

Cosmology challenges brane scenarios in AdS₅

arXiv:1112.4783

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PPC2012 – KIAS, Seoul

VIIth International Workshop on the Interconnection between
Particle Physics and Cosmology

November 8th, 2012

Outline

introduction

at the brane: effective Einstein-like equation

AdS₅ bulk: large-scale structure on the brane?

conclusions

beyond General Relativity

- ▶ ongoing search for a unified description of
 - gravity
 - gauge interactions of the Standard Model
 - ↳ *string theories* amongst the most promising proposals
- ▶ *low-energy effective action* in string theories (only massless modes)
 - depending on the string theory type: includes terms for various particles
 - *dilaton* (ϕ): scalar field accompanying gravity (common for all string theories)
 - at the leading order (when restricted to gravity and the dilaton)
 - ↳ Einstein's gravity coupled to the dilaton
 - in a spacetime with additional spatial dimensions
 - ↳ i.e. an extended, higher-dimensional theory of gravity: *dilaton gravity*
- ▶ **dilaton gravity in a 5D brane scenario**
 - Standard Model localized on a 4D brane embedded in a 5D spacetime

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scalar-tensor theories of gravity & conformal frames

► *dilaton gravity*: a scalar-tensor theory of gravity

↪ can be formulated in various **conformally-related frames**

○ $g_{\mu\nu}$ & $\tilde{g}_{\mu\nu}$ related by a Weyl (conformal) transformation: $g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu}$

○ gravitational Lagrangians differ e.g. in the coefficient of the Ricci scalar

↪ (generically) scalar field dependent coefficients

○ Einstein frame: $\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} + \dots$

(natural in standard gravity & cosmology; coefficient: a constant)

○ Jordan frame: e.g. $\mathcal{L} = \frac{1}{16\pi} \phi \mathcal{R} + \dots$

(“traditional” frame in scalar-tensor theories of gravity;
coefficient: a polynomial function of the scalar field)

○ string frame: e.g. $\mathcal{L} = e^{-\phi} \frac{\alpha_1}{2} \mathcal{R} + \dots$

(natural in string theories; coefficient: an exponential function of the dilaton)

non-minimal matter-dilaton coupling

- ▶ if a matter term \mathcal{L}_m is included into the Lagrangian in one frame
 - ↻ conformal transformation to another frame will change its coefficient
 - ↔ if constant in one frame, it will become dilaton dependent in others
- ▶ which **conformal frame** is the *natural physical* frame?
 - ↻ no clear consensus in the literature
 - ↔ in which frame the matter-dilaton coupling should be minimal?
- ▶ thus: a general **non-minimal coupling** $f(\phi) \mathcal{L}_m$
 - ↻ of the **dilaton**
 - ↔ to the **matter content** of the universe \mathcal{L}_m
(localized on the brane)

the aim of the game

► framework:

○ dilaton gravity in a 5D brane scenario

○ non-minimal matter-dilaton coupling $f(\phi) \mathcal{L}_m$

(\mathcal{L}_m : matter content of the universe localized on the brane)

► take **assumptions** crucial to many models in the modern literature:

○ *bulk*: **exact** anti de Sitter type spacetime (**AdS₅**)

(vital for many higher-dimensional scenarios, allows for a 4D Minkowski foliation)

○ *brane*: matter content of the universe

(as in cosmological considerations)

described by an inhomogeneous **perfect fluid**

► and **answer** the **question**:

*can the observed **large-scale structure of the universe***

exist on the brane in an AdS₅ bulk?

i.e.: is a “sufficiently” inhomogeneous perfect fluid (and thus the large-scale structure) permitted on the brane?

another point of view

- ▶ for standard **Einstein's gravity** & **exact AdS₅** [Shiromizu, Maeda, Sasaki (2000)]
 - i.e. *no dilaton* & its non-minimal coupling to matter
 - matter content of the universe as a perfect fluid on the brane
 - ↳ ONLY perfectly **spatially homogeneous universes** are allowed

- ▶ here: including an “additional degree of freedom” - the **dilaton**
 - and a **non-minimal coupling** to the matter content of the universe
 - ↳ will it be *enough* for a **sufficiently inhomogeneous** perfect fluid on the brane?
 - as a crude description of our world
 - ↳ in terms of the cosmological large-scale structure as is observed today

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dilaton gravity at the brane with general matter-dilaton coupling

- ▶ dilaton gravity in a 5D brane scenario: (Einstein frame)

$$\mathcal{L} = \frac{\alpha_1}{2} \left[\mathcal{R} - \frac{2}{3} \nabla^\sigma \partial_\sigma^{(5)} \phi - \frac{1}{3} (\partial^{(5)} \phi)^2 \right] - V(\phi) + [f(\phi) \mathcal{L}_m + \lambda(\phi)] \delta_B$$

- $\left[\mathcal{R} - \frac{2}{3} \nabla^\sigma \partial_\sigma^{(5)} \phi - \frac{1}{3} (\partial^{(5)} \phi)^2 \right]$: 5D dilaton gravity (α_1 : constant)
- ($V(\phi)$: scalar potential in the bulk)

- \mathcal{L}_m : (brane localized) matter content of the universe

$\lambda(\phi)$: 'cosmological constant'-type term on the brane

↪ position of the co-dimension 1 brane: Dirac delta type distribution δ_B

- $f(\phi) \mathcal{L}_m$: (non-minimal) coupling of the dilaton ϕ to brane localized matter \mathcal{L}_m

- ▶ how does the gravity look like on the brane? (i.e. for us)

↪ its effective 4D description has to be established

derivation of the effective gravitational equations at the brane

- ▶ induced (projected) brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ (covariant approach)
 - n^μ : vector field orthonormal to the brane at its position
 - $g_{\mu\nu}$: $\mathcal{R}_{\mu\nu}{}^{\rho\sigma}$ & ∇_μ vs $h_{\mu\nu}$: $R_{\mu\nu}{}^{\rho\sigma}$ & D_μ
- ▶ assume a \mathbb{Z}_2 symmetry for the bulk (with its fixed point at the brane's position)
 - ↳ usually imposed 'automatically'
 - crucial for the existence of the effective gravitational equations at the brane
- ▶ in the absence of the bulk \mathbb{Z}_2 symmetry
 - only consistency condition (on the brane sources): $D_\lambda (f(\phi)\tau_\mu^\lambda) = f(\phi)\tau_\phi(\partial_\mu\phi)$
(on the brane: "generalized" covariant conservation of the energy-momentum tensor)

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- ▶ induced (projected) brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ (covariant approach)
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- ▶ assume a \mathbb{Z}_2 symmetry for the bulk (with its fixed point at the brane's position)
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- ▶ in the absence of the bulk \mathbb{Z}_2 symmetry
 - only **consistency condition** (on the brane sources): $D_\lambda (f(\phi)\tau_\mu^\lambda) = f(\phi)\tau_\phi(\partial_\mu\phi)$
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at the brane: effective Einstein-like equation

- ▶ consequently, the *effective Einstein-like equation* at the brane reads

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial\phi)^2 - E_{\mu\nu}$$

$$\circ \bar{G}(\phi) = \frac{-1}{48\pi\alpha_1^2} f(\phi)\lambda(\phi) \quad (\text{effective brane Newton's constant})$$

$$\circ \tau_{\mu\nu} = h_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta h^{\mu\nu}}, \tau_\phi = \frac{f'(\phi)}{f(\phi)} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta \phi} \quad (\text{brane localized sources})$$

$$\circ \bar{\Lambda}(\phi) = \frac{1}{2\alpha_1} V - \frac{f^2}{4\alpha_1^2} \left[\frac{3}{4} \tau_\phi^2 + \frac{3\lambda'}{2f} \tau_\phi - \frac{\lambda^2}{3f^2} + \frac{3\lambda'^2}{4f^2} \right] \quad (\text{eff. brane cosmol. const.})$$

- ▶ three types of 'corrections' to the standard Einstein equation

- terms quadratic in the brane energy-momentum tensor:

$$\pi_{\mu\nu} = -\tau_{\mu\rho} \tau_\nu^\rho + \frac{1}{3} \tau \tau_{\mu\nu} + \frac{1}{2} h_{\mu\nu} \tau_\rho^\sigma \tau_\sigma^\rho - \frac{1}{6} h_{\mu\nu} \tau^2 \quad (\text{typical of brane gravity theories})$$

- kinetic terms for the dilaton (typical of scalar-tensor theories of gravity)

- explicit **bulk's influence on the brane gravity**: $E_{\mu\nu} = n^\alpha h_\mu^\beta n^\gamma h_\nu^\delta C_{\alpha\beta\gamma\delta}$
(bulk Weyl tensor projected on the brane: a single term, but generically non-vanishing)

↪ to describe gravity induced on the brane: solution of the e.o.m.'s for the bulk gravity necessary

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on the brane:

constraint on the spatial derivative of the matter energy density

▶ OR: “sufficiently” inhomogeneous perfect fluid on the brane in AdS₅ bulk?

▶ calculus ingredients:

○ effective gravitational (Einstein-like) equation at the brane:

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} \\ + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial\phi)^2 - E_{\mu\nu}$$

○ consistency condition (on the brane sources): $D_\lambda (f(\phi) \tau_\mu^\lambda) = f(\phi) \tau_\phi (\partial_\mu \phi)$

○ **4D Bianchi identity**: $D^\nu (R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R) = 0$

▶ assumptions:

○ bulk: exact anti de Sitter type spacetime: **AdS₅** $\rightarrow E_{\mu\nu} = 0$

(no bulk influence on the brane gravity)

○ brane (matter content of the universe): **perfect fluid** $\rightarrow \tau_{\mu\nu} = \rho_m t_\mu t_\nu + p_m \gamma_{\mu\nu}$

($\gamma_{\mu\nu}$: 3d spatial metric, ρ_m : (dark) matter & radiation)

on the brane: late universe

- ▶ thus, spatial derivative of the matter energy density on the brane reads

$$\rho_{m,i} = - \left(\frac{f'}{f} \rho_m - \frac{\lambda'}{f} \right) \phi_{,i} + \frac{\alpha_1^2}{3f^2(\rho_m + \rho_m)} \left[D^\nu \partial_i \phi - \dot{\phi}^{-1} \phi_{,i} D^\nu \partial_t \phi \right] (\partial_\nu \phi)$$

- ↳ imposes a strict condition on the matter content of the universe
(potentially: a strong constraint on how inhomogeneous the matter distribution can be
- and thus on the cosmological large-scale structure as is observed today)

- ▶ (at least) late universe: terms $\mathcal{O}((\partial\phi)(D\partial\phi))$ can be neglected, as

$$\odot \dot{\phi}_0 \lesssim 2.4 H_0 \simeq 1.8 (10^{10} \text{ yr})^{-1}$$

(derived: model-independent bound set by current observational data)

$$\odot |\ddot{\phi}_0| \ll \dot{\phi}_0^2 \text{ can be assumed / expected}$$

(otherwise: currently observed $\phi_0 \approx \text{const}$ would be yet another coincidence problem)

$$\odot \text{ typical models: } |\phi_{,i}| \lesssim c_1 |\dot{\phi}| \quad (c_1 > 0 \text{ and of order } 1)$$

(any initial inhomogeneities of the dilaton washed out by inflation)

- ▶ hereafter: $\lambda \neq \lambda(\phi)$ ('cosmological constant'-type term in the energy-momentum tensor on the brane)
(only a contribution to the effective brane cosmological constant $\bar{\Lambda}(\phi)$)

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- ▶ (at least) **late universe**: terms $\mathcal{O}((\partial\phi)(D\partial\phi))$ can be neglected, as

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late universe: spatial derivative of the energy density - quantify it!

- ▶ hence for the **late universe** we obtain

$$\rho_{m0,i} \simeq -\frac{f'}{f} \rho_{m0} \phi_{0,i}$$

highly **constrained**: how **inhomogeneous** the **matter** energy density can be

for the common assumptions of **AdS₅** bulk and **perfect fluid** on the brane

- ▶ inhomogeneous perfect fluid ($\rho_{m,i} \neq 0$) on the brane? *only if:*
matter content of the universe coupled **non-minimally** ($f' \neq 0$) to the *dilaton*
- ▶ if dilaton spatially homogeneous: no inhomogeneous matter energy density
 ↪ here already: $\dot{\phi}_0 \lesssim 2.4 H_0 \simeq 1.8 (10^{10} \text{ yr})^{-1}$ (as for *no dilaton* case!)

let's quantify the implications of the constraint on $\rho_{m0,i}$:

- ▶ current observational limits: $|\dot{\bar{G}}_0/\bar{G}_0| < (10^{11} \text{ yr})^{-1}$ ($\bar{G} = \bar{G}(\phi)$)
 (searches for time variation of the Newton's constant:

pulsar timing, solar system, stellar, cosmological constraints)

$$\Leftrightarrow \left| \frac{f'}{f} \phi_{0,i} \right| \lesssim 3.3 c_1 (10^5 \text{ Mpc})^{-1} \quad (\text{for } |\phi_{0,i}| \lesssim c_1 |\dot{\phi}|) \quad \text{resulting in}$$

$$|\rho_{m0,i}| \lesssim 3.3 c_1 \rho_{m0} (10^5 \text{ Mpc})^{-1} \quad \text{a stringent constraint!}$$

late universe: spatial derivative of the energy density - quantify it!

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highly **constrained**: how **inhomogeneous** the **matter** energy density can be for the common assumptions of **AdS₅** bulk and **perfect fluid** on the brane

- ▶ inhomogeneous perfect fluid ($\rho_{m,i} \neq 0$) on the brane? *only if:*
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cosmological large-scale structure (LSS) data

- ▶ LSS of the universe: spatial distribution of galaxies, their groups and clusters
- ▶ galaxy distribution: probed by **galaxy redshift surveys**
 - (addressed: content and statistical properties of the LSS)
(e.g. Sloan Digital Sky Survey (SDSS))
 - ↳ characterized statistically through the two-point correlation function $\xi(x)$
(excess number of galaxy pairs separated by x relative to that expected for a random distribution)
- ▶ galaxy surveys \rightsquigarrow *estimation* of the two-point correlation function of galaxies
 - ↳ data usually presented in the form of the power spectrum of the perturbation field
(i.e. Fourier transformation of the two-point correlation function)

confrontation with LSS data

▶ time to *compare*

- ↻ upper limit on $\rho_{m0,i}$ as *predicted* by the model (dilaton gravity, brane, AdS₅)
 - (constraint on the allowed values of the spatial derivative of the matter energy density on the brane)
 - (combined with current *observational limits* on the **Newton's constant (non-)variation**)
- ↪ with $\rho_{m0,i}$ estimation from the *observational data* on the **LSS of the universe**

▶ approximations:

(*aim*: just an estimation - allowing to compare orders of magnitude)

- ↻ for the spatial derivative: $\rho_{m,i} \simeq \frac{\rho_m(x_1) - \rho_m(x_2)}{|x_1 - x_2|}$
- ↻ LSS surveys probe the overall baryonic matter distribution
- ↻ spatial distributions of baryonic and dark matter similar
 - (typical of most dark matter models)

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confrontation with LSS data

↪ and the outcome is . . .

$$\rho_{m0,i} \text{ [model's prediction - upper limit]} \ll \rho_{m0,i} \text{ [LSS data estimation]}$$

(within the entire range of measured scales)

i.e. **brane scenario of dilaton gravity with AdS₅ bulk**

(or any other spacetime with vanishing Weyl tensor)

(and **matter** content of the universe described by a **perfect fluid**)

means **NO large scale structure** as is *observed today*

⊙ thus including the dilaton & its non-minimal coupling to matter

↪ improved the situation

⊙ but: observations of the **Newton's constant (non-)variation** *too restrictive*

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⦿ dilaton gravity studied in a 5D brane scenario

- ⦿ non-minimal coupling $f(\phi)\mathcal{L}_m$ of dilaton to matter content of the universe
localized on the brane
- ↪ derived: effective gravitational equations at the brane

⦿ *can large-scale structure of the universe exist on the brane?*

(*inhomogeneous* matter content of the universe)

- ↪ investigated for AdS₅ bulk & **perfect fluid** on the brane (matter content of the universe)
- ⦿ employed: effective gravitational eqs. at the brane, 4D Bianchi identity (only!)

↪ **spatial derivative** of the brane **matter** energy density *strongly constrained*

- ⦿ non-minimal **dilaton-matter coupling** essential (in the Einstein frame!)
- ⦿ result quantified with current limits (very constraining!)
from the searches for time variation of the Newton's constant
- ⦿ and confronted with the observational data on LSS from galaxy surveys

↪ within the entire range of measured scales

NO large-scale structure as is observed today

dilaton gravity brane scenario **ruled out?** surely **NOT!** next step: **forget exact AdS₅** (& $E_{\mu\nu} = 0$)