# 2012 KIAS-POSTECH Number Theory Workshop-"$L$-function"

June 11(Monday) – June 14(Thursday), 2012

# 1 Lecture Room

Math. Sci. Bldg., Room 404
Dept of Mathematics, POSTECH, Pohang, Korea

http://pmi.postech.ac.kr;

http://workshop.kias.re.kr/KPW2012/?Home

# 2 Schedule

| | Time | June 11 | Jun 12 | June 13 | Jun 14 | |
|---|---|---|---|---|---|---|
| | 9:00 | Registration | Tea | Tea | Tea | |
| | 9:30-10:30 | Gelbart* | Friedberg | Greenberg | Shahidi | |
| | 11-12 | Hoffstein | Matsumoto | Gelbart | Greenberg | |
| | 14-15 | Friedberg | Hoffstein | Shahidi | Templier | |
| | 15:30-16:30 | Matsumoto | Kohnen | Templier | Lee | |
| | 17 -18 | Kim(17-17:30) Jin(17:40-18:10) | B. Im | Kohnen | | |
| | 18:30- | | Banquet | | | |

- Gelbart* is an expository talk .
- Planning to take a trip to GyeongJu on June 15
  (it will cost about 80 US dollars individually(entrance fee+trasportation) )
  http://guide.gyeongju.go.kr/deploy/eng/

# 3 Speakers

**Main speakers:**

- Solomon Friedberg (solomon.friedberg@bc.edu) : Boston College
- Stephen Gelbart (stephen.gelbart@weizmann.ac.il): The Weizmann Institute of Science
- Ralph Greenberg (greenber@math.washington.edu) : University of Washington
- Jeff Hoffstein (jhoff@math.brown.edu): Brown University
- Bo-Hae Im (imbh@cau.ac.kr) : Chung-Ang University
- Winfried Kohnen (winfried@mathi.uni-heidelberg.de): University of Heidelberg
- Min Lee (minlee@math.brown.edu): Brown University
- Kohji Matsumoto (kohjimat@math.nagoya-u.ac.jp): Nagoya University
- Freydoon Shahidi (shahidi@math.purdue.edu): Purdue University
- Nicolas Templier (templier@math.princeton.edu): Princeton University

**Contribute talks:**

- Seokho Jin (archimed@postech.ac.kr): POSTECH
- Dohyeong Kim (polygon0307@gmail.com): POSTECH

# 4 Organizers

- YoungJu Choie (yjc@postech.ac.kr): POSTECH, Pohang, Korea
- YunSeo Choi (y-choi2@kias.re.kr): KIAS, Seoul, Korea

**Local organizers:**

- Yeonjeong Kim (benoble@postech.ac.kr): Pohang Mathematical Institute , POSTECH , Korea
- Kangwon Lee (kwlee@kias.re.kr) : Korea Institute Advanced Study, Seoul, Korea

# 5 Title and Abstract

## 5.1 Solomon Friedberg (solomon.friedberg@bc.edu): Boston College

**Title**: Automorphic forms on covers of groups

**Abstract**: Automorphic forms on reductive groups give rise to L-functions, whose study is of great interest in number theory. In this series of talks I discuss aspects of the theory where the reductive group is replaced by an n-fold cover. Such a cover is constructed number-theoretically and may be defined when the ground field has enough roots of unity. Describing automorphic forms on such covers is challenging, but has the potential to lead to new families of analytic objects that include and generalize Langlands L-functions.

## 5.2 Stephen Gelbart (stephen.gelbart@weizmann.ac.il): The Weizmann Institute of Science

**Title**: Mysteries of the Riemann Zeta-fnction, and Beyond

**Abstract**: I shall discuss how the mysteries of the Zeta-function of Euler, Riemann and Kummer gave birth to the more modern theories of Langlands and Iwasawa.

**Title**: A $p$-adic Integral for the Reciprocal of $L$-functions

**Abstract**: We introduce an analog of part of the Langlands-Shahidi method to the $p$-adic setting, constructing reciprocals of certain $p$-adic L-functions using the nonconstant terms of the Fourier expansions of Eisenstein series. We carry out the method for the group $SL(2)$ and give explicit $p$-adic measures whose Mellin transforms are reciprocals of Dirichlet $L$-functions. This is joint work with Steve Miller, Alexei Panchishkin, and Freydoon Shahidi.

## 5.3 Ralph Greenberg (greenber@math.washington.edu): University of Washington

**Title**: Trivial zeros of $p$-adic L -functions

**Abstract**: The title refers to a phenomenon where the interpolation property defining a $p$-adic $L$-function forces it to vanish at a certain point. It is then an interesting question to compute the derivative at such a point. Kubota-Leopoldt $p$-adic $L$-functions provide examples of this phenomenon. The derivative formula in that case was proved in the 1970s by Bruce Ferrero and myself. Another example is the $p$-adic $L$-function attached to an elliptic curve. For certain elliptic curves, this function also has a trivial zero. We will discuss the proof of the derivative formula in that case (due to Glenn Stevens and myself) and how one can adapt the approach in that proof to the earlier case of Kubota-Leopoldt $p$-adic $L$-functions (joint work with Benjamin Lundell and Shaowei Zhang).

## 5.4 Jeff Hoffstein (jhoff@math.brown.edu): Brown University

**Title**: An introduction to shifted multiple Dirichlet series

**Abstract**: I'll introduce the subject in the research talk, with little background required other than a modest familiarity with the classical study of modular forms. I'll explain what I think is an interesting collection of questions that can be approached from this point of view.

**Title**: Multiple Dirichlet series and shifted convolutions

**Abstract**: I'll define, and describe how to obtain the meromorphic continuation of, shifted Rankin-Selberg convolutions in one and two variables. I'll also describe some applications, including a mean value estimate for the second moment of twists of $GL(2)$ $L$-series by Dirichlet characters modulo $\mathbb{Q}$.

## 5.5 Bo-Hae Im (imbh@cau.ac.kr): Chung-Ang University

**Title**:Larsen's conjecture for elliptic curves and some applications of the Hales-Jewett Theorem

**Abstract**: Let $K$ be a field whose absolute Galois group is finitely generated. If $K$ neither finite nor of characteristic 2, then every hyperelliptic curve over $K$ with all of its Weierstrass points defined over $K$ has infinitely many $K$-points. If, in addition, $K$ is not locally finite, then every elliptic curve over $K$ with all of its 2-torsion

rational has infinite rank over $K$. These and similar results are deduced from the Hales-Jewett theorem. This is a joint work with Michael Larsen.

## 5.6 Seokho Jin (archimed@postech.ac.kr): POSTECH

**Title**: On the Regularized Imaginary Doi-Naganuma lifting

**Abstract**: This is a joint work with Subong Lim. There is a Doi-Naganuma lifting which sends elliptic modular forms to Hilbert modular forms. Asai and Friedberg studied this lifting for imaginary quadratic fields. In our paper we extend this lifting to weak Maass forms by using regularized integral. The result states the locations of the singularities, their singularity types, and the Fourier expansions of the liftings of weak Maass forms.

## 5.7 Dohyeong Kim (polygon0307@gmail.com): POSTECH

**Title**: On non-commutative $p$-adic $L$-functions attached to modular forms over the false Tate curve extensions

**Abstract**: The non-commutative Iwasawa theory predicts congruences between special values of Artin twists of $L$-functions attached to Hecke eigenforms. We will consider the Artin representations which factor through so called false Tate curve extensions. I will explain the conjectured congruences among them, for which I will give a conditional proof assuming the vanishing of $\mu$-invariant of specialization of Katz-Hida-Tilouine measure. If time permits, I will explain how to obtain an analogue of David Rohrlich's non-vanishing result for an Artin twist of an $L$-function as a by-product of our method.

## 5.8 Winfried Kohnen (winfried@mathi.uni-heidelberg.de): University of Heidelberg

**Title**: A characterization of Siegel cusp forms of degree two

**Abstract**: We give a new characterization of Siegel cusp forms of degree two by means of the growth of their Fourier coefficients. This is recent joint work with Y. Martin.

**Title**: Conic theta functions

**Abstract**: We study a class of polyhedral functions called conic theta functions, which are closely related to classical theta functions. This is recent joint work with A. Folsom and S. Robins.

## 5.9 Min Lee (minlee@math.brown.edu): Brown University

**Title**: Approximate Converse Theorem

**Abstract**: We present an approximate converse theorem which measures how close a given set of irreducible admissible unramified unitary generic local representations of $GL(n)$ is to a genuine cuspidal representation. To get a formula for the measure, we introduce a quasi-Maass form on the generalized upper half plane for a given set of local representations. We also construct an annihilating operator which enables us to write down an explicit cuspidal automorphic function.

## 5.10 Kohji Matsumoto (kohjimat@math.nagoya-u.ac.jp): Nagoya University

**Title**: Zeta-functions of weight lattices of compact connected semisimple Lie groups

**Abstract**: This is a joint work with Yasushi Komori and Hirofumi Tsumura. We define zeta-functions of weight lattices of compact connected (but not necessarily simply-connected) semisimple Lie groups, show the explicit volume formulas of Witten type for these zeta-functions, and further discuss functional relations among them. New parity results are also mentioned.

## 5.11 Freydoon Shahidi (shahidi@math.purdue.edu):Purdue University

**Title**: Stability of root numbers and the local Langlands correspondence

**Abstract**: In this talk we will review the problem of stability of root numbers under highly ramified twists in some generality. Special cases of stability have been central in proofs given for functoriality by means of converse theorems. We then discuss how one may prove the equality of certain Artin root numbers for GL(n) under the local Langlands correspondence (LLC) with those defined by automorphic theory

(Langlands-Shahidi method) by having the stability for supercuspidals.

**Title**: A converse to tempered L-packets conjecture

**Abstract**: The tempered or generic L-packets conjecture requires every tempered L-packet of a quasisplit connected reductive group over a local field to have a member which is generic with respect to some non-degenerate character of the maximal unipotent subgroup. In this talk we address the converse to this by showing that under the validity of LLC for certain Levi subgroups, the converse of this is also true for Arthur packets. More precisely, if the Langlands packet attached to a given Arthur packet, which always contains the most non-tempered members of the A-packet, has a generic element, then it is tempered. We will also discuss its global consequences, e.g., local and global genericity for cusp forms of quasisplit reductive groups over number fields are equivalent up to isomorphism.

## 5.12 Nicolas Templier (templier@math.princeton.edu): Princeton University

**Title**: Harmonic families of automorphic representations

**Abstract**: We consider certain families of automorphic representations of reductive groups over number fields. We establish a general Plancherel equidistribution theorem for the local parameters of these families. The theorem is strong enough to shed new light on the Katz-Sarnak heuristics on zeros of L-functions and eigenvalues of random matrices. In particular we find a criterion for the conjectured symmetry type. This is joint work with Sug-Woo Shin.