# The New Minimal Supersymmetric SO(10) GUT

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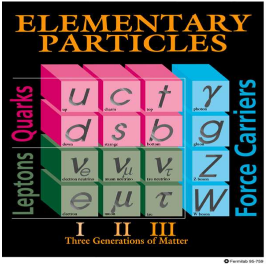
# Outline

- Introduction: Grand Unification
- Supersymmetric SO(10) GUT
- MSGUT: Minimal Susy GUT
- MSGUT SEESAW Failure
- New Minimal Susy GUT
- Fermion Fitting: NMSGUT
- Summary

# Standard Model(SM)

Standard Model(SM) is a renormalizable, spontaneously broken QFT describing the strong and electroweak interactions.



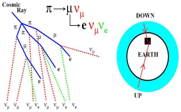

1. Gauge Group: $SU(3) \times SU(2)_L \times U(1)_Y$
2. Higgs field(generation of Masses)

- Symmetry Breaking: $SU(3) \times SU(2)_L \times U(1)_Y \Rightarrow SU(3) \times U(1)_Q$
- Explains all the low energy data successfully so far observed.

# Looking Beyond SM

SM - effective field theory i.e. low energy limit of more fundamental theory.
Look at various extensions(GUT, susy, extra dimensions..etc..)

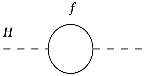
- Strong Gravity: $M_P \sim 10^{19}\,Gev$
  Gauge Coupling unification: $M_{GUT} \sim 10^{16}\,Gev$
- Neutrino Oscillations: No $\nu_R$




1. Seesaw :

$$M_\nu \sim Y\frac{v^2}{M} \Rightarrow M \sim 10^{14\pm1}GeV \approx M_{GUT}!!$$

- Structural unstability in SM: Corrections to Higgs Mass




1. Higgs Corrections: $\Delta m_H^2 \sim \frac{y_t^2 M^2}{16Pi^2}$
   If $M \sim 10^{16}$Gev then $\Delta m_H^2 \sim O(M^2)$

- Recent LHC Higgs Signal $\sim$ 125 GeV
  Excess of events in $\gamma\gamma, ZZ^*, WW^*$ channels

# Grand Unification

One beautiful idea is Grand unification.

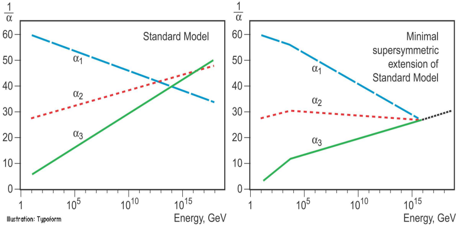
- Idea Grand Unification: J Pati and A. Salam
  $SU(3) \times SU(2)_L \times U(1)_Y \subset SU(4) \times SU(2)_L \times SU(2)_R$
  Quark-Lepton Unification( Phys. Rev. D **10**, 275(1974))
- SU(5) : H.Georgi and S.Glashow
  $SU(5) \rightarrow SM \rightarrow SU(3)_c \times U(1)_Q$
  Strong, Weak and em Unification(PRL. **32**, 438(1974))
- SO(10): H.Fritzsch and P. Minkowski
  $SU(4) \times SU(2)_L \times SU(2)_R, SU(5) \subset SO(10)$
  (Annals Phys.**93**,193(1975))
- Simple models ruled out : proton decay constraints!!
  Minimal SU(5) ($\tau_p > 10^{33}$ years)

Thus look for more refined models(SUSY GUTS!!)

# Motivations: Supersymmetry

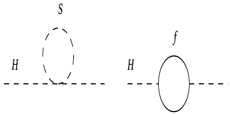
Supersymmetric GUT have a no. of nice features associated with them.

- Gauge coupling Unification




1. SM: No meeting at one point. $M_{GUT} \sim 10^{15}$ Gev
   MSSM: Couplings meet at a point $M_{GUT} \sim 2 \times 10^{16}$ Gev

- Susy solves Hirearchy Problem!:




1. $\Pi^f_{hh}(0) + \Pi^s_{hh}(0)$ free of quadratic divergences if $\lambda_s = -\lambda_f^2$

- Susy predictions: $m_t \sim 200 Gev$ and $\sin^2\theta_w \sim .233$
  Much before their experimental confirmation!!

# SO(10): Generic Features

SO(10) GUT's have a number of remarkable features which make them leading contender for BSM Physics.

- spinor representaion SO(2n): $2^{n-1}$ dimension SO(10)-16 dimensional
- Parity breaking:Left-Right symmetry($SU(4) \times SU(2)_L \times SU(2)_R$ )
- Natural seesaw connection between neutrino mass and GUT scale Seesaw :

$$M_\nu \sim \frac{v_W^2}{M_{B-L}} \Rightarrow M_{B-L} \sim 10^{14\pm1} GeV \approx M_X!!!!!$$

- Gauge unification can be acheived with or without supersymmetry
- Susy SO(10): $R_P = (-1)^{3(B-L)+2s} \subset U(1)_{B-L}$ Certain class of these models R parity conserving

# SO(10): Representations

Under Pati-Salam($SU(4) \times SU(2)_L \times SU(2)_R$):

- Matter Supermultiplets

$$\begin{aligned} 16 &= (4,2,1) + (\bar{4},1,2) \\ 16 \otimes 16 &= 10 \oplus 120 \oplus 126 \Rightarrow 16 \cdot 16 \cdot (10 + 120 + \overline{126}) \end{aligned}$$

- Higgs Supermultiplets

$$\begin{aligned} 10_H &= (\mathbf{1},\mathbf{2},\mathbf{2}) + (6,1,1) \\ \overline{126}_H &= (\mathbf{10},\mathbf{1},\mathbf{3}) + (\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) + (\mathbf{15},\mathbf{2},\mathbf{2}) + (6,1,1) \\ 120_H &= (10,1,1) + (\overline{10},1,1) + (\mathbf{15},\mathbf{2},\mathbf{2}) + (6,1,3) \\ &\quad + (6,3,1) + (\mathbf{1},\mathbf{2},\mathbf{2}) \\ 210_H &= (\mathbf{15},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{15},\mathbf{1},\mathbf{3}) + (15,3,1) \\ &\quad + (6,2,2) + (10,2,2) + (\overline{10},2,2) \end{aligned}$$

# Role Of Different Rep's

Thus a complete viable model can be constructed by choosing different rep's.

- 16 can contain $(Q_L, u_L^c, d_L^c, L_L, e_L \bigoplus \nu_L^c)$
- Charged fermion Masses$(16 \cdot 16 \cdot (10 + \overline{126} + 120))$
  (1,2,2)$\subset$ 10, (15,2,2)$\subset$ $\overline{126}$, (1,2,2),(15,2,2)$\subset$ 120
  Only one not sufficient!!...give bad mass relations!
- $\overline{126}$: Type I + Type II(seesaw masses!!)

$$\begin{aligned} M_{\nu_R} &= <(10,1,3)> Y_{\overline{126}}, M_{\nu_L} =< (\overline{10},3,1) > Y_{\overline{126}} \\ M_\nu &= -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L} \end{aligned} \quad (1)$$

- (15,1,1),(1,1,1),(15,1,3) $\subset$ 210
  Completes symmetry breakdown: SO(10)$\rightarrow$ MSSM by 210.

# Minimal Supersymmetric Grand Unified Theory(MSGUT)

This theory was proposed long ago[1,2] but recently studied in detail

- Superpotential:($\mathbf{10} \oplus \overline{\mathbf{126}} \oplus \mathbf{126} \oplus \mathbf{210}$)

$$\begin{aligned} W_{AM} &= \frac{1}{2} M_H H_i^2 + \frac{m}{4!}\Phi_{ijkl}\Phi_{ijkl} + \frac{\lambda}{4!}\Phi_{ijkl}\Phi_{klmn}\Phi_{mnij} \\ &+ \frac{M}{5!}\Sigma_{ijklm}\overline{\Sigma}_{ijklm} + \frac{\eta}{4!}\Phi_{ijkl}\Sigma_{ijmno}\overline{\Sigma}_{klmno} \\ &+ \frac{1}{4!}H_i\Phi_{jklm}(\gamma\Sigma_{ijklm} + \overline{\gamma}\overline{\Sigma}_{ijklm}) \end{aligned}$$

and

$$W_{FM} = h_{AB}\psi_A^T C_2^{(5)}\gamma_i\psi_B H_i + \frac{1}{5!} f_{AB}\psi_A^T C_2^{(5)}\gamma_{i_1}...\gamma_{i_5}\psi_B\overline{\Sigma}_{i_1...i_5}$$

- 26 Hard Parameters(Minimal theory!!..Aulakh etal..hep-ph/0306242)

[1]C.S. Aulakh and R.N. Mohapatra, Phys.Rev.D28,217(1983).
[2]T.E. Clark etal..Phys. lett. **115B**, 26(1982).

# MSGUT: Symmetry breaking

- 

$$\langle(15,1,1)\rangle_{210} \quad : \quad \frac{a}{2} \tag{2}$$

$$\langle(15,1,3)\rangle_{210} \quad : \quad \omega \tag{3}$$

$$\langle(1,1,1)\rangle_{210} \quad : \quad p \tag{4}$$

$$\langle(10,1,3)\rangle_{\overline{126}} \quad : \quad \bar{\sigma} \tag{5}$$

$$\langle(\overline{10},1,3)\rangle_{126} \quad : \quad \sigma. \tag{6}$$

The standard model vacuum in units of (m/$\lambda$) are $\tilde{\omega} = -x$ and

$$\tilde{a} = \frac{(x^2+2x-1)}{(1-x)} \quad ; \quad \tilde{p} = \frac{x(5x^2-1)}{(1-x)^2} \quad ; \quad \tilde{\sigma}\tilde{\bar{\sigma}} = \frac{2}{\eta}\frac{\lambda x(1-3x)(1+x^2)}{(1-x)^2} \tag{7}$$

where x is a solution of the cubic equation:

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2 \tag{8}$$

with $\xi = \frac{\lambda M}{\eta m}$.(see Aulakh etal refrences!!...)

# MSGUT: RG Analysis

- Superheavy spectrum Calculation !!
  (Aulakh etal, B.Bajc etal and T.Fukuyama etal)
- Hall defines the matching functions $\lambda_i$ in terms of gauge couplings $\alpha_i$ to the grand unified coupling $\alpha_G$:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln\frac{M_X}{M_Z} + 4\pi\sum_j \frac{b_{ij}}{b_j}\ln X_j - 4\pi\lambda_i(M_X) \quad (9)$$

here

$$X_j = 1 + 8\pi b_j \alpha_G(M_X^0) ln\frac{M_X^0}{M_Z} \quad (10)$$

is understood to be evaluated at the values of $M_X^0, \alpha_G(M_X^0)$ determined from the one loop calculations.

$$\lambda_i(\mu) = -\frac{2}{21}(b_{iV} + b_{iGB}) + 2(b_{iV} + b_{iGB}) ln\frac{M_V}{\mu} \quad (11)$$

$$+ \quad 2b_{iS} ln\frac{M_S}{\mu} + 2b_{iF} ln\frac{M_F}{\mu} \quad (12)$$

# MSGUT: RG Analysis

- One loop values $\alpha_G(M_X), Sin^2\theta_w and M_X$ remains stable against superheavy threshold corrections!![1]
- $sin^2\theta_w(M_Z)$ precisely known(0.1%) so usual to choose to predict $\alpha_3(M_Z)$ which carries largest uncertainty
- $$\begin{aligned}
\Delta^{(th)}(ln M_X) &= \frac{\lambda_1(M_X) - \lambda_2(M_X)}{2(b_1 - b_2)} \\
\Delta^{(th)}(\alpha_3(M_Z)) &= \frac{100\pi(b_1 - b_2)\alpha(M_Z)^2}{[(5b_1 + 3b_2 - 8b_3)sin^2\theta_w(M_Z) - 3(b_2 - b_3)]^2} \\
&\quad . \sum_{ijk} \epsilon_{ijk}(b_i - b_j)\lambda_k(M_X) \\
\Delta^{(th)}(\alpha_G^{-1}(M_X)) &= \frac{4\pi(b_1\lambda_2(M_X) - b_2\lambda_1(M_X))}{b_1 - b_2} \qquad (13)
\end{aligned}$$

[1]C.S. Aulakh and A. Girdhar, Nucl. Phys. B **711**, 275(2005).

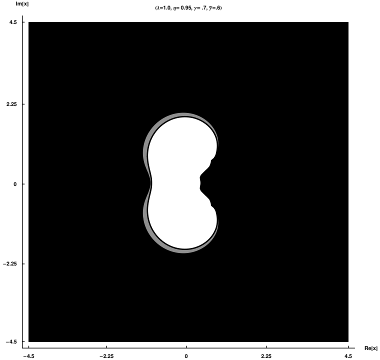
# MSGUT: RG Analysis

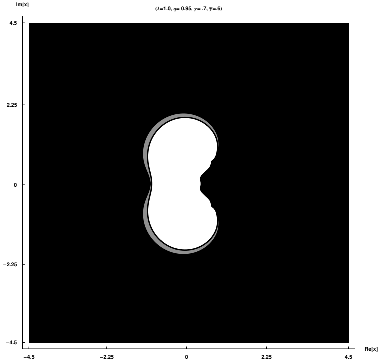
- $$\alpha_s(M_Z) = 0.130 \pm 0.001 + H_{\alpha_s} + \Delta_{\alpha_s} \quad (14)$$
  $$+ \quad 3.1 \times 10^{-7} GeV^{-2} \times [(m_t^{pole})^2 - (172.7 GeV)^2] \quad (15)$$
  $$\Delta^{susy}_{\alpha_s} \approx -\frac{-19\alpha_s^2}{28\pi} ln \frac{M_{susy}}{M_Z} and - 0.003 < H_{\alpha_s}(h_t, h_b) < 0.$$
  For $250 GeV > M_{SUSY} > 20 GeV$ we get $0.005 > \Delta^{Susy}_{\alpha_s} > -0.003$. Thus $-0.006 > \Delta^{GUT}_{\alpha_s} > -0.017$ are required to reconcile with the measured value $\alpha_3(M_Z) = 0.1176 \pm 0.002$.

$$|\Delta_G| \equiv |\Delta(\alpha_G^{-1}(M_X))| \leq 10$$
$$\Delta_X \equiv \Delta(Log_{10} M_X) \geq -1$$
$$-0.017 < \Delta_3 \equiv \Delta\alpha_3(M_Z) < -0.006 \quad (16)$$

# MSGUT: Parameter Space Survey

Contour Plots of $\Delta\alpha_3(M_Z)$ on complex $x$ plane




- 
  - complex parameter x is crucial parameter!!
  - $\lambda, \eta, \gamma, \bar{\gamma} \sim 1.$

# MSGUT: Fermion mass formulae

- Generic mass formulae in MSGUT:

$$\begin{aligned} m^u &= v(\hat{h} + \hat{f}) \\ m^\nu &= v(\hat{h} - 3\hat{f}) \\ m^d &= v(r_1\hat{h} + r_2\hat{f}) \\ m^l &= v(r_1\hat{h} - 3r_2\hat{f}) \end{aligned} \tag{17}$$

Here $v = 174 GeV$

$$\begin{aligned} M_\nu^I &= vr_4 \ \hat{n} \\ M_\nu^{II} &= 2vr_3 \ \hat{f} \end{aligned} \tag{18}$$

where $\hat{n} = (\hat{h} - 3\hat{f})\hat{f}^{-1}(\hat{h} - 3\hat{f})$

# Generic Fermion Fitting IN MSGUT

In generic fits the required relative strength of Type I and Type II is simply assumed and there is no restriction of choosing the values of $r_1, r_2, r_3, r_4$

- Data for GUT To Explain : $m_{q,l}, \theta_i^{CKM}, \delta^{CKM}, M_\nu, \theta_i^{PMNS}$
- Babu and Mohapatra ( 1992 ) : $\mathbf{10} \oplus \overline{\mathbf{126}} \Leftrightarrow \quad m_{q,l}$ Predictive in the Neutrino Sector! : failure (1992)
- Attempts Lavoura(1993) , Lee Mohapatra(1994) , Brahmachari Mohapatra (1998)

* Matsuda, Koide , Fukuyama, Nishiura (2002): Successful Type I , large $\theta^{PMNS}$ fit !

# Generic Fermion Fitting IN MSGUT

- Bajc, Senjanovic, Vissani (2002)

$$M_\nu^{II} \sim f < \Delta_L > \sim (M_d - M_l) \sim m_\tau \begin{pmatrix} & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & & \frac{(m_b - m_\tau)}{m_\tau} \end{pmatrix} \Rightarrow$$

- MSSM : $m_b = m_\tau(M_X)$ BSV : Large PMNS mixing Natural IF

$$\frac{(m_b - m_\tau)}{m_\tau} \sim \epsilon^2$$

- Goh Mohapatra Ng :Type II : 3 generations , Real/Complex : Good Fits except $\delta^{CKM} > \frac{\Pi}{2}$ .
- Bertolini, Malinsky (2004)(Type II, $\oplus$120 ; Babu Macesanu (2005)(Type I and II) **Good Angle and Ratio Fits. !!**
- Magnitude $M_\nu$ and Relative Strength Type I vs Type II INPUT

# MSGUT: Seesaw Mechanism

In fully specified model strength of Type I vs Type II and parameters $r_1, r_2, r_3, r_4$ are fixed in terms of GUT parameters!!

- $$\begin{aligned} m^u &= v(\hat{h} + \hat{f}) \\ m^\nu &= v(\hat{h} - 3\hat{f}) \\ m^d &= v(r_1 \hat{h} + r_2 \hat{f}) \\ m^l &= v(r_1 \hat{h} - 3 r_2 \hat{f}) \end{aligned} \tag{19}$$

  Here $v = 174 GeV$

  $$\begin{aligned} \hat{h} &= 2\sqrt{2} h \alpha_1 \sin\beta; \quad \hat{f} = -4\sqrt{\frac{2}{3}} i f \alpha_2 \sin\beta \\ r_1 &= \frac{\bar{\alpha_1}}{\alpha_1} \cot\beta; \quad r_2 = \frac{\bar{\alpha_2}}{\alpha_2} \cot\beta \end{aligned} \tag{20}$$

  where $\alpha_i, \bar{\alpha}_i$ are components of the null eigenvectors of the doublet mass matrix. superpotential.

# MSGUT: Seesaw Mechanism

The fermion mass formulae in terms of GUT parameters are given by

- $$\begin{aligned} M_\nu^I &= (1.70 \times 10^{-3} eV) \sin\beta F_I \ \hat{n} \\ M_\nu^{II} &= (1.70 \times 10^{-3} eV) \sin\beta F_{II} \ \hat{f} \end{aligned} \tag{21}$$

  where

  $$\begin{aligned} F_I &= \frac{10^{-\Delta_x}}{2\sqrt{2}} \frac{\gamma g}{\sqrt{\lambda\eta}} |p_2 p_3 p_5| \sqrt{\frac{z_2}{z_{16}}} \sqrt{\frac{(1-3x)}{x(1+x^2)}} \frac{q_3'}{p_5} \\ F_{II} &= 10^{-\Delta_x} \frac{2\gamma g}{\sqrt{\eta\lambda}} \frac{|p_2 p_3 p_5|}{(x-1)} \sqrt{\frac{z_2}{z_{16}}} \sqrt{\frac{(x^2+1)}{x(1-3x)}} \frac{(4x-1)q_3^2}{q_3' q_2 p_5} \\ R &= |\frac{F_I}{F_{II}}| = |\frac{(x-1)(3x-1) q_2 q_3'^2}{4\sqrt{2}(4x-1)(x^2+1) q_3^2}| \end{aligned} \tag{22}$$

# Type II seesaw: No dominace

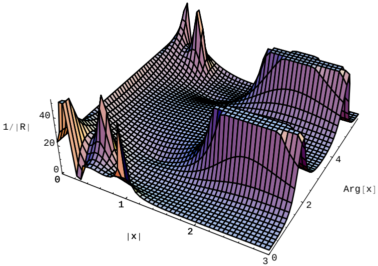



Figure: $R^{-1}$ vs $x$ on complex plane.

- $|x| \to \infty$ it *grows* as $(3/(64\sqrt{2}))|x|$. For $|x| \geq 3 \Rightarrow R \geq 10^{-1}$ or so.

- This is in fact the region which contains the zeros of R namely $x = \{1/3, 1, \{(3 \pm i\sqrt{7})/8\}, \{0.198437, -.0992186 \pm 2.24266i\}\}$ of which the last two sets are the zeros of $q_2$ and $q_3'$ respectively.

# Type II seesaw: No dominace

- $$\begin{aligned} & \{x_i, \epsilon_i, \bar{R}_{min}(x_i), \{USMPs\}\} \\ = & \{1, .014, .01, \{\Delta_X < -9.79, \Delta\alpha_s < -.433\}\} \\ = & \{.3333, .0025, .02, \{\Delta_X > 4.86, \Delta\alpha_s > 0.063\}\} \\ = & \{.198437, .0025, 7 \times 10^{-5}, \{\Delta_X < -3.2, \Delta\alpha_s < -.019\}\} \\ = & \{-.099219 + 2.2426i, .02, 2.4 \times 10^{-5}, \\ = & \{\Delta\alpha_G^{-1} > 15.4, \Delta\alpha_s < -.034\}\} \end{aligned}$$

- $$\begin{aligned} & \{x, \epsilon, \bar{R}_{min}(x_i), \{USMPs\}\} = \\ & \{(3 + i\sqrt{7})/8, .06, .021, \{\Delta\alpha_s > .012, \Delta\alpha_G^{-1} > 18.8\}\} \\ & \{(3 + i\sqrt{7})/8, .005, .002, \{\Delta\alpha_s > .059, \Delta\alpha_G^{-1} > 18.9\}\} \quad (23) \end{aligned}$$

The parameters of the MSGUT can *never* be chosen to ensure Type II domination while maintaining viable USMPs.

# Type I: Not Strong enough

- $$\begin{aligned} M_I^\nu &= 1.7\times 10^{-3} eV F_I\ \hat{n}\sin\beta \\ F_I &= \frac{\gamma g}{2\sqrt{2\eta\lambda}}\sqrt{\frac{((1-3x)}{x(x^2+1)}}\frac{|p_2 p_3|\sqrt{z_2}}{\sqrt{z_{16}}}10^{-\Delta_X} q_3' = \hat{F}_I 10^{-\Delta_X} \end{aligned} \quad (24)$$

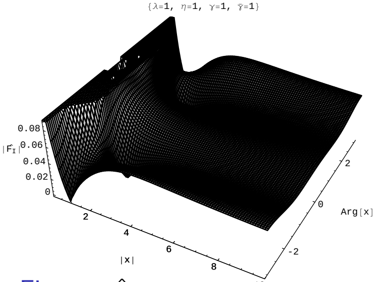



Figure: $|\hat{F}_I|$ vs $x$ on complex plane.

- $m_\nu \sim .05eV\hat{F}_I \sim 10(\hat{n} \sim 0.3\, and\, \Delta_X >$ $-1)$

$$\begin{aligned} |x| &\to \infty \Rightarrow |\hat{F}_I| \to\sim .02 \\ |x| &\to \{0, \pm i\} \Rightarrow |\hat{F}_I| \to \infty \end{aligned} \quad (2$$

# Type I: Not Strong enough

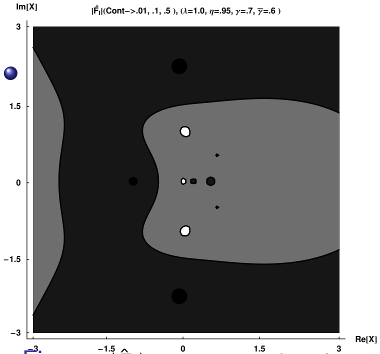



Figure: $|\hat{F}_I|$ vs $x$ on complex plane.

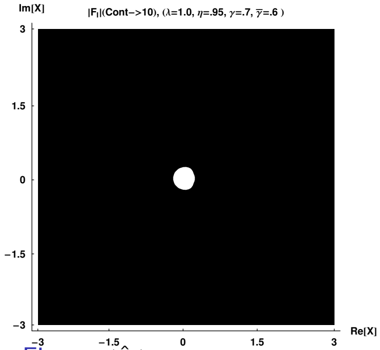



Figure: $|\hat{F}_I|$ vs $x$ on complex plane.

- Around x=0, $\Delta_X < -1$, and near $x \pm i, \Delta_X > 1$ and hence suppress the favourable value of $\hat{F}_I$.
- **Thus even with Type I seesaw there is no possibility of achieving neutrino masses larger than about** $5 \times 10^{-3} eV$ **in the MSGUT !**.

# MSGUT: Conclusions

- Generallically no type II dominance. Even near exceptional points no viable unification.
- Type I seesaw: $(m_\nu)_{max} \leq 5 \times 10^{-3}$eV.
- Thus combined constraints of seesaw fit and stability of one loop gauge unification are enough to ruin the compatibility of the MSGUT with generic Type I and Type II seesaw mechanisms.
- Thus one should investigate the role of missing piece(i.e. **120**-plet).

# New Minimal SUSY GUT

- NO SM VEV: Symmetry Pattern remains same

$$
\begin{aligned}
O_{ijk}(120) &= O^{(s)}_{\mu\nu}(10,1,1) + \overline{O}^{\mu\nu}_{(s)}(\overline{10},1,1) + O_{\nu\alpha\dot{\alpha}}{}^{\mu}(15,2,2) \\
&+ O^{(a)}_{\mu\nu\dot{\alpha}\dot{\beta}}(6,1,3) + O^{(a)}_{\mu\nu}{}_{\alpha\beta}(6,3,1) + O_{\alpha\dot{\alpha}}(1,2,2) \quad (26)
\end{aligned}
$$

- Superpotential:NMSGUT

$$
\begin{aligned}
W_{NMSGUT} &= \frac{m_o}{2(3!)} O_{ijk} O_{ijk} + \frac{k}{3!} O_{ijk} H_m \Phi_{mijk} + \frac{\rho}{4!} O_{ijk} O_{mnk} \Phi_{ijmn} \\
&+ \frac{1}{2(3!)} O_{ijk} \Phi_{klmn} (\zeta \Sigma_{lmnij} + \bar{\zeta} \bar{\Sigma}_{lmnij}) \\
&+ \frac{1}{5!} g_{AB} \Psi_A^T C_2^{(5)} \gamma_{i_1} \gamma_{i_2} \gamma_{i_3} \Psi_B O_{i_1 i_2 i_3} + W_{MSGUT}
\end{aligned}
$$

# NMSGUT: RG Analysis

- Plot NMSGUT Allowed Region over complex $x$ plane.

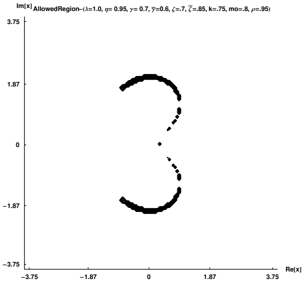



Figure: NMSGUT Allowed Region Over Complex $x$ plane.

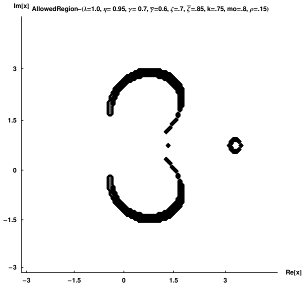



Figure: NMSGUT Allowed Region Over Complex $x$ plane.

- Non diagonal couplings$(\gamma, \bar{\gamma}, \zeta, \bar{\zeta}, k)$ minor influence on unification parameters diagonal$(\lambda, \eta, \rho)$ have mild dependance except when coherent.

# Fermion mass relations

- $$\begin{aligned}
m^u &= v(\hat{h} + \hat{f} + \hat{g}) \quad ; \quad r_1 = \frac{\bar{\alpha}_1}{\alpha_1}\cot\beta \quad ; \quad r_2 = \frac{\bar{\alpha}_2}{\alpha_2}\cot\beta \\
m^\nu &= v(\hat{h} - 3\hat{f} + (r_5 - 3)\hat{g}) \quad ; \quad r_5 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5} \\
m^d &= v(r_1\hat{h} + r_2\hat{f} + r_6\hat{g}); \quad r_6 = \frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}\cot\beta \\
m^l &= v(r_1\hat{h} - 3r_2\hat{f} + (\bar{r}_5 - 3r_6)\hat{g}); \quad \bar{r}_5 = \frac{4i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}\cot\beta \quad (27) \\
\hat{g} &= 2ig\sqrt{\frac{2}{3}}(\alpha_6 + i\sqrt{3}\alpha_5)\sin\beta \quad ; \quad \hat{h} = 2\sqrt{2}h\alpha_1\sin\beta \quad ; (28) \\
\hat{f} &= -4\sqrt{\frac{2}{3}}if\alpha_2\sin\beta
\end{aligned}$$

# NMSGUT Parameter Space

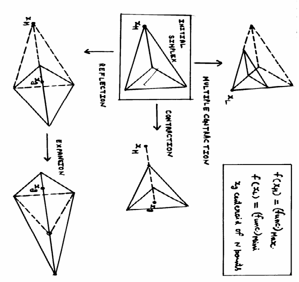
- Superpotential Parameters : $m, M, M_O, \lambda, \eta, \rho, k, \gamma, \bar{\gamma}, \zeta, \bar{\zeta}$
  21-4 (fixed by phase freedom from 5 Higgs fields)=17
  $M_H$ can be fixed by fine tuning to keep two pairs of doublets light.
  Mass parameter $m$ is fixed by RG flow via fixing $M_X$.

$$|m| = 10^{16.25+\Delta_X} \frac{|\lambda|}{g_5\sqrt{2|\tilde{a}+\tilde{w}|^2 + |\tilde{p}+\tilde{\omega}|^2}} GeV = f(x, \lambda, \eta, \gamma ...)$$

- Yukawa sector: 3 (h)+ 12(f)+ 6(g) = 21 parameters.
- Thus in full NMSGUT total 37 parameters are aviliable for fitting of which 15 are phases.
- Data for GUT To Explain : $m_{q,l}, \theta_i^{CKM}, \delta^{CKM}, M_\nu, \theta_i^{PMNS}$

# Fermion Fitting-NMSGUT-I

- Downhill Simplex:




$$\chi^2 = \sum_i \left( \frac{f_i(x) - \bar{f}_i}{\delta f_i} \right)^2$$

- This method does the function evalutions with reflection, contraction, extrapolation etc.to find the best possible minimum.

# Fermion Fitting-Results

- Solution obtained with $\chi^2 \sim 2.7$
- The main features of solutions obtained are:
- Neutrino masses and mixing accurately fit
- $M_R \sim 10^8 - 10^{13} Gev$ ($|f| \sim 10^{-6}$)
- $y_d, y_s$ 2-3 standard deviations below the expected central values.
- Thus the message is clear: unless the SUSY threshold corrections effects lower the $y_{d,s}$ the tree level NMSGUT formulae fail to fit the fermion data.

# Susy Threshold Corrections

After matching SM with MSSM at SUSY scale yukawa couplings are related as[1,2]

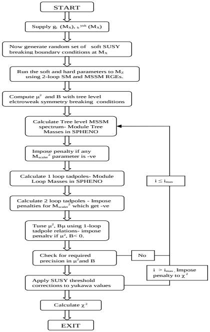
- $$h_i^{MSSM} = \frac{h_i^{SM}}{\cos\beta(1+\eta_i)}$$

  $\eta_i$ govern susy threshold corrections
  trig$\beta$ = sin$\beta$ for $T_{3L} = +1/2$ and cos$\beta$ for $T_{3L} = -1/2$.
  So these corrections can give significant corrections to yukawa couplings in large *tan*$\beta$*limit*.

[1]S. Antusch and M. Spinrath, arXiv:0804.0717
[2]D. M. Pierce etal Nucl. Phys. B **491**,3(1997).

# Inclusion of Susy threshold corrections

- Fitting flow Chart




- $M_1 = M_2 = M_3 = r_1$
  $M^2_{H_1} = r_2, M^2_{H_2} = r_3$
  $M^2_{\tilde{f}} = Diagonal(r_4, r_4, r_4)$
  $(A_e, A_d, A_u) =$
  $Diagonal(r_5, r_5, r_5) \bigotimes (Y_e, Y_d, Y_u)$
- $\chi^2 = \Sigma_i (1 - \frac{y_i'^{MSSM}}{y_i^{SM}})^2$
  $y_i'^{MSSM} = y_i^{MSSM}(trig\beta(1 + \eta_i))$

$$
\begin{aligned}
|\mu|^2 &= \frac{1}{2}[-M_Z^2 + tan2\beta(-(M^2_{H_1} - \frac{t_1}{v_1})cot\beta + (M^2_{H_2} - \frac{t_2}{v_2})tan\beta)] \\
B &= \frac{1}{2}[-M_Z^2 sin2\beta - tan2\beta(M^2_{H_1} - \frac{t_1}{v_1} - M^2_{H_2} + \frac{t_2}{v_2})]
\end{aligned}
$$

# NMSGUT:Conclusions

- The model accurately fit the fermion data.
- only 5 GUT scale soft parameters!!
  (correcting mismatch between SM and MSSM yukawa couplings)
- 3rd generation sfermions are heavy then Ist and 2nd generation (a completely distinct signature of model!!)
- However one should test different yukawa fermion fits so that definite conclusions about soft spectra can be made.
- once it is done one can make predictions about proton decay, B physics etc..for this model

# Susy Threshold Corrections-BackUP

- down quarks suffer corrections from squark/gluino($\tilde{s}\tilde{g}$)

$$\{\frac{\Delta m_q}{m_q}\}^{\tilde{s}\tilde{g}} = -\frac{g_3^2}{12\pi^2}\{B_1(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) + B_1(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2)$$
$$-sin(2\theta_q)(\frac{m_{\tilde{g}}}{m_q})[B_0(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) - B_0(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2)]\}$$

$$\{\frac{\Delta m_q}{m_q}\}^{\tilde{s}\tilde{\chi}^+} = -\frac{y_q^2}{16\pi^2}\mu\frac{A_q^0 tan\beta - mu}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}\{B_1(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2)$$
$$+B_1(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2) - sin(2\theta_q)(\frac{m_{\tilde{g}}}{m_q})[B_0(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2)$$
$$-B_0(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2)]\}$$

- Leptons corrections small colorless particles

$$(\frac{\Delta m_l}{m_l}) = -\frac{g^2}{16\pi^2}\frac{\mu M_2 tan\beta}{\mu^2 - M_2^2}\{B_0(M_2, m_{\nu_l}; M_Z^2) - B_0(M_2, m_{\nu_l}; M_Z^2)\}$$

# Susy Threshold Corrections

- for up quarks no tan$\beta$ enhanced corrections
  squark-gluino contribution is much similar to down quarks
- top quark-gluon correction are given by

$$(\frac{\Delta m_t}{m_t})^{tg} \quad = \quad \frac{g_3^2}{6\pi^2}[2B_0[m_t, m_t; M_Z^2] - B_1[m_t, m_t; M_Z^2]]$$

- Here $B_0(m_1, m_2; Q^2) = -ln(\frac{M^2}{Q^2}) + 1 + \frac{m^4}{m^2 - M^2} ln(\frac{M^2}{m^2})$
  $B_0(m_1, m_2; Q^2) = \frac{1}{2}[\frac{M^2}{Q^2} + \frac{1}{2} + \frac{1}{1-x} + \frac{lnx}{(1-x)^2} - \theta(1-x)lnx]$
  $M = max(m_1, m_2), m = min(m_1, m_2) andx = m_2^2/m_1^2 and\theta$ is unit step function
- for chargino corrections
  $\eta_i = \epsilon_i tan\beta$ so with $\epsilon_i \sim 10^{-2} - 10^{-1}$ and $tan\beta \sim 50 \eta_i \sim 0.5 - 5$