = \epsilon_{ijk}\frac{R}{in}\Big(-v^3(ra'_n - k_v(r)a_n)\frac{x^k x^a}{r} - \delta^a_k a_n k_v(r)\frac{v}{r}\Big), \quad n\neq 0.$$

- As before, we can include a scalar field ϕ as needed in the (2,0) theory, and the charge is found to be the same as the uncompactified string (as it should be by consistency).

Outline

I. We have constructed a non-abelian action of tensor fields with the properties:
   1. the action admits a self-duality equation of motion,
   2. the action has manifest 5d Lorentz symmetry and a modified 6d Lorentz symmetry,
   3. on dimensional reduction, the action gives the 5d Yang-Mills action plus corrections.

   Based on these properties, we propose our action to be the bosonic theory describing the gauge sector of coincident M5-branes in flat space.

II. We have also constructed the non-abelian string solutions of the theory. By including a scalar field, we argue how the solution can be promoted to become a solution of the (2,0) theory.
   This provides further support to our proposed theory.

Further questions

- Covariant PST extension of our model?
- Supersymmetry: (2,0)? (1,0)?
  Scalar potential and BPS equation?
- Other solutions? BPS string junction?
- Integrability?
- Connection with the 5d SYM proposal of Douglas and Lambert-Papageorgakis-Schmidt-Summerfield?