Homework

- Problem 1 – Top Observation paper from D-Zero

> From all seven channels, we observed 17 events with an expected background of 3.8 ± 0.6 events (see Table II). Our measured cross section as a function of the top quark mass

  - Calculate the probability that you will observe 17 events if the mean expected number of events is 3.8
  - Calculate the cross section for integrated luminosity of 50 pb^{-1}, total efficiency times acceptance of $\epsilon \times A = 4.1\%$
  - Calculate the statistical error on the cross section

Appendix

Averaging Measurements

- Best estimate of true value of x is

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}}$$

  - If we knew σ from some model or theory

- If σ is not known, we can take the square root of sample variance

Formal Derivation

- Calculate variance of f

$$\sigma_f^2 = \overline{f^2} - \bar{f}^2 = \frac{1}{n}\sum_{i=1}^{i} f^2(x_i) - \left[\frac{1}{n}\sum_{i=1}^{i} f(x_i)\right]^2$$

- Average of f

$$\frac{1}{n}\sum_{i=1}^{i} f(x_i) = \frac{1}{n}\sum_{i=1}^{i} f\left(\bar{x} + \delta_{x_i}\right) = \frac{1}{n}\sum_{i=1}^{i}\left[f(\bar{x}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_{x_i}\right]$$

$$= f(\bar{x})$$

  - $\delta_{x_i} = x_i - \bar{x}$
  - Assume that f doesn't change too much over the range of values of xi

- Average of f^2

$$\frac{1}{n}\sum_{i=1}^{i} f^2(x_i) \approx \frac{1}{n}\sum_{i=1}^{i}\left[f(\bar{x}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_{x_i}\right]^2$$

$$= \frac{1}{n}\sum_{i=1}^{i} f^2(\bar{x}) + 2f(\bar{x})\left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_{x_i} + \left(\left.\frac{\partial f}{\partial x}\right|_{\bar{x}} \delta_{x_i}\right)^2$$

$$= f^2(\bar{x}) + \left(\frac{\partial f}{\partial x}\right)^2 \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = f^2(\bar{x}) + \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2$$

Error of f(x,y)

- Variance of f

$$\sigma_f^2 = \overline{f^2} - \bar{f}^2$$

- Average value of f

$$\bar{f} = \frac{1}{n}\sum_{i=1}^{n} f(x_i, y_i) = \frac{1}{n}\sum_{i=1}^{n} f\left(\bar{x} + \delta_{x_i}, \bar{y} + \delta_{y_i}\right)$$

  - Taylor expand

$$\bar{f} = \frac{1}{n}\sum_{i=1}^{n}\left[f(\bar{x}, \bar{y}) + \frac{\partial f}{\partial x}\delta_{x_i} + \frac{\partial f}{\partial y}\delta_{y_i}\right] = f(\bar{x}, \bar{y})$$

Formal Derivation

$$\overline{f^2} = \frac{1}{n}\sum_{i=1}^{n}\left[f(\bar{x},\bar{y}) + \frac{\partial f}{\partial x}\delta_{x_i} + \frac{\partial f}{\partial y}\delta_{y_i}\right]^2$$

$$\overline{f^2} = \bar{f}^2 + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x}\right)^2\delta_{x_i}^2 + \left(\frac{\partial f}{\partial y}\right)^2\delta_{y_i}^2 + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\delta_{x_i}\delta_{y_i}$$

- $cov(x,y) \equiv \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}\delta_{x_i}\delta_{y_i}$