PPC2012, KIAS

Constraints on dark energy models from recent observations

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Observations suggest the late-time cosmic acceleration.

Equation of state: $w = P_{\rm DE}/\rho_{\rm DE}$ ($P_{\rm DE}$: pressure, $\rho_{\rm DE}$: energy density)

For constant w models: $w = -1.001^{+0.348}_{-0.398}$ (SN Ia)

(Suzuki et al, 2011) $w = -1.013^{+0.068}_{-0.073}$ (SNIa+CMB+BAO+H0)

Dark energy candidates

- **The simplest candidate: Cosmological constant**

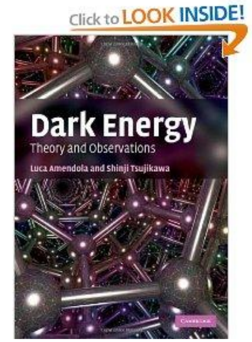
Equation of state $w = -1$

→ If the cosmological constant originates from the vacuum energy, its energy scale is enormously larger than the dark energy scale.

- **Dynamical dark energy models**

Quintessence, k-essence, chaplygin gas, coupled dark energy, f (R) gravity, scalar-tensor theories, DGP model, Galileon,...

Dynamical dark energy models



Amendola and S.T., Cambridge University Press (2010)

1. Modified matter models

- Quintessence: Acceleration driven by the potential energy $V(\phi)$ of a field ϕ

$$\mathcal{L} = X - V(\phi) \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

- K-essence: Acceleration driven by the kinetic energy X of a field ϕ

$$\mathcal{L} = K(\phi, X)$$

e.g. Dilatonic ghost condensate:

$$K = -X + ce^{\lambda\phi}X^2$$

2. Modified gravity models

- $f(R)$ gravity: The Lagrangian is the function of a Ricci scalar R.
- Scalar-tensor gravity: $\mathcal{L} = F(\phi)R + K(\phi, X)$
- DGP model: Acceleration by the gravitational leakage to extra dimensions.
- Galileon gravity: The Lagrangian is constructed to satisfy the Galilean symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in the flat spacetime.

Such as $X\Box\phi$

Most general single-field scalar-tensor theories with second-order equations of motion

Horndeski (1973)
Deffayet et al (2011)
Charmousis et al (2011)
Kobayashi et al (2011)

$$S = \int d^4x\sqrt{-g}\,[K(\phi, X) - G_3(\phi, X)\Box\phi + \mathcal{L}_4 + \mathcal{L}_5]$$

$$\mathcal{L}_4 = G_4(\phi, X)\,R + G_{4,X}\,[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)]$$

$$\mathcal{L}_5 = G_5(\phi, X)\,G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]$$

This action covers most of the dark energy models proposed in literature.

- LCDM: $K = -\Lambda, \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- Quintessence and K-essence: $K = K(\phi, X), \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- f(R) gravity and scalar-tensor gravity: $G_4 = F(\phi), \quad G_3 = 0, \quad G_5 = 0$
- Galileon: $K = -c_2X, \quad G_3 = \frac{c_3}{M^3}X, \quad G_4 = \frac{1}{2}M_{\rm pl}^2 - \frac{c_4}{M^6}X^2, \quad G_5 = \frac{3c_5}{M^9}X^2$
- Gauss-Bonnet coupling $\xi(\phi)\mathcal{G}$:

$$K = 8\xi^{(4)}(\phi)X^2(3 - \ln X), \quad G_3 = 4\xi^{(3)}(\phi)X(7 - 3\ln X),$$

$$G_4 = 4\xi^{(2)}(\phi)X(2 - \ln X), \quad G_5 = -4\xi^{(1)}(\phi)\ln X$$

Horndeski's paper in 1973

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

Gregory Walter Horndeski

MathSciNet

Ph.D. University of Waterloo 1973 

Dissertation: *Invariant Variational Principles and Field Theories*

Advisor: David Lovelock

Friedmann equations on the flat FLRW background

$$S = \int d^4x\sqrt{-g}\,[K(\phi,X) - G_3(\phi,X)\Box\phi + \mathcal{L}_4 + \mathcal{L}_5] + \underline{S_m} + \underline{S_r}$$

(Horndeski's action)

Non-relativistic matter   Radiation

The background equations of motion are

$$3M_{\rm pl}^2 H^2 = \rho_{\rm DE} + \rho_m + \rho_r$$

$$-2M_{\rm pl}^2 \dot{H} = \rho_{\rm DE} + P_{\rm DE} + \rho_m + 4\rho_r/3$$

$\rho_{\rm DE}$ and $P_{\rm DE}$ are the density and pressure of the "dark" component.

$$\rho_{\rm DE} = 2XK_{,X} - K - 2XG_{3,\phi} + 6X\dot{\phi}HG_{3,X} - 6H^2G_4 + 3M_{\rm pl}^2H^2 + 24H^2X(G_{4,X} + XG_{4,XX})$$
$$-12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX})$$

The equation of state of dark energy is given by

$$w = P_{\rm DE}/\rho_{\rm DE}$$

→ The evolution of w is different depending on dark energy models.

Dark energy equation of state: modified matter models

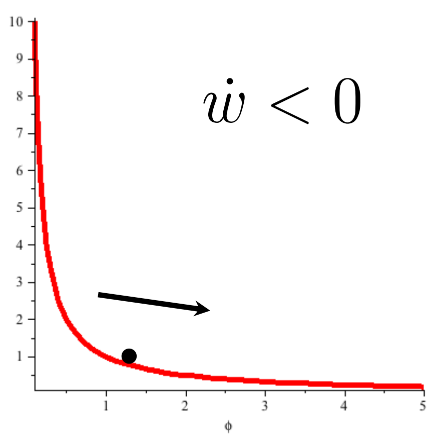
(1) LCDM → $w = -1$

(2) Quintessence → w depends on the potential.

(a) Freezing models

$$V(\phi) = M^{4+n}\phi^{-n}, \quad (n > 0)$$

Caldwell and Linder (2005)

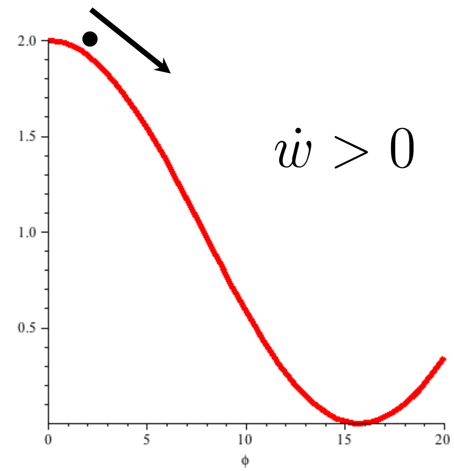


w decreases toward -1

(b) Thawing models

e.g. PNGB boson

$$V(\phi) = \mu^4 \left[1 + \cos(\phi/f)\right]$$



w increases from -1

(3) k-essence

Typically the evolution of w is similar to that in thawing models.

Quintessence equation of state: Freezing and Thawing models

The field equation of state satisfies

$$w' = (1-w)\left[-3(1+w) + \lambda\sqrt{3(1+w)\Omega_\phi}\right]$$

where $\lambda = -\frac{M_{\rm pl}V_{,\phi}}{V}$, $\Omega_\phi = \frac{\rho_\phi}{3M_{\rm pl}^2 H^2}$

There are two distinct cases where w is constant.

(i) Tracker solutions (freezing models): $\Omega_\phi = \frac{3(1+w)}{\lambda^2}$

Tracking (the decrase of λ) occurs for $\Gamma = VV_{,\phi\phi}/V_{,\phi}^2 > 1$

Along the tracker w is nearly constant: $w = w_{(0)} = -\frac{2(\Gamma-1)}{2\Gamma-1}$

Steinhardt et al (1998)

For the potential $V(\phi) = M^{4+p}\phi^{-p}$, we have $w_{(0)} = -2/(p+2)$. (matter era)

(ii) Thawing models: $w \approx -1$

Initially w is close to -1, but w deviates from -1 at late times.

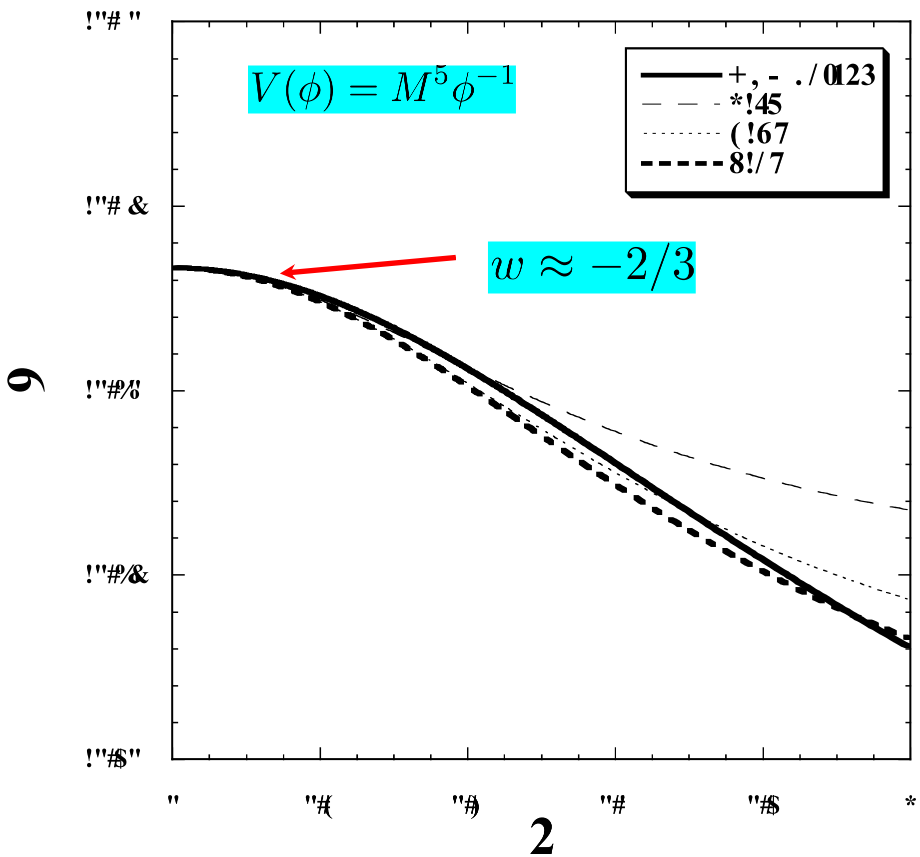
Tracker solutions

Chiba (2010)

Considering a homogeneous perturbation around $w = w_{(0)}$, it follows that

$$w(a) = w_{(0)} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} w_{(0)} (1 - w_{(0)}^2)}{1 - (n+1) w_{(0)} + 2n(n+1) w_{(0)}^2} \left(\frac{\Omega_\phi(a)}{1 - \Omega_\phi(a)} \right)^n$$

Two parameters $w_{(0)}$ and $\Omega_{\phi 0}$



$$\Omega_\phi(a) = \frac{\Omega_{\phi 0} a^{-3 w_{(0)}}}{\Omega_{\phi 0} a^{-3 w_{(0)}} + 1 - \Omega_{\phi 0}}$$

$$w_{(0)} = -\frac{2(\Gamma - 1)}{2\Gamma - 1} = -\frac{2}{p + 2}$$

for $V(\phi) = M^{4+p} \phi^{-p}$

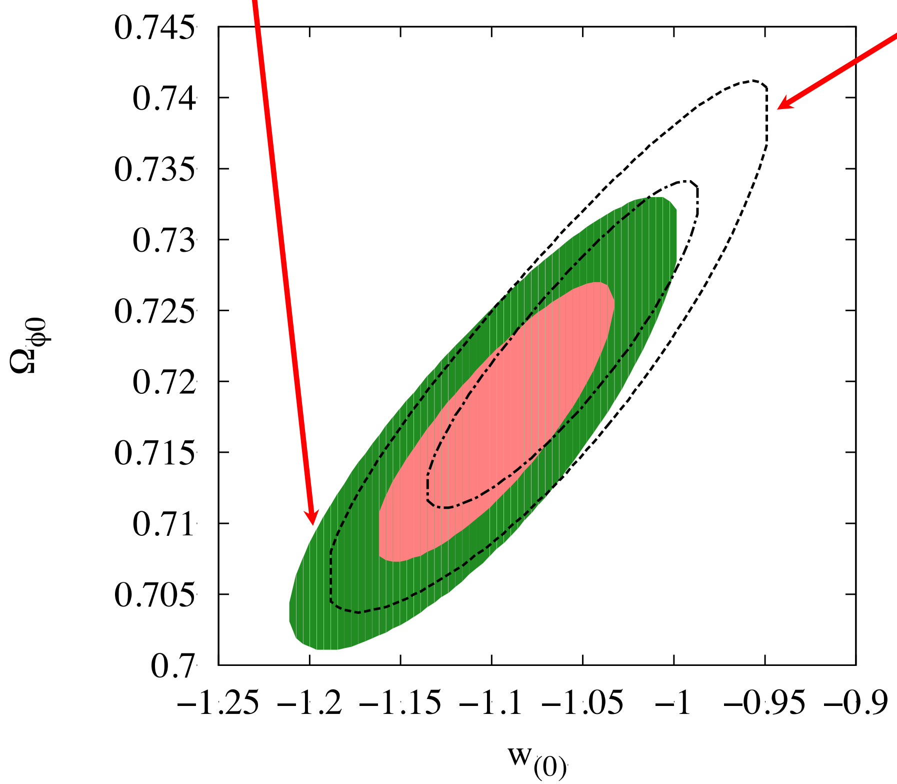
The analytic solution shows good agreement with numerical results.

Likelihood analysis for tracker solutions (SNIa +CMB+BAO)

Chiba, De Felice, S.T.,
1210.3859 [astro-ph.CO]

With the BAO BOSS data: $D_V/r_s = 13.67 \pm 0.22$ at $z = 0.57$

Without the BAO BOSS data



Without the quintessence prior on $w_{(0)}$, $w_{(0)}$ is bounded to be

$$-1.211 < w_{(0)} < -0.998 \quad (2\sigma)$$

Even with the prior $w_{(0)} > -1$, $w_{(0)}$ is tightly bounded to be

$$w_{(0)} < -0.95 \quad (2\sigma)$$

⇨ $p < 0.1$ for $V(\phi) = M^{4+p}\phi^{-p}$

With the BAO data, even the ΛCDM model is disfavored over the models with $w < -1$.

Approximate formula for thawing models

Dutta and Scherrer (2008)

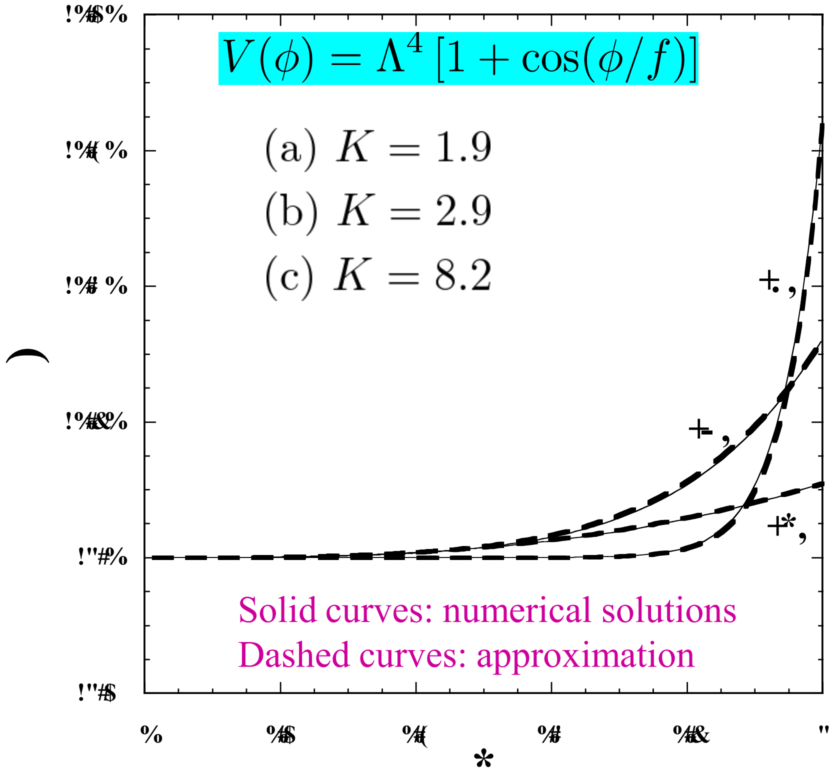
Using the approximation $|1+w| \ll 1$ around the initial field value ϕ_i, we have

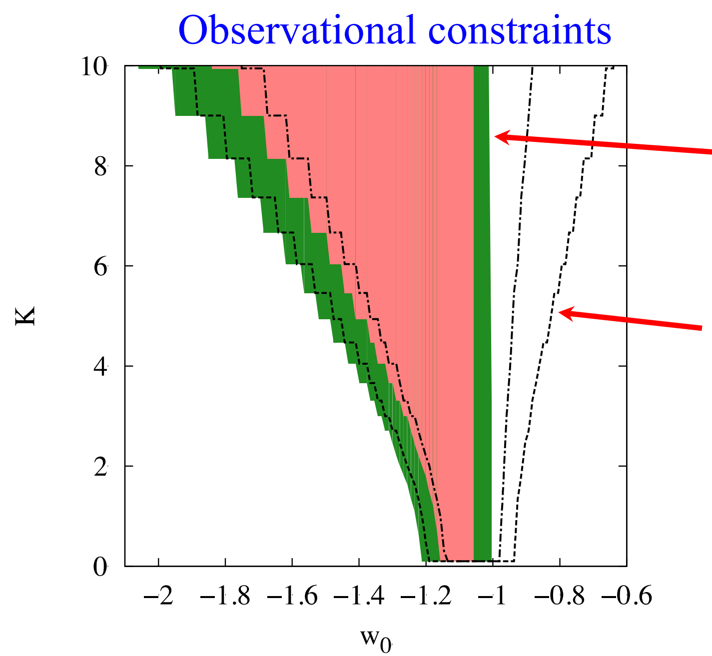
$$w(a) = -1 + (1+w_0)a^{3(K-1)} \left[\frac{(K-F(a))(F(a)+1)^K + (K+F(a))(F(a)-1)^K}{(K-\Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2}+1)^K + (K+\Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2}-1)^K} \right]^2$$

w_0 is the value of w today.

where $K = \sqrt{1 - \frac{4M_{\rm pl}^2 V_{,\phi\phi}(\phi_i)}{3V(\phi_i)}}$, $\quad F(a) = \sqrt{1 + (\Omega_{\phi 0}^{-1} - 1)a^{-3}}$

Three parameters $w_0, K, \Omega_{\phi 0}$





Chiba, De Felice, S.T., 1210.3859

With the BOSS data

Without the BOSS data

With the BAO data the upper bound on w_0 is very close to -1.

Dark energy equation of state: modified gravity models

(1) f(R) gravity

Viable dark energy models were proposed by Amendola et al, Hu and Sawicki, Starobinsky, Appleby and Battye, and S.T. in 2007.

Hu and Sawicki model

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad (n>0)$$

$$\Rightarrow \quad f(R) \simeq R - \lambda R_0 [1-(R/R_0)^{-2n}]$$

for $R \gg R_0$ (close to LCDM)

$$R_0 \sim H_0^2 \quad \text{for } \lambda = \mathcal{O}(1)$$

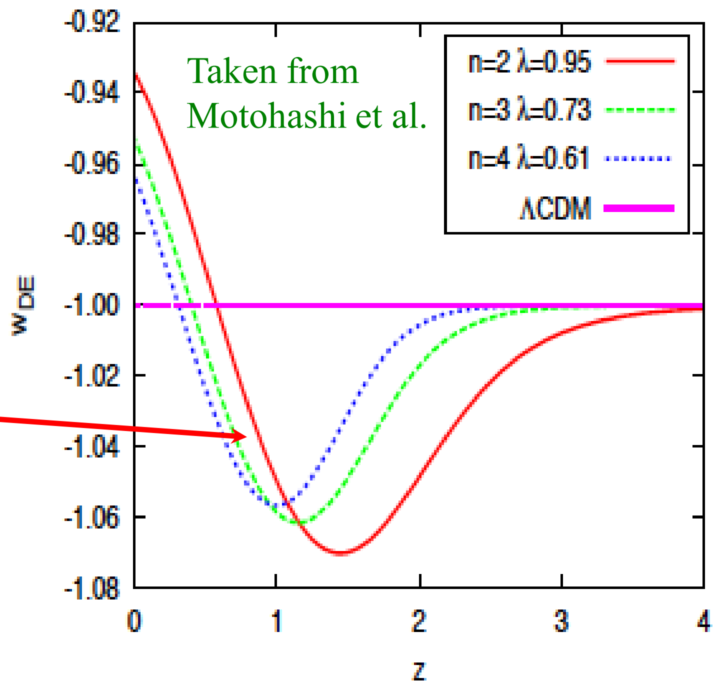
Dark energy equation of state

$$w_{\rm DE} = \frac{w_{\rm eff}}{1 - f_{,R}\Omega_m}$$

where

$$w_{\rm eff} = -1 - 2\dot{H}/(3H^2), \quad \Omega_m = \rho_m/(3f_{,R}H^2)$$

$w_{\rm DE} < -1$ without ghosts



At the background level, $f(R)$ gravity is consistent with observations.

(2) Covariant Galileon

Nicolis et al., Deffayet et al. (2008)

In the DGP braneworld model a brane-bending mode ϕ gives rise to a field self-interaction of the form $\Box\phi(\partial^\mu\phi\partial_\mu\phi)$

- However the DGP model is plagued by the ghost problem.
- This problem can be evaded by considering more general field self interactions respecting the Galilean symmetry: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$

$$K = -c_2 X, \qquad G_3 = \frac{c_3}{M^3}X, \qquad G_4 = \frac{1}{2}M_{\rm pl}^2 - \frac{c_4}{M^6}X^2, \qquad G_5 = \frac{3c_5}{M^9}X^2$$

(Horndeski's action)

There is a tracker solution with

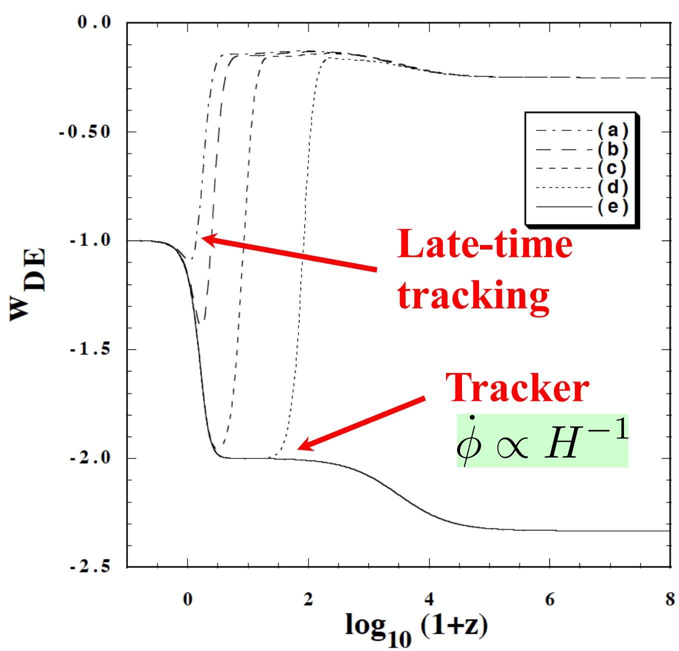
$$w_{\rm DE} = -2 \ (\text{matter era})$$

De Felice, S.T. (2010)

However the tracker is disfavored from the joint data analysis of SNIa, CMB, BAO.

Only the late-time tracking solution is allowed observationally.

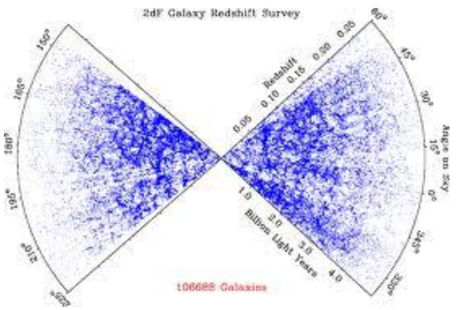
Nesseris, De Felice, S.T. (2010)



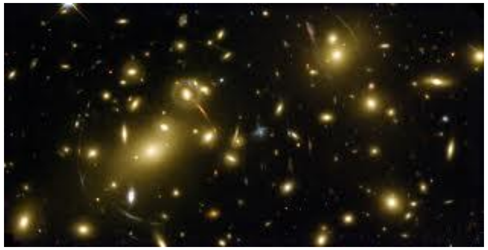
Discrimination of models from density perturbations

In order to place constraints on dark energy models from the observations of large-scale structure, weak lensing, CMB (ISW effect) etc, we need to study the evolution of density perturbations.

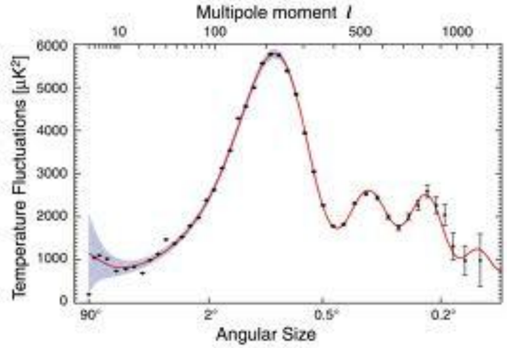
LSS



Weak lensing



CMB



Perturbed metric: $ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$

Non-relativistic matter: $\rho_m = \rho_m(t) + \delta\rho_m(t, \mathbf{x})$

with the four velocity $u^\mu = (1-\Psi, \nabla^i v)$

v is the rotational-free velocity potential.

Density perturbations in the Horndeski's theory

$$S=\int d^4x\sqrt{-g}\,[K(\phi,X)-G_3(\phi,X)\Box\phi+\mathcal{L}_4+\mathcal{L}_5]+S_m+S_r$$

$\delta\equiv\delta\rho_m/\rho_m$ and $\theta\equiv\nabla^2 v$ obey

$$\dot{\delta}=-\theta/a-3\dot{\Phi}$$
$$\dot{\theta}=-H\theta+(k^2/a)\Psi$$

The growth rate of matter perturbations is related with the peculiar velocity.

We introduce the gauge-invariant density contrast: $\delta_m\equiv\delta+\frac{3aH}{k^2}\theta$

Under the quasi-static approximation on sub-horizon scales, it follows that

$$\ddot{\delta}_m+2H\dot{\delta}_m+\frac{k^2}{a^2}\Psi\simeq 0$$ and $$\frac{k^2}{a^2}\Psi\simeq-4\pi G_{\rm eff}\rho_m\delta_m$$ (Modified Poisson equation)

where

$$G_{\rm eff}=\frac{2M_{\rm pl}^2[(B_6D_9-B_7^2)\,(k/a)^2-B_6M^2]}{(A_6^2B_6+B_8^2D_9-2A_6B_7B_8)\,(k/a)^2-B_8^2M^2}G$$

The evolution of Ψ and δ_m is generally different from that in GR.

$$A_6=-2XG_{3,X}-4H\left(G_{4,X}+2XG_{4,XX}\right)\dot{\phi}+2G_{4,\phi}+4XG_{4,\phi X}$$
$$+4H\left(G_{5,\phi}+XG_{5,\phi X}\right)\dot{\phi}-2H^2X\left(3G_{5,X}+2XG_{5,XX}\right)\quad ,\dots$$
$$M^2=-K_{,\phi\phi}$$

See De Felice, Kobayashi, S.T. (2011) for full perturbation equations.

Constraints from redshift-space distortions (RSD)

The galaxy perturbation δ_g is related with δ_m via the bias factor b, i.e., $\delta_g = b\delta_m$.

$\theta = \nabla^2 v$ is related with $f_m \equiv \dot{\delta_m}/(H\delta_m)$ via

$$\theta/(aH) \simeq -f_m \delta_m$$

which comes from $\dot{\delta} = -\theta/a - 3\dot{\Phi}$

The galaxy power spectrum in the redshift space can be modelled as

$$\mathcal{P}_g^s(\mathbf{k}) = \mathcal{P}_{gg}(\mathbf{k}) + 2\mu^2 \mathcal{P}_{g\theta}(\mathbf{k}) + \mu^4 \mathcal{P}_{\theta\theta}(\mathbf{k})$$

μ is the cosine of the angle of the $\boldsymbol{k}$ vector and along the line of sight.

The real space galaxy power spectrum: $(b\sigma_8)^2$

The cross power spectrum: $(b\sigma_8)(f_m\sigma_8)$

The real space velocity power spectrum: $(f_m\sigma_8)^2$

σ_8 is the rms mass fluctuations in spheres within the radius $8\,h^{-1}$ Mpc.

The redshift-space distortions are known as an additive component by observing $b\sigma_8$ and $f_m\sigma_8$.

Quintessence and perfect fluid models in GR

Unless $c_s^2 \ll 1$, it is possible to obtain an analytic formula of $f_m \sigma_8$.

Expansion of w: $w = w_0 + \sum_{n=1}^{\infty} w_n (\Omega_x)^n$

Ω_x is the dark energy density parameter.

The growth index γ defined from $f_m = (1-\Omega_x)^\gamma$ is given by

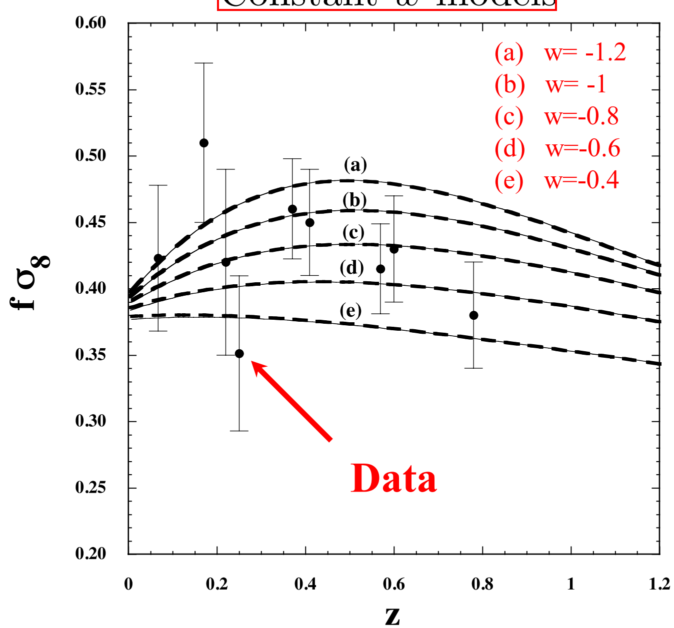
$$\gamma = \frac{3(1-w_0)}{5-6w_0} + \frac{3}{2}\frac{(1-w_0)(2-3w_0)+2w_1(5-6w_0)}{(5-6w_0)^2(5-12w_0)}\Omega_x + \cdots$$

We obtain

$$f_m \sigma_8 = (1-\Omega_x)^\gamma \, \sigma_8(z=0) \, \exp\left\{ \frac{1}{3w_0} \left[\ln \frac{\Omega_{x0}}{\Omega_x} + \sum_{n=1}^{\infty} \frac{c_n}{n} \left((\Omega_{x0})^n - (\Omega_x)^n \right) \right] \right\}.$$

S.T., De Felice, Alcaniz, 1210.4239.

Constant w models



The full numerical solutions (solid curves) show excellent agreement with the above analytic estimation (dashed curves).

We carried out the likelihood analysis by using the recent RSD data and derived the bound

$$-1.245 < w < -0.347 \quad (1\sigma)$$

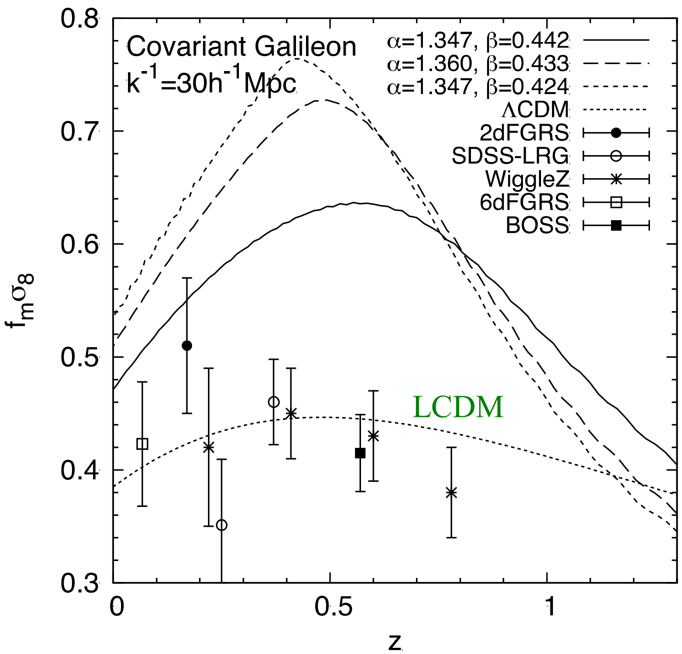
Still weak in current observations.

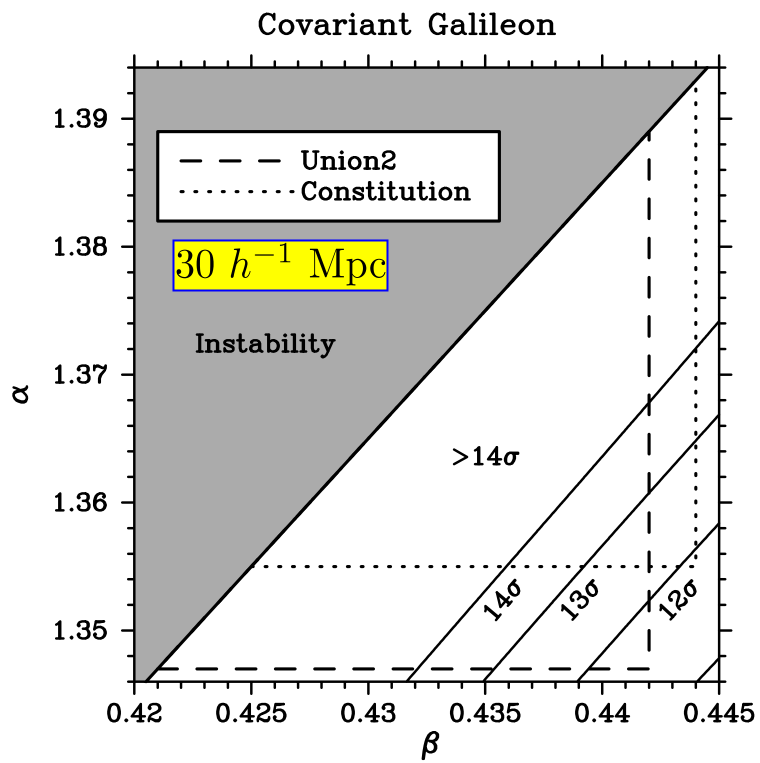
Constraints on Galileons from redshift-space distortions

The covariant Galileon corresponds to the choice

$$K = -c_2 X, \qquad G_3 = \frac{c_3}{M^3} X, \qquad G_4 = \frac{1}{2} M_{\rm pl}^2 - \frac{c_4}{M^6} X^2, \qquad G_5 = \frac{3c_5}{M^9} X^2$$

α and β are parameters related to c_4 and c_5.





Because of large growth rate of matter perturbations, the covariant Galileon is excluded at more than 10σ CL.

Okada, Totani, S.T. , arXiv:1208.4681

See also Appleby and Linder, arXiv: 1204.4314.

Conclusions and Outlook

The cosmological constant is still consistent with observational data, but it seems that the recent BAO data from BOSS favor the dark energy equation of state less than -1.

↓

This may imply some modifications of gravity (because it is possible to explain $w < -1$ without having ghosts).

↓

On the other hand, recent LSS observations such as RSD started to constrain modified gravity models tightly.

Let's see how future observations constrain dark energy!