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Modified gravity theory and Gravitational waves

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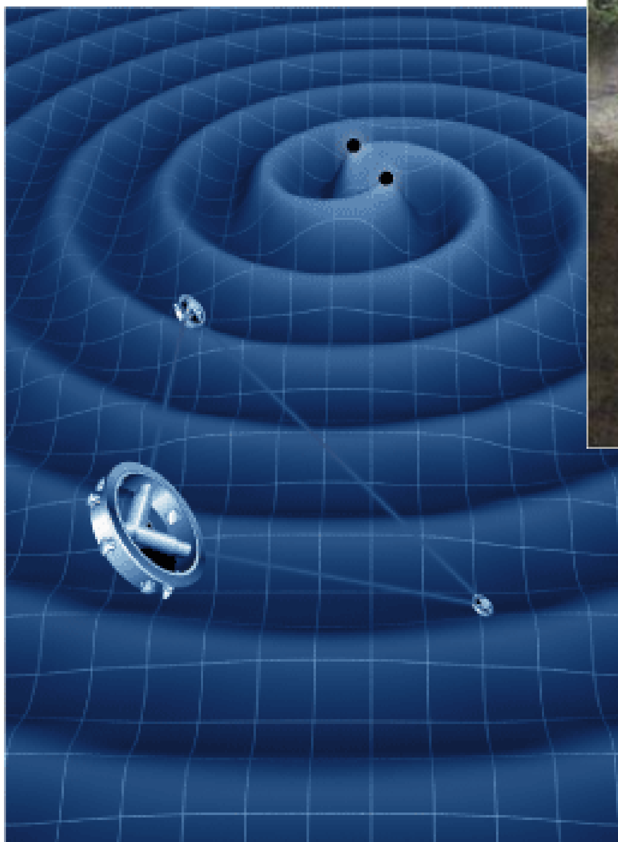
Gravitation wave detectors



eLISA(NGO)
⇒DECIGO/BBO

LIGO⇒adv LIGO

Coalescing binaries

- Inspiral phase (large separation)

  (Cutler et al, PRL **70** 2984(1993))

  Clean system

  Point particle approximation neglecting internal structure is valid.

  Accurate wave form is predictable

  - for detection
  - for parameter extraction(direction, mass, spin, …)
  - for precision test of general relativity

- Merging phase

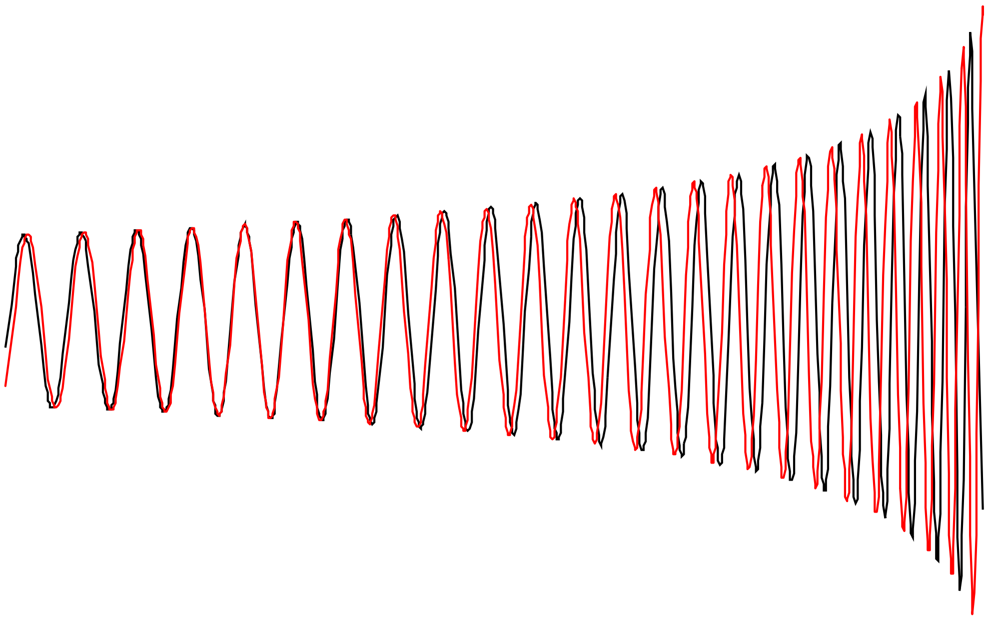
  Recent progress in numerical relativity

  - EOS of nuclear matter
  - Electromagnetic counterpart

- Ringing tail - quasi-normal oscillation of BH

General relativity is correct even in strong gravity regime？

Many cycles before plunge



Easy to detect phase difference of O(1cycle)

- Accurate orbital parameter determination
- Mapping the black hole spacetime

Predicted wave form

Standard post-Newtonian approximation

∼ (v/c) expansion

Complete up to 3.5PN=$(v/c)^7$ order

(Ref. Blanchet, Living Rev.Rel.9:4,2006)

Wave form in Fourier space

$$h(f) \approx A f^{-7/6} e^{i\Psi(f)}$$

For quasi-circular orbit

$$A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}^{5/6}}{D_L}, \quad \mathcal{M} = \mu^{3/5} M^{2/5}, \quad \eta = \frac{\mu}{M}$$

$$\Psi = 2\pi f t_c - \phi_c + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \left[1 + \underbrace{\frac{20}{9} \left(\frac{743}{331} + \frac{11}{4} \eta \right) u^{2/3}}_{\text{1PN}} \underbrace{- (16\pi - \beta) u}_{\text{1.5PN}} + \cdots \right]$$

$$u \equiv \pi M f = O(v^3)$$

Space interferometer is a powerful tool for the test of GR

Gravity Modification of Scalar-tensor type

(Berti et al, gr-qc/0411129)

$$\Psi = \cdots + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[bu^{-2/3} + 1 + \frac{20}{9}\left(\frac{743}{331} + \frac{11}{4}\eta\right)u^{2/3} - (16\pi - \beta)u + \cdots \right]$$

$$b \propto \frac{1}{2+\omega}$$

-1 PN-like frequency dependence

Current constraint on dipole radiation: (Bhat et al. arXiv:0804.0956)

$\omega_{BD} > 1.5\text{x}10^5$, J1141-6545 (NS(young pulsar)-WD)

(Yagi & TT, arXiv:0908.3283)

LISA(NGO)– $1.4M_{\odot}$NS+$1000M_{\odot}$BH: $\omega_{BD} > 5 \times 10^3$

@ 40Mpc corresponding to $SNR = \sqrt{200}$

Decigo–$1.4M_{\odot}$NS+$10M_{\odot}$BH: $\omega_{BD} > 8 \times 10^7$

assuming 10^4 events at cosmological distances

Modified propagation speed

(Berti & Will, PRD71 084025(2005))

phase velocity of massive graviton

$$c_{phase}(f) = \frac{k}{\omega} \approx 1 - \frac{m^2}{2\omega^2} = 1 - \frac{1}{2\lambda_g^2 f^2}$$

$$D = \int d\eta\, a^2$$

$$\Rightarrow \quad \Delta\Psi = 2\pi f \Delta t = 2\pi f D \Delta c_{phase}(f) \approx -\frac{\pi D}{\lambda_g^2 f}$$

Frequency dependent phase shift

$$\Psi = \cdots + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[\alpha u^{-2/3} + 1 + \left(\frac{3715}{756} + \frac{55}{9}\eta - \frac{128}{3}\eta\beta_g\right)u^{2/3} - (16\pi - \beta)u + \cdots\right]$$

$$\beta_g = \frac{\pi^2 DM}{\lambda_g^2}$$

Effect of graviton mass

(e)LISA(NGO)– $10^7 M_\odot$BH+$10^6 M_\odot$BH at3Gpc:

graviton Compton wavelength

$\lambda_g > 4$kpc

(Yagi & TT, arXiv:0908.3283)

Chern-Simons Modified Gravity

(Sopuerta & Yunes, arXiv:0904.4501)

$$S \supset \frac{\alpha}{4}\int d^4x\sqrt{-g}\,\phi\,{}^*R\,R - \frac{\beta}{2}\int d^4x\sqrt{-g}\left[(\partial\phi)^2 + 2V(\phi)\right]$$

$${}^*R\,R = \varepsilon^{\alpha\beta}{}_{\sigma\chi}R^{\sigma\chi}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

$$G_{\mu\nu} + \frac{\alpha}{\kappa}C_{\mu\nu} = \frac{1}{2\kappa}\left(T^{(mat)}_{\mu\nu} + T^{(\phi)}_{\mu\nu}\right) \qquad T^{(\phi)}_{\mu\nu} = \beta\left[\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu}V\right]$$

$$\beta\Box\phi = \beta\frac{dV}{d\theta} - \frac{\alpha}{4}\,{}^*R\,R \qquad C^{\mu\nu} = \left(\partial_\gamma\phi\right)\varepsilon^{\gamma\delta\varepsilon(\mu}\nabla_\varepsilon R^{\nu)}{}_\delta + \left(\nabla_\gamma\nabla_\delta\phi\right){}^*R^{\delta(\mu\nu)\gamma}$$

Small back ground gradient $\nabla\theta$ is hard to detect

$$\left|M_{pl}^{-2}\alpha\dot{\phi}\right| < 3\times10^{-6}\,\mathrm{cm}$$:J0737-3039(double pulsar)

(Yunes & Spergel, arXiv:0810.5541)

Right handed and left handed gravitational waves are magnified differently during propagation, depending on the frequencies.

$$\boldsymbol{h}^{(L,R)} = \frac{1}{\sqrt{2}}\left(\boldsymbol{h}^{(+)} + i\boldsymbol{h}^{(\times)}\right)$$

$$\boldsymbol{h}^{(L,R)} = \boldsymbol{h}^{(L,R)}_{GR}\exp\left(\pm 64\pi^2\alpha\, f\, H_0\int_0^z dz(1+z)^{5/2}\left[\frac{7}{2}\frac{d\phi}{dz} + (1+z)\frac{d^2\phi}{dz^2}\right]\right)$$

$$\approx \boldsymbol{h}^{(L,R)}_{GR}\exp\left(\pm f\, O\!\left(10^3\alpha\dot{\phi}/M_{pl}^2\right)\right)$$

Deviation from Kerr metric

(Konno, Matsuyama & Tanda, arXiv:0706.3080arXiv:0902.4767)
(Yunes & Pretorius, arXiv:0902.4669)

known only for slow rotation

$$\phi = \frac{5}{8}\frac{\alpha}{\beta}\frac{a}{M}\frac{\cos\theta}{r^2}\left(1+\frac{2}{7}\frac{M}{r}+\frac{18}{5}\frac{M^2}{r^2}\right) \qquad g_{t\varphi} = \frac{5}{8}\frac{\alpha^2 aM}{\kappa\beta r^4}\left(1+\frac{12}{7}\frac{M}{r}+\frac{27}{10}\frac{M^2}{r^2}\right)$$

Leading correction to the binary orbit

～2PN frequency dependence

(Yagi, Yunes, T.T. arXiv:1206.6130, 1208.5102)

Scalar dipole-dipole interaction force
Scalar radiation induced by dipole moment
Gravitational tidal force by induced quadrupole moment

$$g_{tt} = -\frac{M}{r}\left(1+\frac{603}{3584}\frac{\alpha^2 a^2}{\kappa\beta M}\frac{\cos^2\theta}{r^2}\right)$$

KAGRA, adv LIGO Virgo can constrain

$$\left(\frac{\alpha^2}{\kappa\beta}\right)^{\frac{1}{4}} < 10 \sim 100km$$

more stringent by ～10^6 than solar system constraint

Bi-gravity theory

$$L = \frac{\sqrt{-g}R}{16\pi G_N} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi G_N \kappa} + L_{matter}(g, \phi) + \cdots$$

Both massive and massless gravitons exist.

$\rightarrow \nu$ oscillation-like phenomena?

First question is if we can construct a viable model.

Ghost free bi-gravity

Special interaction between g and $\tilde{g}$ does not have BD ghost.

$$L_{mass} = \sqrt{-g} \sum_{n=0}^{4} c_n V_n$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots \quad \tau_n \equiv Tr\left[\gamma^n\right] \ \gamma^\alpha_\mu \equiv \sqrt{g^{\alpha\xi}\tilde{g}_{\xi\mu}}$$

We set $8\pi G_N = 1$

(de Rham-Gabadadze-Tolley (2011))
(Hassan, Rosen (2011,2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

$$ds^2 = a^2(t)\left(-dt^2 + dx^2\right)$$

$$T_{\mu\nu}^{(mass)} = 2\frac{\delta S^{(mass)}}{\delta g^{\mu\nu}}$$

$$d\tilde{s}^2 = b^2(t)\left(-c^2(t)dt^2 + dx^2\right)$$

$$\xi \equiv b/a$$

$$\nabla^\mu T_{\mu\nu}^{(mass)} = 0 \Rightarrow \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 :

At linear level, expected scalar and vector modes are absent. Strong coupling? Unstable for anisotropic perturbation.

branch 2 :

All perturbation modes are equipped.

Branch 2 background

$$\underbrace{\left(6c_3\xi^2+4c_2\xi+c_1\right)}_{\text{branch 1}}\underbrace{\left(cba'-ab'\right)}_{\text{branch 2}}=0 \qquad \xi\equiv b/a$$

branch 2 :

$$\rho-\frac{c_1}{\kappa\xi}+\left(c_0-\frac{6c_2}{\kappa}\right)+\left(3c_1-\frac{18c_3}{\kappa}\right)\xi+\left(6c_2-\frac{24c_4}{\kappa}\right)\xi^2+6c_3\xi^3=0$$

ξ becomes a function of ρ. $\xi\rightarrow\xi_c$ for $\rho\rightarrow 0$.

$$H^2=\frac{\rho+\rho_{mass}}{3} \qquad \rho_{mass}:=c_0+3c_1\xi+6c_2\xi^2+6c_3\xi^3$$

effective energy density due to mass term

$$\frac{1}{c-1}\frac{\xi'}{\xi}=\frac{a'}{a} \Rightarrow c-1=\frac{3(\rho+P)\kappa\xi}{c_1+(3c_1\kappa-18c_3)\xi^2+(12c_2\kappa-48c_4)\xi^3+18c_3\kappa\xi^4}$$

$$\approx 3(\rho+P)/m_1^2$$

Automatic tuning to c=1 for $\rho\rightarrow 0$.

EOM of Gravitational waves (Comelli, Crisostomi, Pilo (2012))

$$h'' + 2aHh'' - \Delta h + a^2 m_g^2 \left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' + \left(2aH + 2\xi'/\xi - c'/c\right)\tilde{h}' - c^2 \Delta \tilde{h} + a^2 m_g^2 \frac{c}{\kappa \xi^2}\left(h - \tilde{h}\right) = 0$$

$$m_g^2 = \xi\left(c_1 + 2c_2(c+1)\xi + 6cc_3\xi^2\right)$$

Short wavelength approximation: $k >> m_g >> H$

we take limit $c \to 1$

$$\begin{pmatrix} -\omega^2 + k^2 + m_g^2 & -m_g^2 \\ -\frac{c}{\kappa\xi^2} m_g^2 & -\omega^2 + c^2 k^2 + \frac{c}{\kappa\xi^2} m_g^2 \end{pmatrix} \begin{pmatrix} h \\ \tilde{h} \end{pmatrix} = 0$$

⇨ $h_A = h + \tilde{h}$ $\qquad$ $h_B = \tilde{h} - \kappa\xi^2 h$

Eigen modes $\quad k_A^2 = \omega^2 \qquad k_B^2 = \omega^2 + m_g^2\left(1 + \frac{1}{\kappa\xi^2}\right)$

⇨ Graviton oscillation scale $(k_B - k_A)^{-1} \approx \omega / m_g^2$

becomes cosmological distance scale for $m_g^{-1} \sim 1\mathrm{kpc}$

On the other hand, dispersion relation of the scalar type perturbations

$$\omega^2 = c_s^2 k^2 \qquad \omega^2 = k^2 + m_2^2$$

matter oscillation massive scalar graviton

Graviton mass can be very different from $\rho_{\mathrm{mass}}^{1/2}$, m_1 or m_2.

$$m_g^2 = \xi\left(c_1 + 2c_2(c+1)\xi + 6cc_3\xi^2\right)$$

If the scalar mass m_2 is large enough, the infamous fifth force (force mediated by helicity 0 mode) is suppressed and the model should pass solar system tests.

Summary

Gravitational wave observations give us a new probe to the modified gravity theory.

Constraints by gravitational waves are sometimes a few orders of magnitude more stringent than the current bound.

Even graviton oscillation is not immediately denied, and hence we may find something similar to the case of solar neutrino experiment in near future.