Form of the Primordial Spectrum & Cosmological Parameter Estimation

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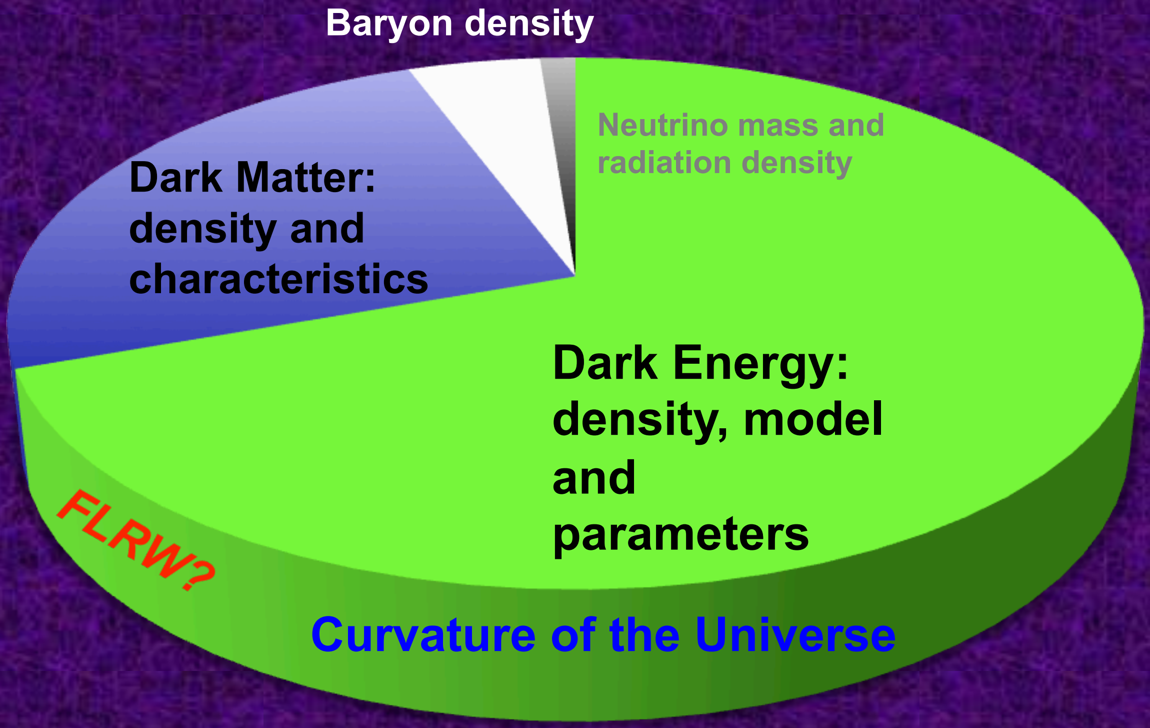
Asia Pacific Center for Theoretical Physics

4th-9th November 2012 , KIAS, Seoul-Korea

Conference on Particle Physics and Cosmology

Era of Precision Cosmology

Combining theoretical works with new measurements and using statistical techniques to place sharp constraints on cosmological models and their parameters.



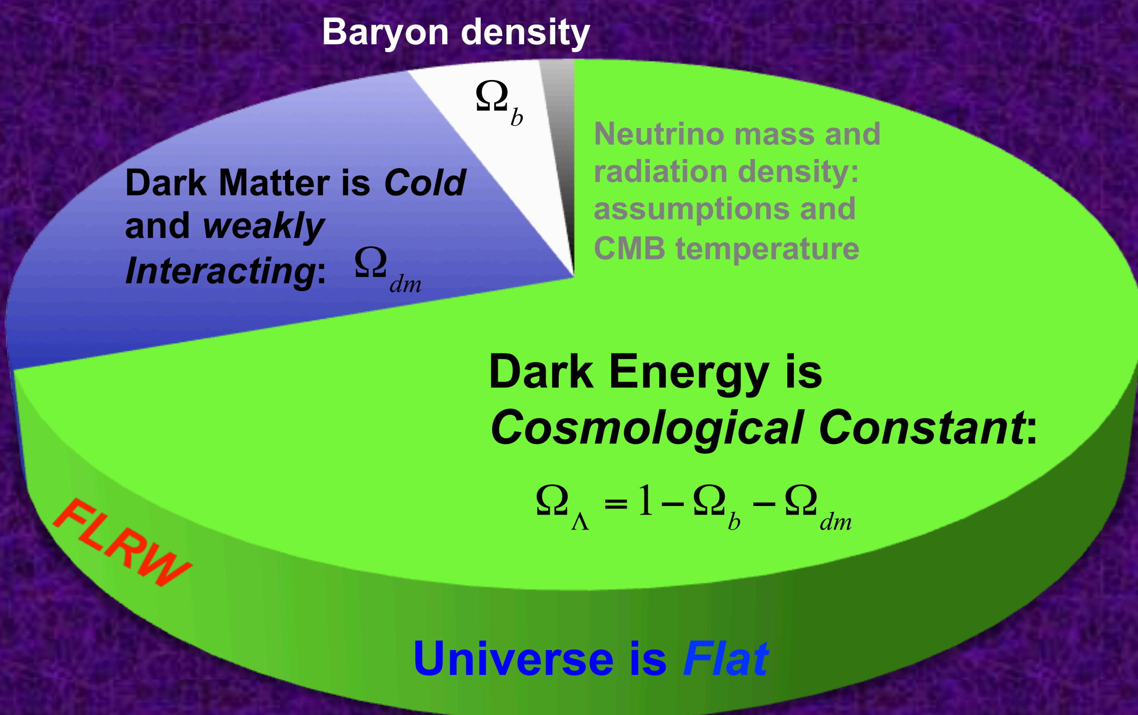
Initial Conditions: Form of the Primordial Spectrum and Model of Inflation and its Parameters

Epoch of reionization

Hubble Parameter and the Rate of Expansion

Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological models.



Initial Conditions: Form of the Primordial Spectrum is *Power-law*

n_s, A_s

Epoch of reionization

τ

Hubble Parameter and the Rate of Expansion

H_0

Inflation

- Extreme accelerated expansion of the early universe.
- It can be realized by scalar fields (or some other mechanisms).
- So far the best theory that can resolve the magnetic monopole problem (absence of relics), flatness problem, horizon problem and explain the initial perturbations from quantum fluctuations.
- It has many many models.
- These models are different in their statistical properties and we may be able to distinguish between them using cosmological observations.

Models of Inflation

**old, new, pre-owned,
chaotic, quixotic, ergodic,
ekpyrotic, autoerotic,
faith-based, free-based,
D-term, F-term, summer-term,
brane, braneless, brainless,
supersymmetric, supercilious,
natural, supernatural, *au natural,*
hybrid, low-bred, white-bread,
one-field, two-field, left-field,
eternal, internal, infernal,
self-reproducing, self-promoting,
dilaton, dilettante, …….**

[shamelessly stolen from Rocky Kolb]

[Shamelessly stolen from Jan Hamann]

Constraints on inflationary scenarios from cosmological observations:

- Form of the primordial spectrum ***(degenerate with other cosmological quantities).***

- Tensor-to-scalar ratio of perturbation amplitudes
  (near future potential probe)

- Primordial non-Gaussianities
  (near future potential probe)

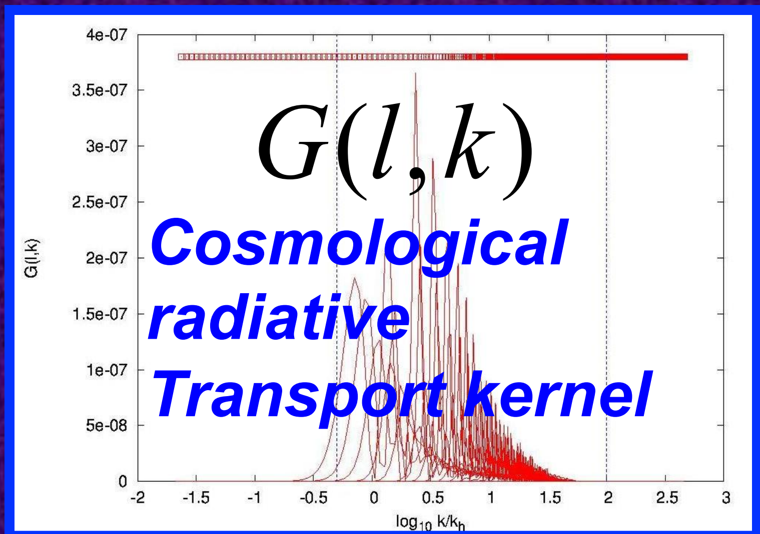
$$P(k)$$

Primordial power Spectrum

?

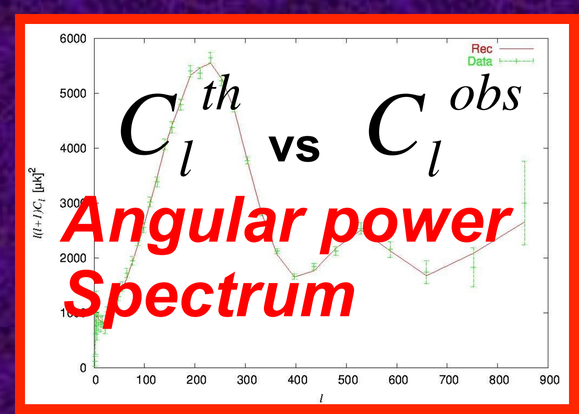
Assumption suggested by inflation

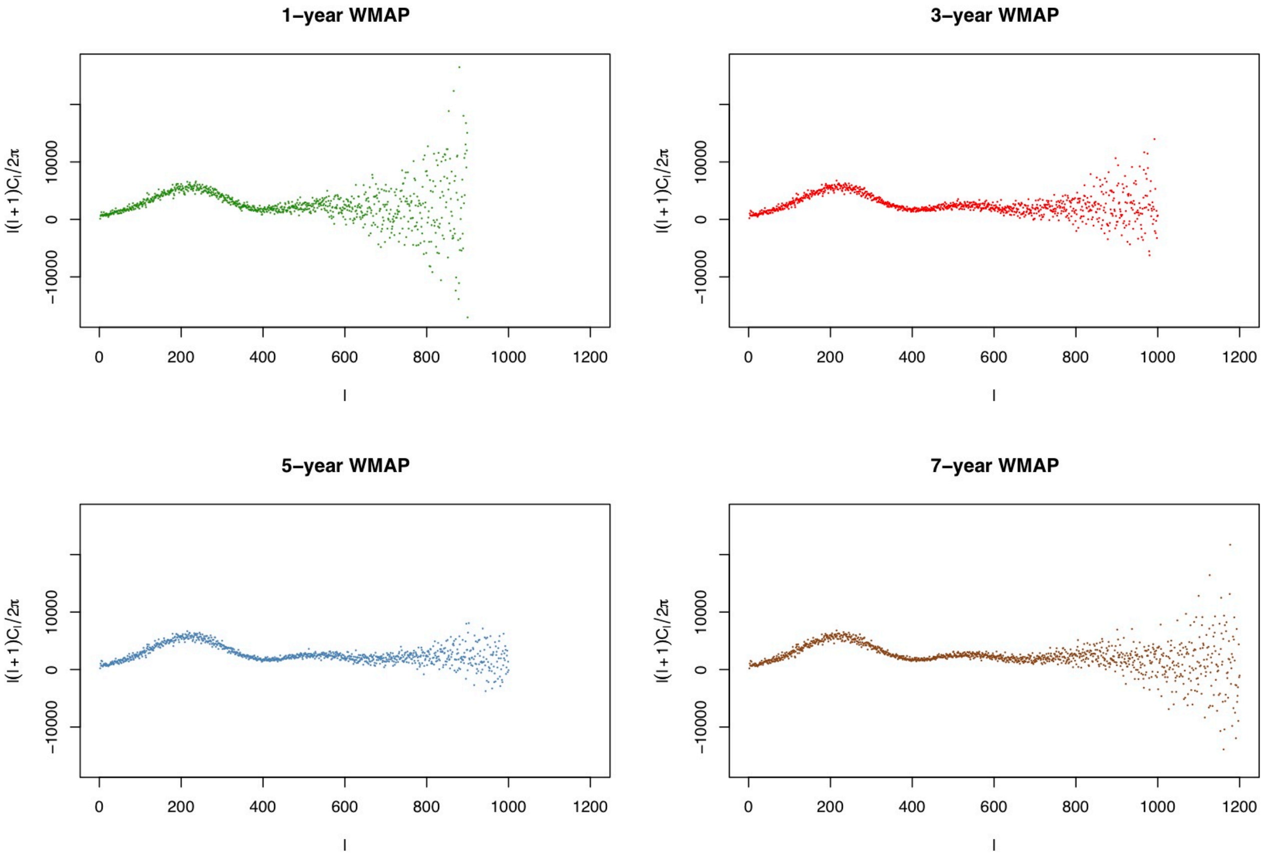
$$C_l = \sum G(l,k)P(k)$$



Determined by background model and cosmological parameters

Detected by observation

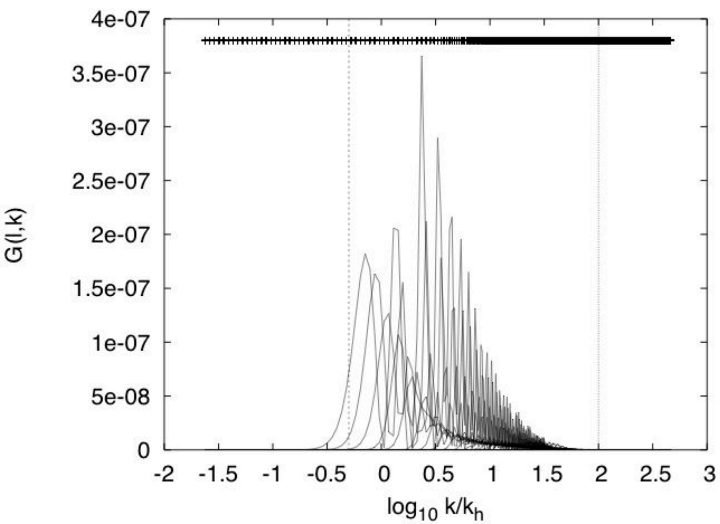




Observed angular power spectrum from WMAP

Shafieloo & Souradeep PRD 2004

Plot from Aghamousa et al, APJ 2011



Shape of the transfer kernel, G(l,k)

Standard Model of Cosmology- ***Vanilla Model***

- Flat Lambda Cold Dark Matter Universe (LCDM) with power–law form of the primordial spectrum
- It has 6 main parameters:

$$\Omega_b$$

$$\Omega_m$$

$$H_0$$

$$\tau$$

$$\boxed{\begin{matrix} A_s \\ n_s \end{matrix}}$$

Cosmological Parameters from WMAP

WMAP Cosmological Parameters

Model: lcdm+sz+lens

Data: wmap5

$10^2\Omega_b h^2$	2.273 ± 0.062	$1 - n_s$	$0.037^{+0.015}_{-0.014}$
$1 - n_s$	$0.0081 < 1 - n_s < 0.0647$ (95% CL)	$A_{\rm BAO}(z = 0.35)$	0.457 ± 0.022
C_{220}	5756 ± 42	$d_A(z_{\rm eq})$	14279^{+186}_{-189} Mpc
$d_A(z_*)$	14115^{+188}_{-191} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.41 \pm 0.11) \times 10^{-9}$
h	$0.719^{+0.026}_{-0.027}$	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc
$k_{\rm eq}$	0.00968 ± 0.00046	$\ell_{\rm eq}$	136.6 ± 4.8
ℓ_*	$302.08^{+0.83}_{-0.84}$	n_s	$0.963^{+0.014}_{-0.015}$
Ω_b	0.0441 ± 0.0030	$\Omega_b h^2$	0.02273 ± 0.00062
Ω_c	0.214 ± 0.027	$\Omega_c h^2$	0.1099 ± 0.0062
Ω_Λ	0.742 ± 0.030	Ω_m	0.258 ± 0.030
$\Omega_m h^2$	0.1326 ± 0.0063	$r_{\rm hor}(z_{\rm dec})$	286.0 ± 3.4 Mpc
$r_s(z_d)$	153.3 ± 2.0 Mpc	$r_s(z_d)/D_v(z = 0.2)$	0.1946 ± 0.0079
$r_s(z_d)/D_v(z = 0.35)$	0.1165 ± 0.0042	$r_s(z_*)$	146.8 ± 1.8 Mpc
R	1.713 ± 0.020	σ_8	0.796 ± 0.036
A_{SZ}	$1.04^{+0.96}_{-0.69}$	t_0	13.69 ± 0.13 Gyr
τ	0.087 ± 0.017	θ_*	0.010400 ± 0.000029
θ_*	0.5959 ± 0.0017 °	t_*	380081^{+5843}_{-5841} yr
$z_{\rm dec}$	1087.9 ± 1.2	z_d	1020.5 ± 1.6
$z_{\rm eq}$	3176^{+151}_{-150}	$z_{\rm reion}$	11.0 ± 1.4
z_*	1090.51 ± 0.95		

Table from NASA - LAMBDA website

Parameter estimation within a cosmological framework

Harisson-Zel'dovich (HZ)

WMAP Cosmological Parameters Model: lcdm+ns=1 Data: wmap	
$10^2\Omega_b h^2$	$2.405^{+0.046}_{-0.047}$
$\Delta^2_{\mathcal{R}}(k=0.002/\mathrm{Mpc})$	$(23.1 \pm 1.2) \times 10^{-10}$
h	0.778 ± 0.032
H_0	77.8 ± 3.2 km/s/Mpc
$\Omega_b h^2$	$0.02405^{+0.00046}_{-0.00047}$
Ω_Λ	0.788 ± 0.031
Ω_m	0.212 ± 0.031
$\Omega_m h^2$	$0.1271^{+0.0086}_{-0.0087}$
σ_8	$0.796^{+0.053}_{-0.054}$
A_{SZ}	$0.92^{+0.63}_{-0.61}$
t_0	13.353 ± 0.096 Gyr
τ	0.141 ± 0.029
θ_A	0.5986 ± 0.0017 °
z_r	14.6 ± 2.0

Power-Law (PL)

WMAP Cosmological Parameters Model: lcdm Data: wmap	
$10^2\Omega_b h^2$	2.229 ± 0.073
$\Delta^2_{\mathcal{R}}(k=0.002/\mathrm{Mpc})$	$(23.5 \pm 1.3) \times 10^{-10}$
h	$0.732^{+0.031}_{-0.032}$
H_0	$73.2^{+3.1}_{-3.2}$ km/s/Mpc
$\log(10^{10} A_s)$	3.156 ± 0.056
$\boxed{n_s(0.002)}$	$\boxed{0.958 \pm 0.016}$
$\Omega_b h^2$	0.02229 ± 0.00073
$\Omega_c h^2$	$0.1054^{+0.0078}_{-0.0077}$
Ω_Λ	0.759 ± 0.034
Ω_m	0.241 ± 0.034
$\Omega_m h^2$	$0.1277^{+0.0080}_{-0.0079}$
σ_8	$0.761^{+0.049}_{-0.048}$
τ	0.089 ± 0.030
θ_A	0.5952 ± 0.0021 °
z_r	$11.0^{+2.6}_{-2.5}$

PL with Running (RN)

WMAP Cosmological Parameters Model: lcdm+run Data: wmap	
$10^2\Omega_b h^2$	2.10 ± 0.10
$\Delta^2_{\mathcal{R}}(k=0.002/\mathrm{Mpc})$	$(23.9 \pm 1.3) \times 10^{-10}$
$\boxed{dn_s/d\ln k}$	$\boxed{-0.055^{+0.030}_{-0.031}}$
h	$0.681^{+0.042}_{-0.041}$
H_0	$68.1^{+4.2}_{-4.1}$ km/s/Mpc
$\boxed{n_s(0.002)}$	$\boxed{1.050^{+0.059}_{-0.058}}$
$\Omega_b h^2$	0.0210 ± 0.0010
Ω_Λ	$0.703^{+0.056}_{-0.055}$
Ω_m	$0.297^{+0.055}_{-0.056}$
$\Omega_m h^2$	$0.1350^{+0.0099}_{-0.0097}$
σ_8	$0.771^{+0.051}_{-0.050}$
A_{SZ}	$1.06^{+0.62}_{-0.65}$
t_0	13.97 ± 0.20 Gyr
τ	0.101 ± 0.031
θ_A	0.5940 ± 0.0021 °
z_r	12.8 ± 2.8

Tables from NASA - LAMBDA website

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Tables from NASA - LAMBDA website

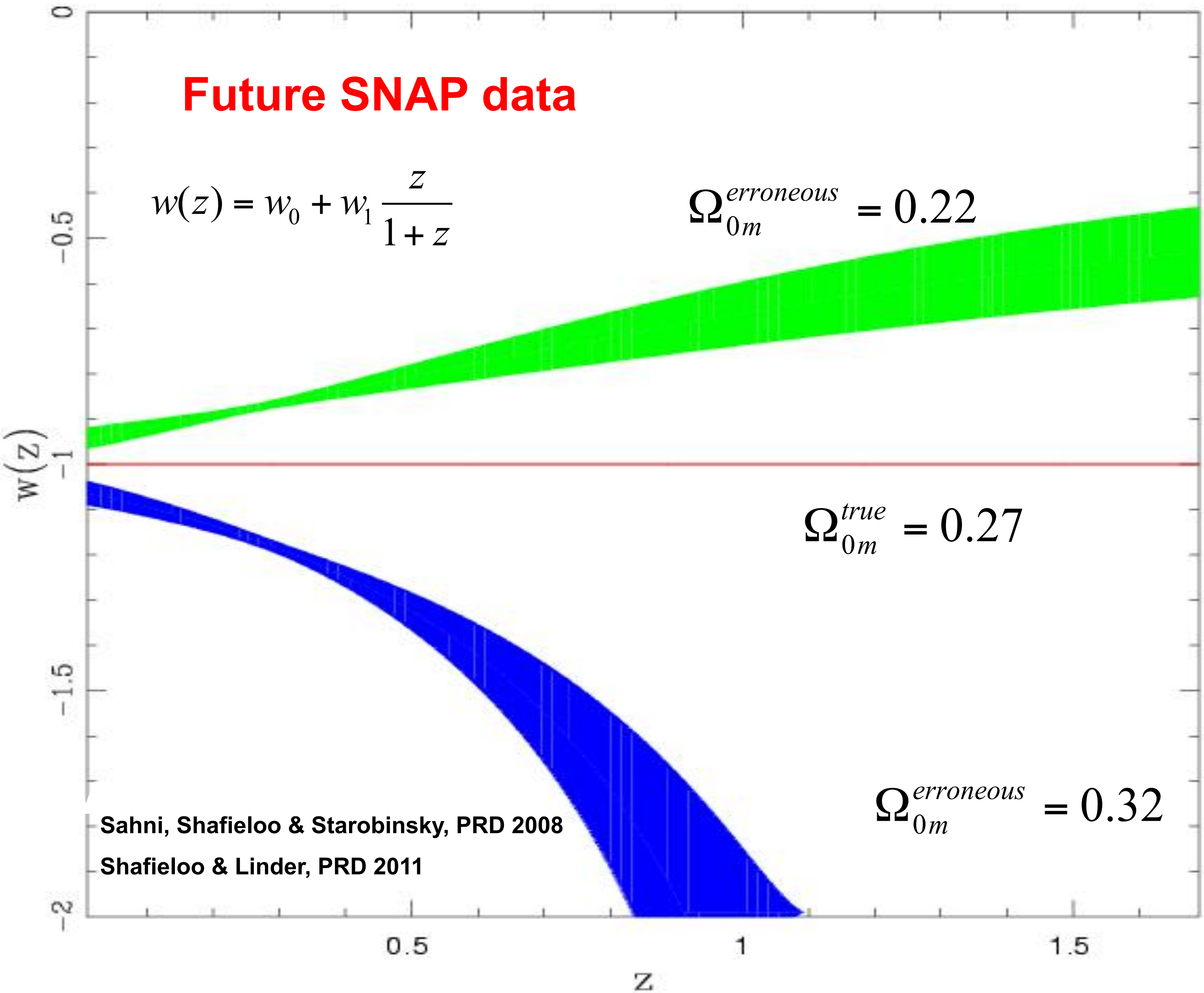
Dark Energy Reconstruction

- Any uncertainties in matter density is bound to affect the reconstructed w(z).

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}$$

$$\omega_{DE} = \frac{\left(\frac{2(1+z)}{3}\frac{H'}{H}\right) - 1}{1 - \left(\frac{H_0}{H}\right)^2 \boxed{\Omega_{0M}} (1+z)^3}$$

Shafieloo et al, MNRAS 2006 ; Shafieloo MNRAS 2007



Future SNAP data
$w(z) = w_0 + w_1 \frac{z}{1+z}$
$\Omega_{0m}^{erroneous} = 0.22$
$\Omega_{0m}^{true} = 0.27$
$\Omega_{0m}^{erroneous} = 0.32$
Sahni, Shafieloo & Starobinsky, PRD 2008
Shafieloo & Linder, PRD 2011
w(z)
z
0
−0.5
−1
−1.5
−2
0.5
1
1.5

Model Independent Estimation of Primordial Spectrum

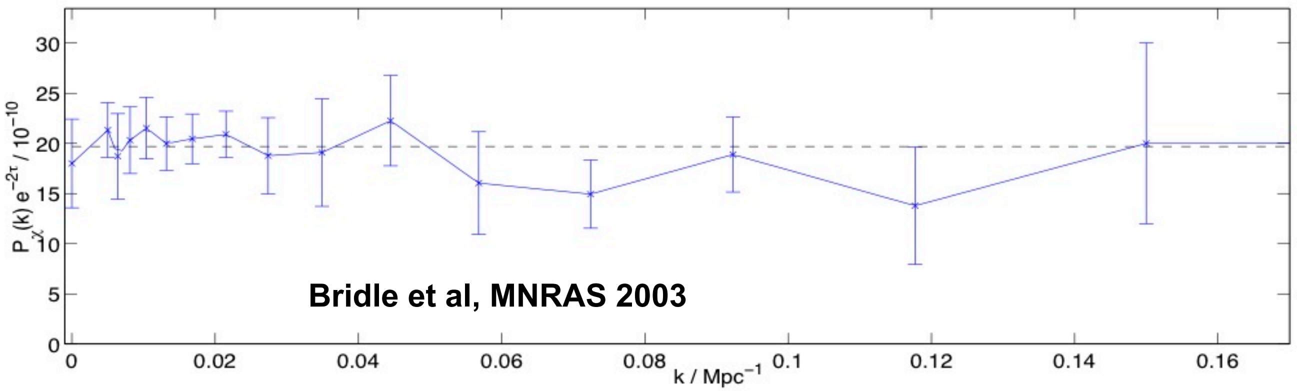
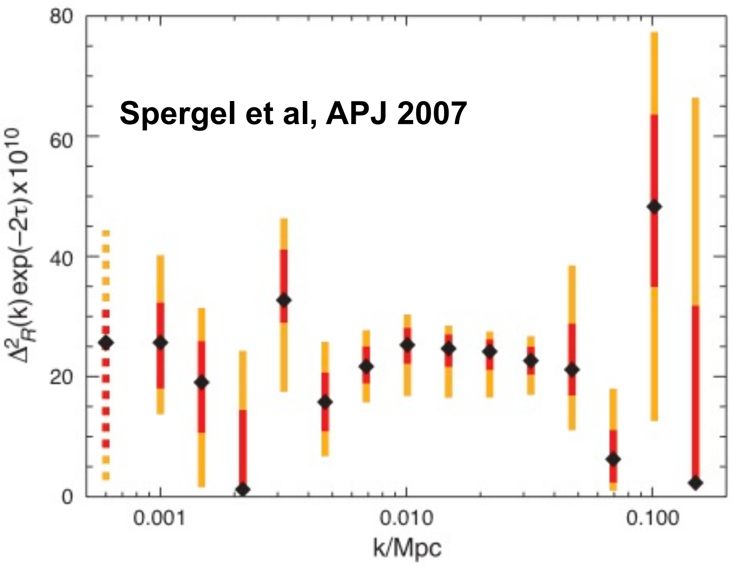


Figure 4. Reconstruction of the shape of the primordial power spectrum in 16 bands after marginalising over the Hubble constant, baryon and dark matter densities, and the redshift of reionization.

Binning Primordial Spectrum



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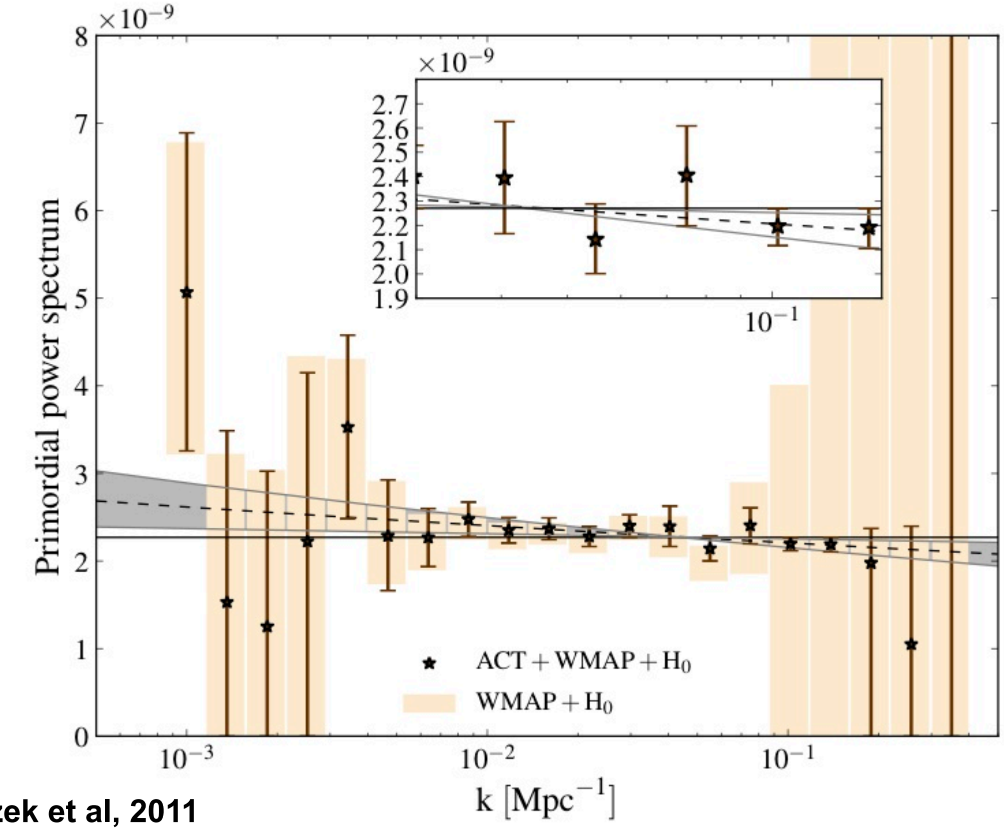


Hlozek et al, 2011

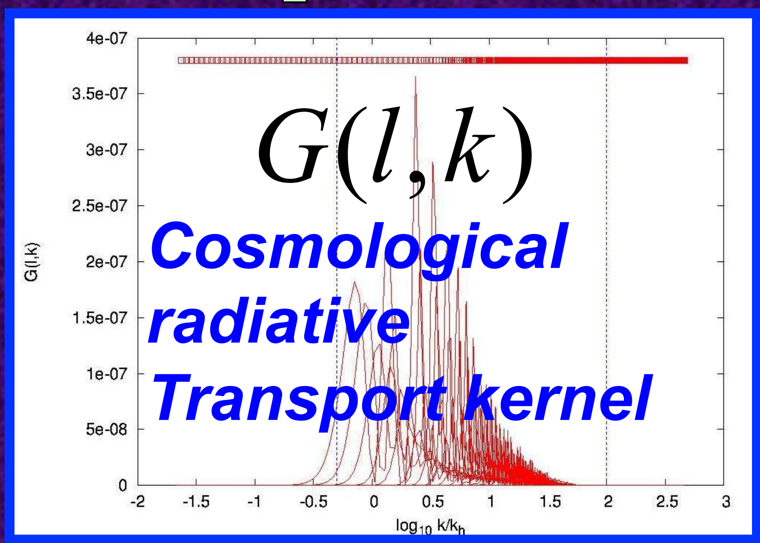
Binning Primordial Spectrum

$$P(k)$$

Primordial power Spectrum

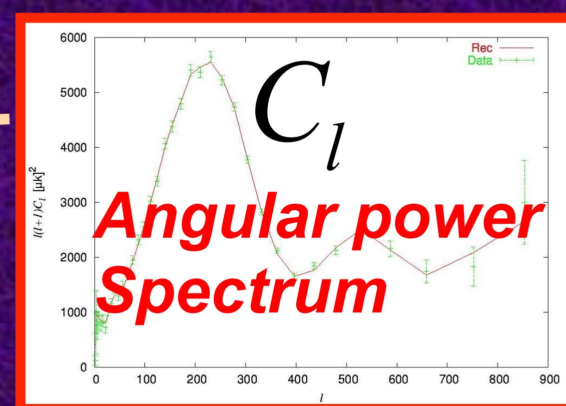
DIRECT RECONSTRUCTION

$$C_l = \sum G(l,k)P(k)$$



Determined by cosmological parameters

Detected by observation



Richardson-Lucy Deconvolution

➔ Iterative algorithm.
➔ Not sensitive to the initial guess.
➔ Enforce positivity of P(k).
[$G(l,k)$ is positive definite and C_l is positive]

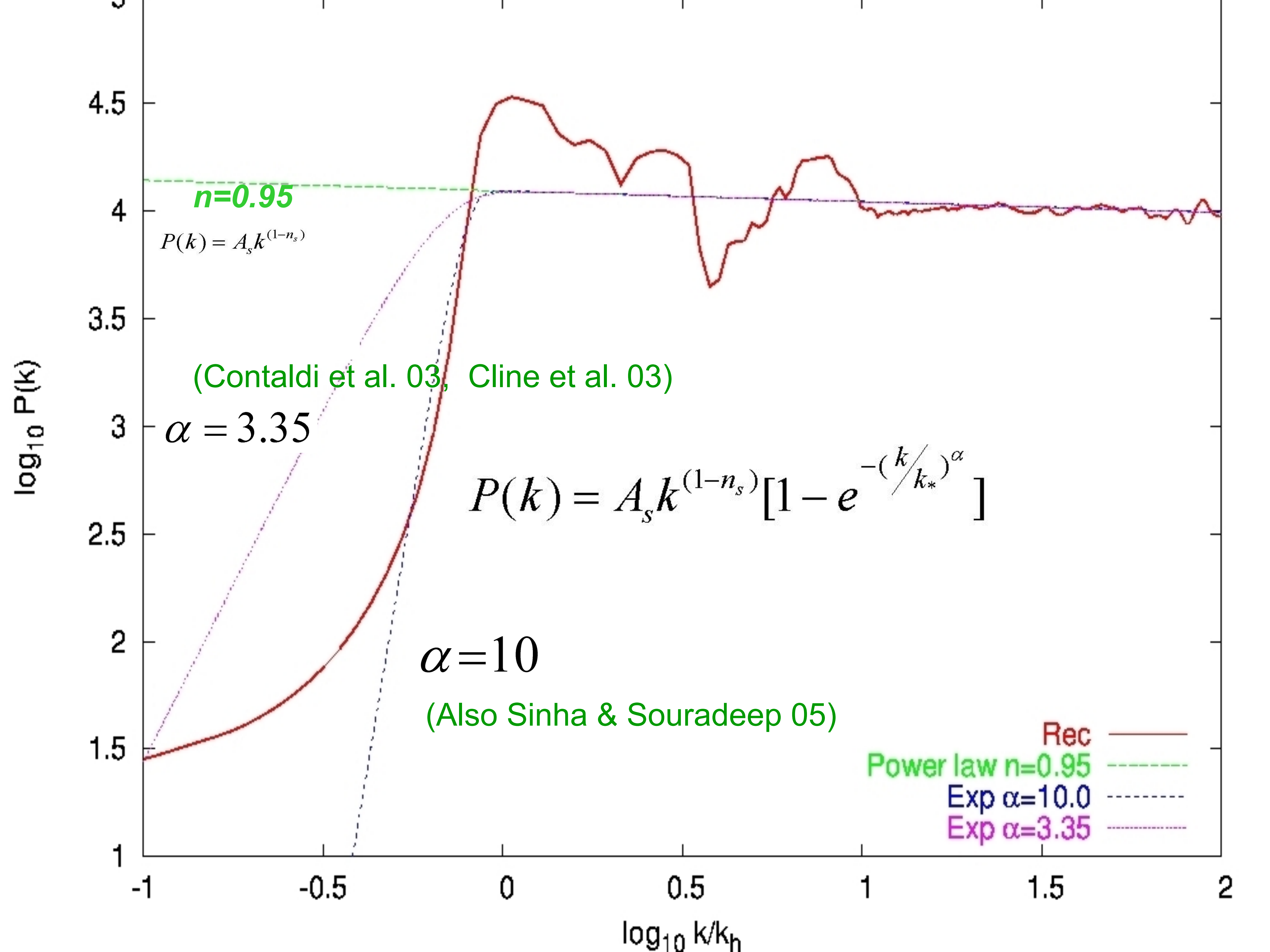
$$C_l^{(i)} = \sum_l G(l,k) P^{(i)}(k)$$

$$P^{(i+1)}(k) - P^{(i)}(k) = P^{(i)}(k) \sum_l G(l,k) \frac{C_l^D - C_l^{(i)}}{C_l^{(i)}} \tanh^2 \frac{(C_l^D - C_l^{(i)})^2}{\sigma_l^2}$$

C_l^D has some finite error bars.

Regularizing function

Shafieloo & Souradeep PRD 2004 ;
Shafieloo et al, PRD 2007;
Shafieloo & Souradeep, PRD 2008;
Nicholson & Contaldi JCAP 2009


n=0.95
$P(k) = A_s k^{(1-n_s)}$
(Contaldi et al. 03, Cline et al. 03)
$\alpha = 3.35$
$P(k) = A_s k^{(1-n_s)}[1 - e^{-(k/k_*)^\alpha}]$
$\alpha = 10$
(Also Sinha & Souradeep 05)
Rec
Power law n=0.95
Exp α=10.0
Exp α=3.35
$\log_{10} P(k)$
$\log_{10} k/k_h$
4.5
4
3.5
3
2.5
2
1.5
1
-1
-0.5
0
0.5
1
1.5
2

Inflationary scenarios

Is the recovered spectrum unusual for inflationary scenarios?

- Starobinsky (1992): sharp changes in the slope in the inflation potential.
- Vilenkin and Ford (1982): pre-inflationary radiation dominated epoch.

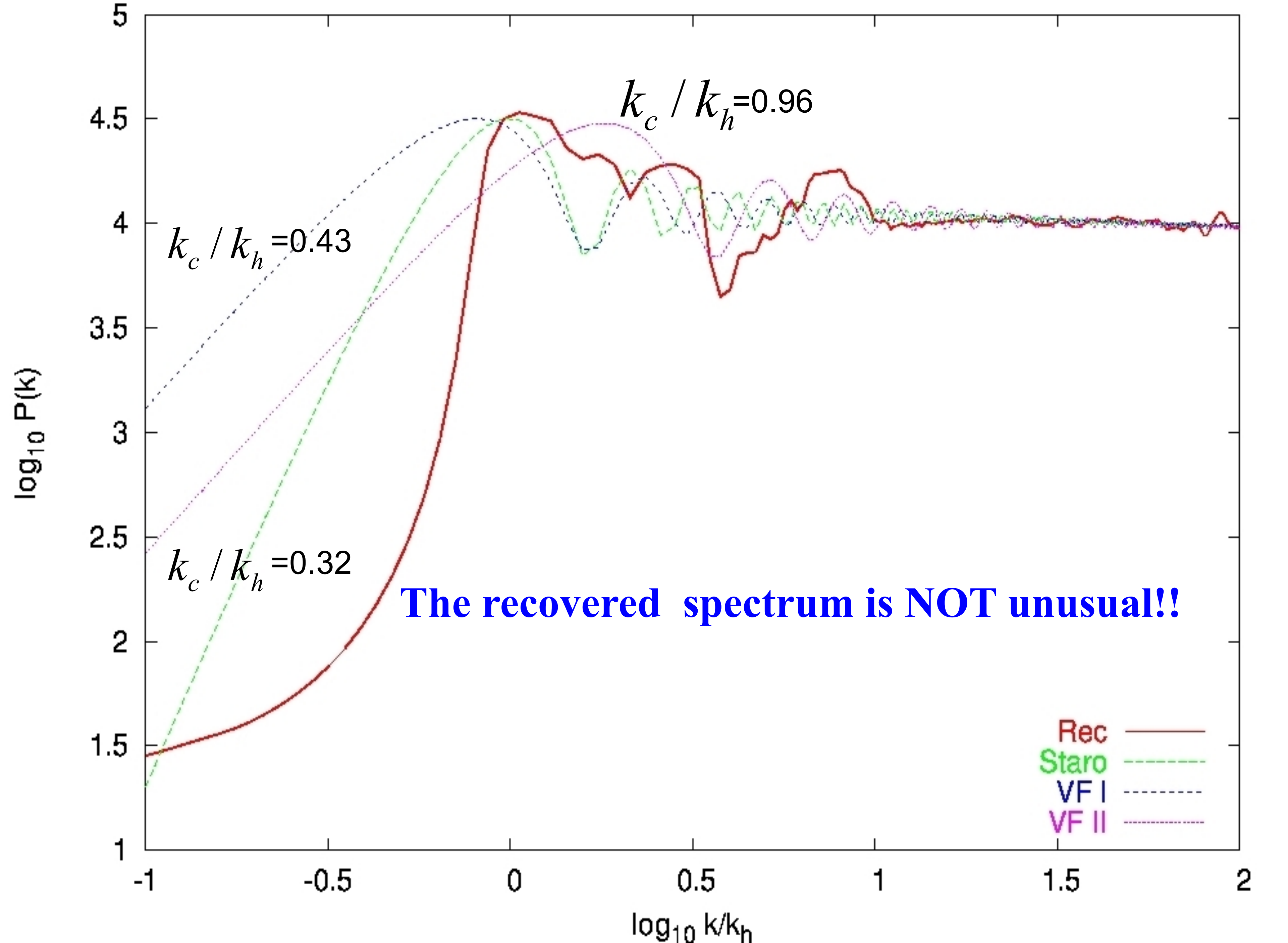
$$P(k) = P_0(k)D(k,k_c,r) = A_s k^{1-n_s}[1-3(r-1)\frac{1}{y}((1-\frac{1}{y^2})\sin 2y + \frac{2}{y}\cos 2y + \frac{9}{2}(r-1)^2\frac{1}{y^2}((1+\frac{1}{y^2})\cos 2y - \frac{2}{y}\sin 2y)]$$

Starobinsky

$$y = k/k_c$$

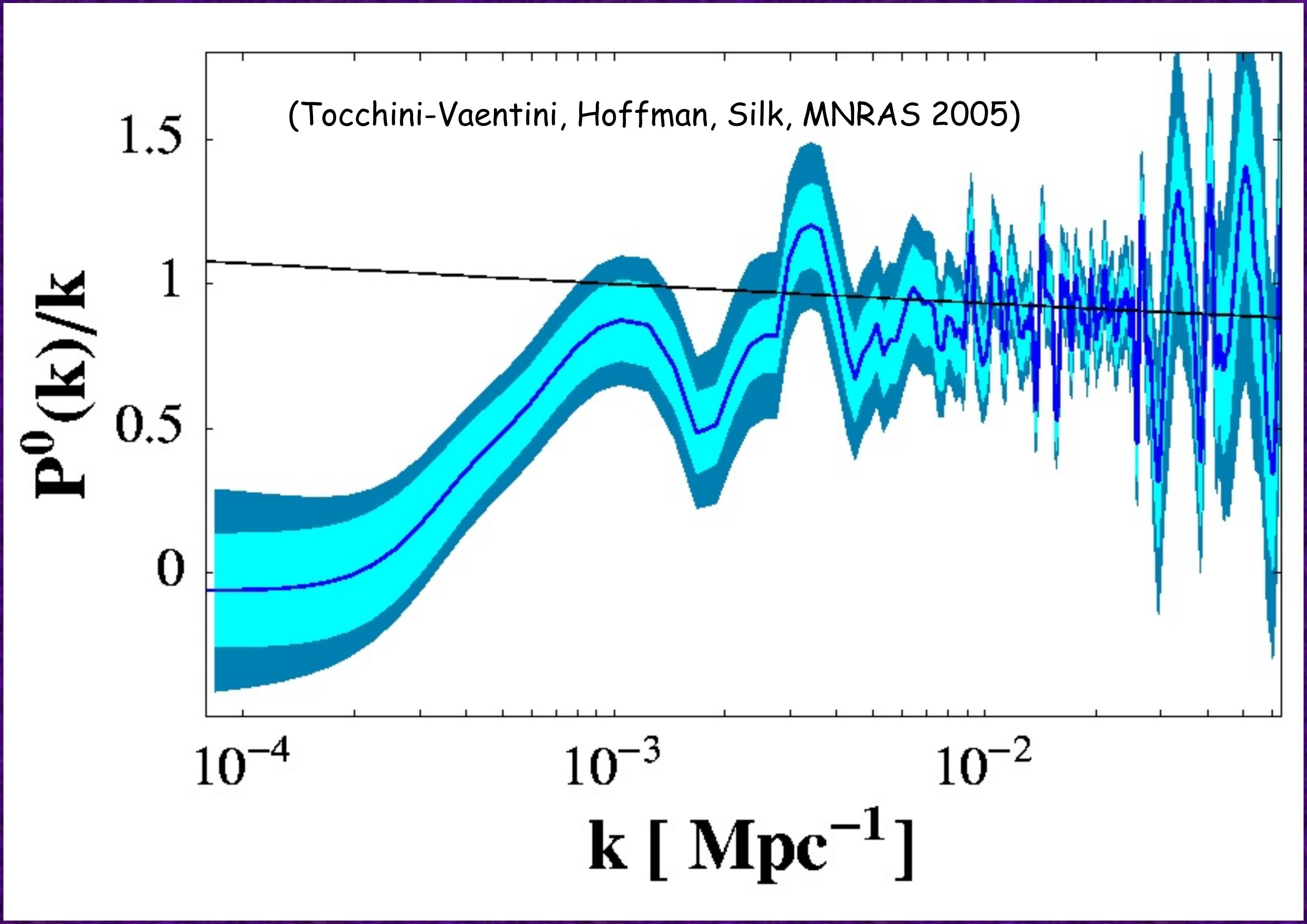
$$P(k) = A_s k^{1-n_s}\frac{1}{4y^4}|e^{-2iy}(1+2iy)-1-2y^2|^2$$

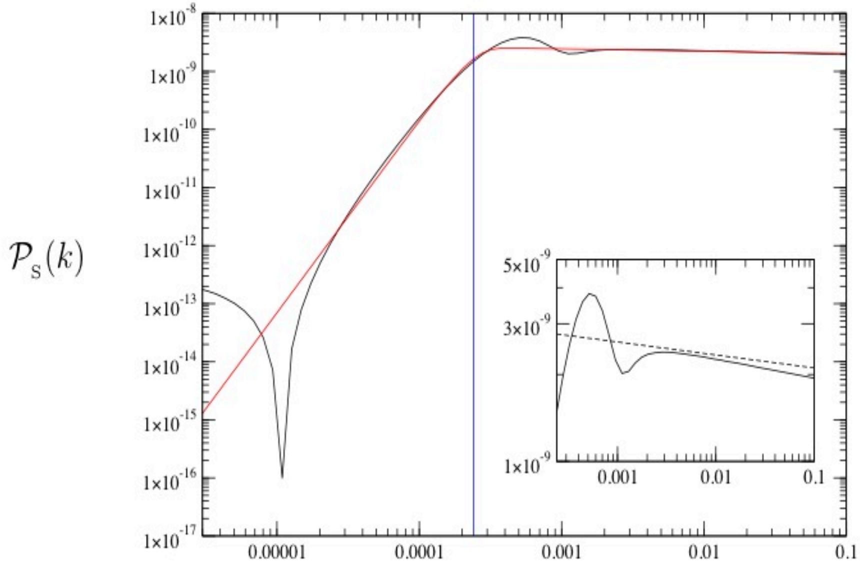
Vilenkin and Ford



k_c / k_h=0.96
k_c / k_h =0.43
k_c / k_h =0.32
The recovered spectrum is NOT unusual!!
Rec
Staro
VF I
VF II
$\log_{10} P(k)$
$\log_{10} k/k_h$

Regularized Least Square Method



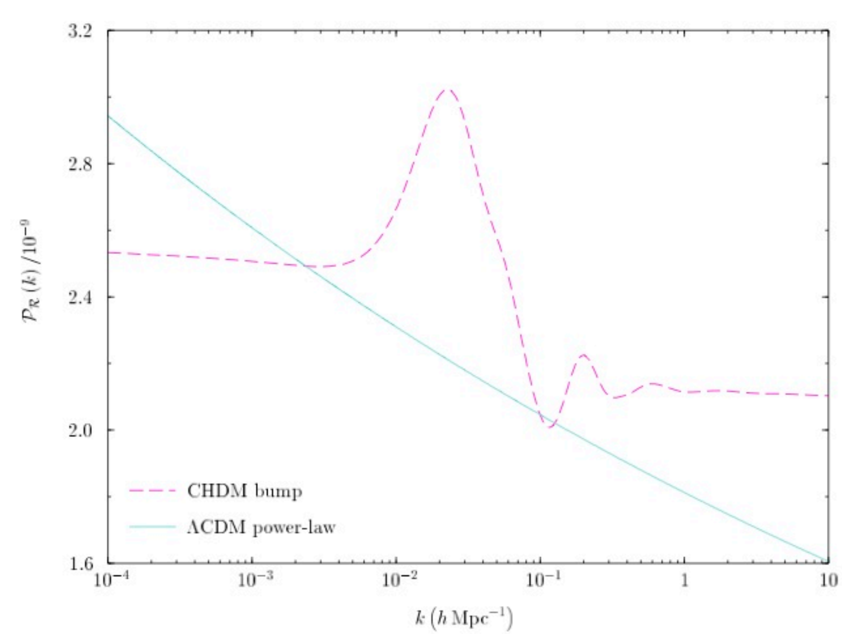


Jain et al, (2008)

Punctuated Inflation

LCDM Model

Parameter	Reference model	Our model
$\Omega_{\rm b}\, h^2$	$0.02242^{+0.00155}_{-0.00127}$	$0.02146^{+0.00142}_{-0.00108}$
$\Omega_{\rm c}\, h^2$	$0.1075^{+0.0169}_{-0.0126}$	$0.12051^{+0.02311}_{-0.02387}$
θ	$1.0395^{+0.0075}_{-0.0076}$	$1.03877^{+0.00979}_{-0.00931}$
τ	$0.08695^{+0.04375}_{-0.03923}$	$0.07220^{+0.04264}_{-0.02201}$
$\log\,[10^{10}\, A_{\rm S}]$	$3.0456^{+0.1093}_{-0.1073}$	—
$n_{\rm S}$	$0.9555^{+0.0394}_{-0.0305}$	—
$\log\,[10^{10}\, m^2]$	—	$-8.3509^{+0.1509}_{-0.1473}$
ϕ_0	—	$1.9594^{+0.00290}_{-0.00096}$
a_0	—	$0.31439^{+0.02599}_{-0.02105}$



Hunt & Sarkar, (2008)

Bump model

CHDM Model

Hunt & Sarkar, PRD 2008

	WMAP	+*SDSS*	+LRG	+*SDSS*+LRG
$\Omega_b h^2$	$0.01748^{+0.00073}_{-0.00071}$	$0.01762^{+0.00080}_{-0.00078}$	$0.01692^{+0.00047}_{-0.00047}$	$0.01688^{+0.00044}_{-0.00045}$
θ	$1.0365^{+0.0051}_{-0.0051}$	$1.0378^{+0.0049}_{-0.0049}$	$1.0300^{+0.0040}_{-0.0040}$	$1.0300^{+0.0039}_{-0.0039}$
τ	$0.078^{+0.012}_{-0.011}$	$0.079^{+0.012}_{-0.012}$	$0.071^{+0.011}_{-0.011}$	$0.071^{+0.012}_{-0.011}$
f_ν	$0.096^{+0.017}_{-0.023}$	$0.103^{+0.011}_{-0.011}$	$0.1360^{+0.0092}_{-0.0092}$	$0.1353^{+0.0075}_{-0.0067}$
$10^4 k_1/\mathrm{Mpc}^{-1}$	86^{+15}_{-13}	$82^{+11}_{-9.8}$	77^{+12}_{-10}	$77^{+11}_{-9.5}$
$10^4 k_2/\mathrm{Mpc}^{-1}$	527^{+78}_{-78}	539^{+84}_{-82}	380^{+24}_{-24}	379^{+22}_{-22}
$\ln\left(10^{10}\mathcal{P}_{\mathcal{R}}{}^{(0)}\right)$	$3.282^{+0.047}_{-0.047}$	$3.276^{+0.045}_{-0.046}$	$3.270^{+0.046}_{-0.046}$	$3.270^{+0.046}_{-0.047}$
b_{LRG}			$2.99^{+0.16}_{-0.16}$	$2.99^{+0.16}_{-0.16}$
$\Omega_c h^2$	$0.155^{+0.012}_{-0.011}$	$0.1539^{+0.0084}_{-0.0083}$	$0.1387^{+0.0041}_{-0.0044}$	$0.1387^{+0.0037}_{-0.0036}$
$\Omega_d h^2$	$0.1712^{+0.0063}_{-0.0062}$	$0.1715^{+0.0061}_{-0.0059}$	$0.1605^{+0.0030}_{-0.0031}$	$0.1604^{+0.0030}_{-0.0030}$
Age/GYr	$15.01^{+0.27}_{-0.27}$	$14.99^{+0.26}_{-0.27}$	$15.48^{+0.14}_{-0.14}$	$15.48^{+0.14}_{-0.14}$
σ_8	$0.668^{+0.093}_{-0.089}$	$0.648^{+0.053}_{-0.054}$	$0.565^{+0.032}_{-0.033}$	$0.565^{+0.029}_{-0.028}$
z_{reion}	$13.6^{+3.1}_{-3.1}$	$13.6^{+3.0}_{-3.1}$	$12.7^{+3.1}_{-3.1}$	$12.7^{+3.1}_{-3.2}$
h	$0.4344^{+0.0078}_{-0.0077}$	$0.4348^{+0.0079}_{-0.0076}$	$0.4212^{+0.0037}_{-0.0038}$	$0.4211^{+0.0038}_{-0.0038}$
Δm_1^2	$0.07476^{+0.00070}_{-0.00071}$	$0.07468^{+0.00068}_{-0.00069}$	$0.07459^{+0.00068}_{-0.00069}$	$0.07459^{+0.00068}_{-0.00070}$
Δm_2^2	$0.1510^{+0.0013}_{-0.0013}$	$0.1508^{+0.0012}_{-0.0013}$	$0.1507^{+0.0012}_{-0.0013}$	$0.1507^{+0.0012}_{-0.0013}$
$\widetilde{t}_2 - \widetilde{t}_1$	$1.82^{+0.16}_{-0.18}$	$1.89^{+0.15}_{-0.19}$	$1.62^{+0.12}_{-0.17}$	$1.62^{+0.11}_{-0.17}$
χ^2	11247	11265	11297	11315
Δ_{AIC}	0	0	28	29

Combining different CMB data sets

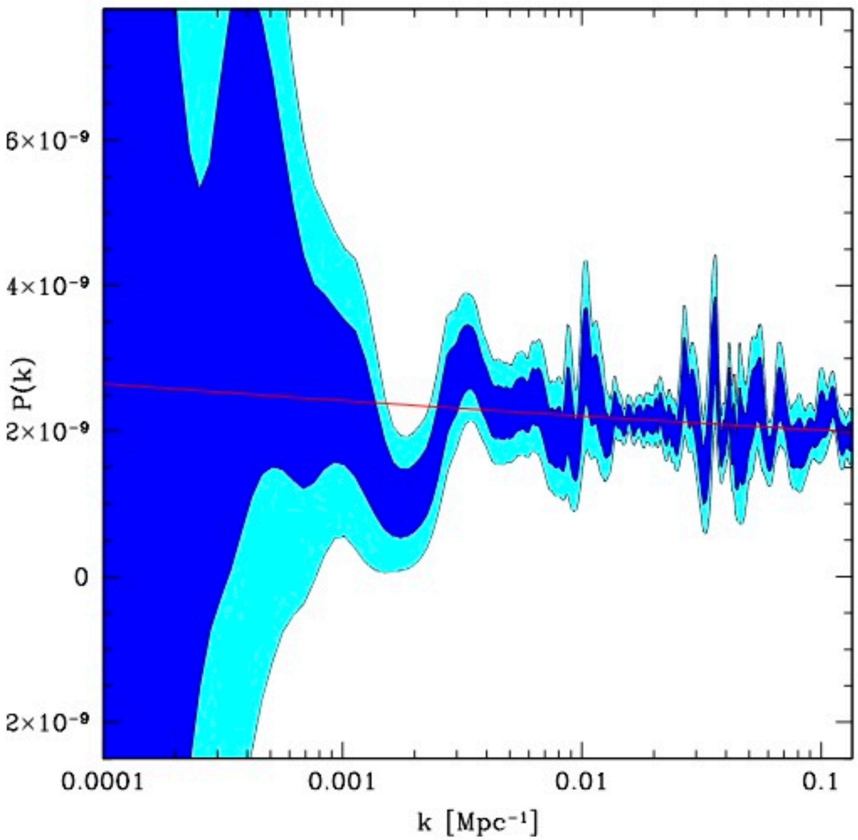


FIG. 3: Current limits from a combination of CMB data sets (WMAP, ACBAR, QUaD, BOOMERanG and CBI). There is some evidence of a dip in power at around $k \approx 0.002$ below the best fit power law model. Shaded regions are defined as in Fig. 2.

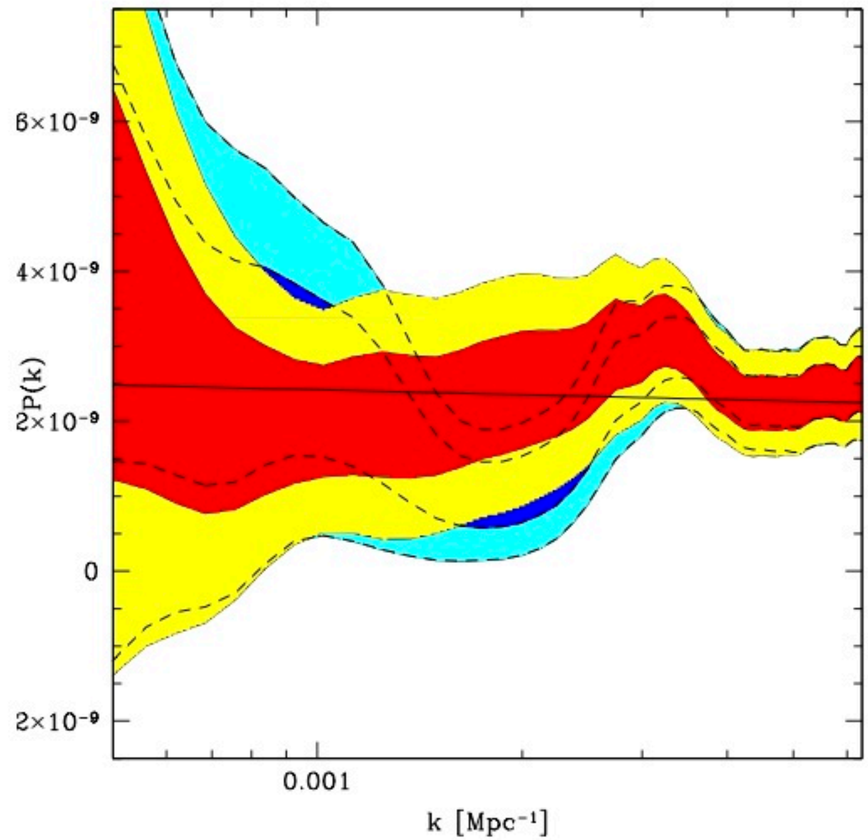


FIG. 4: As an indication of the origin of the dip at $k \approx 0.002$ Mpc^{-1} we remove the data between $\ell = 18$ and $\ell = 26$ and re-run the estimator. The red/yellow contours show the effect of the removal over the original estimate (blue/cyan).

Nicholson & Contaldi, JCAP 2009

Model A: SDSS

Model B: 2df

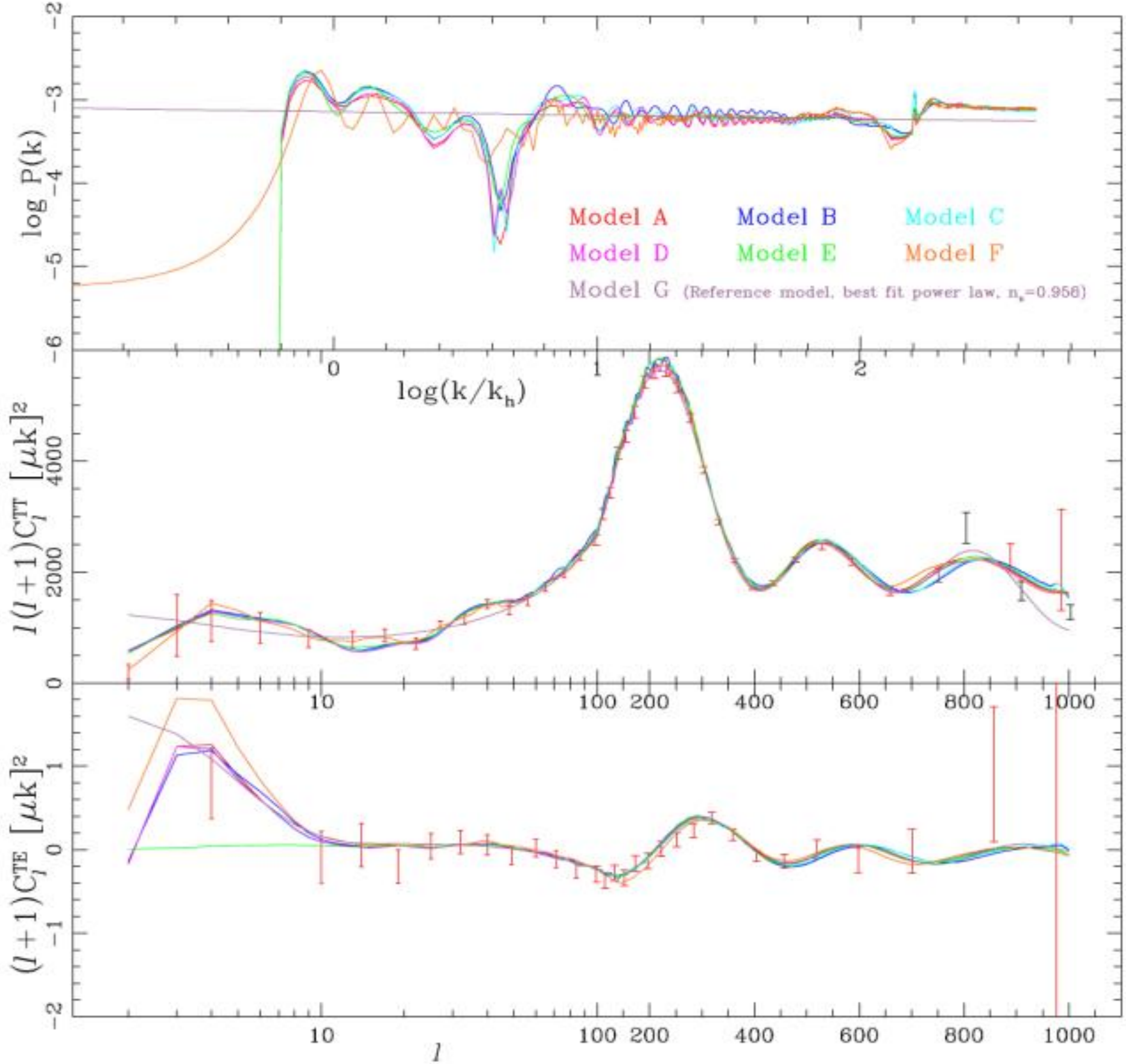
Model C: BAO

Model D: SN +BAO

Model E: WMAP1

Model F: SCDM

Model G: PL

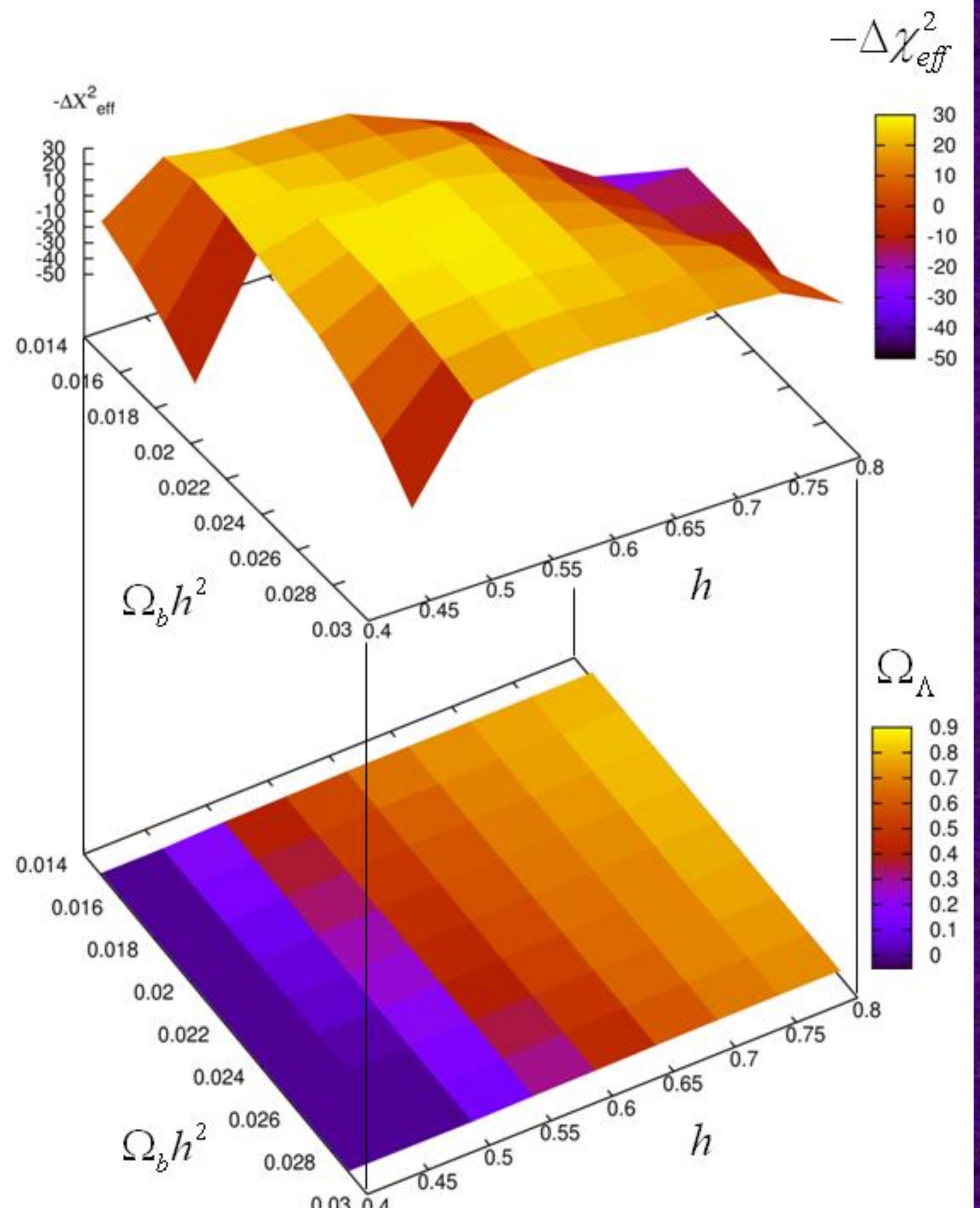


Shafieloo & Souradeep, PRD 2008

$$\Omega_b = 0.058$$
$$\Omega_c = 0.416$$
$$\Omega_\Lambda = 0.526$$
$$H_0 = 60$$

$$\Delta\chi^2 = -29.014$$

A. Shafieloo & T. Souradeep PRD 2008



If . . . Actual Universe

$$\begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Just as an EXAMPLE

Primordial Spectrum

Initial condition from inflation or any alternative scenario

Transfer Function

Related to the model of the universe and its parameters

Angular Spectrum

Observed Cosmic Microwave Background Radiation

Actual Universe

$$\begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum **Transfer Function** **Angular Spectrum**

Initial condition from inflation or any alternative scenario

Related to the model of the universe and its parameters

Observed Cosmic Microwave Background Radiation

Just as an EXAMPLE

Observed Universe

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum **Transfer Function** **Angular Spectrum**

Actual Universe

$$\boxed{\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 4 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum — Initial condition from inflation or any alternative scenario

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Just as an EXAMPLE

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Primordial Spectrum **Transfer Function** **Angular Spectrum**

$$\boxed{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

assuming a form for the initial condition

Actual Universe

$$\boxed{\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}} \begin{bmatrix} 2 & 1 & \boxed{3} \\ \boxed{0} & 1 & 4 \\ 1 & \boxed{2} & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum — Initial condition from inflation or any alternative scenario

Transfer Function — Related to the model of the universe and its parameters

Angular Spectrum — Observed Cosmic Microwave Background Radiation

Just as an EXAMPLE

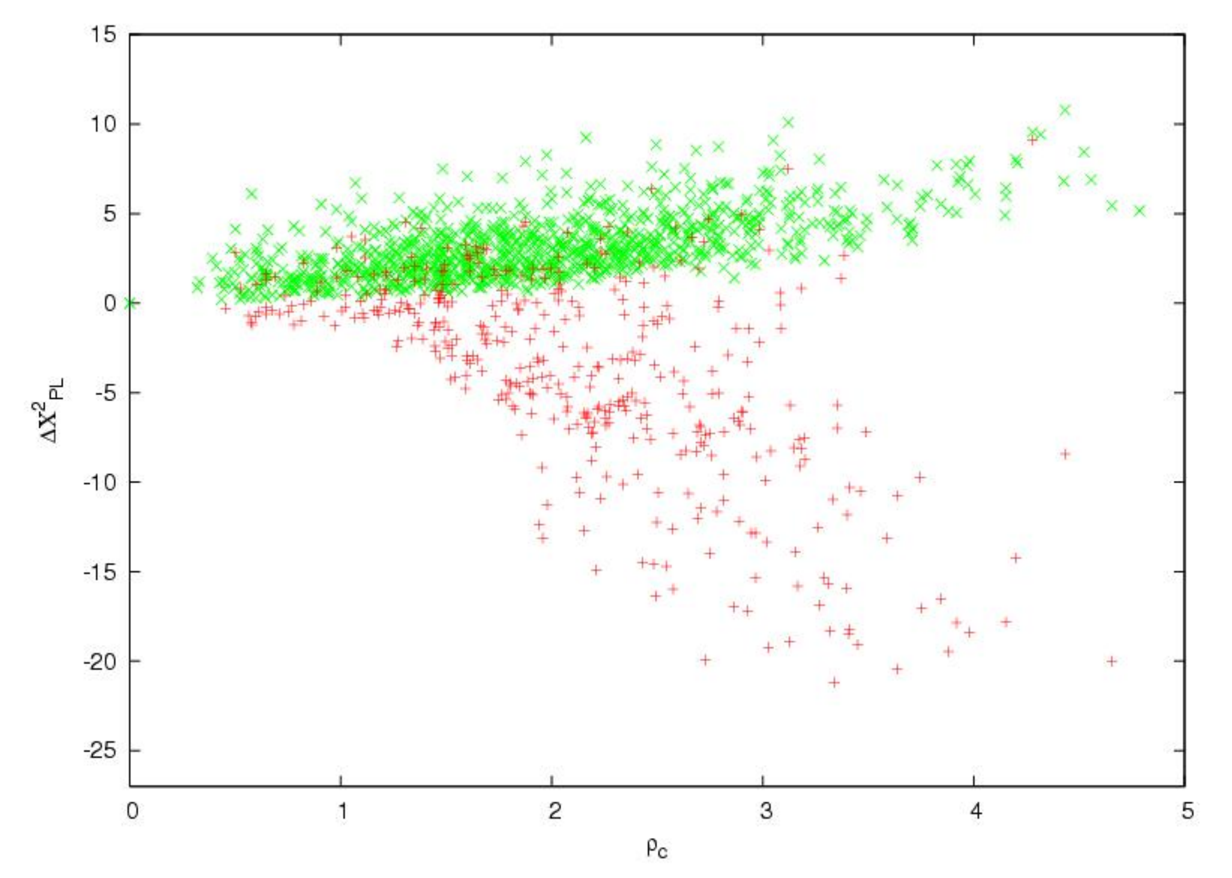
Observed Universe

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum **Transfer Function** **Angular Spectrum**

$$\boxed{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix} \Rightarrow \boxed{X_1 = -1} \quad \boxed{X_2 = 2} \quad \boxed{X_3 = 5}$$

assuming a form for the initial condition



$$\rho(a,b) = \sqrt{\sum_i \frac{(P_i^a - P_i^b)^2}{(\sigma_i^b)^2}}$$

$$\rho(a,b) \neq \rho(b,a)$$

$\Omega_b h^2$ h

$\Omega_{0m} h^2$ τ

$\rho(HZ, PL) = 6.89$

$\rho(RN, PL) = 4.85$

$\rho(HZ, RN) = 11.09$

$\rho(PL, RN) = 2.44$

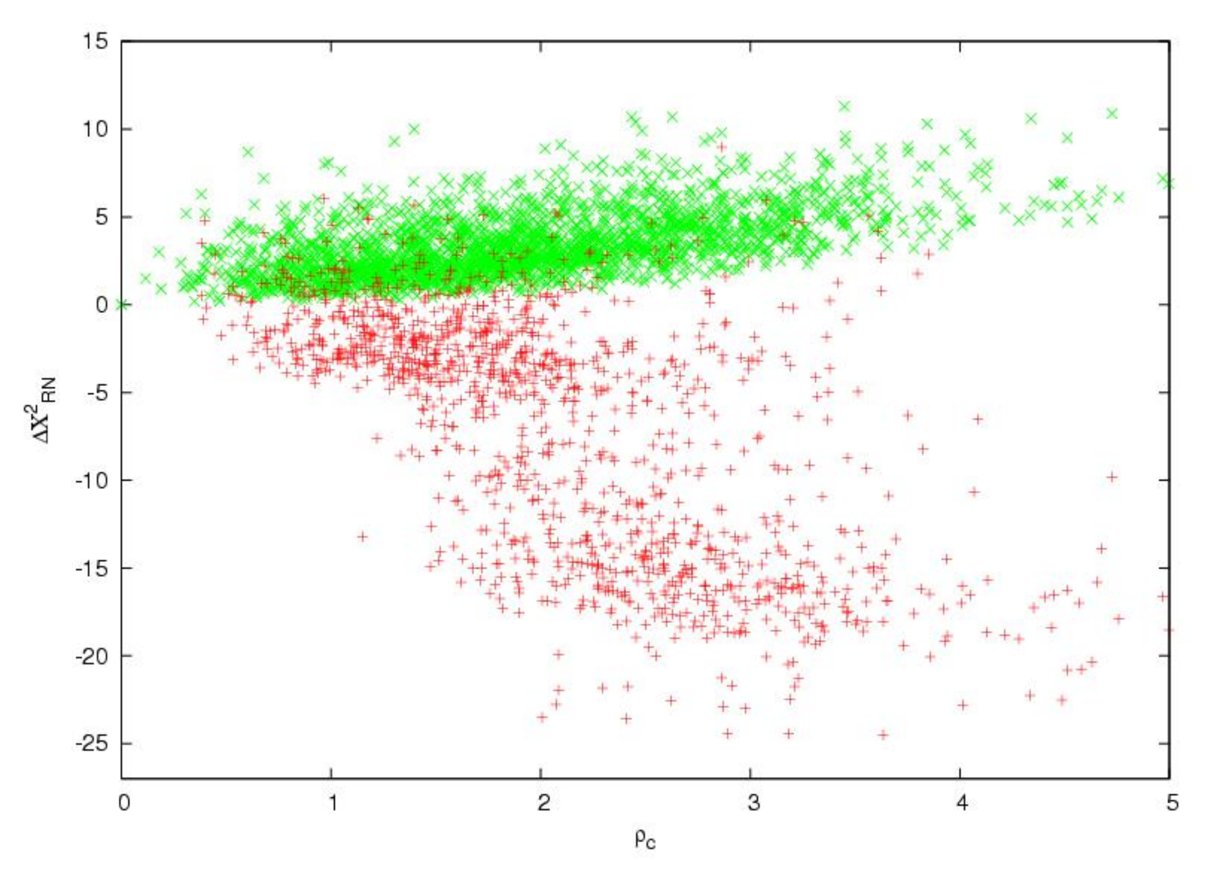
$\rho(PL, HZ) = 14.56$

$\rho(RN, HZ) = 43.79$

Power Law Assumption

Optimized over Primordial Spectrum

Shafieloo & Souradeep, NJP 2012



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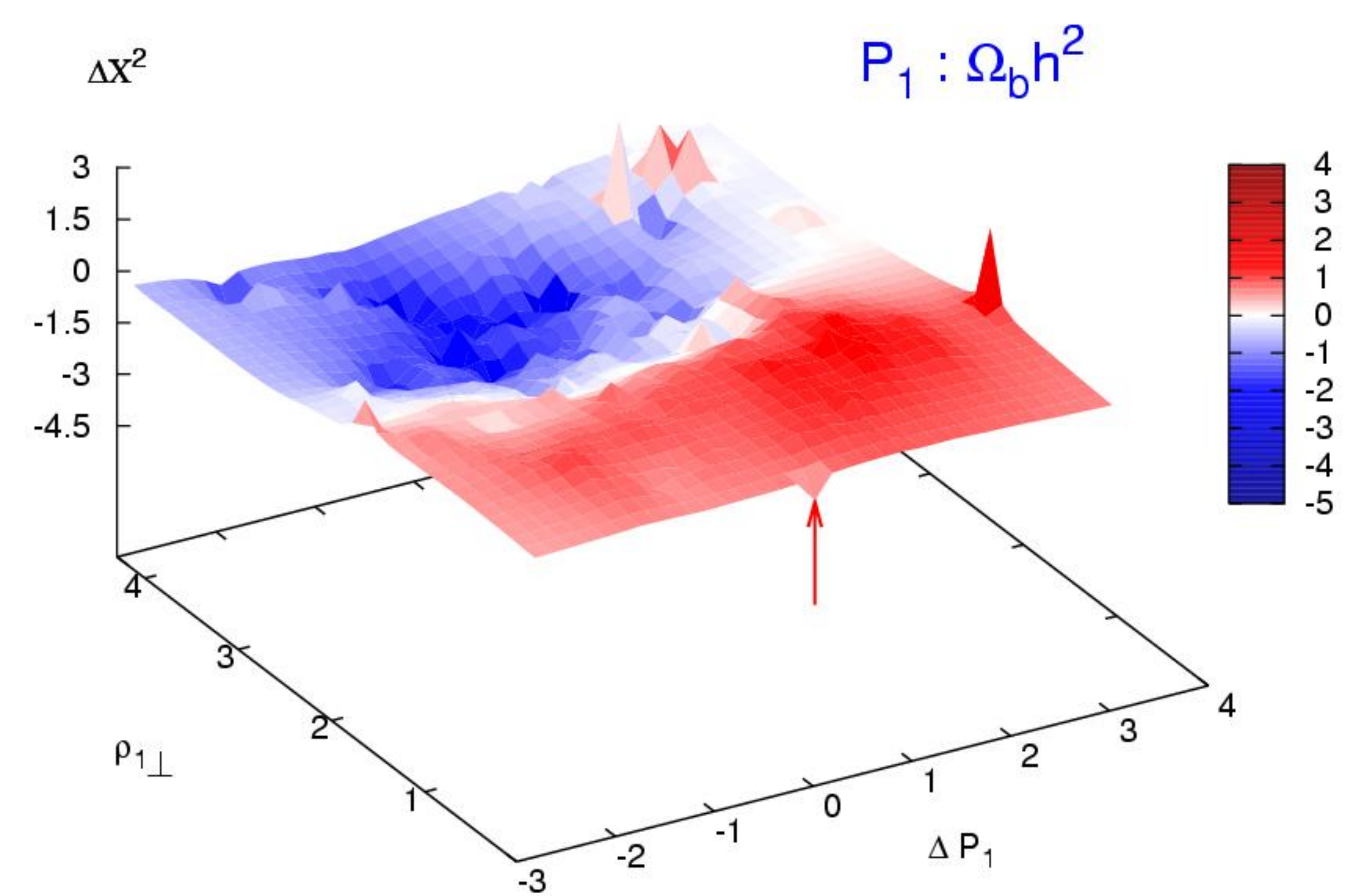
$\rho(RN, HZ) = 43.79$

Power Law with Running Assumption

Optimized over Primordial Spectrum

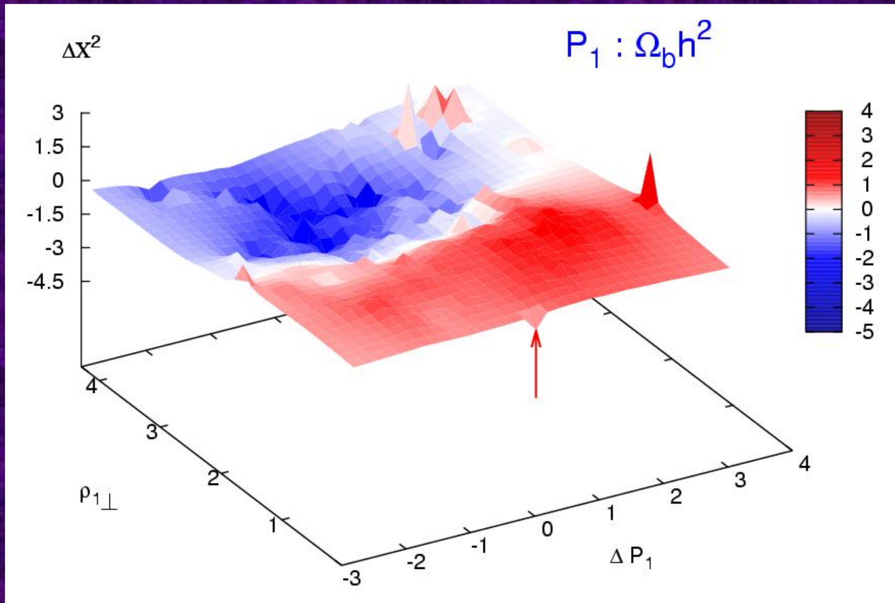


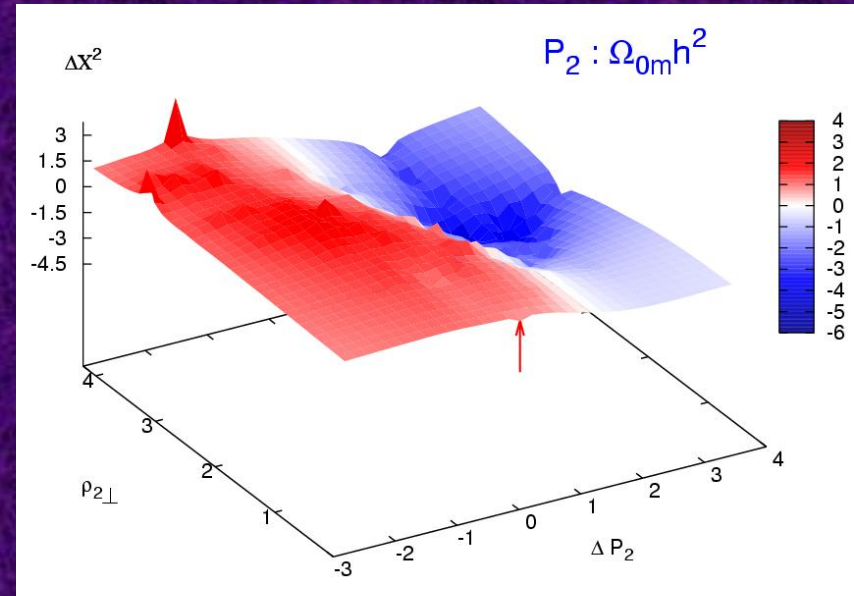
Using Power Law Sample

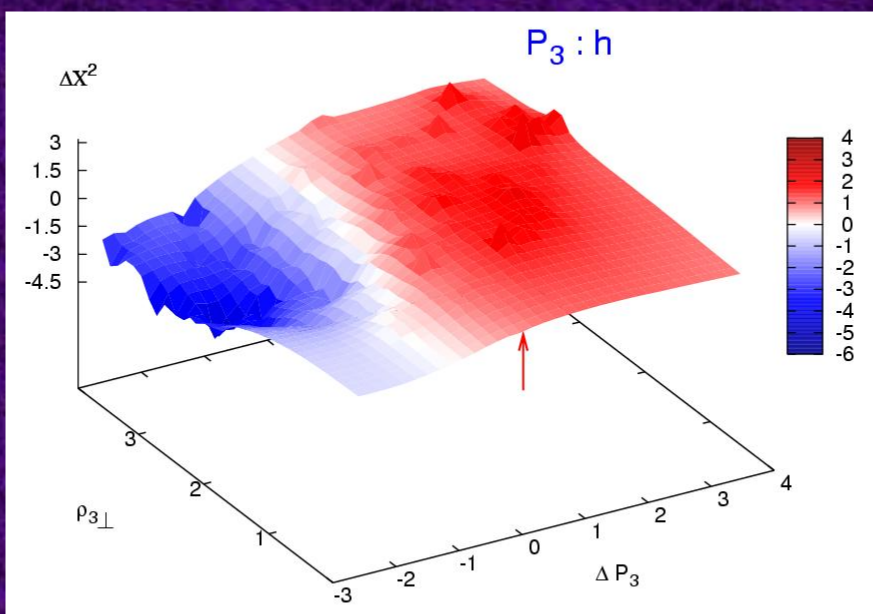
Shafieloo & Souradeep, NJP 2011

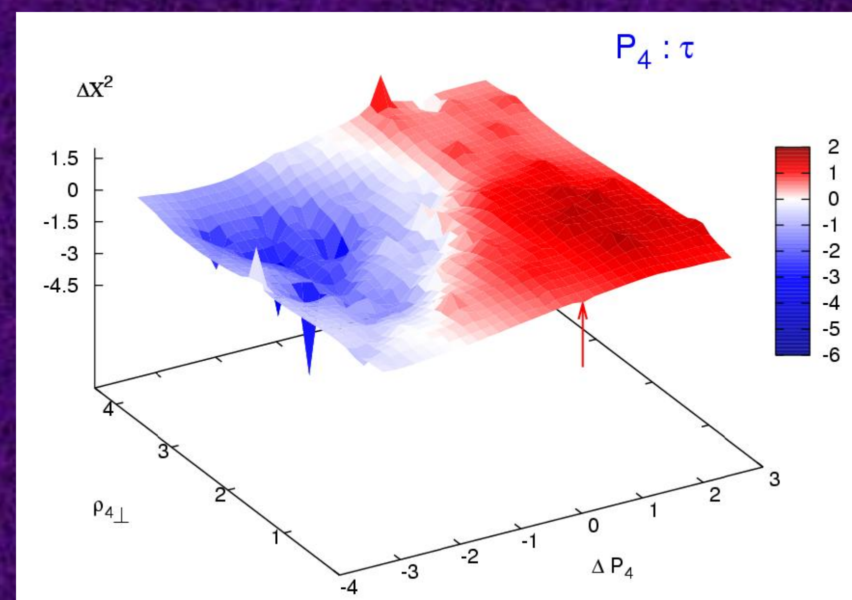
$$\Delta P_i = \frac{P_i^a - P_i^b}{\sigma_i^b}$$

$$\rho_{i\perp} = \sqrt{\sum_{j \neq i} \frac{(P_j^a - P_j^b)^2}{(\sigma_j^b)^2}}$$









Shafieloo & Souradeep, NJP 2011

Actual Universe

$$\boxed{\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}} \begin{bmatrix} 2 & 1 & \boxed{3} \\ \boxed{0} & 1 & 4 \\ 1 & \boxed{2} & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum

Initial condition from inflation or any alternative scenario

Transfer Function

Related to the model of the universe and its parameters

Angular Spectrum

Observed Cosmic Microwave Background Radiation

Just as an EXAMPLE

Observed Universe

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum **Transfer Function** **Angular Spectrum**

$$\boxed{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix} \Rightarrow \boxed{X_1 = -1} \quad \boxed{X_2 = 2} \quad \boxed{X_3 = 5}$$

assuming a form for the initial condition

Actual Universe

$$\boxed{\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}} \begin{bmatrix} 2 & 1 & \boxed{3} \\ \boxed{0} & 1 & 4 \\ 1 & \boxed{2} & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum | **Transfer Function** | **Angular Spectrum**

Initial condition from inflation or any alternative scenario

Related to the model of the universe and its parameters

Observed Cosmic Microwave Background Radiation

Just as an EXAMPLE

Observed Universe

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 2 & 1 & X_1 \\ X_2 & 1 & 4 \\ 1 & X_3 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

Primordial Spectrum | **Transfer Function** | **Angular Spectrum**

$$\boxed{\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix}} \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & 4 \\ 1 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

$$P_1 = 1$$
$$P_2 = 1$$
$$P_3 = 1$$

assuming a form for the initial condition

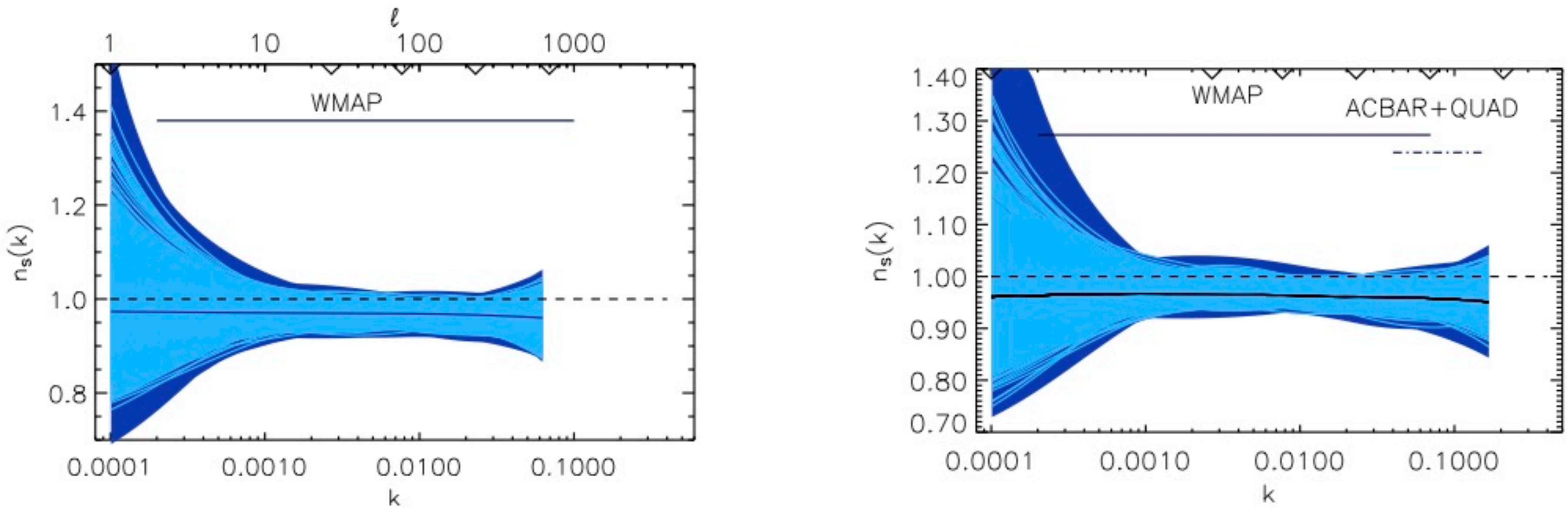
At the current status of cosmology one cannot tell with high certainty what is the actual form of the primordial spectrum, however, we can test the consistency of different forms to the data.

Note, that while there is only **ONE** actual model of the universe, depends on the precision of the observations many models can be consistent to the data.

If a model of the universe is consistent to the data, it does not mean that it is **THE** actual model of the universe.

Testing the Standard Power-Law form of PPS

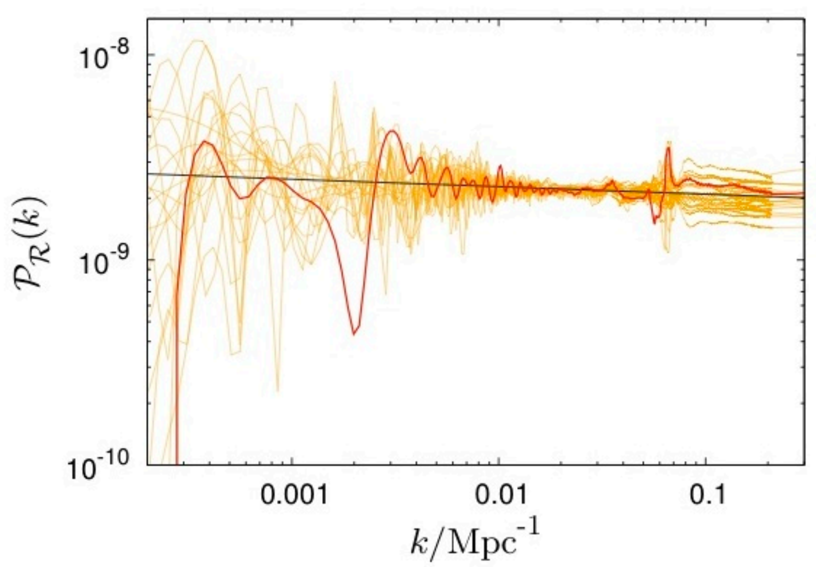
Smoothing Spline Method along with Cross Validation



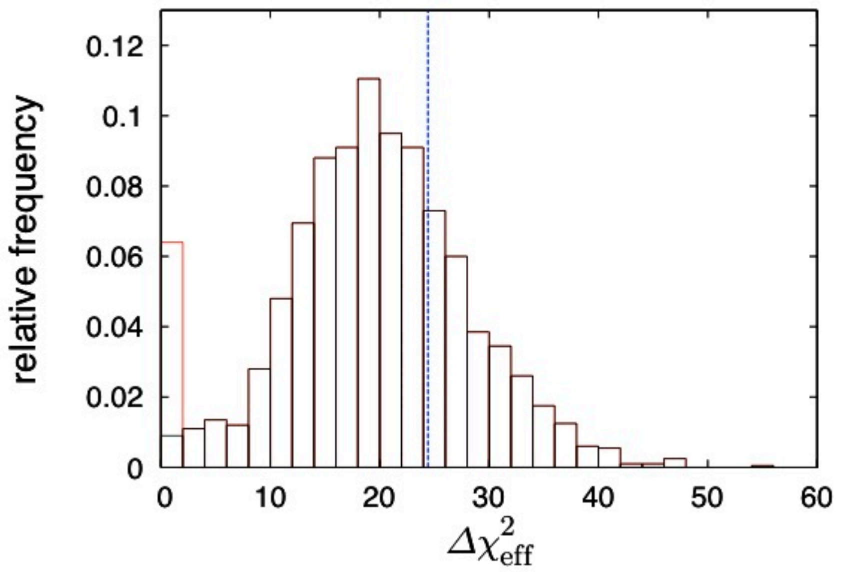
Peiris & Verde PRD 2010

Testing the Standard Power-Law form of PPS

Frequentist test using IRL deconvolution method



It is evident that the spectrum reconstructed from real data does not have an unusual amount of features. The apparent feature at 0.05 Mpc < k < 0.07 Mpc is caused by the noise term becoming dominant at the corresponding multipoles in the WMAP data.



P-value = 26%

Hamann, Shafieloo & Souradeep, JCAP 2010

Full picture

$$C_l^{TT} = \int \frac{dk}{k} P(k) \quad G_l^{TT}(k)$$

$$C_l^{EE} = \int \frac{dk}{k} P(k) \quad G_l^{EE}(k)$$

$$C_l^{BB} = \int \frac{dk}{k} P(k) \quad G_l^{BB}(k)$$

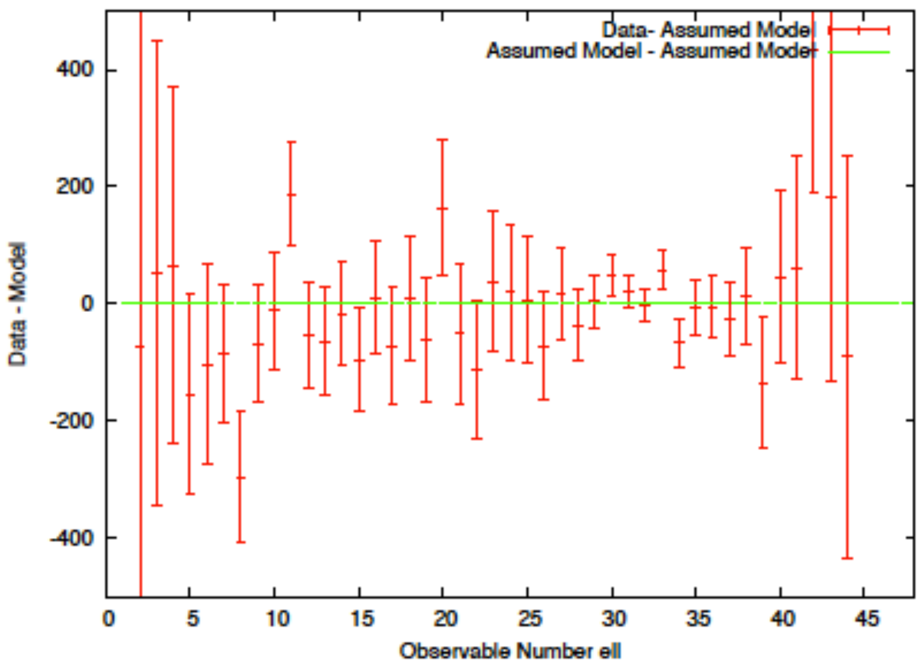
$$C_l^{TE} = \int \frac{dk}{k} P(k) \quad G_l^{TE}(k)$$

$$P_S(k), P_T(k), P_{iso}(k)$$

Primordial power spectra from Early universe

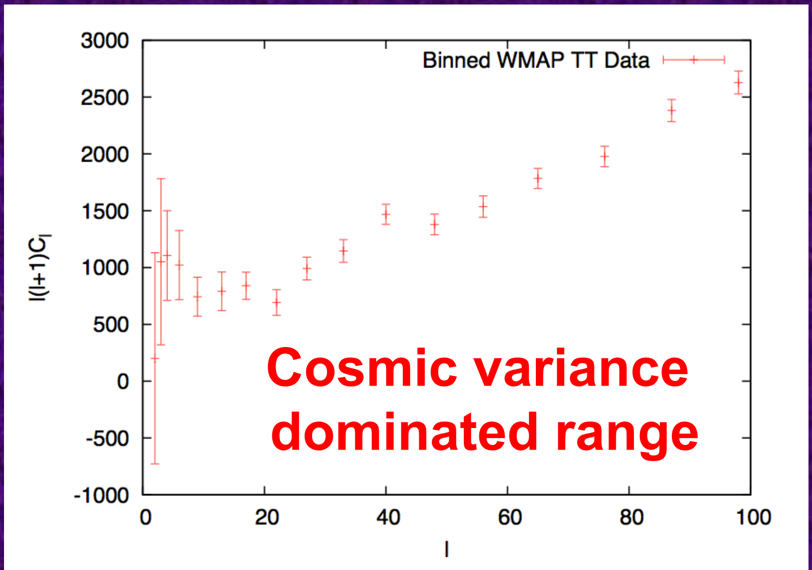
$$G_l^{TT}(k), G_l^{EE}(k), G_l^{BB}(k), G_l^{TE}(k)$$

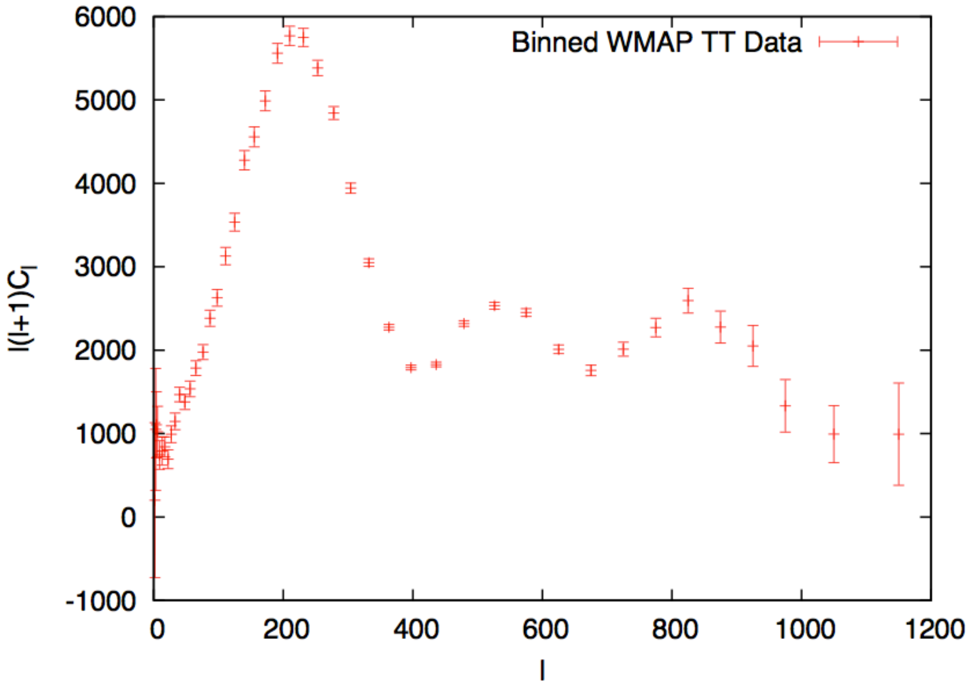
Post recombination Radiative transport kernels in a **given** cosmology



Cosmic Variance & Observational Limits

Relative fluctuations of the WMAP7 data in comparison with best fit LCDM-PL standard model





Gaussian Process & Detecting Features

- ➔Efficient in statistical modeling of stochastic variables
- ➔Derivatives of Gaussian Processes are Gaussian Processes
- ➔Provides us with all covariance matrices

A. Shafieloo. A. Kim & E. Linder, PRD 2012

Data

Mean Function

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \\ \mathbf{m}''(\mathbf{Z_1}) \end{bmatrix}, \begin{bmatrix} \Sigma_{00}(Z,Z) & \Sigma_{00}(Z,Z_1) & \Sigma_{01}(Z,Z_1) & \Sigma_{02}(Z,Z_1) \\ \Sigma_{00}(Z_1,Z) & \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z) & \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z) & \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} \right),$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}K}{dz_i^\alpha dz_j^\beta},$$

$$\begin{bmatrix} \overline{\mathbf{f}} \\ \overline{\mathbf{f}'} \\ \overline{\mathbf{f}''} \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \\ \mathbf{m}''(\mathbf{Z_1}) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\,\mathbf{y}$$

Kernel

$$k(z,z') = \sigma_f^2 \exp\left(-\frac{|z-z'|^2}{2l^2}\right),$$

$$\mathrm{Cov}\left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix}\right) = \begin{bmatrix} \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} - \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\left[\Sigma_{00}(Z,Z_1), \Sigma_{01}(Z,Z_1), \Sigma_{02}(Z,Z_1)\right].$$

Note: Results shown in the talk are removed as they are not yet published.

Summary

- By assuming any form of PPS, we in fact find a region in the parameter space which prefers these specific forms.
- The regions where considerably better likelihoods are obtained allowing free PPS lie outside these basins.
- The current cosmological parameters estimates are strongly prejudiced by the assumed form of PPS.
- Our results strongly motivate approaches toward simultaneous estimation of the cosmological parameters and the shape of primordial spectrum from upcoming cosmological data.
- Though standard power-law form of the PPS is very well consistent to the data, It is also important to keep an open mind towards early universe scenarios that produce features in the PPS.