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Inflation using non-canonical scalar fields[a]

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[a]Based on arXiv:1205.0786 (JCAP)

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The role of inflation

- **INFLATION** $\Rightarrow$
  - **Explains Large Scale Homogenity**
  - **A mechanism for generating density perturbations**
  - **Explanation for nearly flat universe**

$\star$ **What drives inflation $\Rightarrow$ Scalar fields**

Classification of scalar field models

- **Canonical Scalar Field** $\Longrightarrow \mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

  * $V(\phi) = V_0\phi^n \rightarrow$ **Chaotic inflation models (Linde 1983)**
  * $V(\phi) = V_0 \exp\left[-\sqrt{2/p}(\phi/M_{_{pl}})\right] \rightarrow$ **Power law inflation** $a(t) \propto t^p$

- **Non canonical Scalar Field** $\Longrightarrow \mathcal{L}_\phi = \mathcal{L}(X,\phi)$ where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

$$\Downarrow$$

  * $\mathcal{L}(X,\phi) = F(X) - V(\phi)$

  or

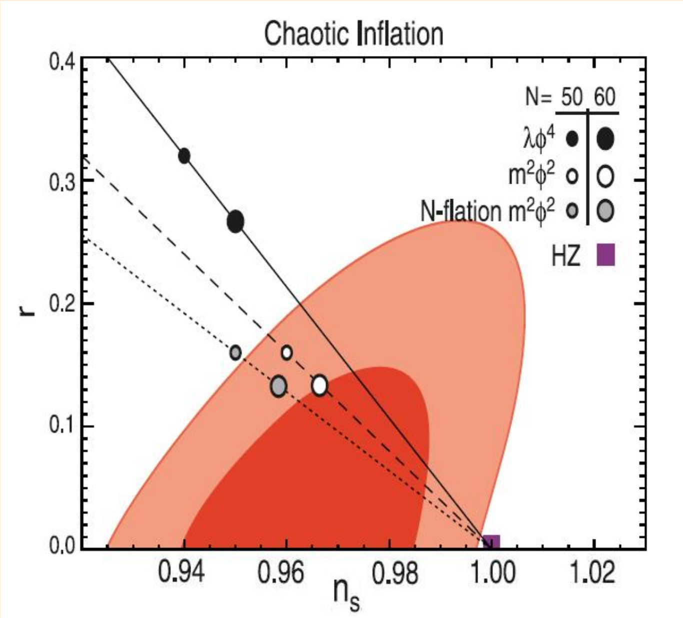
  * $\mathcal{L}(X,\phi) = V(\phi)F(X)$

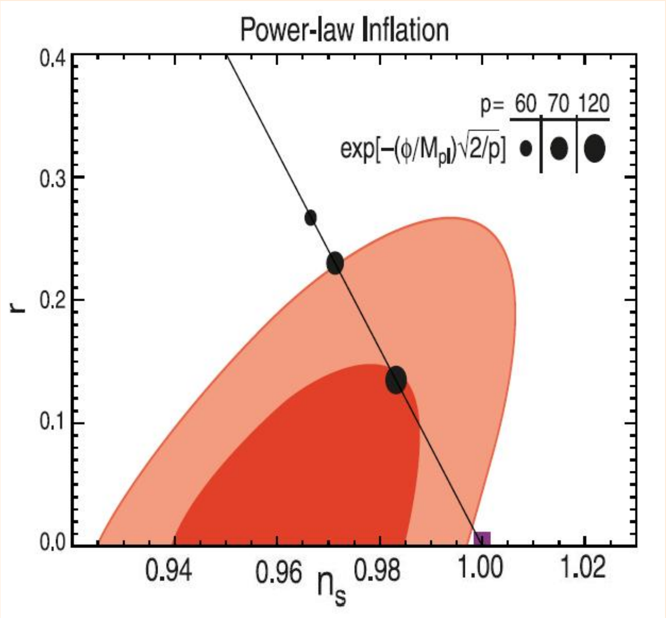
  * **These class of models are also known as K-inflation models**

Inflation using canonical scalar fields

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

- **Chaotic inflation models (Linde 1983)** $\rightarrow V(\phi) = V_0\phi^n$
- **Power law inflation** $a(t) \propto t^p \Rightarrow V(\phi) = V_0 \exp[-\sqrt{2/p}(\phi/M_{pl})]$





Figures from Komatsu et al (2011)

- From COBE normalization $\Rightarrow$ $m \sim 10^{-6}\mathrm{M_{pl}}$ and $\lambda \sim 10^{-13}$

Inflation using non canonical scalar fields

A specific model

$$\mathcal{L}(X, \phi) = X \left(\frac{X}{M^4} \right)^{\alpha - 1} - V(\phi)$$

where $X = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$

- M **is a constant with dimension of mass**
- α **is the dimensionless parameter of the theory.**
- $\alpha = 1$ **corresponds to canonical scalar field.**

The above Lagrangian can be viewed as a generalization of the usual Lagrangian for the canonical scalar field

Slow roll parameters

- **The slow roll parameters ϵ and δ are defined as**

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \qquad \text{and} \qquad \delta \equiv \varepsilon - \frac{\dot{\varepsilon}}{2\,H\,\varepsilon}$$

- **It follows from the Friedmann equation that**

$$\frac{\ddot{a}}{aH^2} = 1 - \varepsilon$$

- **Therefore, inflation ($\ddot{a} > 0$) occurs when $\varepsilon < 1$ and ends at $\varepsilon = 1$**
- **EOS parameter w_ϕ is related to ε as** $\rightarrow w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \left(\frac{2\varepsilon}{3}\right) - 1$
- **Slow roll inflation occurs when $\varepsilon << 1$ which gives $p_\phi \simeq -\rho_\phi$**
- **Slow roll approximation is defined as**

$$\varepsilon << 1 \quad \text{and} \quad |\delta| << 1$$

Solution in the slow roll limit

The slow roll assumptions ($\varepsilon \ll 1$ and $|\delta| \ll 1$) leads to

$$\dot{\phi} = -\theta \left\{ \left(\frac{M_{pl}}{\alpha\,\sqrt{3}}\right) \left(\frac{\theta\, V'(\phi)}{\sqrt{V}}\right) \left(2\, M^4\right)^{\alpha-1} \right\}^{\frac{1}{2\alpha-1}}$$

where

$$\theta = +1 \text{ when } V'(\phi) > 0$$
$$\theta = -1 \text{ when } V'(\phi) < 0.$$

- **In which regime of the potential $V(\phi)$ is the above solution valid ?**

Potential slow roll parameter

- **The slow roll condition $\varepsilon \ll 1$ and $|\delta| \ll 1$ implies that**

$$\varepsilon_{\rm V} \ll 1 \quad \text{and} \quad \delta_{\rm V} \ll 1$$

$$\begin{aligned}
\varepsilon_{\rm V} &\equiv \left\{ \left(\frac{1}{\alpha}\right) \left(\frac{3\,M^4}{V(\phi)}\right)^{\alpha-1} \left(\frac{M_{pl}\,V'(\phi)}{\sqrt{2}\,V(\phi)}\right)^{2\alpha} \right\}^{\frac{1}{2\alpha-1}} \\
\delta_{\rm V} &\equiv \left(\frac{\alpha}{2\alpha-1}\right) (\eta_{\rm V} - \varepsilon_{\rm V})
\end{aligned}$$

where

$$\eta_{\rm V} \equiv 2\,\varepsilon_{\rm V} \left(\frac{V(\phi)V''(\phi)}{V'(\phi)^2}\right)$$

In the canonical limit

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

- $\alpha = 1 \;\Rightarrow$ **Canonical Scalar field** $\rightarrow \mathcal{L}(X,\phi) = X - V(\phi)$
- $\varepsilon_{\rm V}$ **and** $\delta_{\rm V}$ **becomes**

$$\begin{aligned} \varepsilon_{\rm V} &= \left(\frac{M_{_{pl}}^2}{2}\right)\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \\ \delta_{\rm V} &= \eta_{\rm V} - \varepsilon_{\rm V} \end{aligned}$$

where

$$\eta_{\rm V} = M_{_{pl}}^2\left(\frac{V''(\phi)}{V(\phi)}\right)$$

PSR parameter for non canonical scalars

For the non-canonical model $\mathcal{L} = X^{\alpha} - V(\phi)$

$$\varepsilon_{\rm V} \equiv \left\{ \left(\frac{1}{\alpha}\right) \left(\frac{3\,M^4}{V(\phi)}\right)^{\alpha-1} \left(\frac{M_{_{pl}}\,V'(\phi)}{\sqrt{2}\,V(\phi)}\right)^{2\alpha} \right\}^{\frac{1}{2\alpha-1}}$$

- $\varepsilon_{\rm V}$ **can be expressed as**

$$\varepsilon_{_V} = \left(\frac{1}{\alpha}\right)^{\frac{1}{2\alpha-1}} \left(\frac{3\,M^4}{V}\right)^{\frac{\alpha-1}{2\alpha-1}} \left[\varepsilon_{_V}^{(c)}\right]^{\frac{\alpha}{2\alpha-1}}$$

  where $\varepsilon_{_V}^{(c)}$ corresponds to the canonical value of $\varepsilon_{_V}$

- **For $3M^4 \ll V \Rightarrow \varepsilon_{_V} < \varepsilon_{_V}^{(c)}$**

- $\varepsilon_{_V}$ **evolves from $\varepsilon_{_V} << 1$ towards $\varepsilon_{_V} \simeq 1$ for a wider class of potentials.**

- **For exponential potential $V(\phi) = V_{_0}\, \exp\left[-\lambda(\phi/M_{_{pl}})\right]$ also it turns out that $\varepsilon_{\rm V}$ evolves from $\varepsilon_{\rm V} << 1$ to $\varepsilon_{\rm V} \simeq 1$.**

Scalar and Tensor perturbations

- **FRW line element with scalar and tensor perturbations**

$$\mathrm{d}s^2 = (1 + 2\,A)\,\mathrm{d}t^2 - 2\,a(t)\,(\partial_i B)\;\mathrm{d}t\;\mathrm{d}x^i - a^2(t)\;\left[(1 - 2\,\psi)\;\delta_{ij} + 2\;(\partial_i\,\partial_j E) + h_{ij}\right]\mathrm{d}x^i\,\mathrm{d}x^j$$

- **Curvature perturbation is defined as**

$$\mathcal{R} \equiv \psi + \left(\frac{H}{\dot{\phi}}\right)\delta\phi$$

- **From $\delta G^{\mu}_{\nu} = \kappa\,\delta T^{\mu}_{\nu}$ and equation of motion for $\delta\phi$**

$$\mathcal{R}_k'' + 2\left(\frac{z'}{z}\right)\mathcal{R}_k' + c_s^2 k^2 \mathcal{R}_k = 0$$

where

$$z \equiv \frac{a\;\left(\rho_\phi + p_\phi\right)^{1/2}}{c_s\,H} \qquad \text{and} \qquad c_s^2 \equiv \left[\frac{(\partial\mathcal{L}/\partial X)}{(\partial\mathcal{L}/\partial X) + (2\,X)\;(\partial^2\mathcal{L}/\partial X^2)}\right]$$

Mukhanov Sasaki Equation

- **In terms of the Mukhanov-Sasaki variable** $u_k \equiv z\,\mathcal{R}_k$,

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) u_k = 0$$

- **For tensor perturbations** $\rightarrow v_k \equiv (h/a)$ **where** h **is the amplitude of the tensor perturbation**

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

- **Scalar and tensor power spectra are defined as**

$$\mathcal{P}_S(k) \equiv \left(\frac{k^3}{2\pi^2}\right) |\mathcal{R}_k|^2 = \left(\frac{k^3}{2\pi^2}\right) \left(\frac{|u_k|}{z}\right)^2$$

$$\mathcal{P}_T(k) \equiv 2\left(\frac{k^3}{2\pi^2}\right) |h_k|^2 = 2\left(\frac{k^3}{2\pi^2}\right) \left(\frac{|v_k|}{a}\right)^2$$

Power spectra for non canonical model

- **For the model**

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

in the slow roll limit it turns out that

$$\mathcal{P}_{_S}(k) = \left(\frac{1}{72\pi^2 c_{_s}}\right)\left\{\left(\frac{\alpha\, 6^\alpha}{\mu^{4(\alpha-1)}}\right)\left(\frac{1}{M_{_{pl}}^{14\alpha-8}}\right)\left(\frac{V(\phi)^{5\alpha-2}}{V'(\phi)^{2\alpha}}\right)\right\}^{\frac{1}{2\alpha-1}}$$

$$\mathcal{P}_{_T}(k) = \left(\frac{2\,V(\phi)}{3\,\pi^2 M_{_{pl}}^4}\right)$$

where

$$c_{_S}^2 = \frac{1}{2\,\alpha - 1}$$

- $\alpha \geq 1$ **ensures that** $c_{_S}^2 \leq 1$

Scalar spectral index and T-to-S ratio

- **Scalar spectral index $n_{_S}$ is defined as**

$$n_{_S} - 1 \equiv \frac{\mathrm{d}\ln\mathcal{P}_{_S}}{\mathrm{d}\ln k}$$

- **Tensor to scalar ratio**

$$r \equiv \frac{\mathcal{P}_{_T}}{\mathcal{P}_{_S}}$$

- **For chaotic inflationary model $V(\phi) = V_{_0}\,\phi^n$, it turns out that**

$$n_{_S} = 1 - 2\left(\frac{\gamma + n}{2N\gamma + n}\right) \quad \text{and} \quad r = \left(\frac{1}{\sqrt{2\,\alpha - 1}}\right)\left(\frac{16\,n}{2\,N\gamma + n}\right)$$

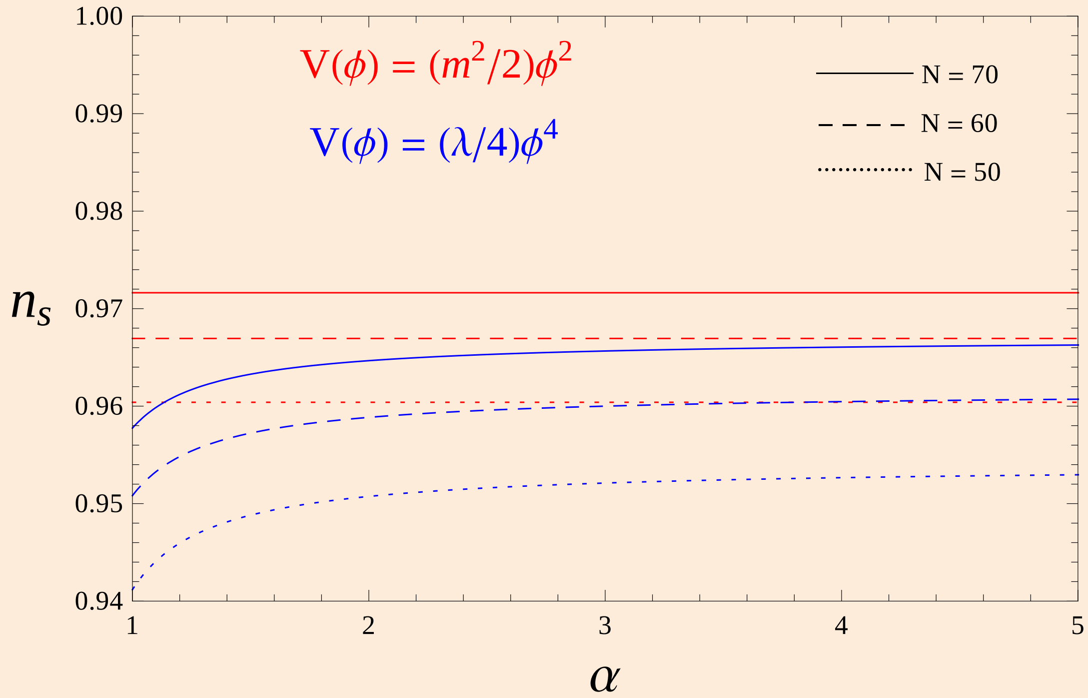
where

$$\gamma \equiv \frac{2\alpha + n\,(\alpha - 1)}{2\alpha - 1}$$

⋆ This result was also independently obtained by Sheng and Liddle (arXiv:1204.6214) !

Scalar spectral index n_S

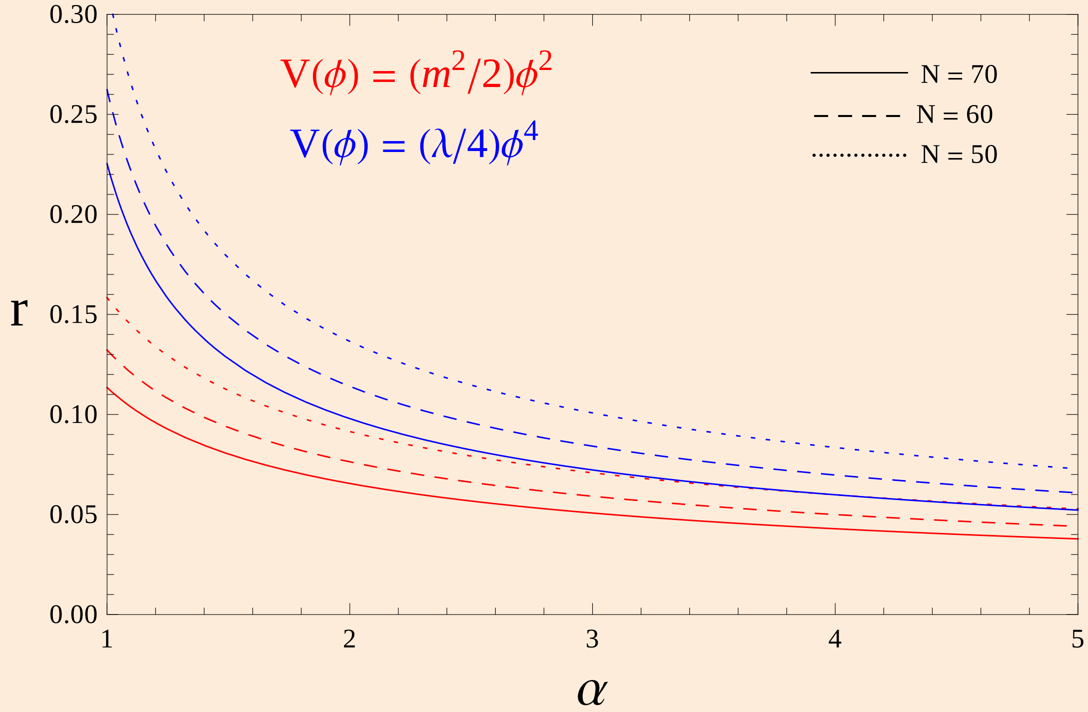
$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$



- **The value of n_S for $m^2\phi^2$ potential is independent of α !**

Tensor-to-Scalar ratio r

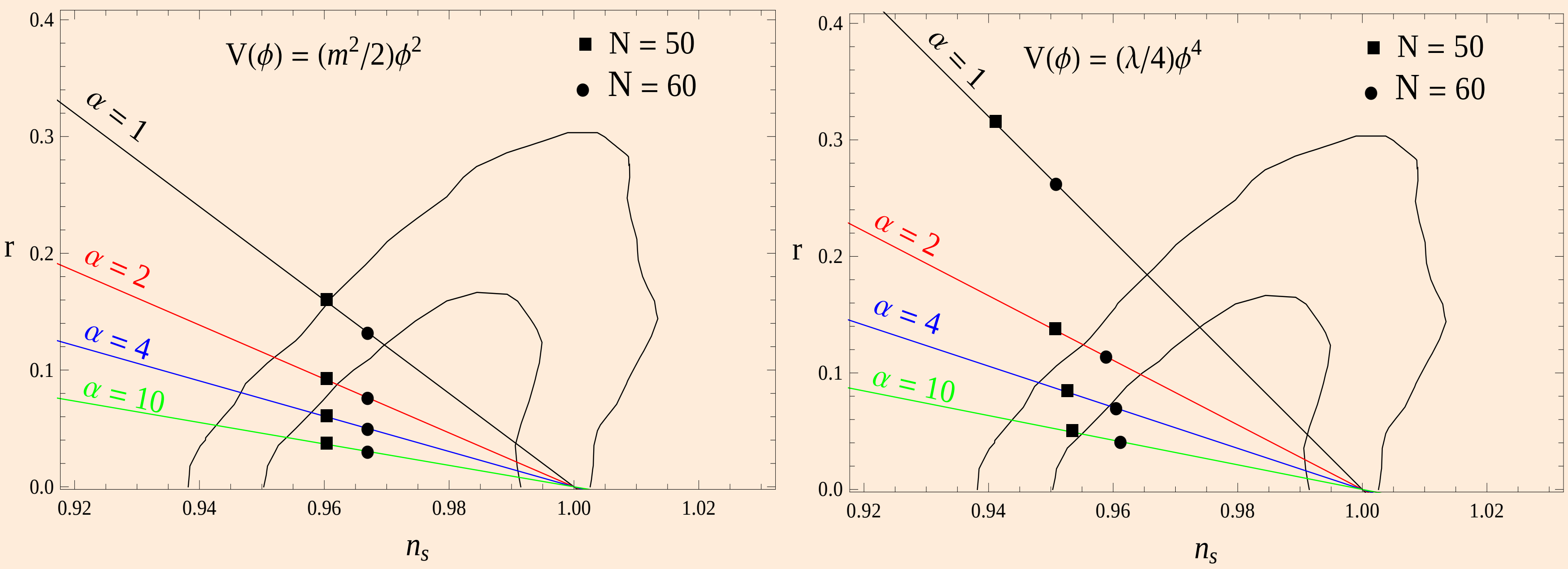
$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$



- **Tensor-to-scalar ratio decreases as the parameter α is increased.**

n_s-r Plain

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

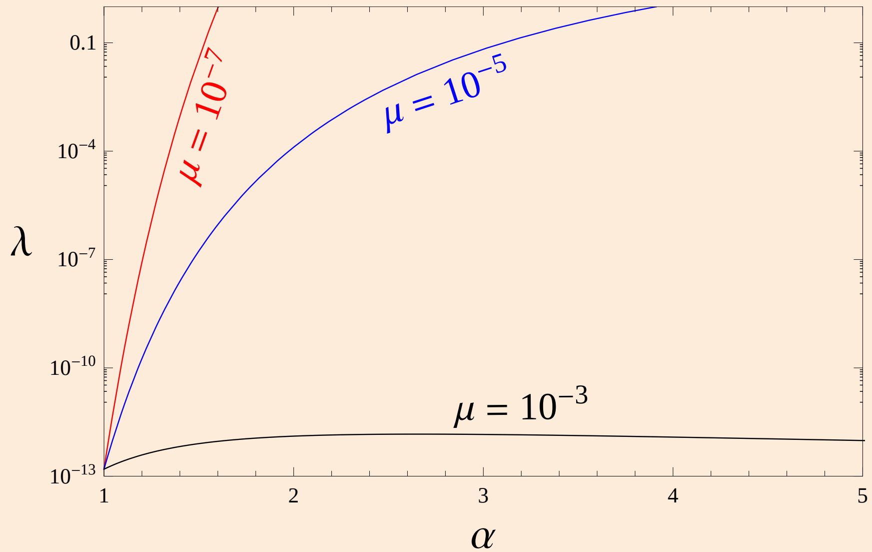


CMB Normalization

- $\lambda\phi^4$ **Potential**

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - \frac{\lambda}{4}\phi^4$$

- **CMB Normalization** $\rightarrow \mathcal{P}_S \simeq 2.4\times 10^{-9}$ **at** $k = 0.002\,\mathrm{Mpc}^{-1}$ **(pivot scale) (Komatsu et al 2011)**



where $\mu \equiv M/M_{pl}$

Inflationary consistency relation

- **For canonical scalar field**

$$r = -8n_T$$

where

$$n_T \equiv \frac{\mathrm{d}\ln\mathcal{P}_T}{\mathrm{d}\ln k}$$

- **For the model**

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

It turns out that

$$r = -\frac{8\,n_T}{\sqrt{2\alpha-1}}$$

- **For** $\alpha > 1 \Rightarrow r < -8\,n_T$

$\Rightarrow$ **Non-canonical scalar fields violates the standard consistency relation**

Summary and Conclusions

We considered a non-canonical model of inflation with

$$\mathcal{L}_\phi = \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)^\alpha - V(\phi)$$

- **The tensor-to-scalar ratio decreases considerably as the parameter α is increased.**
- **Therefore non-canonical scalars can accommodate a wider class of potentials for driving inflation.**
- **The non-canonical version of $V(\phi) \sim \lambda\phi^4$ inflation model, is found to agree with observations for values of $\lambda \simeq 1$!**
- **This model violates the standard consistency relation $r = -8n_T$.**
- **When $\alpha >> 1$, it turns out that $f_{NL}^{equil} \simeq 0.65 \times \alpha \Rightarrow$ it can lead to large non-Gaussianity**

* **Thank you** *