Towards more precise estimates of the primordial bispectrum

Jinn-Ouk Gong

APCTP, Pohang 790-784, Korea

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Based on

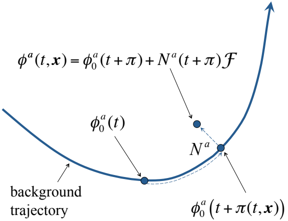
- C. T. Byrnes and JG, arXiv:1210.1851 [astro-ph.CO]
- A. Achucarro, JG, G. A. Palma and S. P. Patil, to appear
- JG, K. Schalm and G. Shiu, to appear

Outline

General single field inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm Pl}^2}{2}R + P(X,\phi)\right] \quad \text{with} \quad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Originated from multi-field setup: light $\mathscr{R}$ and heavy $\mathscr{F}$



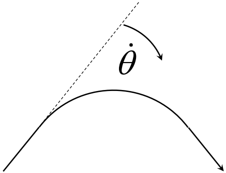
- Trajectory along the lightest direction
- Effects of heavy physics in curved traj
- Can we apply EFT to find universal features of "heavy" physics?

1. Write the action in terms of $\mathscr{R}$ (along traj) and $\mathscr{F}$ (off traj)
2. Integrate out $\mathscr{F}$: $e^{S_{\rm eff}[\mathscr{R}]} = \int[D\mathscr{F}]e^{S[\mathscr{R},\mathscr{F}]}$
   $\left[= \text{equiv to plugging linear sol: } \left(-\Box + M_{\rm eff}^2\right)\mathscr{F} = -2\dot\theta\left(\dot\phi_0/H\right)\dot{\mathscr{R}}\right]$
3. Effective single field action $S_{\rm eff}[\mathscr{R}]$

Effects of heavy physics as non-trivial c_s

Effects of heavy physics in "speed of sound"

$$c_s^{-2} \equiv 1 + \frac{4\dot{\theta}^2}{M_{\rm eff}^2}$$ ($\dot{\theta}$: angular velocity of traj)



Single field theory with non-trivial c_s^2: Footprint of heavy physics

(Achucarro et al. 2012a)

$\mathscr{F}$ borrows kinetic energy of $\mathscr{R}$ → propagation speed c_s reduced

- EFT in $\Box/M_{\rm eff}^2$: universal footprint of heavy physics
- Many scalar fields in BSM, e.g. moduli
- New observables poorly constrained → to be tested in next decades

Splitting canonical action

EFT = canonical ($c_s = 1$) + (occasional) departure from $c_s = 1$

$$S = \underbrace{\int d^4x a^3 \epsilon m_{\rm Pl}^2 \left[\frac{\dot{\mathscr{R}}^2}{c_s^2} - \frac{(\nabla\mathscr{R})^2}{a^2} \right]}_{= S_2,\ \text{“free” part}} + S_3 + \cdots$$

$$= \underbrace{S_{2,\text{canonical}}}_{c_s=1\ \text{part}} + \underbrace{\int d^4x a^3 \epsilon m_{\rm Pl}^2 \left(\frac{1}{c_s^2} - 1 \right) \dot{\mathscr{R}}^2}_{\equiv S_{2,\text{int}}} + S_3 + \cdots$$

- Well known, accurate Green's function

  (For example, JG & Stewart 2001, Choe, JG & Stewart 2004)

- Interaction valid for a limited interval (c.f. Chen & Wang 2010)

c.f. Using $dy \equiv c_s d\tau = c_s dt/a$, $q^2 \equiv a^2\epsilon/c_s$ and $v = \sqrt{2} q \mathscr{R}$ (Baumann, Senatore & Zaldarriaga 2011)

$$S_2 = \int d^4x \frac{m_{\rm Pl}^2}{2} \left[(v')^2 - (\nabla v)^2 + \frac{q''}{q} v^2 \right]$$

But see later parts of this presentation

Features in the power spectrum

Interaction Hamiltonian at quadratic order

$$H_{\rm int}^{(2)}(t) = \int d^3x \left(\frac{\partial \mathscr{L}_{\rm int}^{(2)}}{\partial \dot{\mathscr{R}}} \dot{\mathscr{R}} - \mathscr{L}_{\rm int}^{(2)} \right) = \int d^3x a^3 \epsilon m_{\rm Pl}^2 \underbrace{\left(\frac{1}{c_s^2} - 1 \right)}_{\equiv -u(t)} \dot{\mathscr{R}}^2$$

Features in the power spectrum

$$\left\langle \widehat{\mathscr{R}}_{\boldsymbol{k}}(\tau)\widehat{\mathscr{R}}_{\boldsymbol{q}}(\tau) \right\rangle = -i\int_{\tau_{\rm in}}^{\tau} a(\tau')d\tau' \left\langle 0 \left| \left[\widehat{\mathscr{R}}_{\boldsymbol{k}}(\tau)\widehat{\mathscr{R}}_{\boldsymbol{q}}(\tau), H_{\rm int}^{(2)}(\tau') \right] \right| 0 \right\rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}+\boldsymbol{q}) \frac{2\pi^2}{k^3} \Delta\mathscr{P}_{\mathscr{R}}$$

$$\rightarrow \frac{\Delta\mathscr{P}_{\mathscr{R}}}{\mathscr{P}_{\mathscr{R}}} = \kappa \int_0^\infty dt u(t) \sin(2\kappa t) \quad \text{with} \quad \mathscr{P}_{\mathscr{R}} = \frac{H^2}{8\pi^2 m_{\rm Pl}^2 \epsilon}, t \equiv \frac{\tau}{\tau_\star}, \kappa \equiv \frac{k}{k_\star}$$

Inverting this relation to write u in terms of observable $\Delta\mathscr{P}_{\mathscr{R}}/\mathscr{P}_{\mathscr{R}}$

$$u(t) = \frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{d\kappa}{\kappa} \frac{\Delta\mathscr{P}_{\mathscr{R}}}{\mathscr{P}_{\mathscr{R}}} \left(\frac{\kappa}{2} \right) e^{i\kappa\tau}$$

Correlated bispectrum and power spectrum: $B_{\mathscr{R}} = \int (\cdots \Delta\mathscr{P}_{\mathscr{R}}/\mathscr{P}_{\mathscr{R}})$

Leading bispectrum for varying c_s

Leading order action in terms of $u(t)$

$$S_3 \supset \int d^4x a^3 m_{\rm Pl}^2 \epsilon \left[3u\dot{\mathcal{R}}^2\mathcal{R} - (u+2s)\mathcal{R}(\nabla\mathcal{R})^2 \right] \qquad \left(s \equiv \frac{\dot{c}_s}{Hc_s} \right)$$

Assumption: H, ϵ and $\eta_\parallel$ approximately constant ($K \equiv k_1 + k_2 + k_3$)

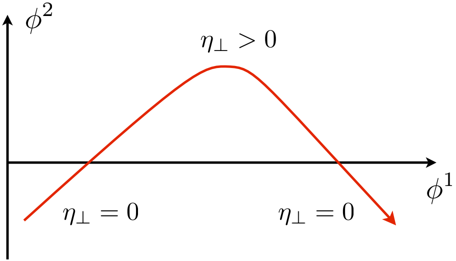
$$B_\mathcal{R}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2\Re\left\{ 2i\widehat{\mathcal{R}}_{k_1}(0)\widehat{\mathcal{R}}_{k_2}(0)\widehat{\mathcal{R}}_{k_3}(0) \left[3\epsilon\frac{m_{\rm Pl}^2}{H^2}\int_{-\infty}^0 d\tau \frac{u}{\tau^2}\frac{d\widehat{\mathcal{R}}_{k_1}^*(\tau)}{d\tau}\frac{d\widehat{\mathcal{R}}_{k_2}^*(\tau)}{d\tau}\widehat{\mathcal{R}}_{k_3}^*(\tau) + 2\text{ perm} \right.\right.$$
$$\left.\left. +\epsilon\frac{m_{\rm Pl}^2}{H^2}(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + 2\text{ perm})\int_{-\infty}^0 d\tau\frac{u+2s}{\tau^2}\widehat{\mathcal{R}}_{k_1}^*(\tau)\widehat{\mathcal{R}}_{k_2}^*(\tau)\widehat{\mathcal{R}}_{k_3}^*(\tau) \right]\right\}$$
$$= \frac{(2\pi)^4\mathcal{P}_\mathcal{R}^2}{(k_1k_2k_3)^3}\left[\frac{3}{2}(k_1k_2)^2\left\{ \frac{1}{K}\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{K}{2k_\star}\right) - k_3\frac{d}{dk}\left[\frac{1}{k}\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{k}{2k_\star}\right)\right]\bigg|_{k=K} \right\} + 2\text{ perm} \right.$$
$$+\frac{1}{2}(\boldsymbol{k}_1\cdot\boldsymbol{k}_2 + 2\text{ perm})\left\{ \frac{K^2 - (k_1k_2 + 2\text{ perm})}{K}\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{K}{2k_\star}\right) + k_1k_2k_3\frac{d}{dk}\left[\frac{1}{k}\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{k}{2k_\star}\right)\right]\bigg|_{k=K} \right.$$
$$\left.\left. -(k_1k_2 + 2\text{ perm})\frac{d}{dk}\left[\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{k}{2k_\star}\right)\right]\bigg|_{k=K} + k_1k_2k_3\frac{d^2}{dk^2}\left[\frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}}\left(\frac{k}{2k_\star}\right)\right]\bigg|_{k=K} \right\}\right]$$

(Achucarro, JG, Palma & Patil, to appear)

Correlation between spectra is manifest!

Modeling curvilinear trajectory

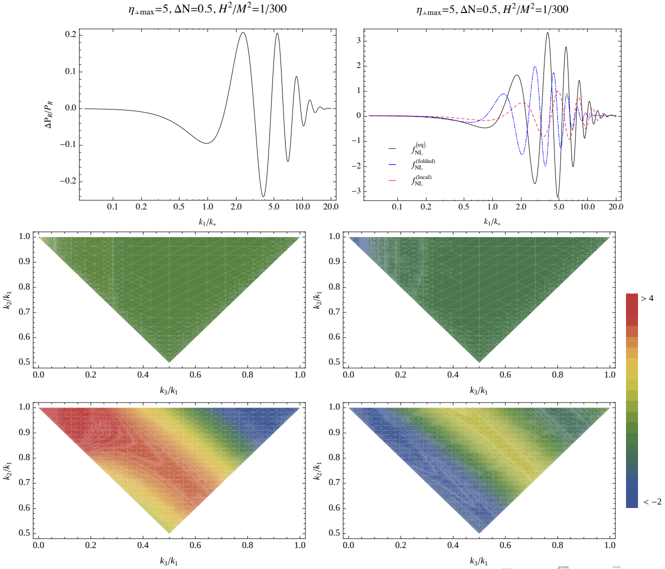
A cosh turn in otherwise straight trajectory in 2-field system



$$\eta_\perp = \frac{\dot{\theta}}{H} = \frac{\eta_{\perp,\max}}{\cosh^2\left[2(N-N_\star)/\Delta N\right]}$$

(Equations of motion: see Achucarro et al. 2011)

Features from smooth curvilinear trajectory



General slow-roll approximation

- $\widehat{\mathscr{R}}_k(\tau)$ = de Sitter piece + higher order corrections
- No guarantee for the hierarchy between slow-roll parameters
- Up to 1st order corrections in the standard SR known (Chen et al. 2007)
- Consistent account in more general contexts

Mode equation: $z^2 \equiv 2a^2 m_{\rm Pl}^2 \epsilon$, $y \equiv \sqrt{2k} z \widehat{\mathscr{R}}_k$, $dx = \equiv -kc_s dt/a$, $f \equiv 2\pi x z/k$

$$\underbrace{\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right)y}_{\text{de Sitter solution}} = \underbrace{\frac{1}{x^2} \underbrace{\frac{f'' - 3f'}{f}}_{\equiv g(\log x)} y}_{\text{departure from dS}} \quad \left(f' \equiv \frac{df}{d\log x}\right) \quad \rightarrow \quad y_0(x) = \left(1 + \frac{i}{x}\right) e^{ix}$$

Green's function solution (JG & Stewart 2001)

$$\begin{aligned} y(x) =& y_0(x) + \frac{i}{2}\int_x^\infty \frac{du}{u^2} g(\log u) \left[y_0^*(u) y_0(x) - y_0^*(x) y_0(u)\right] y(u) \\ \equiv& y_0(x) + L(x,u) y(u) \\ =& y_0(x) + L(x,u) y_0(u) + L(x,u) L(u,v) y_0(v) + \cdots \end{aligned}$$

3rd order action reprocessed

$\dot{\mathscr{R}}^3$ and $\dot{\mathscr{R}}^2\mathscr{R}$: cumbersome to compute with many derivatives

$$\int \dot{\mathscr{R}}^3 \sim \int \left(\dot{y}_0 + \dot{L}y_0 + L\dot{y}_0 + \cdots\right)^3 \sim ☹$$

Using partial int and linear eq to reduce the # of derivatives

$$\left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \equiv \frac{a^3\epsilon}{c_s^2}\left\{\ddot{\mathscr{R}} + \underbrace{\left[\frac{c_s^2}{a^2\epsilon}\frac{d}{dt}\left(\frac{a^2\epsilon}{c_s^2}\right) + H\right]}_{\equiv C = H(3+\eta-2s)}\dot{\mathscr{R}} - \frac{\Delta}{a^2}\mathscr{R}\right\}$$

$$\int A\dot{\mathscr{R}}^3 = \int \frac{\ddot{A} - 3\dot{A}C - 2A\dot{C} + 2AC^2}{2}\frac{d}{dt}\frac{\left(\mathscr{R}^3\right)}{3} + \cdots + \left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \frac{c_s^2}{a^3\epsilon}\left(\frac{\dot{A} - 2AC}{2}\mathscr{R}^2 + \cdots\right)$$

$$\int B\dot{\mathscr{R}}^2\mathscr{R} = \int \frac{-\dot{B} + BC}{2}\frac{d}{dt}\frac{\left(\mathscr{R}^3\right)}{3} + \cdots + \left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \frac{c_s^2}{a^3\epsilon}\frac{B}{2}\mathscr{R}^2$$

Field redefinition with more terms involved (JG, Schalm & Shiu, to appear)

$$S_3 = \int d\tau d^3x \underbrace{\frac{m_{\rm Pl}^2}{3}\frac{a^2\epsilon}{c_s}\left[-c_s aH\left(3s + \frac{\epsilon\eta}{2} + \epsilon s + 9us - 2s^2\right) - \frac{1}{2}\frac{d}{d\tau}\left(\frac{\eta}{c_s^2}\right)\right]}_{\equiv \mathfrak{C}}\frac{d}{d\tau}\left(\mathscr{R}^3\right) + (\text{higher SR terms})$$

1st order bispectrum in GSR

"Source" for the bispectrum

$$g_B(\log\tau) = \frac{c_s}{a^2 m_{\rm Pl}^2 \epsilon}\frac{-\tau}{f}\mathfrak{C} = \frac{1}{f}\left[c_s aH\left(3s+\frac{\epsilon\eta}{2}+\epsilon s+9us-2s^2\right)+\frac{1}{2}\frac{d}{d\log\tau}\left(\frac{\eta}{s}\right)\right]$$

Window functions constructed from homogeneous solution

$$y_0(k_1\tau)y_0(k_2\tau)y_0(k_3\tau) = W_B(k_1,k_2,k_3;\tau)+iX_B(k_1,k_2,k_3;\tau)$$
$$y_0(k_1\tau)y_0(k_2\tau)y_0^*(k_3\tau) = W_B(k_1,k_2,-k_3;\tau)+iX_B(k_1,k_2,-k_3;\tau) \equiv W_{B3}+iX_{B3}$$

Bispectrum up to 1st correction [i.e. $\mathcal{O}(g)$] (c.f. Adshead et al. 2011)

$$\begin{aligned}
B_{\mathscr{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = &\frac{(2\pi)^4}{4}\frac{\sqrt{\mathscr{P}_{\mathscr{R}}(k_1)}}{k_1^2}\frac{\sqrt{\mathscr{P}_{\mathscr{R}}(k_2)}}{k_2^2}\frac{\sqrt{\mathscr{P}_{\mathscr{R}}(k_3)}}{k_3^2}\int_0^\infty\frac{d\tau}{\tau}g_B(\log\tau)\\
&\times\left\{\left(d_\tau-3\frac{f'}{f}\right)W_B+\frac{1}{3}d_\tau\left(X_B+X_{B3}\right)\int_0^\infty\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)X(k_3\tilde\tau)+2\text{ perm}\right.\\
&-\frac{1}{3}d_\tau W_{B3}\int_\tau^\infty\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)W(k_3\tilde\tau)-\frac{1}{3}d_\tau X_{B3}\int_0^\tau\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)X(k_3\tilde\tau)+2\text{ perm}\\
&\left.-\frac{1}{2}d_\tau\left(X_B+X_{B3}\right)\int_\tau^\infty\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)\left(\frac{1}{k_3\tilde\tau}+\frac{1}{k_3^3\tilde\tau^3}\right)+2\text{ perm}\right\}\quad\left(d_\tau\equiv\frac{d}{d\log\tau}+3\right)
\end{aligned}$$

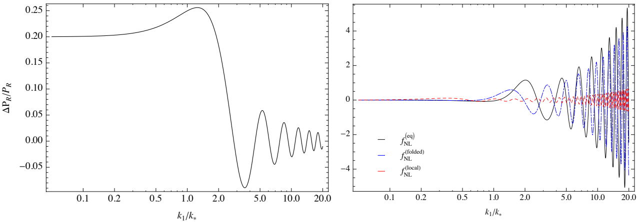
(JG, Schalm & Shiu, to appear)

Example: Starobinsky model

Starobinsky model: linear $V(\phi)$ + sudden slope change (Starobinsky 1992)

$$V(\phi) = V_0 \times \begin{cases} \left[1 - A(\phi-\phi_0)\right] & \text{for} \quad \phi < \phi_0 \\ \left[1 - (A+\Delta A)(\phi-\phi_0)\right] & \text{for} \quad \phi > \phi_0 \end{cases}$$

de Sitter approx: $\dfrac{f'}{f} = -\dfrac{\ddot\phi}{H\dot\phi}$, $g = -3\dfrac{V''}{V}$, $g_B = \dfrac{1}{f}\left(\dfrac{\ddot\phi}{H\dot\phi}\right)'$ (Choe, JG & Stewart 2004)



(cf. Arroja & Sasaki 2012)

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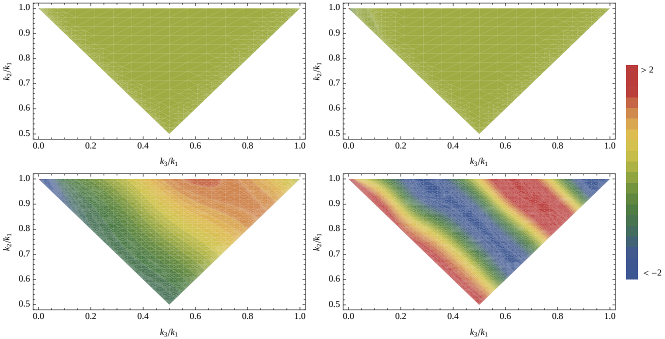
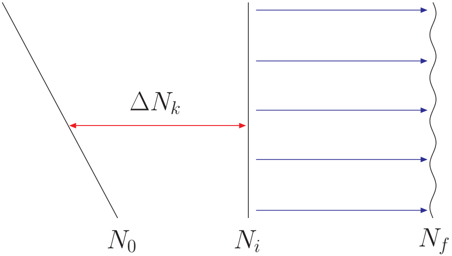


(cf. Arroja & Sasaki 2012)

Evolution of field fluctuations



1. N_i: initial slice (flat) for the δN formalism, $\delta\phi^a_{\rm flat} \equiv Q^a$
2. N_f: final slice (comoving) for the δN formalism
3. N_0: horizon crossing of a mode k

$Q^a(N_0)$ =Gaussian $\rightarrow$ $Q^a(N_i = N_0 + \Delta N_k)$ =non-Gaussian

$$\Delta N_k = \log\left(\frac{a_i}{a_0}\right) \approx \log\left[\frac{(aH)_i}{k}\right] \quad \rightarrow \quad k\text{-dependence}$$

Power spectrum and its running

Evolution equation of Q^a on large scales (Elliston, Seery & Tavakol 2012)

$$D_N Q^a = w^a{}_b Q^b + \frac{1}{2} w^a{}_{bc} Q^b Q^c + \cdots$$

$$w_{ab} = u_{(a;b)} + \frac{R_{c(ab)d}}{3} \frac{\dot\phi_0^c}{H} \frac{\dot\phi_0^d}{H} \qquad \left(u_a = -\frac{V_{;a}}{3H^2} \right)$$

$$w_{abc} = u_{(a;bc)} + \frac{1}{3} \left[R_{(a|de|b;c)} \frac{\dot\phi_0^d}{H} \frac{\dot\phi_0^e}{H} - 4R_{a(bc)d} \frac{\dot\phi_0^d}{H} \right]$$

$$Q^a(N_i = N_0 + \Delta N_k) = Q^a(N_0) + \Delta N_k \left(w^a{}_b Q^b + \frac{1}{2} w^a{}_{bc} Q^b Q^c + \cdots \right) + \cdots$$

Power spectrum and the spectral index

$$\left\langle \mathcal{R}_{\boldsymbol{k}}(t_f) \mathcal{R}_{\boldsymbol{q}}(t_f) \right\rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}+\boldsymbol{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k) = N_a(t_i) N_b(t_i) \left\langle Q^a_{\boldsymbol{k}}(t_i) Q^b_{\boldsymbol{q}}(t_i) \right\rangle$$

$$\left\langle Q^a_{\boldsymbol{k}}(t_i) Q^b_{\boldsymbol{q}}(t_i) \right\rangle = \left\langle Q^a_{\boldsymbol{k}}(t_0) Q^b_{\boldsymbol{q}}(t_0) \right\rangle + 2\Delta N_k w^a{}_c \left\langle Q^b_{\boldsymbol{k}}(t_0) Q^c_{\boldsymbol{q}}(t_0) \right\rangle$$

$$\left\langle Q^a_{\boldsymbol{k}} Q^b_{\boldsymbol{q}} \right\rangle = \frac{H^2}{2k^3} \delta^{(3)}(\boldsymbol{k}+\boldsymbol{q}) \left(\gamma^{ab} + \epsilon^{ab} \right)$$

$$n_{\mathcal{R}} - 1 = \frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k} = -2\epsilon - 2 \frac{N_a N_b w^{ab}}{N_c N^c}$$

(Sasaki & Stewart 1996)

General formula for the running of $f_{\rm NL}$

$$\left\langle \mathcal{R}_{\boldsymbol{k}_1}(t_f)\mathcal{R}_{\boldsymbol{k}_2}(t_f)\mathcal{R}_{\boldsymbol{k}_3}(t_f)\right\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})B_{\mathcal{R}}(k_1,k_2,k_3)$$

$$= N_aN_bN_c\left\langle Q^a_{\boldsymbol{k}_1}Q^b_{\boldsymbol{k}_2}Q^c_{\boldsymbol{k}_3}\right\rangle + \frac{1}{2}\left\{N_{ab}N_cN_d\left\langle \left[Q^a\star Q^b\right]_{\boldsymbol{k}_1}Q^c_{\boldsymbol{k}_2}Q^d_{\boldsymbol{k}_3}\right\rangle + 2\text{ perm}\right\}$$

1. 1st term: NL evolution between horizon crossing & initial slice

$$N_a(t_i)N_b(t_i)N_c(t_i)\left\langle Q^a_{\boldsymbol{k}_1}(t_i)Q^b_{\boldsymbol{k}_2}(t_i)Q^c_{\boldsymbol{k}_3}(t_i)\right\rangle$$

$$= (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})N_a(t_i)N_b(t_i)N_c(t_i)\frac{H^4(t_0)}{4k_1^3k_2^3k_3^3}w^{abc}\left(k_1^3\Delta N_{k_1}+2\text{ perm}\right)$$

2. 2nd term: NL evolution between initial & final slices

$$\frac{1}{2}N_{ab}(t_i)N_c(t_i)N_d(t_i)\left\langle\left[Q^a(t_i)\star Q^b(t_i)\right]_{\boldsymbol{k}_1}Q^c_{\boldsymbol{k}_2}(t_i)Q^d_{\boldsymbol{k}_3}(t_i)\right\rangle$$

$$= (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})N_{ab}(t_i)N_c(t_i)N_d(t_i)\frac{H^4(t_0)}{4k_1^3k_2^3}\left(\gamma^{ac}\gamma^{bd}+2\Delta N_{k_1}w^{ac}\gamma^{bd}+2\Delta N_{k_2}\gamma^{ac}w^{bd}\right)$$

$$\frac{6}{5}f_{\rm NL} = \frac{N_{ab}N^aN^b}{(N_cN^c)^2}\left\{1-\Delta N_k\underbrace{\left[-\frac{N_aN_bN_cw^{abc}}{N_{de}N^dN^e}+4w^{ab}\left(\frac{N_aN_b}{N_dN^d}-\frac{N_{ac}N_bN^c}{N_{de}N^dN^e}\right)\right]}_{\equiv n_{f_{\rm NL}}}\right\}$$

(Byrnes & JG 2012)

Summary

- General single field inflation
  1. From multi-field setup: by integrating out heavy field
  2. Non-trivial c_s: footprint of heavy physics
- Features in the power spectrum ($S_{2,\rm int}$) and bispectrum (S_3)
  1. Heavy quanta extract kinetic energy
  2. Non-trivial, oscillatory, correlated features
- General slow-roll scheme
  1. Terms with more derivatives → field redefinition
  2. More complete 1st order bispectrum
- Running of $f_{\rm NL}$
  1. Sensitive probe of early universe physics
  2. Non-trivial evolution after horizon crossing