Nonzero θ_{13} and Models for Neutrino Masses and Mixing

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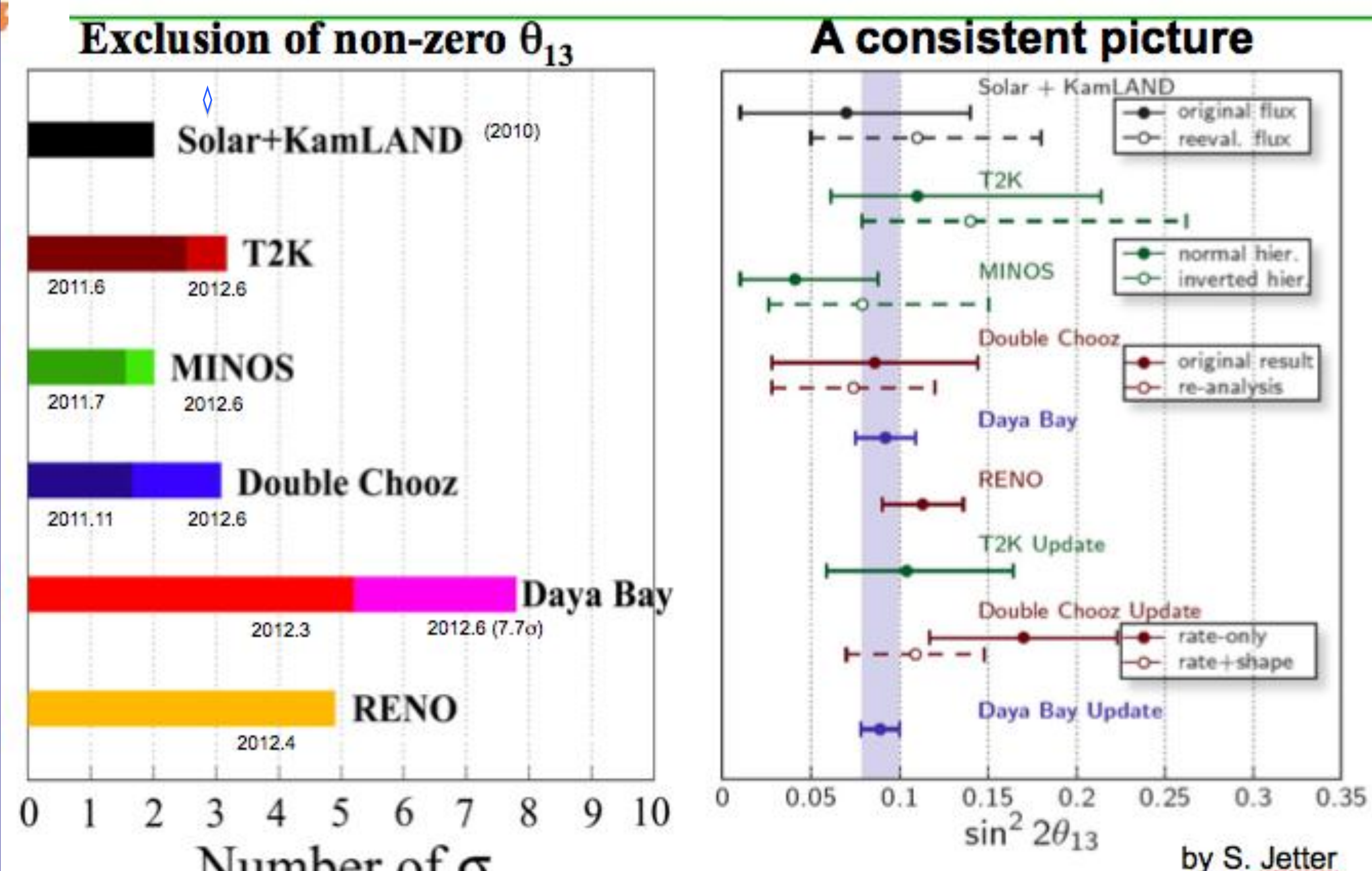
θ_{13} has been measured

Global Picture



Shown by Jun Cao @ ICHEP2012

θ_{13} summary

two extreme assumptions on reactor fluxes:

- use fluxes from Huber, 1106.0687 without SBL reactor data

$$\sin^2\theta_{13} = 0.025 \pm 0.0023 \quad \theta_{13} = (9.2^{+0.42}_{-0.45})^\circ \quad \sin^2 2\theta_{13} = 0.099 \pm 0.009$$

- leave react flux free and include SBL data

$$\sin^2\theta_{13} = 0.023 \pm 0.0023 \quad \theta_{13} = (8.6^{+0.44}_{-0.46})^\circ \quad \sin^2 2\theta_{13} = 0.088 \pm 0.009$$

T. Schwetz

Quark mixing the Cabibbo -Kobayashi-Maskawa (CKM) matrix V_{CKM},

lepton mixing the Pontecorvo -Maki-Nakawaga-Sakata (PMNS) matrix U_{PMNS}

$$L = -\frac{g}{\sqrt{2}} \overline{U}_L \gamma^\mu V_{\mathrm{CKM}} D_L W_\mu^+ - \frac{g}{\sqrt{2}} \overline{E}_L \gamma^\mu U_{\mathrm{PMNS}} N_L W_\mu^- + H.C.\,,$$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For n-generations, $V = V_{\mathrm{CKM}}$ or U_{PMNS} is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.

Neutrinos masses, m_1, m_2, m_3.

Forero et al, arXiv:1205.4018

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\,\pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

Fogli et al, arXiv:1205.3204

Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

NuFIT 1.0 (2012)

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	0.30 ± 0.013	$0.27 \to 0.34$	0.31 ± 0.013	$0.27 \to 0.35$
$\theta_{12}/^\circ$	33.3 ± 0.8	$31 \to 36$	33.9 ± 0.8	$31 \to 36$
$\sin^2\theta_{23}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$	$0.34 \to 0.67$	$0.41^{+0.030}_{-0.029} \oplus 0.60^{+0.020}_{-0.026}$	$0.34 \to 0.67$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$	$36 \to 55$	$40.1^{+2.1}_{-1.7} \oplus 50.7^{+1.1}_{-1.5}$	$36 \to 55$
$\sin^2\theta_{13}$	0.023 ± 0.0023	$0.016 \to 0.030$	0.025 ± 0.0023	$0.018 \to 0.033$
$\theta_{13}/^\circ$	$8.6^{+0.44}_{-0.46}$	$7.2 \to 9.5$	$9.2^{+0.42}_{-0.45}$	$7.7 \to 10.$
$\delta_{\rm CP}/^\circ$	300^{+66}_{-138}	$0 \to 360$	298^{+59}_{-145}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	7.50 ± 0.185	$7.00 \to 8.09$	$7.50^{+0.205}_{-0.160}$	$7.04 \to 8.12$
$\frac{\Delta m^2_{31}}{10^{-3}\ {\rm eV}^2}$ (N)	$2.47^{+0.069}_{-0.067}$	$2.27 \to 2.69$	$2.49^{+0.055}_{-0.051}$	$2.29 \to 2.71$
$\frac{\Delta m^2_{32}}{10^{-3}\ {\rm eV}^2}$ (I)	$-2.43^{+0.042}_{-0.065}$	$-2.65 \to -2.24$	$-2.47^{+0.073}_{-0.064}$	$-2.68 \to -2.25$

Neutrino oscillations

Status

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric | reactor | solar

Remaining unknowns in the 3-flavor picture

Masses

$$m_1,\ m_2,\ m_3 \leftrightarrow \Delta m^2_{12}, |\Delta m^2_{23}|, sign(\Delta m^2_{23}), m_i$$

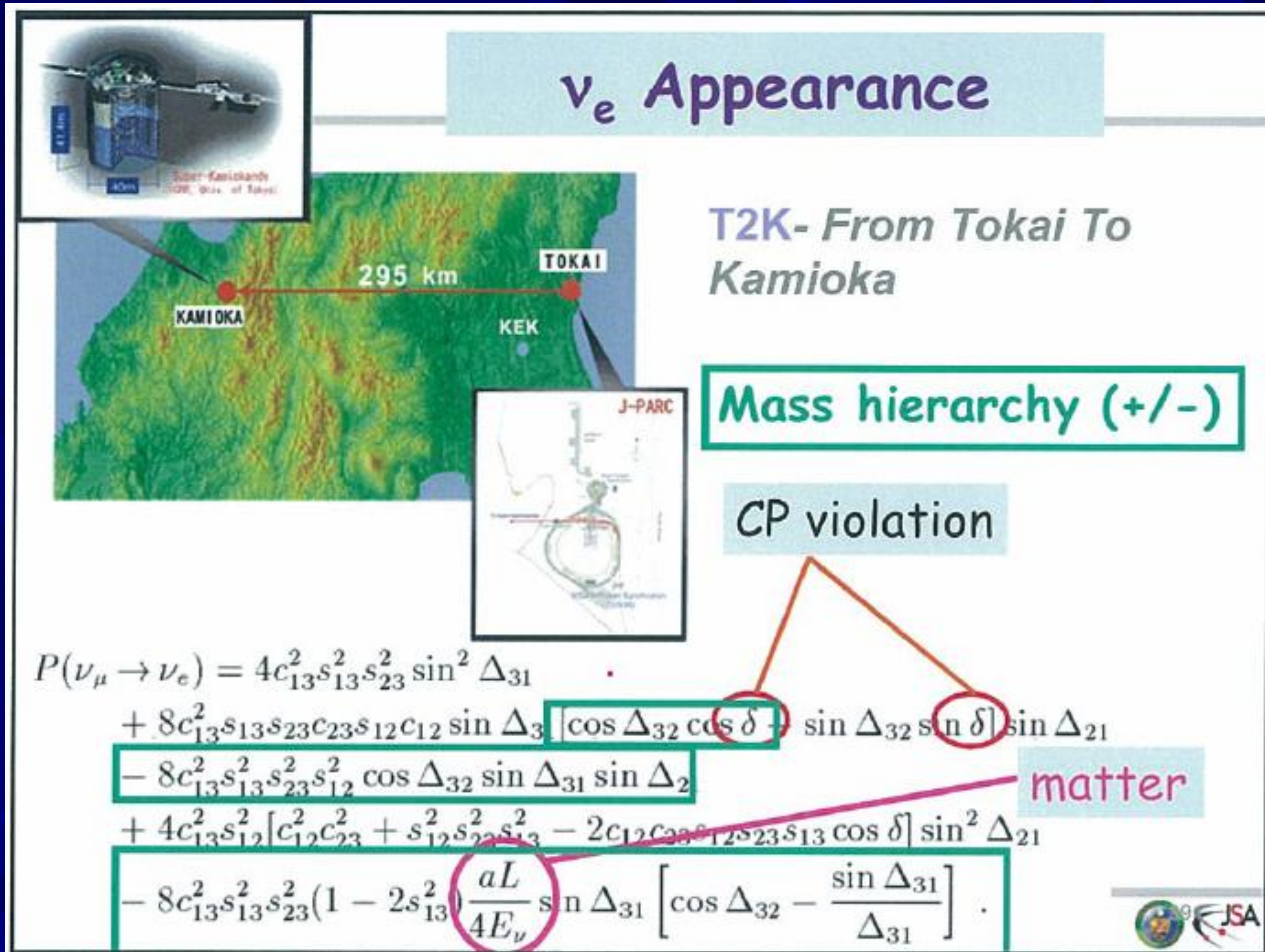
✔ ✔ ? ?

Angles

(plus Majorana phases)

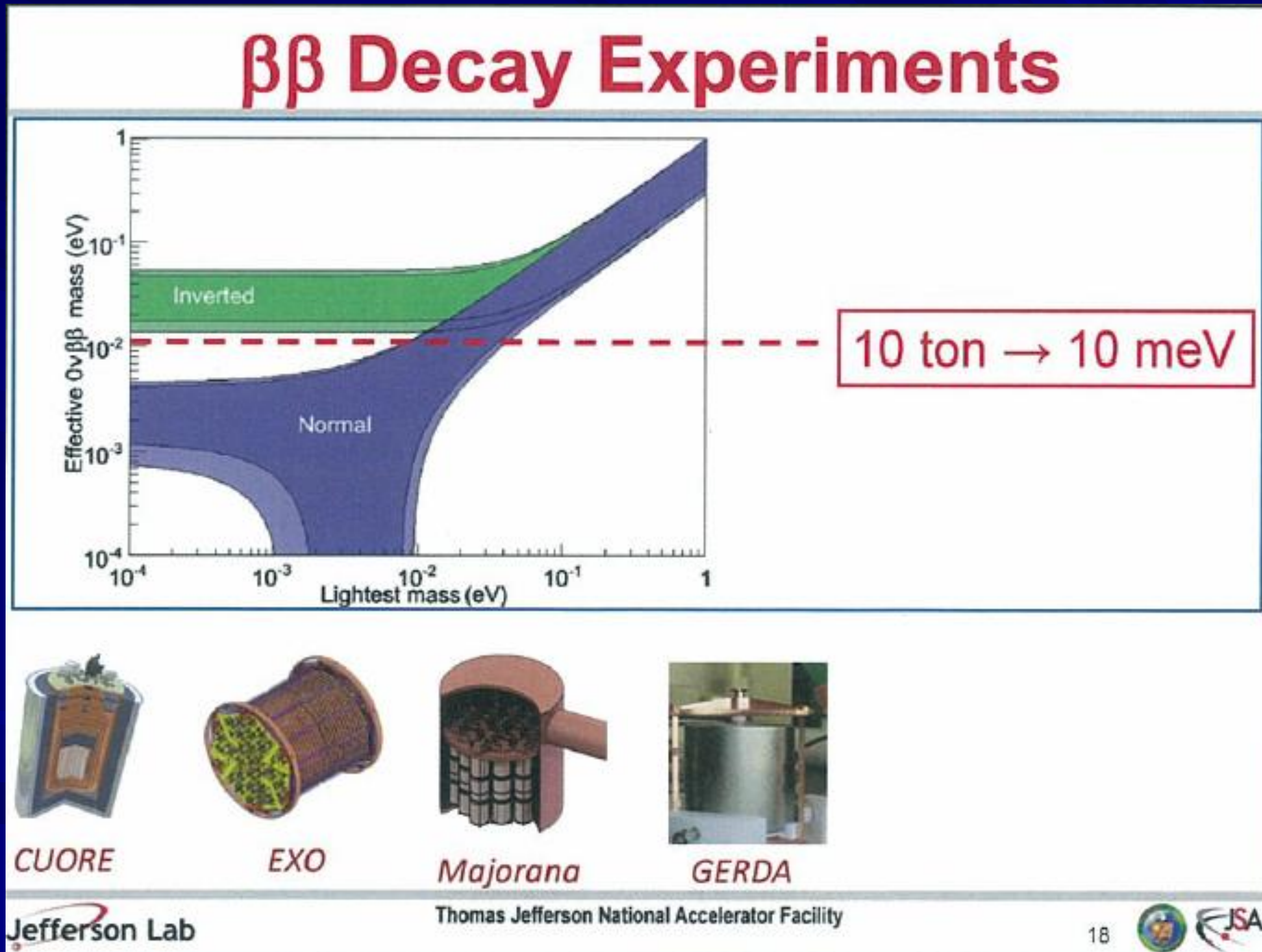
$$\theta_{12},\ \theta_{23},\ \theta_{13},\ \delta$$

Known to good precisions ?

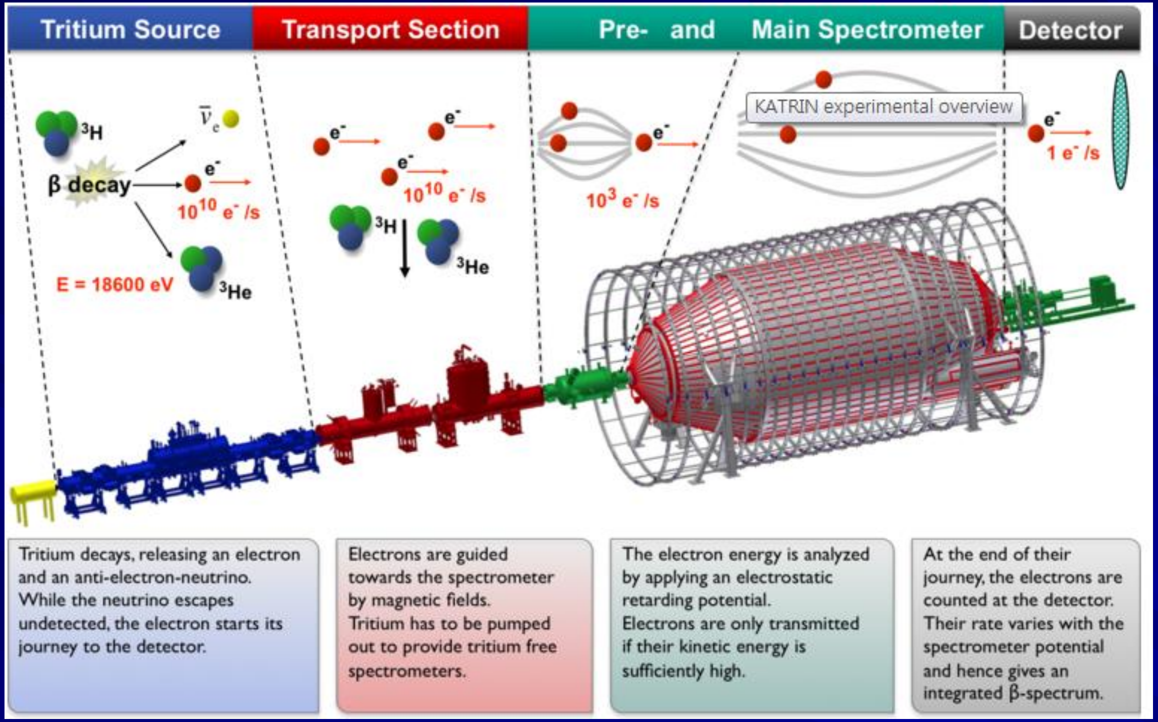


Possible way of measuring mass hierarchy and CP phase

Need to find out whether neutrinos are Dirac or Majorana particles.



Absolute neutrino mass measurement
Kartrin experiment: sensitivity 0.2 eV



Cosmological constraints

Cosmological datasets and corresponding 2σ bounds on $\Sigma = m_1 + m_2 + m_3$.

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + HST + SN-Ia	< 0.75 eV
3	CMB + HST + SN-Ia + BAO	< 0.60 eV
4	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

T. Kajino, talk On 7th for more on cosmology and astrophysics

Theory before and after Daya-Bay/Reno results

Before: popular mixing -The Tribimaximal Mixing

Harrison, Perkins, Scott (2002) , Z-Z. Xing (2002), He& Zee (2003)

The mixing pattern is consistent, within 2σ, with the tri-bimaximal mixing

$$V_{tri-bi} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A4 a promising model (Ma&Ranjasekara, 2001) and realizations (Altarelli&Feruglio 2005, Babu&He 2005). Later many realizations: S4, D3, S3,D4, D7,A5,T',S4, Δ(27, 96), $PSL_2(7)$ … discrete groups Altarelli&Feruglio for review. (H. Lam; Mohapatra et al), T. Mahanthanpa&M-C. Chen; Frampton&Kephart; Y-L Wu, ….

After: Need to have a nonzero θ_{13}

Modification to tri-bimaximal mixing pattern need to be made. (Keum&He&Volkas; He&Zee, 2006).

In fact, more generically, A4 symmetry leads to

$$V = \begin{pmatrix} \frac{2c}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \\ -\frac{c}{\sqrt{6}} - \frac{se^{-i\delta}}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{se^{i\delta}}{\sqrt{6}} \end{pmatrix} .$$

A 4 realization

Tov&He&Zee arXiv:1208.1062

Hint of how to get tri-bimaximal mixing from: $V_{PMNS} = V_{eL}^{\dagger} V_{\nu L}$.

$$\text{If} \quad V_{eL} = U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} , \quad V_{\nu L} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} ,$$

$$\text{then} \quad V_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & \frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{i\pi/6}}{\sqrt{2}} \end{pmatrix} ,$$

where $\omega^3 = 1$. Up to some phases rotation, the above is the tri-bimaximal mixing!

$$M_\nu = \begin{pmatrix} \alpha - \varepsilon & 0 & \beta \\ 0 & \gamma & 0 \\ \beta & 0 & \alpha + \varepsilon \end{pmatrix}$$

$\Phi \sim (2, -\frac{1}{2}; 3, 0)$ and the lepton doublets $\ell \sim (2, -\frac{1}{2}; 3, +1)$. Consider the interactions

$$\mathcal{L}_{\text{dim-5}} = c_1 \mathcal{O}_1 + c_{1'} \mathcal{O}_{1'} + c_{1''} \mathcal{O}_{1''} + c_3 \mathcal{O}_3 + h.c.$$

where:

$$\begin{aligned} \mathcal{O}_1 &= (\Phi_1^\dagger \Phi_1^\dagger + \Phi_2^\dagger \Phi_2^\dagger + \Phi_3^\dagger \Phi_3^\dagger)(\ell_1 \ell_1 + \ell_2 \ell_2 + \ell_3 \ell_3) , \\ \mathcal{O}_{1'} &= (\Phi_1^\dagger \Phi_1^\dagger + \omega^* \Phi_2^\dagger \Phi_2^\dagger + \omega\, \Phi_3^\dagger \Phi_3^\dagger)(\ell_1 \ell_1 + \omega\, \ell_2 \ell_2 + \omega^* \ell_3 \ell_3) , \\ \mathcal{O}_{1''} &= (\Phi_1^\dagger \Phi_1^\dagger + \omega\, \Phi_2^\dagger \Phi_2^\dagger + \omega^* \Phi_3^\dagger \Phi_3^\dagger)(\ell_1 \ell_1 + \omega^* \ell_2 \ell_2 + \omega\, \ell_3 \ell_3) , \\ \mathcal{O}_3 &= (\Phi_2^\dagger \Phi_3^\dagger, \Phi_3^\dagger \Phi_1^\dagger, \Phi_1^\dagger \Phi_2^\dagger) \cdot (\ell_2 \ell_3, \ell_3 \ell_1, \ell_1 \ell_2) . \end{aligned}$$

Consider the case $c_3 = 0$ and $\langle \Phi_1^0 \rangle = \langle \Phi_3^0 \rangle \neq \langle \Phi_2^0 \rangle$. Defining for convenience the overall scale of the VEVs and the small splitting as

$$a \equiv 3 \langle \Phi_1^{0\dagger} \rangle^2 \ , \quad \delta \equiv \langle \Phi_2^{0\dagger} \rangle^2 - \langle \Phi_1^{0\dagger} \rangle^2 \tag{V.8}$$

we find:

$$\begin{aligned} c_1 \mathcal{O}_1 + c_{1'} \mathcal{O}_{1'} + c_{1''} \mathcal{O}_{1''} = & [a c_1 + (c_1 + \omega^* c_{1'} + \omega\, c_{1''}) \delta]\, \ell_1 \ell_1 \\ & + [a c_1 + (c_1 + c_{1'} + c_{1''}) \delta]\, \ell_2 \ell_2 \\ & + [a c_1 + (c_1 + \omega\, c_{1'} + \omega^* c_{1''}) \delta]\, \ell_3 \ell_3 \ . \end{aligned} \tag{V.9}$$

The perturbation by δ recovers a neutrino mass matrix in the form of Eq. (V.1) with:

$$\varepsilon = i \, \frac{\sqrt{3}}{2} (c_{1'} - c_{1''}) \delta \ . \tag{V.10}$$

Other variations: Ahn et al, M-C. Chen; Z-Z Xing et al; B-Q. Ma et al; S. Kim….

Simple modifications: keeping one of the column or a row unchanged. Albright&Rodejohan, 2008; He&Zee, 2011

One of the Columns in V_{TB} unchanged

Leaving one of the columns in V_{TB} unchanged, we have the three possibilities:

$$V^a = V_{TB}\begin{pmatrix} \cos\tau & \sin\tau & 0 \\ -\sin\tau & \cos\tau & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad V^b = V_{TB}\begin{pmatrix} \cos\tau & 0 & \sin\tau e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\tau e^{-i\delta} & 0 & \cos\tau \end{pmatrix},$$

$$V^c = V_{TB}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\tau & \sin\tau e^{i\delta} \\ 0 & -\sin\tau e^{-i\delta} & \cos\tau \end{pmatrix}.$$

$$W^a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} V_{TB}\,, \quad W^b = \begin{pmatrix} c & 0 & se^{i\delta} \\ 0 & 1 & 0 \\ -se^{-i\delta} & 0 & c \end{pmatrix} V_{TB}\,,$$

$$W^c = \begin{pmatrix} c & se^{i\delta} & 0 \\ -se^{-i\delta} & c & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{TB}\,.$$

Tri-bimximal at higher scales?

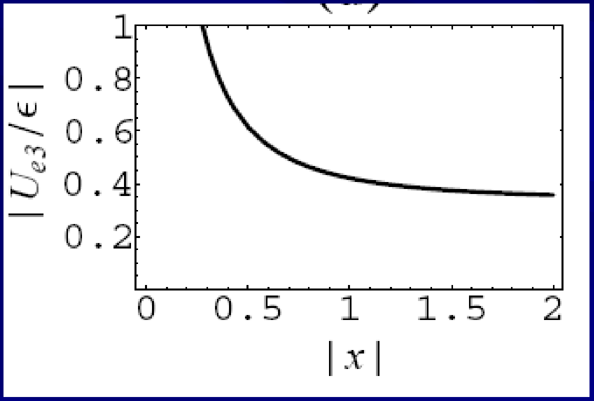
Baub and He, arxiv:0507217(hep-ph): A susy A4 model

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P\ .$$

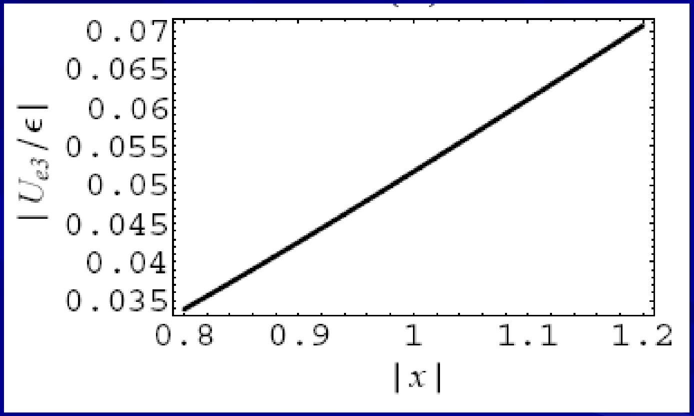
one–loop RGE $\frac{dM_\nu^e}{d\ln t} = \frac{1}{32\pi^2}[M_\nu^e Y_e^\dagger Y_e + (Y_e^\dagger Y_e)^T M_\nu^e] + \ldots$

leading to the entries $M_{13,23}(1-\epsilon)$ and $M_{33}(1-2\epsilon)$

$\epsilon \simeq Y_\tau^2 \ln(M_{\rm GUT}/M_{EW})/32\pi^2$.



Inverted hierarchy



Normal hierarchy

Susy model, Y♦ ~O(1), U_{e3} for inverted hierarchy, can be as large a 0.1, with RG effects!

Many other matrix form come back

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

An interesting coincidence:

$\theta_{13} = \theta_C/\sqrt{2}$

$V_{PMNS} = UV_{KM}$; $V_{Tr}V_{KM}$

B-Q. Ma et al: Ramond et al.; King et al.…

Bi-Maximal Mixing

(Barger, Pakvasa, Weiler, Whisnant)

Tetra Maximal Mixing Z-Z Xing

$$U = \begin{pmatrix} \frac{2+\sqrt{2}}{4} & \frac{1}{2} & \frac{2-\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} + \frac{i\left(\sqrt{2}-1\right)}{4} & \frac{1}{2} - \frac{i\sqrt{2}}{4} & \frac{\sqrt{2}}{4} + \frac{i\left(\sqrt{2}+1\right)}{4} \\ -\frac{\sqrt{2}}{4} - \frac{i\left(\sqrt{2}-1\right)}{4} & \frac{1}{2} + \frac{i\sqrt{2}}{4} & \frac{\sqrt{2}}{4} - \frac{i\left(\sqrt{2}+1\right)}{4} \end{pmatrix}$$

Texture Zeros

Neutrino mass matrix symmetric, more than two zeros, cannot fit data.

non-vanishing elements of similar order:

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} 0 & 0 & 1 \\ \cdot & 3 & 2 \\ \cdot & \cdot & 2 \end{pmatrix}.$$

This mass matrix leads to normal hierarchy and the mixing parameters take the form

$$\sin^2\theta_{12} \simeq \frac{1}{3}, \qquad \sin^2\theta_{13} = \frac{1}{3} - \frac{5}{6\sqrt{7}} \simeq 0.018,$$

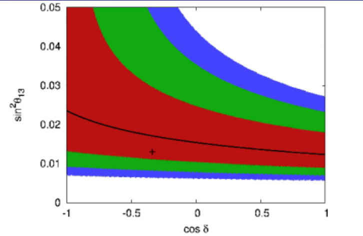
$$\sin^2\theta_{23} \simeq \frac{1}{3} + \frac{2}{3\sqrt{7}} \simeq 0.59, \qquad \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = \frac{1}{2} - \frac{5}{4\sqrt{7}} \simeq 0.027,$$

which are valid at 2σ.

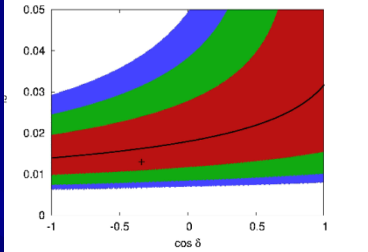
Araki et al., arXiv:1203.4951

Ludl et al., arXiv:1109.3393

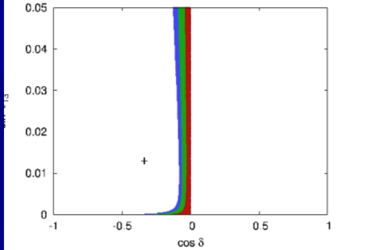
case	texture zeros
A_1	$(\mathcal{M}_\nu)_{ee} = (\mathcal{M}_\nu)_{e\mu} = 0$
A_2	$(\mathcal{M}_\nu)_{ee} = (\mathcal{M}_\nu)_{e\tau} = 0$
B_1	$(\mathcal{M}_\nu)_{\mu\mu} = (\mathcal{M}_\nu)_{e\tau} = 0$
B_2	$(\mathcal{M}_\nu)_{\tau\tau} = (\mathcal{M}_\nu)_{e\mu} = 0$
B_3	$(\mathcal{M}_\nu)_{\mu\mu} = (\mathcal{M}_\nu)_{e\mu} = 0$
B_4	$(\mathcal{M}_\nu)_{\tau\tau} = (\mathcal{M}_\nu)_{e\tau} = 0$
C	$(\mathcal{M}_\nu)_{\mu\mu} = (\mathcal{M}_\nu)_{\tau\tau} = 0$



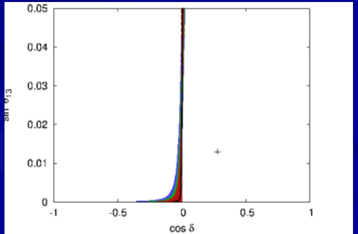
The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case A_1 (normal spectrum).



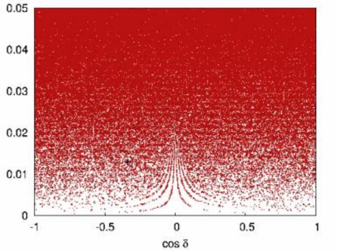
The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case A_2 (normal spectrum).



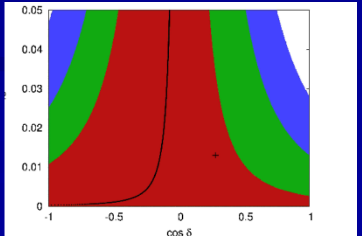
The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case B_1 (normal spectrum).



The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case B_1 (inverted spectrum).



The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case C (normal spectrum).



The relation between $\sin^2\theta_{13}$ and $\cos\delta$ for case C (inverted spectrum).

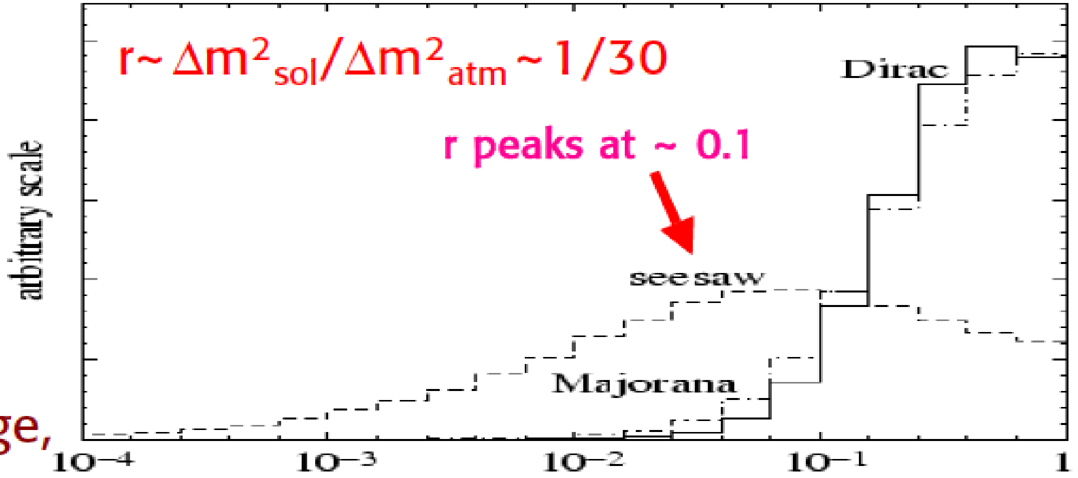
Anarchy neutrino mass matrix

Anarchy (or accidental hierarchy):
No structure in the neutrino sector

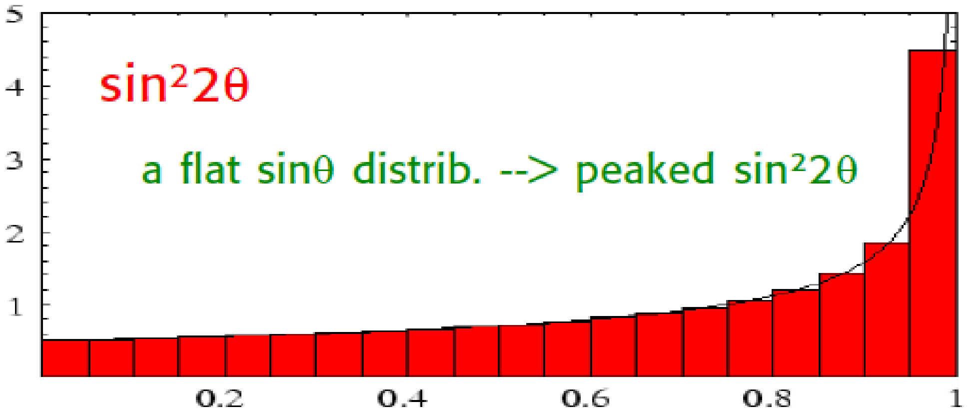
Hall, Murayama, Weiner '00

See-Saw:
$m_\nu \sim m^T M^{-1} m$
produces hierarchy
from random m, M

could fit the data on r



All mixing angles
should be not too large,
not too small



Predicts θ_{13} near old
bound and
θ_{23} sizably non maximal
successful!

But what is the theory? No understanding at all!

Final Thoughts

1. The Anarchy Hypothesis fits the lepton mixing data very well.

2. This does NOT mean that the Anarchy Hypothesis is true. It just means that the data are unable to falsify it. This statement is most likely to remain true for the foreseeable future.

3. The Anarchy Hypothesis may or may not fit the data "better" than different Order Hypotheses. I certainly hope that any worthwhile Order Hypothesis fits the data better than the Anarchy Hypothesis — after all, this is what Order Hypotheses are built to do! Comparisons, however, are very tricky! [Altarelli, next talk]

4. The "flavor problem" has been around for 40 years or so. It has frustrated generations of particle physicists. I am not implying that we don't need to worry about flavor in the lepton mixing sector. Symmetry models, however, have the "burden of proof." They need to do better than the Anarchy Hypothesis. Qualitatively better!

CP violation?

δ the same as V_{KM}: Spontanenous CP phase responsible for V_{PMNS} and V_{KM}. Lu-Hsing Tsai& He 2011

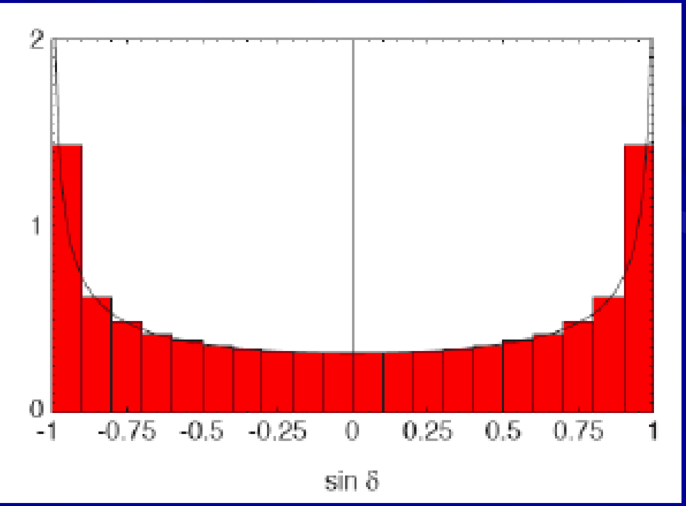
Predicts: J = 0.02

Postulating in KM parametrization phase I 90 degree, also for PMNS, then using modules of elements fix all parameters. Ma&Zhang

Predicts: J = 0.0345

Anarchy prediction

Haba&Murayama



Group theoretical considerations correlate mixing angles

$$\sin^2\theta_{13} = \frac{2+\cos(2\pi(m-n)/N)+\sqrt{3}\sin(2\pi(m-n)/N)}{6},$$

$$\sin^2\theta_{12} = \frac{2}{4-\cos(2\pi(m-n)/N)-\sqrt{3}\sin(2\pi(m-n)/N)},$$

$$\sin^2\theta_{23} = 1-\frac{4\sin^2(\pi(m-n)/N)}{4-\cos(2\pi(m-n)/N)-\sqrt{3}\sin(2\pi(m-n)/N)}.$$

The Dirac CP phase is always vanishing. When $N=8$, the smallest $|U_{e3}|$ is obtained as

$$|U_{e3}| = \sqrt{(2-\sqrt{2+\sqrt{3}})/6},$$

which is about 0.107. In this case, we find[2]

$$\sin\theta_{12} = \frac{2}{\sqrt{8+\sqrt{2}+\sqrt{6}}}, \qquad \sin\theta_{23} = \frac{\sqrt{5-3\sqrt{2}-\sqrt{3}+\sqrt{6}}}{\sqrt{7-3\sqrt{2}-\sqrt{3}+\sqrt{6}}}.$$

Ishimori&Kobayashi, arXiv:1201.3429, Based on Z^3_NxZ3

Henandez & Smirnov, arXiv:1204.0445

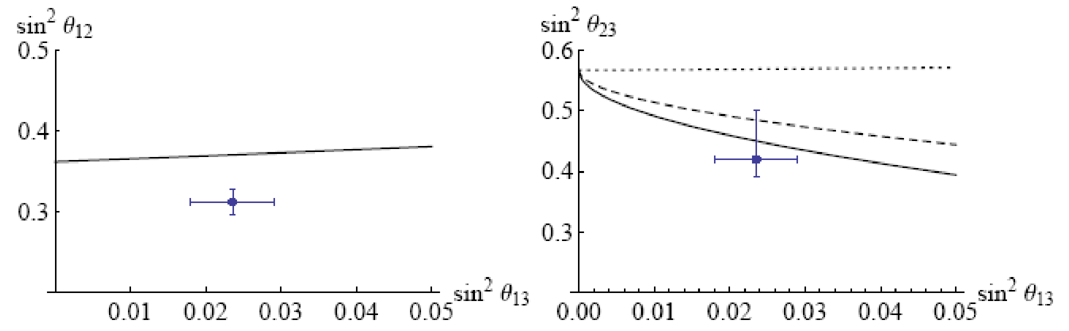
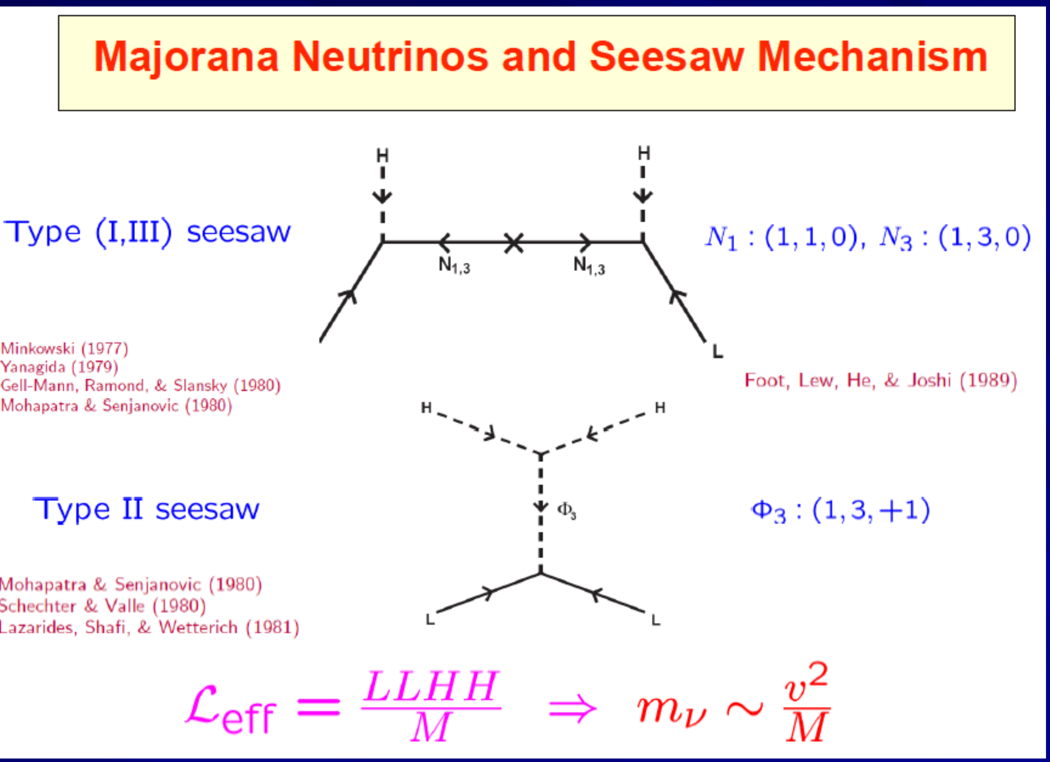


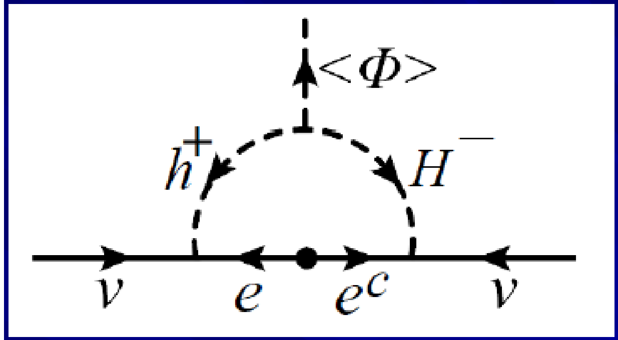
Figure 2: Symmetry relations between mixing parameters. Shown are $\sin^2\theta_{12}$ (left panel) and $\sin^2\theta_{23}$ (right panel) as functions of $\sin^2\theta_{13}$ for the model with $\{T_\mu; a=0, k=2, m=5\}$. The curves in the right panel correspond to $\delta=0$ (thick), $\pi/4$ (dashed), $\pi/2$ (dotted).

Still, no idea about underlying theory!

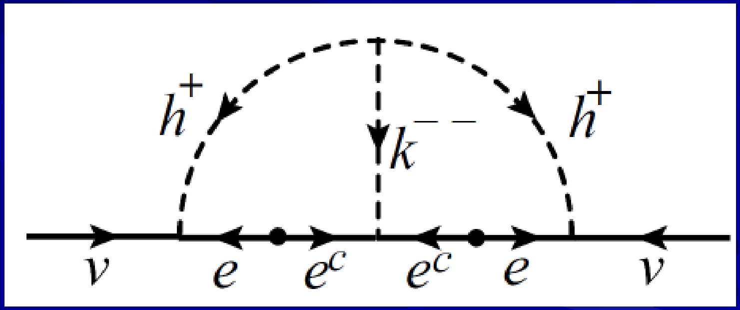
Theoretical Models



Loop generation neutrino masses



Zee, 1980



Zee, 1985, Babu 1988

Provide understand why neutrino masses are small, but no information about mixing!

Without flavor symmetries, the mixing pattern cannot be completely fixed.

Models with symmetries have been discussed earlier.

One can also try to see if some other physical reasons can also make some models predictive. There are models of this kind.

Zee model without additional large hierarchy in Yukawa couplings

SM Higgs H, add: h^+ and a new doublet φ. if coupling f_2 is zero, the model is ruled out by data. If Keep both f (h^+ coupling) and f_2, but assum the elements of each are the same order of magnitudes, keep only f m^2_τ and $f_2 m_\tau$ terms (neglect terms proportional to f m^2_μ, m^2_e and f_2 ... e

He, ... 7,
He and Majee, 2011

$$M_\nu = A[(fm^2 + m^2 f^T) - \frac{v}{\cos\beta}(fmf_2^\phi + f_2^{\phi T} m f^T)],$$

$$A = \sin(2\theta_Z) log(M_2^2/M_1^2)/(16\pi^2 v \tan\beta)$$

$$M_\nu = a \begin{pmatrix} 1 & (ye^{i\delta}+x)/2 & z \\ (ye^{i\delta}+x)/2 & xye^{i\delta} & xz \\ z & xz & 0 \end{pmatrix},$$

where a is the absolute value of the 11 entry M_{11}, $M_{11} = -2Avm_\tau f^{e\tau} f_2^{\tau e}/\cos\beta$,

z is the absolute value of 13 entry M_{13} divided by a $M_{13} = Af^{e\tau} m_\tau (m_\tau - v f_2^{\tau\tau}/\cos\beta)$,

$x = |f^{\mu\tau}|/|f^{e\tau}|$ is the absolute value of the ratio of M_{23} to M_{13}, and $y = |M_{22}|/xa$ with

$$M_{22} = -2Avm_\tau f^{\mu\tau} f_2^{\tau\mu}/\cos\beta.$$

One can choose a convention where all the above parameters are real except $f_2^{\tau\mu}$. We will write it as $f_2^{\tau\mu} e^{i\delta}$.

Rank 2. Predicts one neutrino mass to be zero. Only inverted mass hierarchy can fit data

Parameter	$\delta m^2/10^{-5}\mathrm{eV}^2$	$\Delta m^2/10^{-3}\mathrm{eV}^2$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
Best fit	7.58	2.35	0.312	0.42	0.025
1σ range	7.32 - 7.80	2.26 - 2.47	0.296 - 0.329	0.39 - 0.50	0.018 - 0.032
2σ range	7.16 - 7.99	2.17 - 2.57	0.280 - 0.347	0.36 - 0.60	0.012 - 0.041
3σ range	6.99 - 8.18	2.06 - 2.67	0.265 - 0.364	0.34 - 0.64	0.005 - 0.050

The two mass-square differences are defined as $\delta m^2 = m_2^2 - m_1^2$ and $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$

These best-fit values of the mass matrix parameters are:

$$x = 0.2550, \quad y = 4.1000, \quad z = 1.7900, a = 0.017 \text{ eV}, \quad \delta = 180^\circ,$$

corresponding out puts for the mixing angles and mass-squared differences are given as

$$\sin^2\theta_{12} = 0.3163, \quad \sin^2\theta_{23} = 0.4033, \quad \sin^2\theta_{13} = 0.0256,$$
$$\delta m^2 = 7.51 \times 10^{-5}\mathrm{eV}^2 \quad \Delta m^2 = -2.36 \times 10^{-3}\mathrm{eV}^2.$$

SO(10)Grand Unification

SO(10) Yukawa couplings: $16_F(Y_{10}10_H + Y_{\overline{126}}\overline{126}_H + Y_{120}120_H)16_F$

Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\,\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

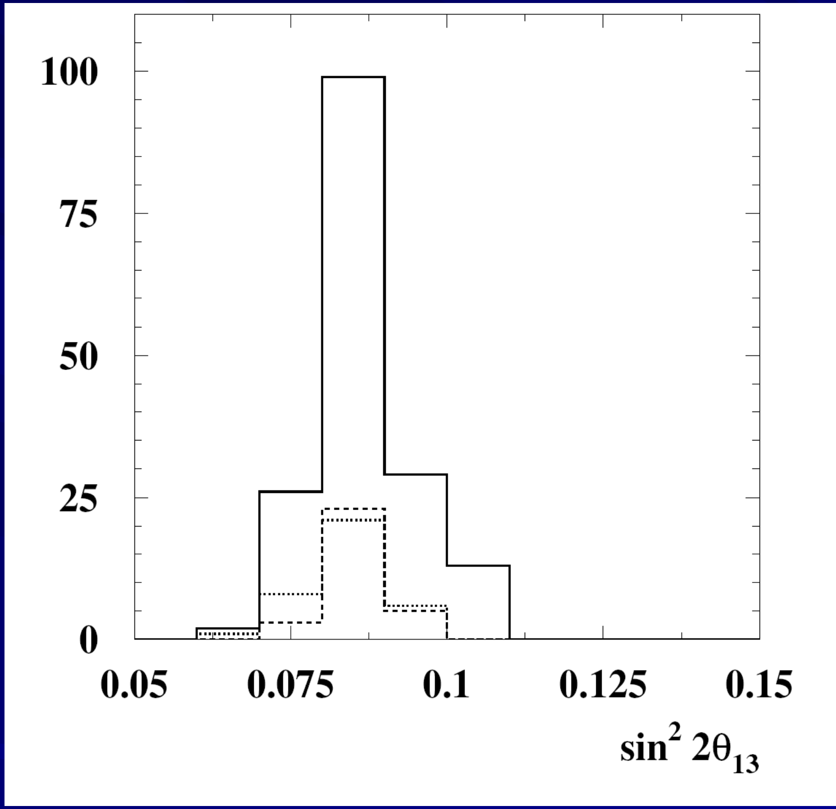
$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

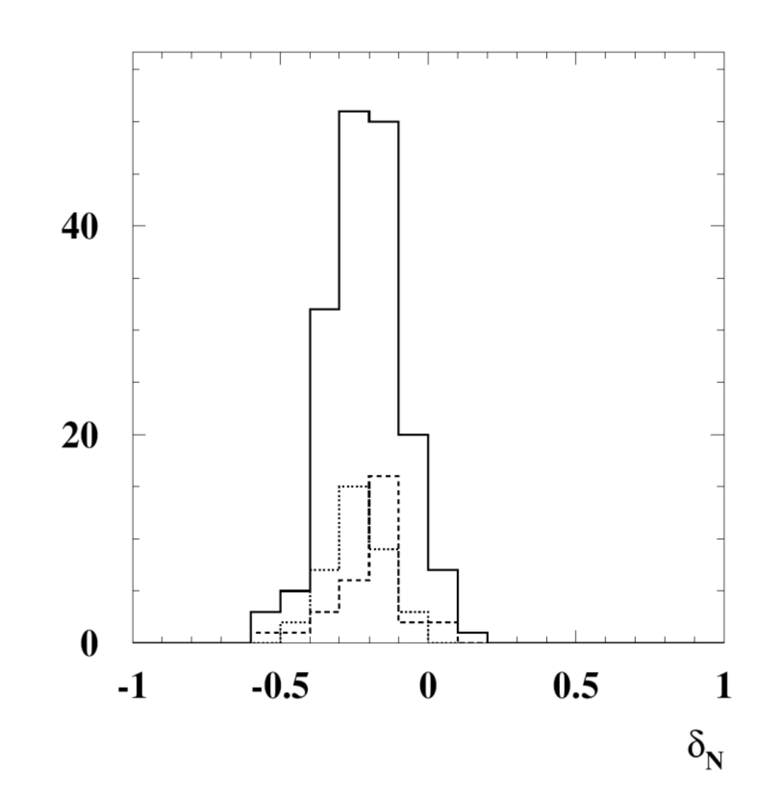
Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)

Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek

θ_{13} in Minimal SO(10)





$\sin^2 2\theta_{13}$ and CP violating phase δ_N

K.S. Babu and C. Macesanu (2005)

Joshipura&Patel: arXiv: 1102.5148

Non-SUSY Minimal S)(10) $r_L = \langle \Delta_L \rangle / \kappa'_d$ $r_R = \kappa'_d / \langle \Delta_R \rangle$

	Type-I		Type-II	
Observables	Fitted value	pull	Fitted value	pull
m_d	0.000810163	−0.687161	0.00101285	−0.264898
m_s	0.0208099	−0.198354	0.0225915	0.0844982
m_b	0.999667	−0.00831657	1.08201	2.05031
m_u	0.000495023	0.0751133	0.000507336	0.13668
m_c	0.237348	0.0670883	0.237096	0.0598882
m_t	73.9427	−0.0154941	74.3006	0.075144
m_e	0.000469652	−	0.000469652	−
m_μ	0.0991466	−	0.0991466	−
m_τ	1.68558	−	1.68558	−
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.030526	0.127968	0.0297114	−0.235285
$\sin\theta^q_{12}$	0.224651	0.0464044	0.224499	−0.0916848
$\sin\theta^q_{23}$	0.0420499	0.0392946	0.0421308	0.103004
$\sin\theta^q_{13}$	0.00349369	−0.0974312	0.00353053	0.0389979
$\sin^2\theta^l_{12}$	0.323245	0.148134	0.3108	−0.610792
$\sin^2\theta^l_{23}$	0.435096	−0.369178	0.113306	−7.02461
$\sin^2\theta^l_{13}$	**0.0244287**	−	**0.0176863**	−
$\delta_{\rm CKM}[^\circ]$	69.5262	−0.0314447	69.2051	−0.128759
$\delta_{\rm MNS}[^\circ]$	**318.465**	−	**14.5386**	−
$\alpha_1[^\circ]$	**21.5053**	−	**345.645**	−
$\alpha_2[^\circ]$	**215.128**	−	**141.905**	−
$r_{R(L)}$	**5.62×10^{-14}**	−	**2.09×10^{-10}**	−
χ^2		**0.710777**		**54.1197**

TABLE VII. Best fit solutions for fermion masses and mixing obtained assuming the type-I and type-II seesaw dominance in the minimal non-SUSY $SO(10)$ model. Various observables and their pulls at the minimum are shown. All the masses shown are in GeV units. The bold faced quantities are predictions of the respective solutions.

Minimal SUSY SO(10)

	A	B	C	D	C1	C2
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	−0.22	+0.22
γ_d	0	0	0	0	−0.21	+0.21
γ_t	0	0	0	0	0	−0.44
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)

TABLE I. The input values of various observables of quark sector and charged lepton masses obtained at GUT-scale M_X for various values of $\tan\beta$ and threshold corrections $\gamma_{t,b,d}$ assuming an effective SUSY scale $M_S = 500$ GeV (see [23] for details).

	A	B	C	D	C1	C2
Observables	Pulls obtained for best fit solution					
(m_u/m_c)	−0.00668428	0.0276825	0.0259467	0.120767	−0.0212532	0.0356043
(m_c/m_t)	0.56521	0.157569	0.0201093	0.0730136	0.130288	0.320944
(m_d/m_s)	−1.21642	−0.891034	−0.27664	−1.36265	−1.04724	−1.57673
(m_s/m_b)	0.112798	0.440678	0.163272	0.752408	0.884723	0.789053
(m_e/m_μ)	0.0590249	−0.00627804	0.3944	0.0396087	0.0297987	0.0555931
(m_μ/m_τ)	0.182548	0.103214	0.821485	0.0192305	0.26316	0.121145
(m_b/m_τ)	0.87282	2.20829	2.79368	2.34331	0.26656	0.407798
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.256292	0.116314	−0.14908	0.230056	0.0188227	−0.0140039
$\sin\theta^q_{12}$	0.0730813	0.0702755	0.0399788	0.105989	0.0779176	0.127757
$\sin\theta^q_{23}$	−0.0311676	−0.172792	−0.471738	−0.0960437	−0.757038	−0.945821
$\sin\theta^q_{13}$	1.33502	−0.0354198	0.494732	0.606606	0.890741	1.17758
$\sin^2\theta^l_{12}$	0.00836789	−0.106439	−0.599727	−0.27881	−0.63356	−0.510182
$\sin^2\theta^l_{23}$	−1.53367	−4.97038	−4.95673	−4.70944	−2.56294	−1.84412
$\delta_{\rm CKM}[^\circ]$	−0.345931	−0.163765	−0.600814	−0.214459	−0.650554	−0.75885
χ^2_{min}	**6.9367**	**30.70**	**34.52**	**30.68**	**10.804**	**9.3559**
Observables	Corresponding Predictions at GUT scale					
$\sin^2\theta^l_{13}$	0.0226508	0.0190847	0.0206716	0.0196974	0.0239619	0.0209208
$\delta_{\rm MNS}[^\circ]$	19.9399	18.9784	19.5619	11.92	358.789	1.78569
$\alpha_1[^\circ]$	337.171	346.627	344.795	350.595	12.4786	349.711
$\alpha_2[^\circ]$	147.364	151.912	146.886	161.702	194.023	168.156
$r_L m_\tau$[GeV]	8.37×10^{-10}	6.0×10^{-10}	6.49×10^{-10}	6.94×10^{-10}	7.15×10^{-10}	9.1×10^{-10}

TABLE II. Best fit solutions for fermion masses and mixing obtained assuming the type-II seesaw dominance in the minimal SUSY $SO(10)$ model. Pulls of various observables and predictions obtained at the minimum are shown for six different data sets.

	A	B	C	D	C1	C2
Observables	Pulls obtained for best fit solution					
(m_u/m_c)	0.0486938	−0.180782	0.0653101	0.0053847	0.0467579	−0.0119661
(m_c/m_t)	1.22599	0.130589	0.246294	0.146932	0.297256	0.273346
(m_d/m_s)	−0.229546	−0.730641	0.223201	−0.748148	−2.2904	−0.689684
(m_s/m_b)	−0.932536	−0.886438	−0.977249	−1.05766	0.735548	0.000467775
(m_e/m_μ)	0.0340323	0.442759	0.103692	−0.476364	0.0649144	−0.0648856
(m_μ/m_τ)	0.310305	−0.526529	0.881934	0.938701	0.705648	0.0178824
(m_b/m_τ)	−0.486477	−0.194215	0.0172182	−0.34079	0.789868	−0.734937
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.122267	−0.10063	−0.00563647	−0.120429	−0.180164	0.158557
$\sin\theta^q_{12}$	0.0432634	0.227948	0.0186715	0.084149	0.130301	0.0922391
$\sin\theta^q_{23}$	−0.281221	−0.0401177	−0.167224	0.0649082	−0.273222	−1.17651
$\sin\theta^q_{13}$	1.37864	−0.275689	0.926186	0.559003	1.48675	0.248759
$\sin^2\theta^l_{12}$	−0.0528379	−0.0598219	−0.38133	−0.172148	−0.746107	0.0694831
$\sin^2\theta^l_{23}$	−1.22555	−1.27077	−1.43475	0.0548963	−1.99485	−0.946001
$\delta_{\rm CKM}[^\circ]$	−0.291137	0.397159	−0.350422	−0.755859	−0.956628	−0.3197
χ^2_{min}	**6.3479**	**3.7962**	**5.0715**	**3.8665**	**14.789**	**3.4746**
Observables	Corresponding Predictions at GUT scale					
$\sin^2\theta^l_{13}$	0.0223307	0.0194886	0.0218753	0.0186789	0.0253152	0.0205366
$\delta_{\rm MNS}[^\circ]$	2.41793	4.52493	6.08769	335.07	357.142	14.7651
$\alpha_1[^\circ]$	347.106	8.42838	7.64991	28.0261	14.5679	1.13126
$\alpha_2[^\circ]$	163.759	191.241	188.713	218.586	196.273	177.828
$r_R\left(\frac{m_t^2}{m_\tau}\right)$ [GeV]	1.77×10^{-10}	2.63×10^{-10}	2.50×10^{-10}	4.02×10^{-10}	7.3×10^{-11}	2.82×10^{-10}

TABLE III. Best fit solutions for fermion masses and mixing obtained assuming the type-I seesaw dominance in the minimal SUSY $SO(10)$ model. Pulls of various observables and predictions obtained at the minimum are shown for six different data sets.

Neutrino Physics at the LHC

What LHC can do for neutrino physics?

Production of new particles in the neutrino models,

The most direct test!

T.~Han and B.~Zhang,]; M. Nebot et al..; R.~Franceschini et al.;

F.~del Aguila etal.; P.~Fileviez Perez et al.; A.~Arhrib et al.,;

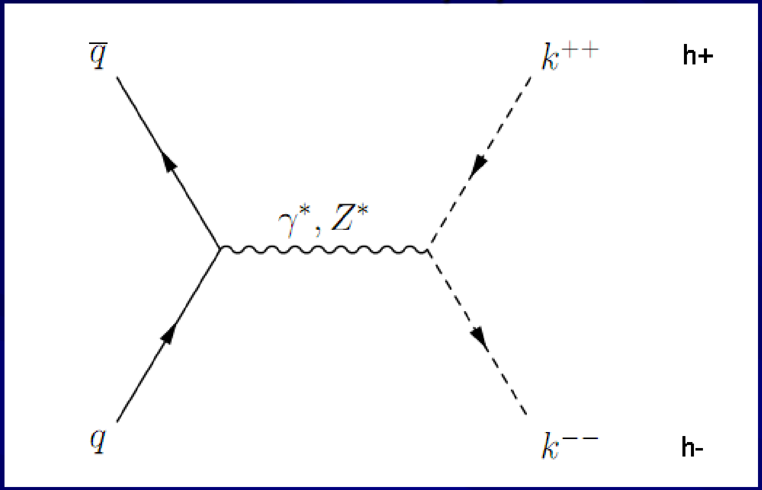
W.~Chao et al.; X.-G. He et al., ; T. Li and X.-G. He.;

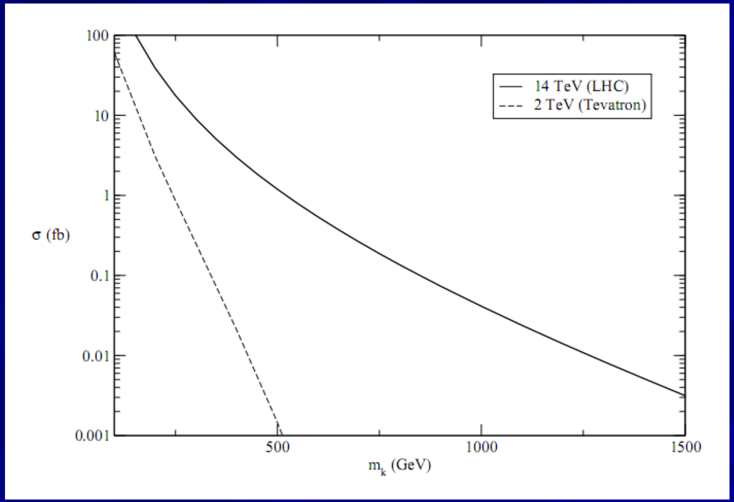
Eung-Jin Cheung; S.K. Kang….

Both Zee, Babu-Zee models have q q-bar -> h^+ h^-

Zee-Babu new: q q-bar -> k^{++} k^{--}

Nebot et al

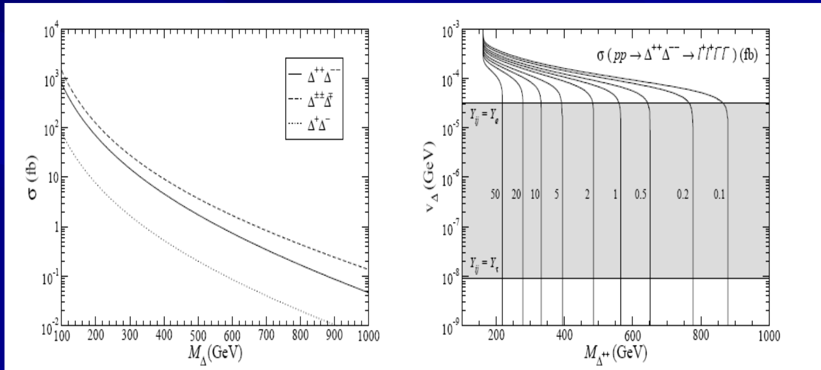




Type II seesaw at the LHC

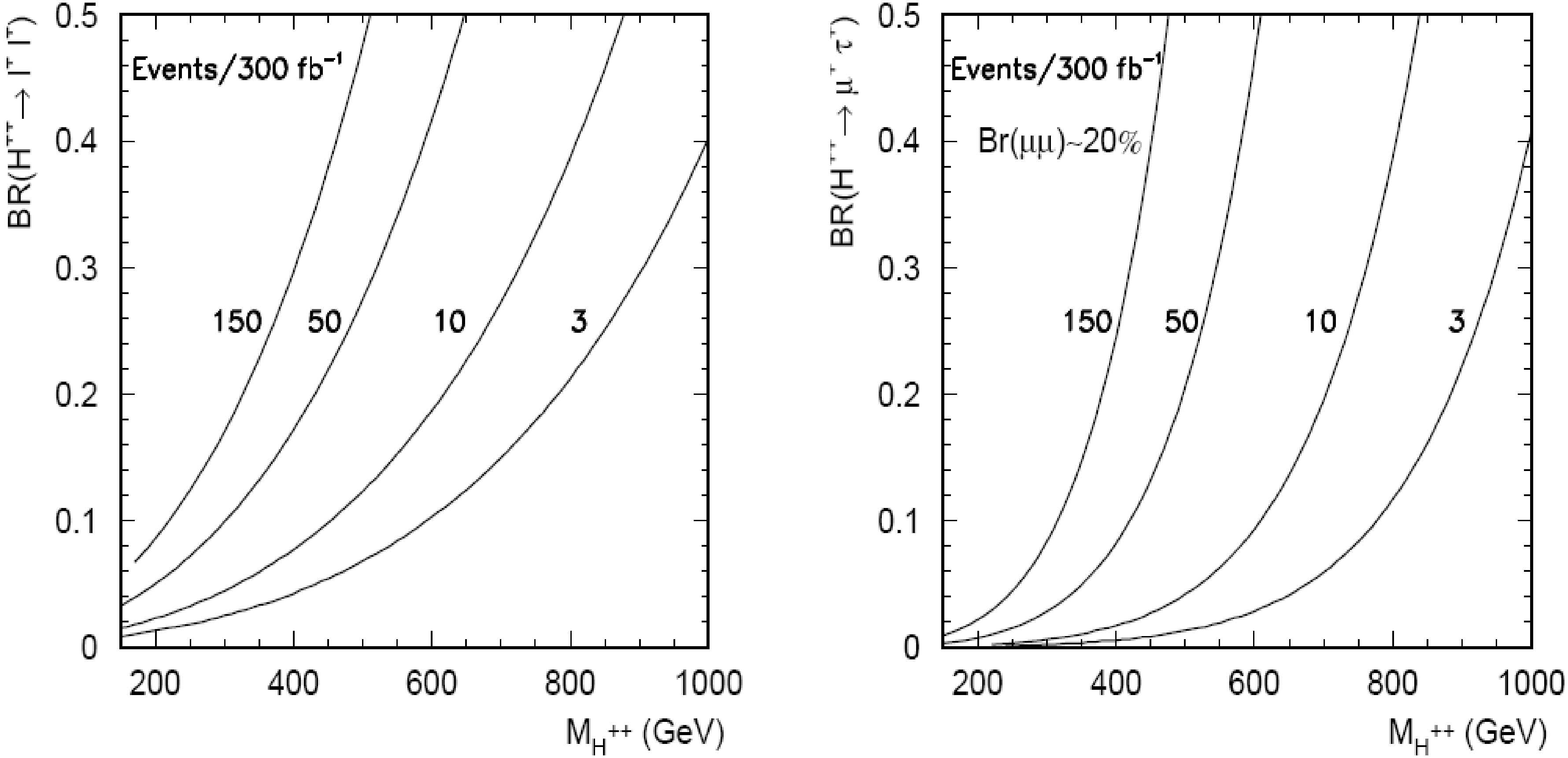
A new triplet Δ contains:

0, +, ++ charged scalars.



Left: Cross section for production of charged scalar pairs $\Delta^{++}\Delta^{--}$, $\Delta^{\pm\pm}\Delta^{\mp}$ and $\Delta^{+}\Delta^{-}$ at LHC. Right: cross section for $\Delta^{++}\Delta^{--}$ decaying into four lepton final states.

Del Aguila and Aguilar-Saavedra



Event contours in the BR$-M_{H++}$ plane for the doubly charged Higgs decay at the LHC with an integrated luminosity 300 fb^{-1} for $\mu^+\mu^+\mu^-\mu^-$ (left) and for $\mu^+\mu^+\mu^-\tau^-$ (right), assuming BR$(H^{++} \to \mu^+\mu^+) = 20\%$.

Discovery of doubly charged scalar as heavy as 1 TeV can be achieved at the LHC

Type-I seesaw at LHC (also Type III seesaw)

Light-light, Heavy-light and heavy-heavy lepton gauge and Yukawa interactions

$$L_{ll} = -\frac{g}{2c_W}\bar{\nu}_{mL}\gamma^\mu U^\dagger_{\nu\nu}U_{\nu\nu}\nu_{mL}Z_\mu - [\frac{g}{\sqrt{2}}\bar{l}_L\gamma^\mu U_{\nu\nu}\nu_L W^-_\mu + \bar{\nu}^c_{mL}\hat{m}_\nu U^\dagger_{\nu\nu}U_{\nu\nu}\nu_{mL}\frac{h}{v}] + h.c.$$

$$L_{lH} = -\frac{g}{2c_W}\bar{\nu}_{mL}\gamma^\mu U_{\nu N}N_{mL}Z_\mu - \frac{g}{\sqrt{2}}\bar{l}_L\gamma^\mu U_{\nu N}N_L W^-_\mu + h.c.$$

$$= \bar{\nu}^c_{mL}(\hat{m}_\nu U^\dagger_{\nu N}U_{\nu\nu} + U^T_{\nu\nu}U^*_{\nu N}\hat{M}_N)N_{mL}\frac{h}{v} + h.c.$$

$$L_{HH} = -\frac{g}{2c_W}\bar{N}_{mL}\gamma^\mu U^\dagger_{\nu N}U_{\nu N}N_{mL}Z_\mu - [\bar{N}^c_{mL}U^T_{\nu N}U^*_{\nu N}\hat{M}_N N_{mL}\frac{h}{v} + h.c.]$$

$U_{\nu\nu}$ is the effective V_{PMNS} mixing matrix.

$$\begin{pmatrix} \nu_L \\ (N_R)^c \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix}, \qquad U \equiv \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}.$$

Production of N at the LHC

$$q\bar{q}' \to W^* \to lN \;, \quad q\bar{q} \to Z^* \to \nu N \;, \quad q\bar{q} \to h^* \to \nu^c N \;.$$

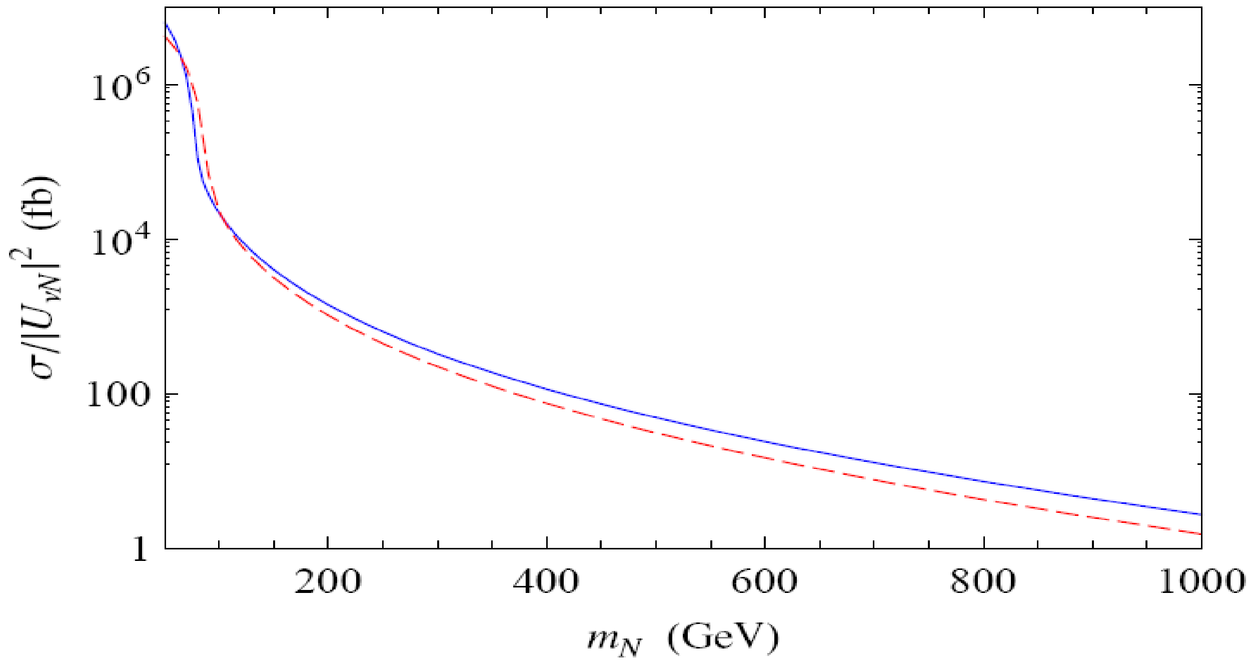
The cross sections are all proportional to $|U_{\nu N}^{ij}|^2 = |V_{lN}^{ij}|^2$.

$$q\bar{q} \to Z^* \to N^c N \;, \quad q\bar{q} \to h^* N^c N \;.$$

The cross sections are all proportional to $|V_{lN}^{im} V_{lN}^{*nj}|^2$.
Important to know V_{lN}! An interesting relation

$$V_{lN} \hat{M}_N V_{lN}^T = -V_{PMNS} \hat{m}_\nu V_{PMNS}^T \;.$$

$V_{lN} \sim (m_\nu/m_N)^{1/2} < 10^{-6}$ for one generation. Too small!



Cross sections σ of $pp \to lNX$ in types-I and -III seesaw (solid curve) and $pp \to lEX$ in type- III seesaw (dashed curve) as functions of $m_N = m_E$ for $|U_{\nu N}| = 1$ and pp center-of-mass energy of $\sqrt{s} = 14\,\text{TeV}$.

$|V^{ij}_{lN}|^2 M_j/100\text{GeV}$ small: $< 10^{-12}$.

Cross section is too small to have significant production of N at LHC!
Any non-trivial indication at LHC for a heavy neutral particle is beyond Type I seesaw.

Large V_{lN} for Type-I and III seesaw possible

X.-G. He et al., arXiv:0907.1607[hep-ph]

Even with small V_{lN} Type-III seesaw can be tested at the LHC

Tong Li and Xiao-Gang He, arXiv:0907.4193[hep-ph]

Production of N and E

Singlet E and N productions

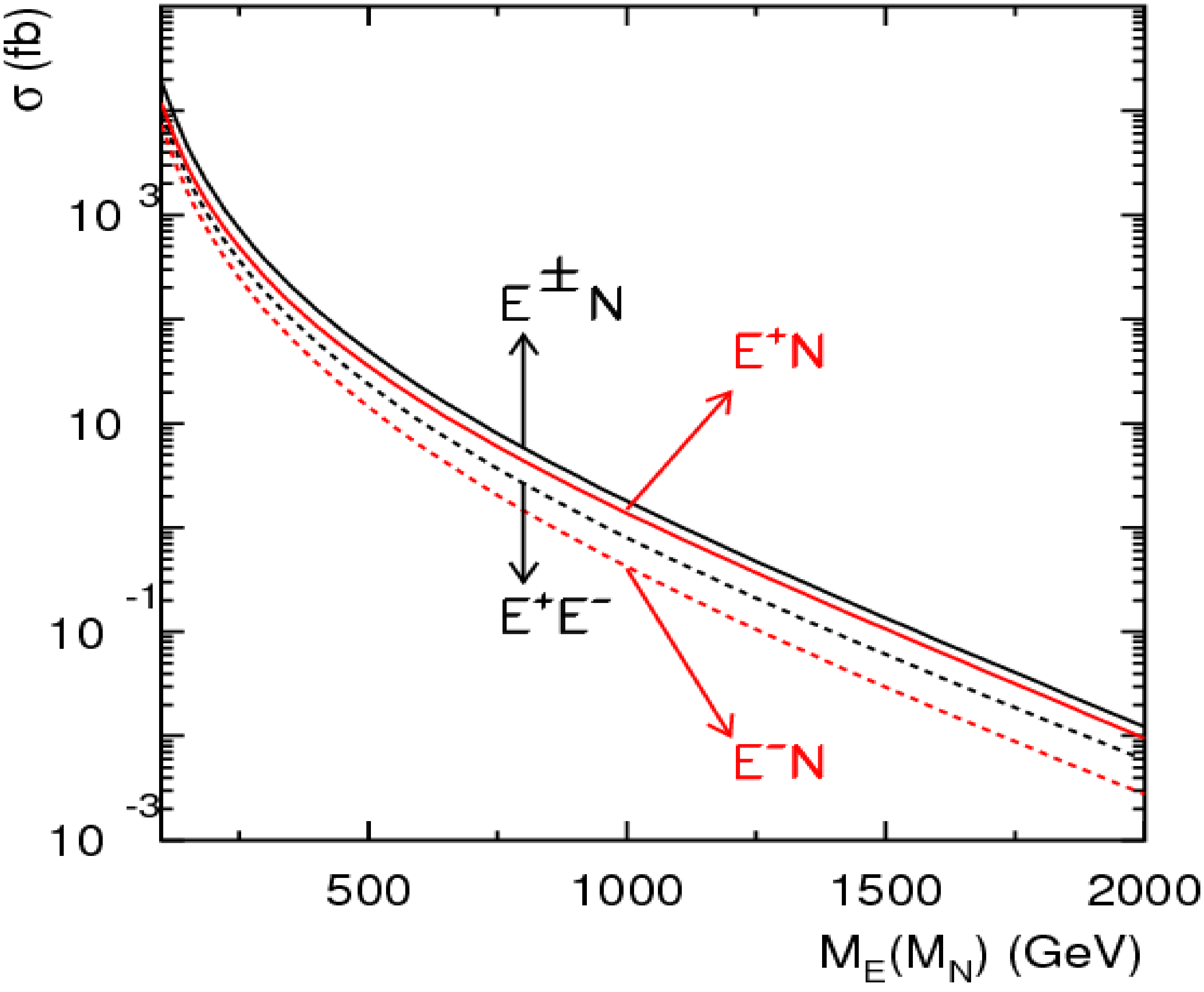
$$q\bar{q} \to Z^* \to \nu N, lN \ , \quad q\bar{q}' \to W^* \to lN, \nu E \ , q\bar{q} \to h^* \to \nu N, lN \ .$$

Cross section too small to be of interesting, just like seesaw Type I.

The main production channels of $E^\pm, N$ are

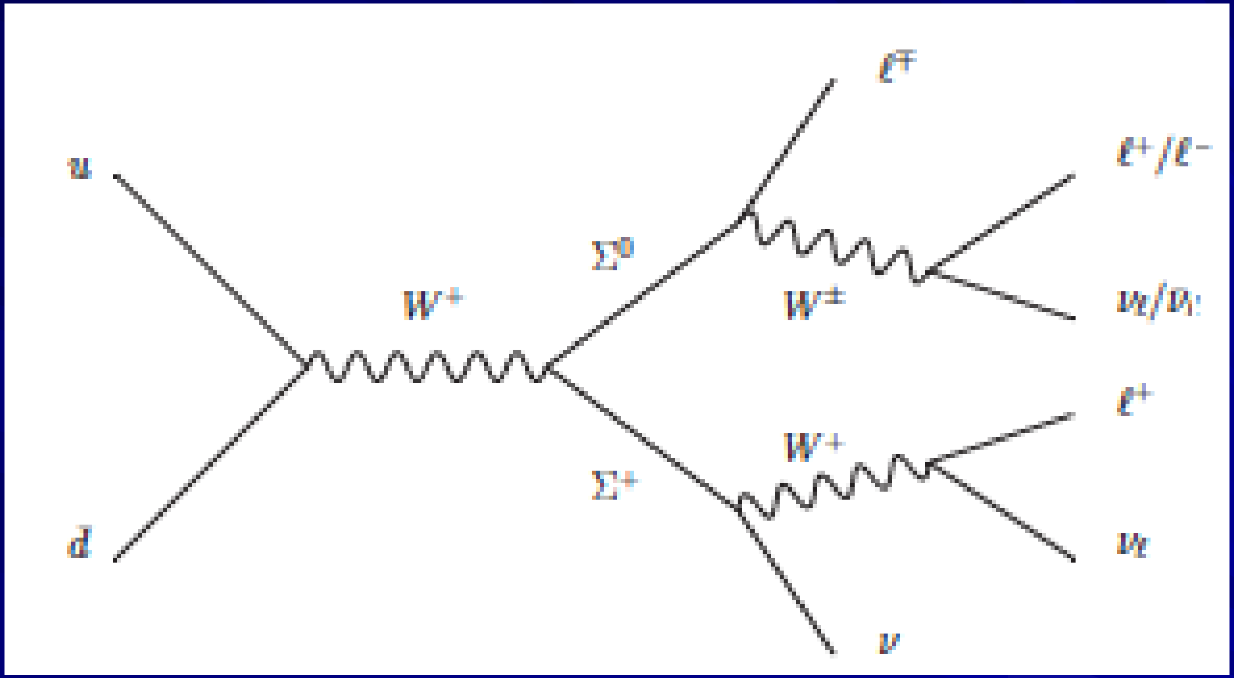
$$q\bar{q} \to \gamma^*/Z^* \to E^+E^-, \quad q\bar{q}' \to W^* \to E^\pm N \ .$$

The relevant total production cross sections are large enough to be probed at LHC.



σ (fb)
10^3
10
10^-1
10^-3
E±N
E+N
E+E−
E−N
500
1000
1500
2000
M_E(M_N) (GeV)

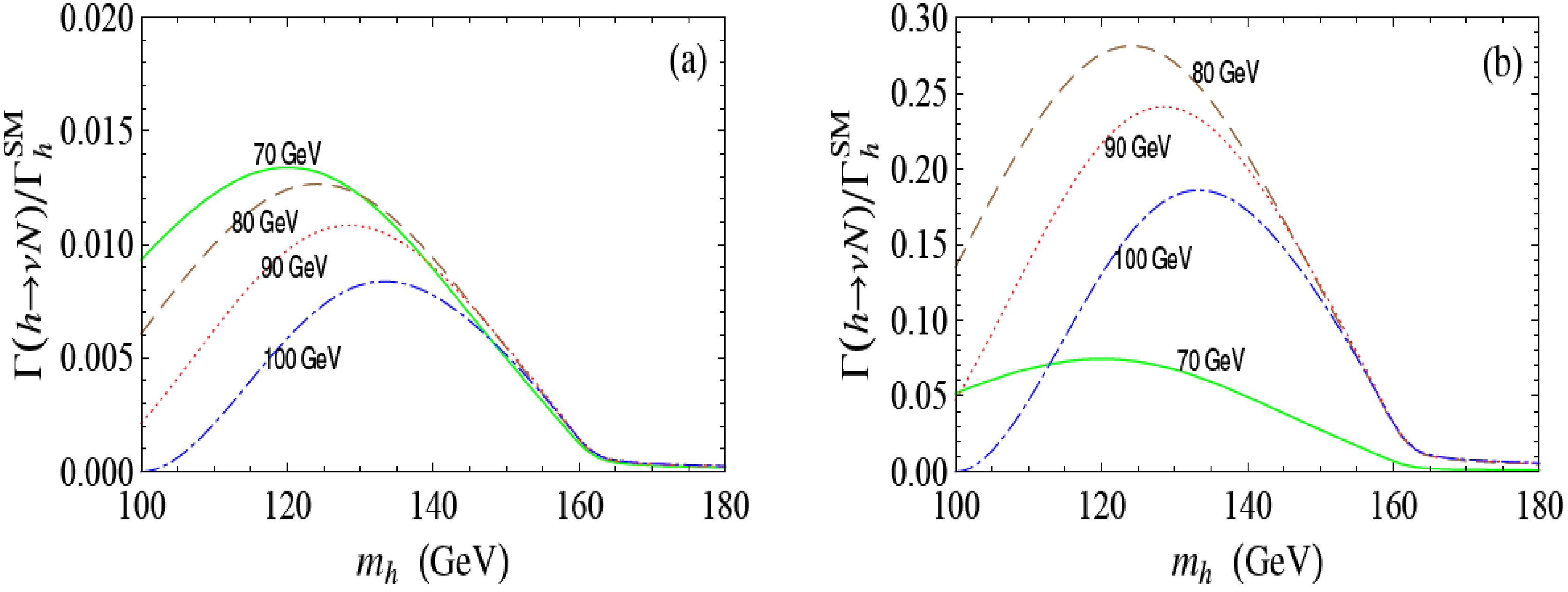
CMS search limit for Type III seesaw particles arXiv:1210.1797



Depending on the considered scenarios, lower limits are obtained on the mass of the heavy partner of the neutrino that range from 180 to 210 GeV.These are the first limits on the production of type III seesaw fermionic triplet states reported by an experiment at the LHC.

If $m_N < m_h$, affect Higgs decay Br.

Park, Wang and Yanagida, arXiv:0909.2937; Chen et al. arXiv:1001.5215



Ratios of width of $h \to \nu N$ in type-I seesaw to the total Higgs width in the SM as functions of the Higgs mass m_h for heavy-neutrino mass values $m_N = 70, 80, 90, 100$ GeV and different choices of $U_{\nu N}$.

A lot more for theoretical neutrino models,

Carlo Giunti talk this afternoon for more.

Sterile neutrinos LSND&MiniBoon, alive???

Lepton number violating FCNC connection

Dark matter connection,

Leptogenesis connection.

…