Probing the early Universe with Primordial Fluctuations: beyond $f_{\rm NL}$

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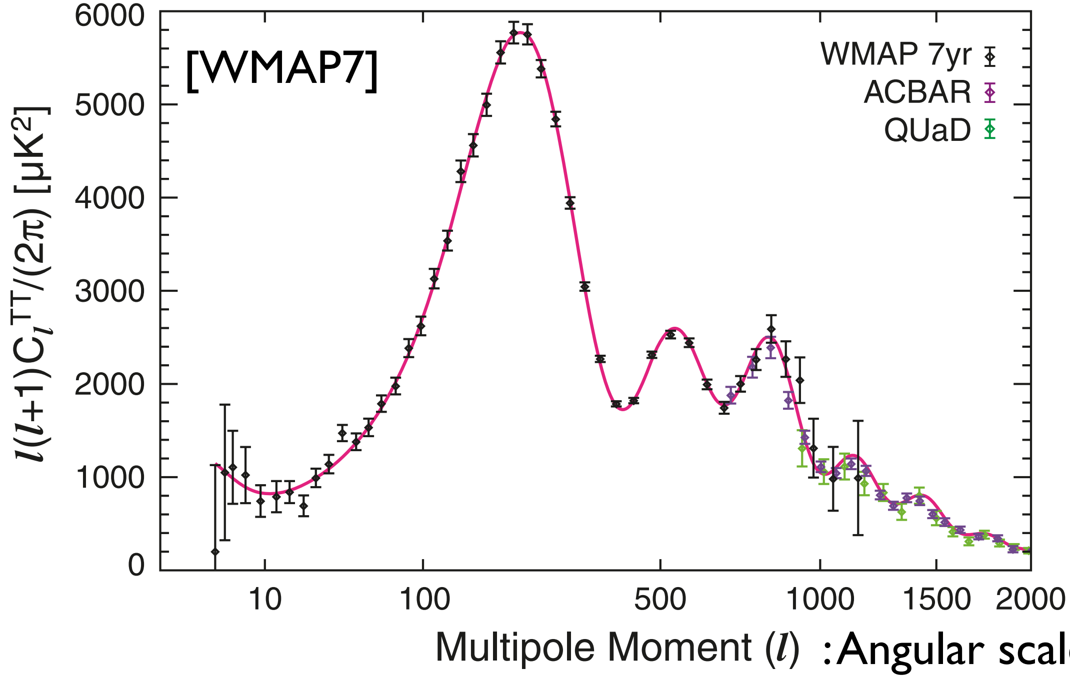
Tomo Takahashi

Department of Physics, Saga University, Japan

Power spectrum

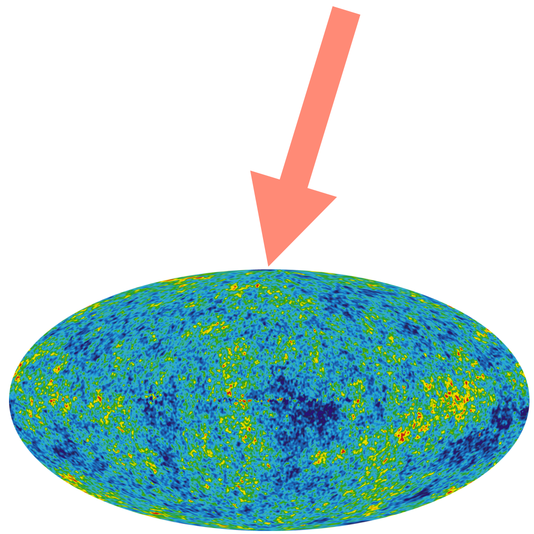
Primordial curvature perturbation

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \right\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_\zeta(k_1)$$



:Angular scales $l \sim 180^\circ/\theta$ $\left(l \sim 14000\frac{k}{\mathrm{Mpc}^{-1}}\right)$

Initial condition



Primordial fluctuations

(almost) adiabatic

(almost) scale invariant (slightly red-tilted)

amplitude ~ 10^{-5}

Models generating primordial fluctuations

- Inflation (fluctuations of the inflaton)

  [various potentials, various models]

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- Curvaton model [Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]

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- Inhomogeneous end of hybrid inflation [Bernardeau, Uzan 2003, Bernardeau et al, 2004, Lyth 2005]

- Inhomogeneous phase transition (e.g., end of thermal inflation)
  [Matsuda,2009; Kawasaki, TT, Yokoyama, 2009]

- Multi-brid inflation [Sasaki 2008; Naruko, Sasaki 2009]

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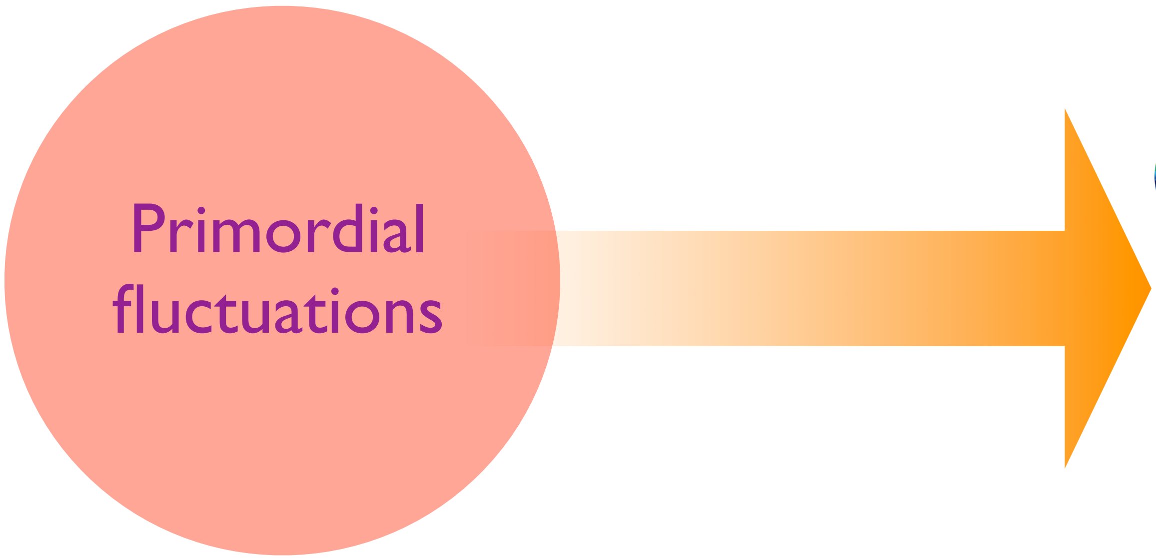
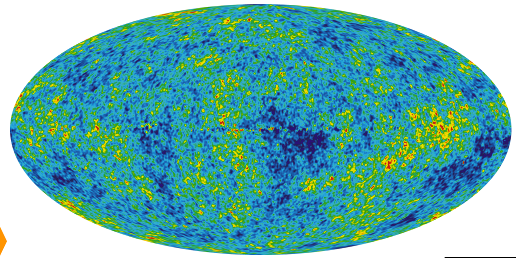
Primordial fluctuations as a probe of the early Universe

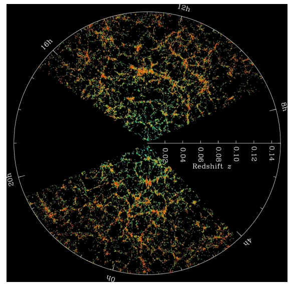
- Primordial density fluctuations (the origin of cosmic structure) are considered to be generated in the very early Universe.

➡ They should give the information of the early Universe.





CMB



Large scale structure

Models generating primordial fluctuations

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  [various potentials, various models]
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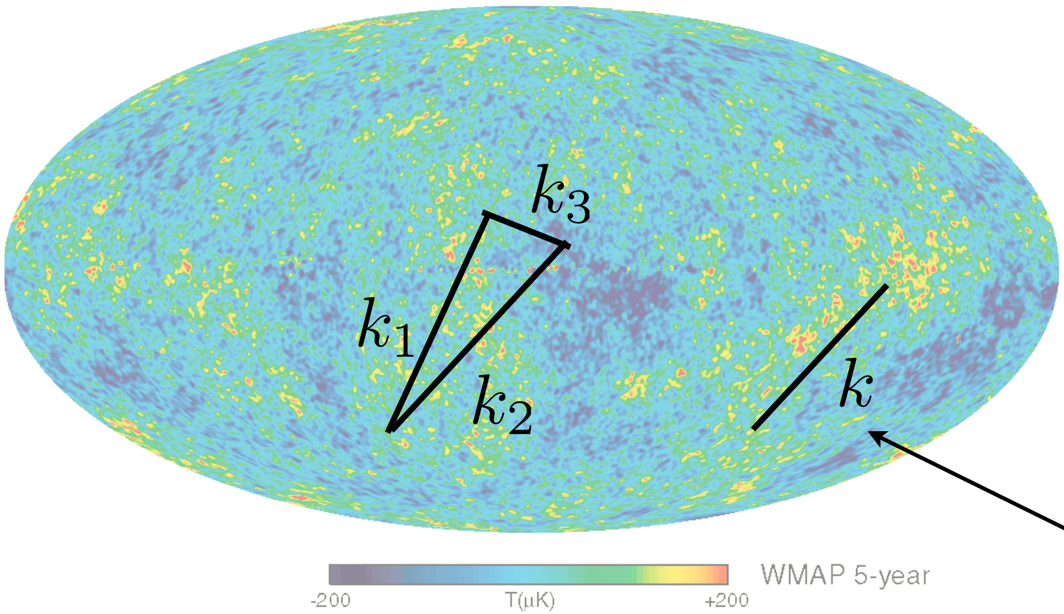
⋮

consistent with power spectrum measurements
(at least, at some parameter space)

3 point function: a measure of non-Gaussianity

$$\left\langle \frac{\Delta T}{T}(\vec{k_1})\frac{\Delta T}{T}(\vec{k_2})\frac{\Delta T}{T}(\vec{k_3}) \right\rangle$$



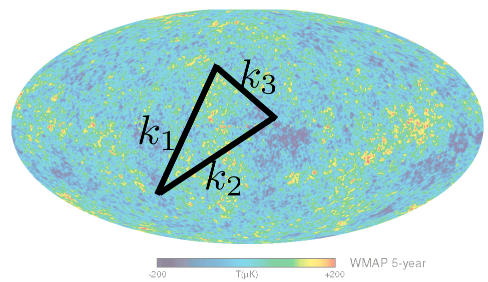
2-point function
(power spectrum)

3 point function: a measure of non-Gaussianity

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \quad = \quad (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$$

$$B_\zeta(k_1, k_2, k_3) \quad = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

- Non-Gaussianity is usually characterized by fNL



  - Amplitude of the bispectrum (3-point function)
  - If fluctuations are Gaussian, $f_{\rm NL} = 0$
  - A critical test of inflation: For (most) inflation models, $f_{\rm NL} \ll \mathcal{O}(1)$

Bispectrum: Observables

- Non-linearity parameter f_{NL}

Bispectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

▶ Depending on momentum distribution ("shape"), there are some types

- local type: $-10 < f_{\rm NL}^{\rm local} < 74$ [WMAP7 (Komatsu et al 2010)]

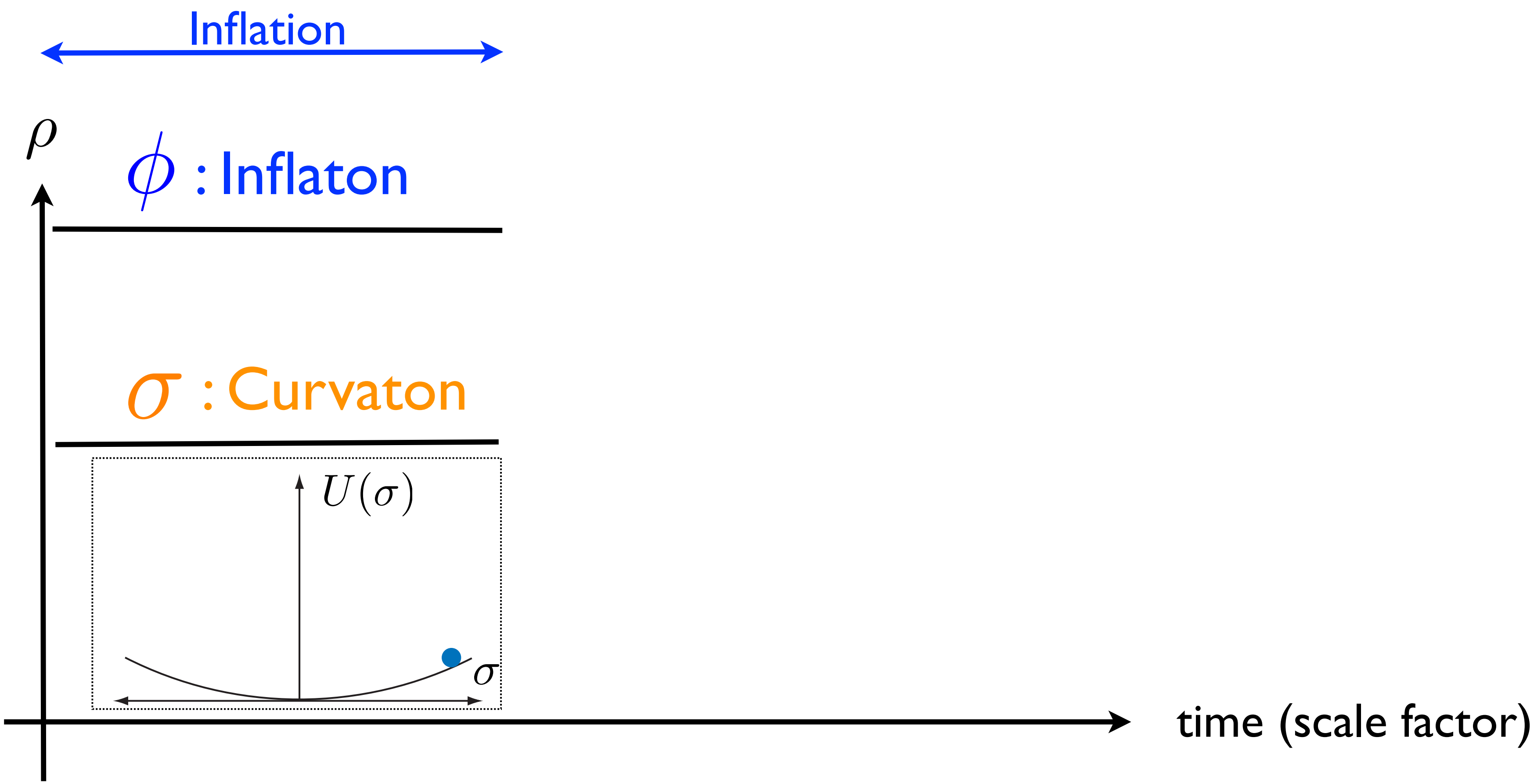
  $25 < f_{\rm NL}^{\rm local} < 117$ [NRAO VLA SKY SURVEY (Xia et al 2010)]

- equilateral type: $-214 < f_{\rm NL}^{\rm equil} < 266$ [WMAP7 (Komatsu et al 2010)]

Example: Curvaton model

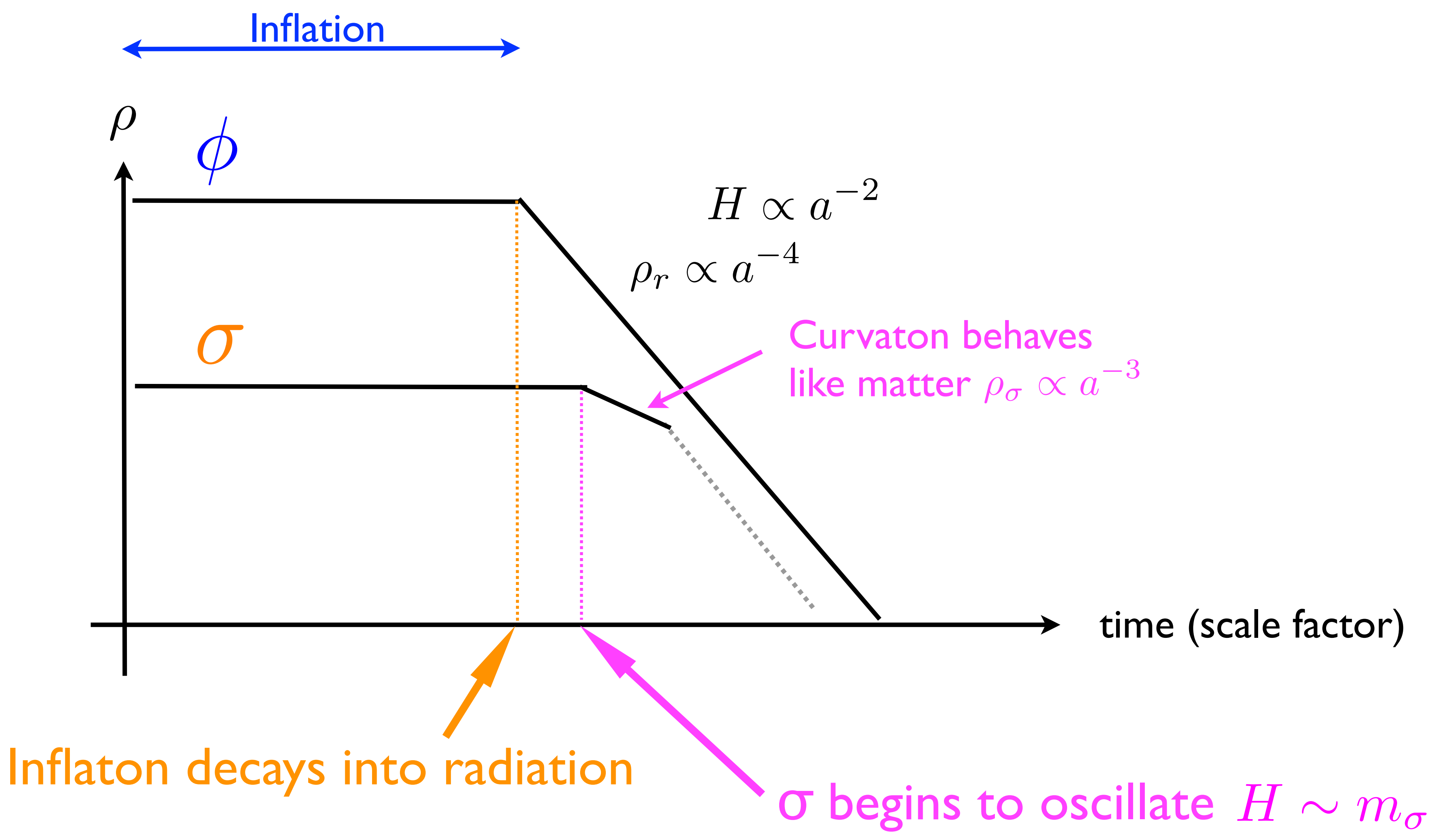
[Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]

Thermal history with curvaton

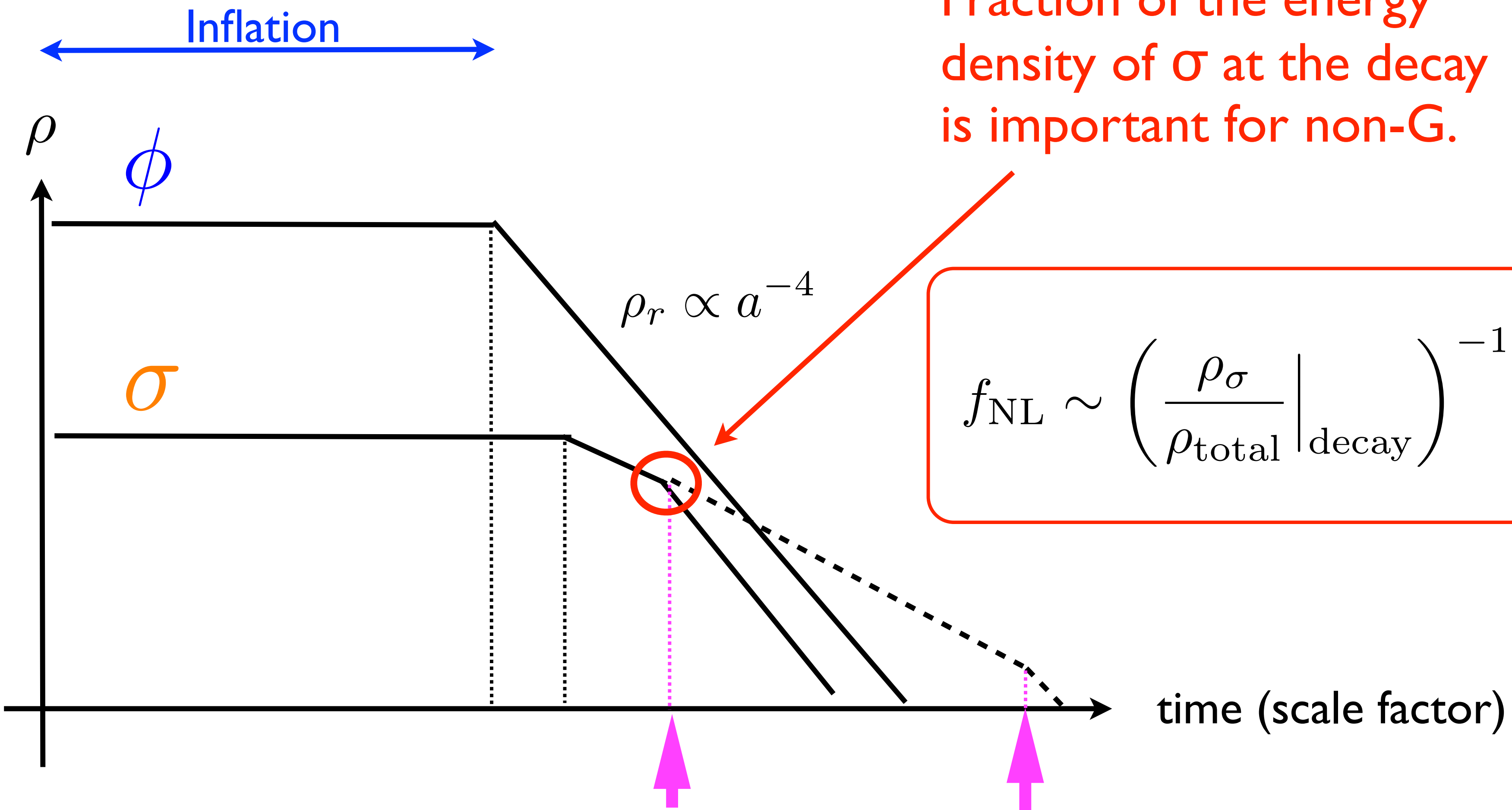


- Inflation is driven by the inflaton $(\rho_\phi \gg \rho_\sigma)$
- σ field can also acquire fluctuations

Thermal history with curvaton



Thermal history with curvaton



Fraction of the energy density of σ at the decay is important for non-G.

$$f_{\rm NL} \sim \left(\frac{\rho_\sigma}{\rho_{\rm total}} \Big|_{\rm decay} \right)^{-1}$$

- σ decays (into radiation)
- Fluctuations of σ gives adiabatic perturbations

Models generating primordial fluctuations

- Inflation (fluctuations of the inflaton)
- Curvaton model [Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]
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- Multi-brid inflation [Sasaki 2008; Naruko, Sasaki 2009]

⋮

f_{NL} can be large...
(even just for local type)

We need to something beyond f_{NL}....

“beyond” $f_{\rm NL}$

- Information of trispectrum (4-pt. function)

  [For a comprehensive discussion for local type, see e.g., Suyama, TT, Yamaguchi, Yokoyama, 1009.1979]

- Scale-dependence of $f_{\rm NL}$

  [Byrnes et al, 2009, 2010; for the curvaton model, Byrnes, Enqvist, TT 2010; Byrnes, Enqvist, Nurmi, TT 2011; Kobayashi, TT 2012]

- Isocurvature fluctuations

  [for nonG, Kawasaki et al 2008; Langlois et al 2008; Hikage, Koyama, Matusbara, TT, Yamaguchi 2008; Kawakami et al 2009; Langlois, Lepidi 2010; Langlois, TT 2010; Hikage, Kawasaki, Sekiguchi, TT 2012.].

“Consistency relations” among f_{NL}, τ_{NL} and g_{NL}

(we consider the local type model.)

Bispectrum and Trispectrum for local type

- 3-point function:

Bispectrum

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

Trispectrum

- 4-point function:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

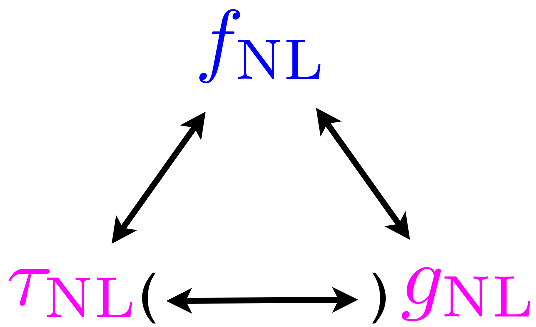
$$T_\zeta(k_1, k_2, k_3, k_4) = \tau_{\rm NL} \left(P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms.} \right)$$

$k_{13} = k_1 + k_3$

$$+ \frac{54}{25} g_{\rm NL} \left(P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms.} \right)$$

Consistency relation between f_{NL}, τ_{NL} and g_{NL}

- We can find some relation between the non-linearity parameters which depend on the model:



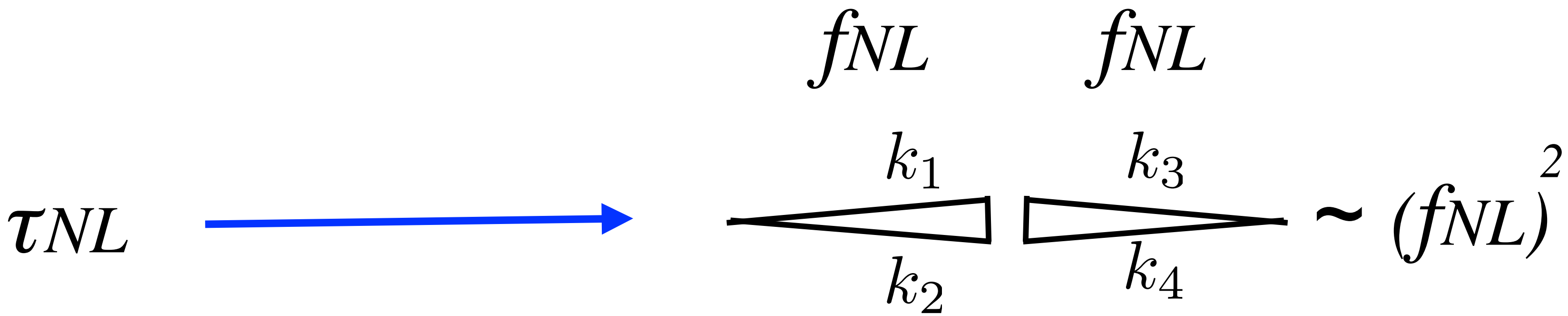
:Bispectrum

:Tripectrum

By using "consistency relation" between these parameters, we can divide the models into some categories.

Relation between $f_{\rm NL}$ and $\tau_{\rm NL}$

- Case with one source (field)



$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2$$

Relation between $f_{\rm NL}$ and $\tau_{\rm NL}$

- Case with multi-source (fields)

➡ No definite relation between $f_{\rm NL}$ and $\tau_{\rm NL}$,

(a general situation)

$$\tau_{\rm NL} \neq \left(\frac{6}{5} f_{\rm NL}\right)^2$$

But, they should satisfy Suyama-Yamaguchi inequality

$$\tau_{\rm NL} > \left(\frac{6}{5} f_{\rm NL}\right)^2$$

[Suyama, Yamaguchi 2008]

Can we differentiate the model?

f_{NL} - τ_{NL} relation

Single-source

Multi-source

- There are still some (many) possibilities for each categories....

Can we differentiate the model?

f_{NL} - τ_{NL} relation

Single-source

- (pure) curvaton model
- (pure) modulated reheating
- Inhomogeneous end of hybrid inflation
- Modulated trapping

⋮

Multi-source

- Mixed inflaton+curvaton
- Mixed inflaton+modulated reh.
- Multi-brid
- Multi-curvaton
- Ungaussiton

⋮

Can we differentiate the model?

f_{NL} - τ_{NL} relation

Single-source

Multi-source

... but, we can further divide models by using f_{NL}-g_{NL}

- "Linear" g_{NL} Type
- "Suppressed" g_{NL} Type
- "Enhanced" g_{NL} Type

Three categories

- "Linear" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}$$

(with O(1) coefficient)

- "Suppressed" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim (\text{suppression factor}) \times f_{\rm NL}$$

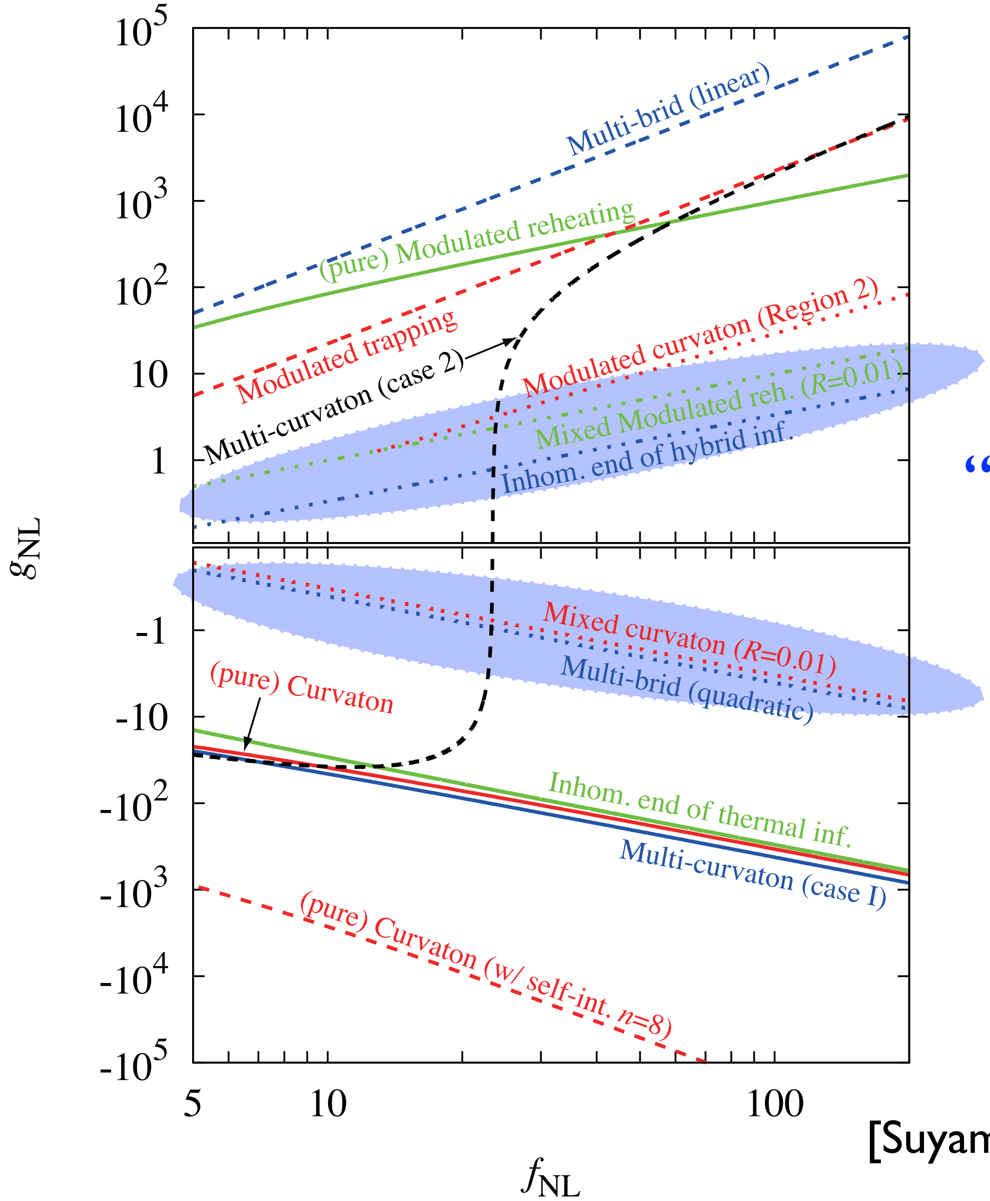
(e.g., suppressed by the slow-roll params. ε, η)

- "Enhanced" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}^{n}$$

(n > 1, in many models n=2)

f_{NL} - g_{NL} diagram

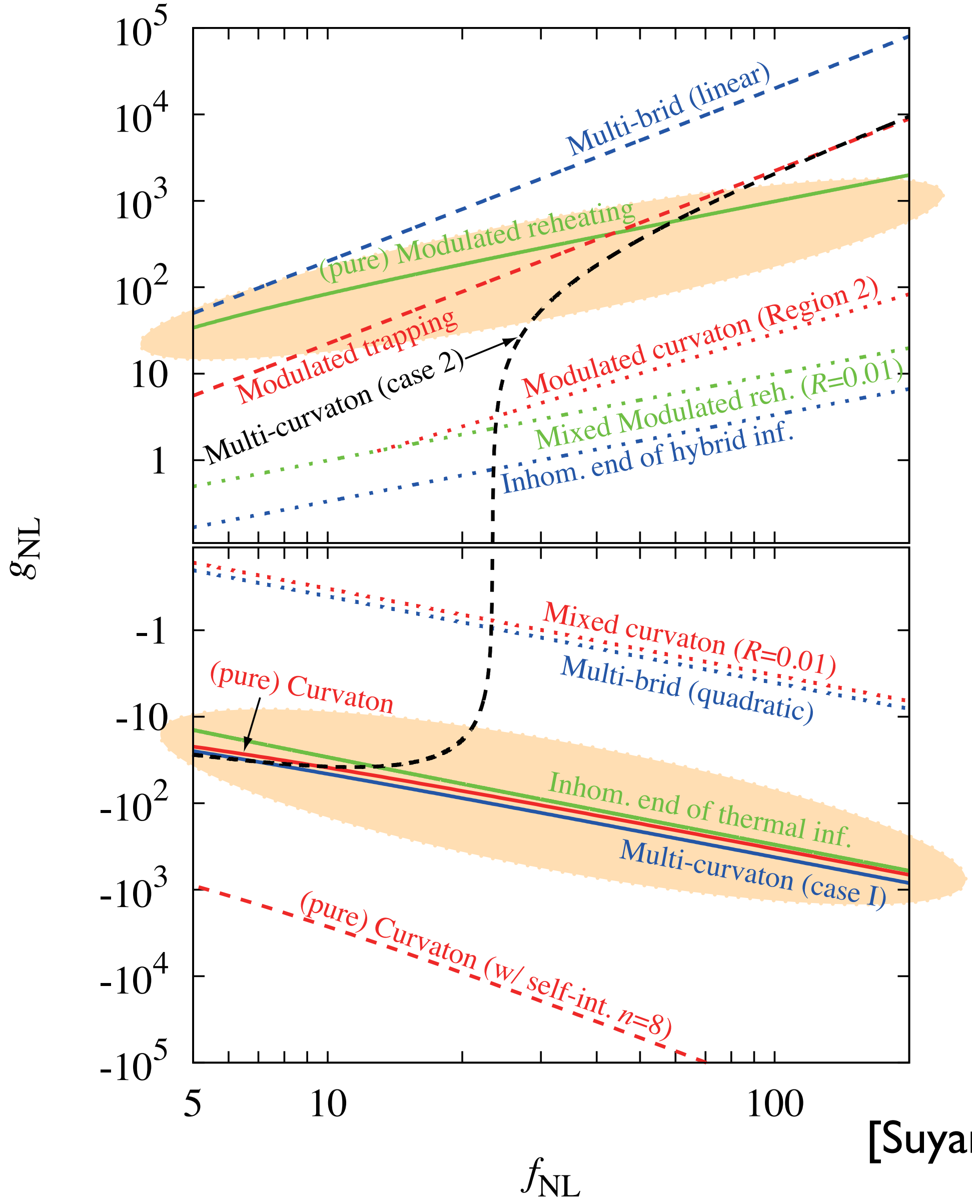


"Suppressed" g_{NL} Type

$g_{NL} \sim (\text{suppression factor}) \times f_{NL}$

[Suyama, TT, Yamaguchi, Yokoyama, 2010].

f_{NL} - g_{NL} diagram

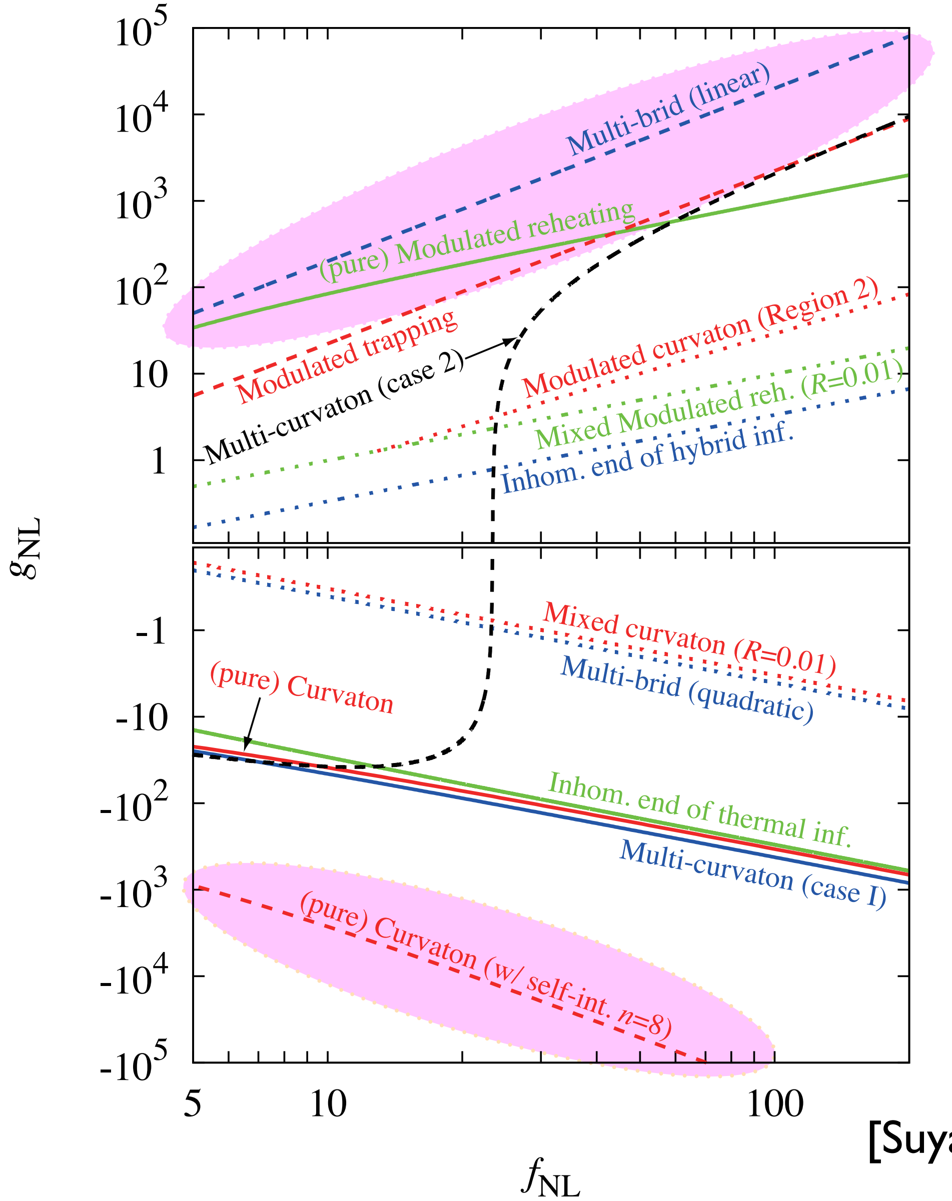


"Linear" g_{NL} Type

$g_{NL} \sim f_{NL}$

[Suyama, TT, Yamaguchi, Yokoyama, 2010].

$f_{\rm NL}$ - $g_{\rm NL}$ diagram



"Enhanced" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}^n$$

[Suyama, TT, Yamaguchi, Yokoyama, 2010].

A strategy to pin down the model

f_{NL} - τ_{NL} relation → Single-source / Multi-source

f_{NL} - g_{NL} relation → "Linear" g_{NL} Type / "Suppressed" g_{NL} Type / "Enhanced" g_{NL} Type

- Once f_{NL} is confirmed to be large, by looking at τ_{NL}, g_{NL}, we can pick up (some) promising model(s).
- However, details of the model may not be probed well...

Even just for the Curvaton model.....

- (simple) Curvaton model with $V = \frac{1}{2}m^2\sigma^2$
- Curvaton with self-interaction [Enqvist, Nurmi 2005; Enqvist, TT, 2008; Enqvist, Nurmi, Taanila, TT, 2009]
- pseudo-Nambu-Goldstone curvaton [Dimopoulos et al 2003; Kawasaki et al 2008]
- Mixed inflaton + curvaton scenario [Langlois, Vernizzi, 2004; Moroi, TT, Toyoda, 2005; Ichikawa, Suyama, TT, Yamaguchi, 2008]
- Multi-Curvaton model (two curvatons) [Assadullahi, Valiviita, Wands, 2007]

Scale-dependence of $f_{\rm NL}$

n_{fNL} :Scale-dependence of $f_{\rm NL}$

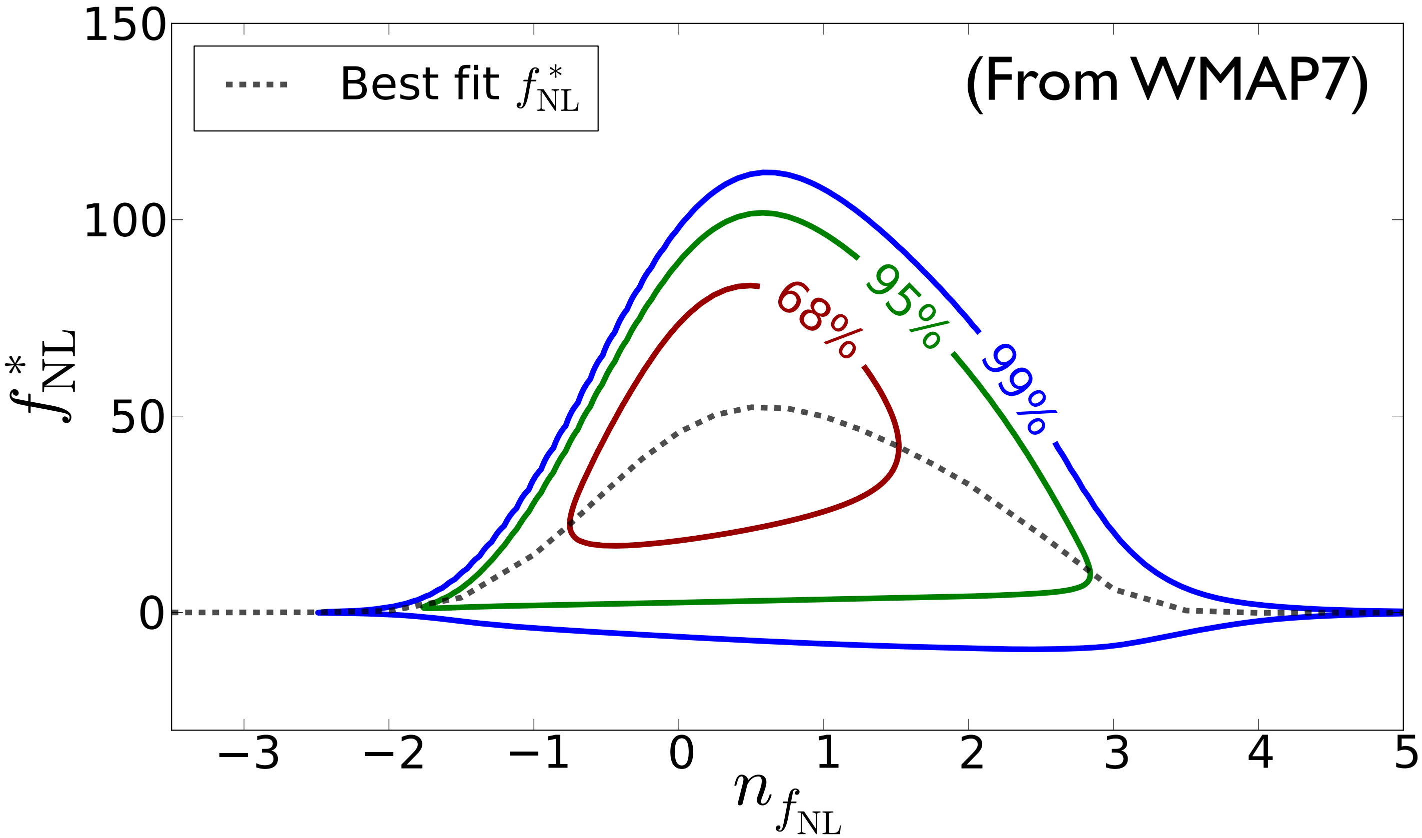
- Definition: $n_{f_{\rm NL}} \equiv \dfrac{d \ln |f_{\rm NL}|}{d \ln k}$

$$\longrightarrow \quad f_{\rm NL}(k) = f_{\rm NL}(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_{f_{\rm NL}}}$$

In the following, we consider "local type": $\zeta = \zeta_G + \dfrac{3}{5} f_{\rm NL} \zeta_G^2$

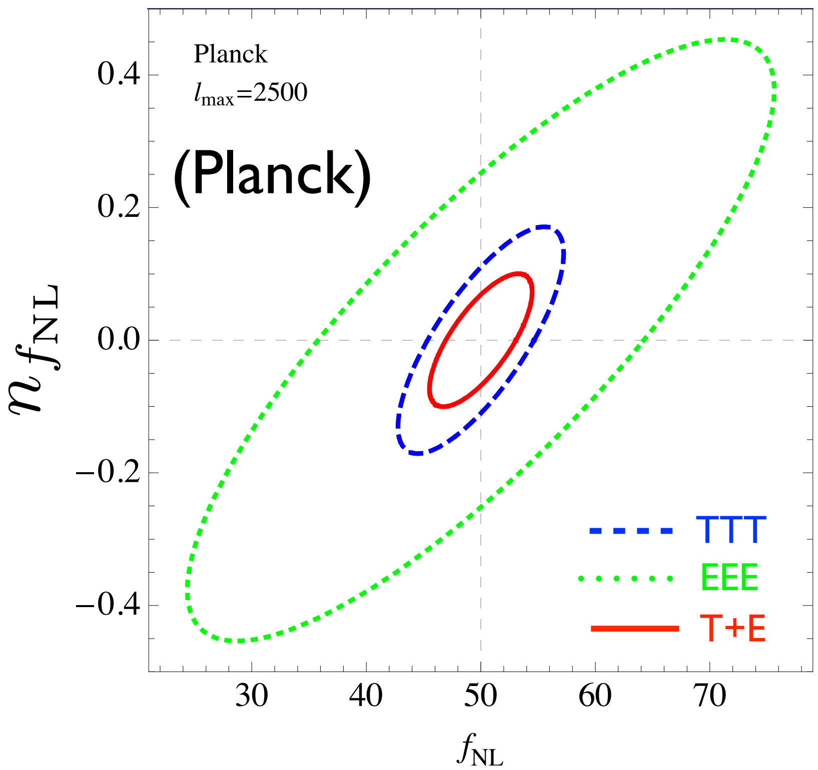
Current limit on n_{fNL}

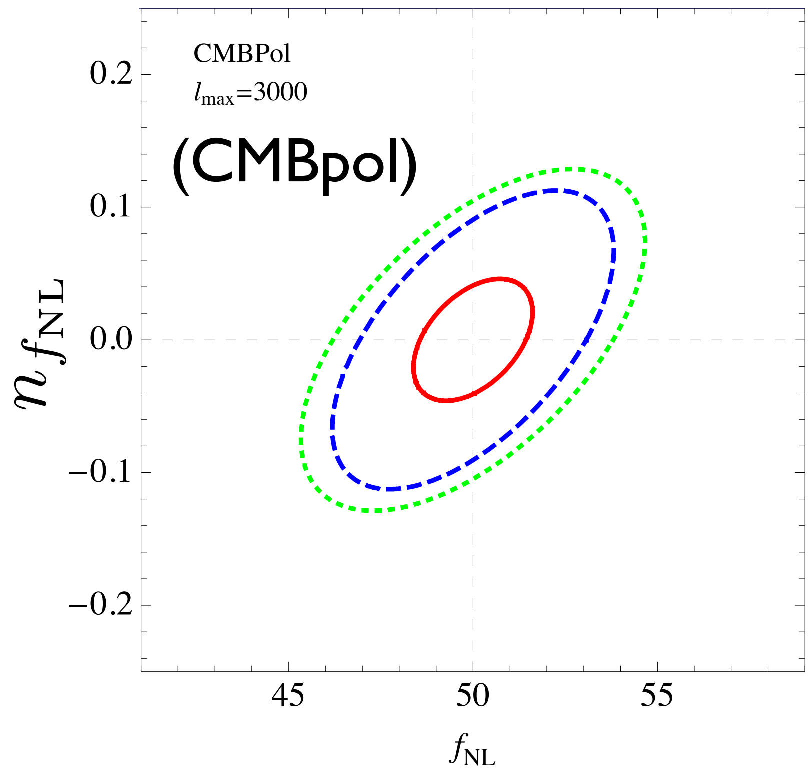
$$f_{\rm NL}(k) = f^*_{\rm NL}\left(\frac{k}{k_*}\right)^{n_{f_{\rm NL}}} \qquad (k_* \simeq 0.064\ h{\rm Mpc}^{-1})$$



[Becker, Huterer 1207.5788]

Projected limit on n_{fNL}





[Sefusatti et al. 2009]

$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}} \quad \text{(CMBpol)}$$

n_{fNL} probes some aspects of models of large f_{NL}

[Byrnes et al, 2009, 2010]

- f_{NL} can be (strongly) scale-dependent when:
  - the potential deviates from the quadratic form.
  - multi-fields are responsible for the perturbations.

$n_{f_{\rm NL}}$ from non-quadratic potential

[Byrnes et al, 2009, 2010]

- When the potential for a light field deviates from a quadratic form, f_{NL} can be scale dependent.

$$f_{\rm NL} n_{f_{\rm NL}} \sim \frac{V'''}{3H^2}$$

cf. for power spectrum

$$\left(n_s - 1 = -2\epsilon + \frac{2V''}{3H^2} \right)$$

- When the potential is quadratic, no scale-dependence

- Non-zero $n_{f_{\rm NL}}$ can give important information on the potential.

$n_{f_{\rm NL}}$ in curvaton with non-quadratic potential

- Self-interacting curvaton [Byrnes, Enqvist, TT 2010; Byrnes, Enqvist, Nurmi, TT 2011; Kobayashi, TT 2012]

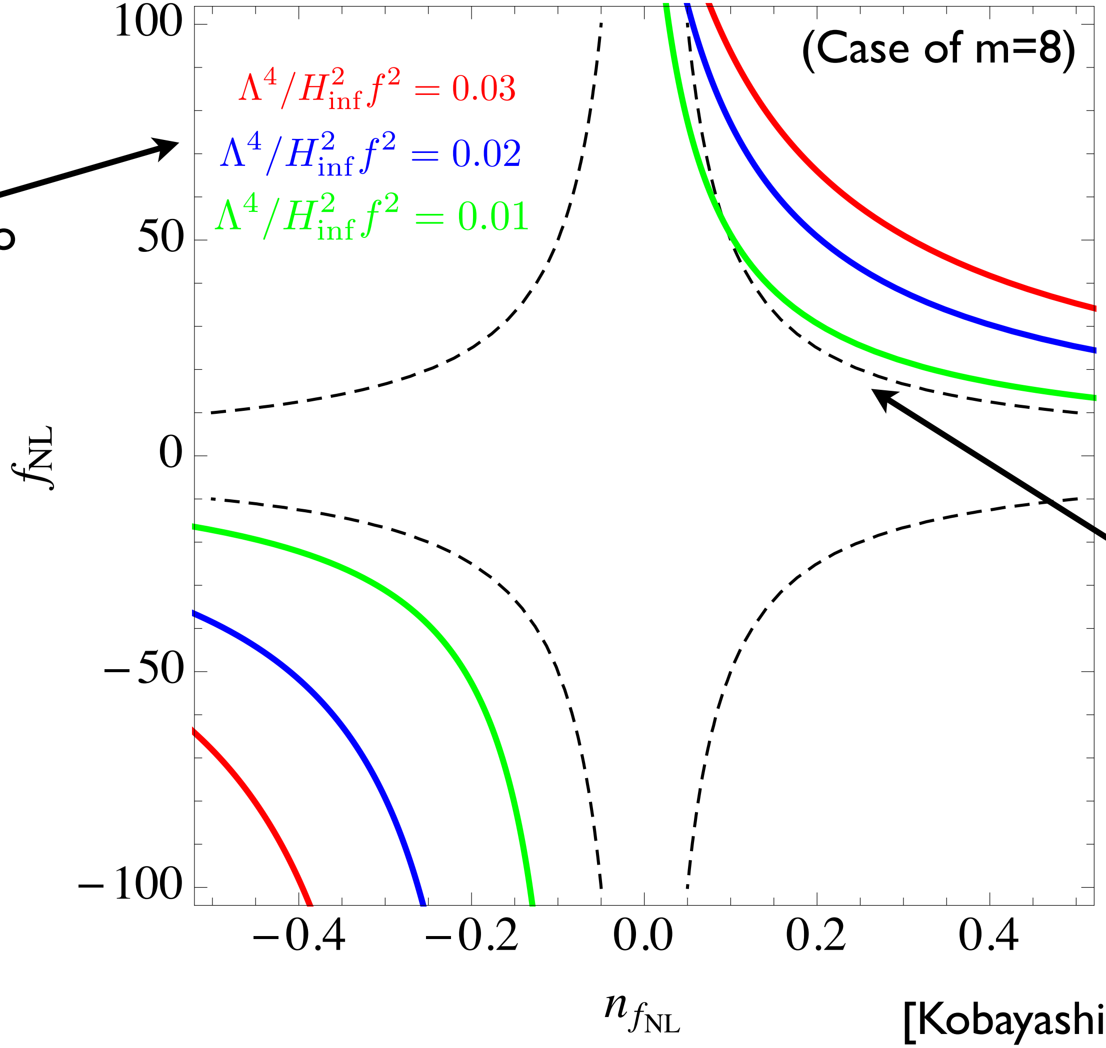
$$V(\sigma) = \Lambda^4 \left[\left(\frac{\sigma}{f} \right)^2 + \left(\frac{\sigma}{f} \right)^m \right]$$

- pseudo-Nambu-Goldstone (NG) curvaton [Huang 2010, Kobayashi, TT 2012]

$$V(\sigma) = \Lambda^4 \left[1 - \cos \left(\frac{\sigma}{f} \right) \right]$$

$n_{f_{\rm NL}}$ in the self-interacting curvaton

corresponding to mass parameter

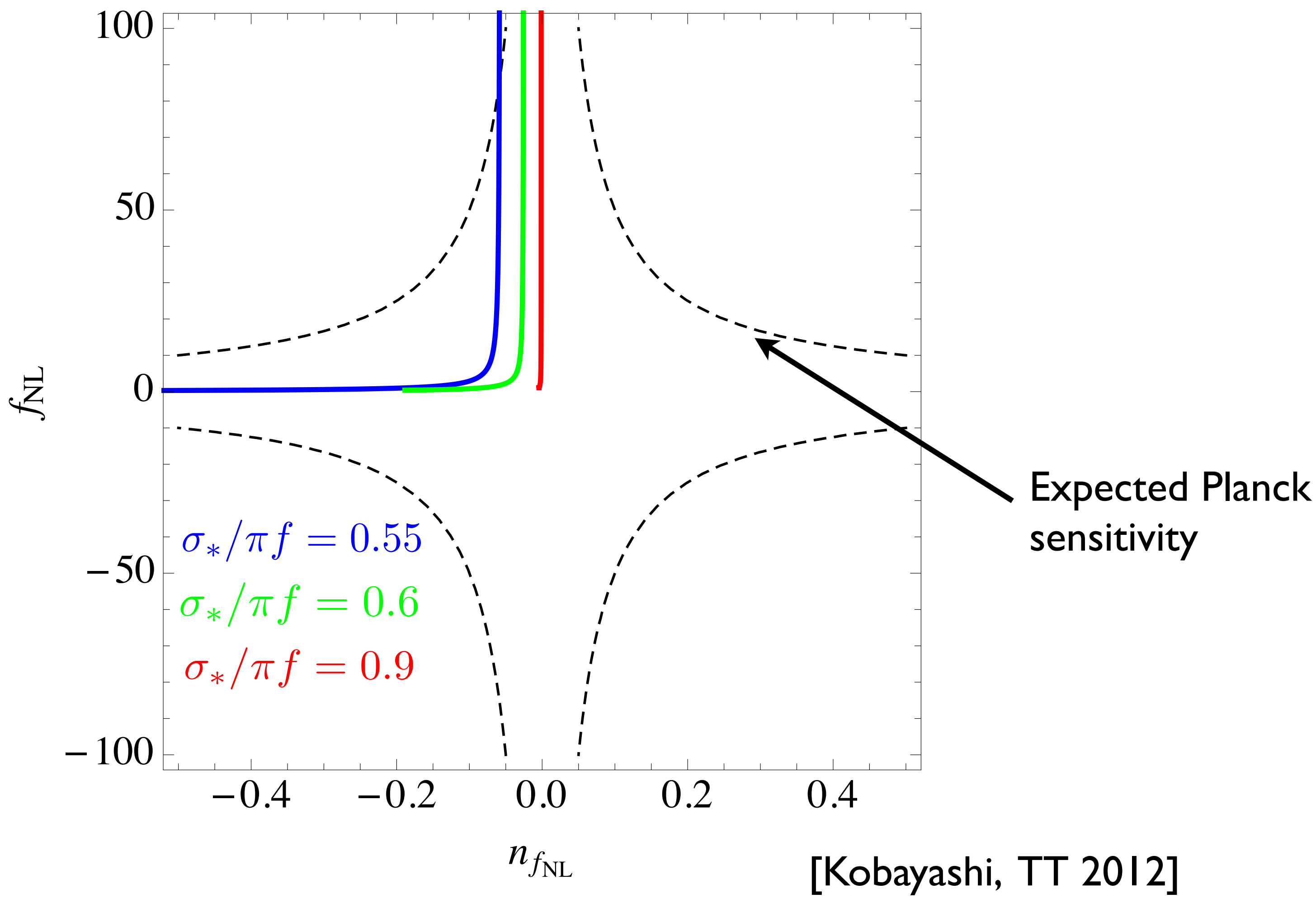


$$V(\sigma) = \Lambda^4 \left[\left(\frac{\sigma}{f}\right)^2 + \left(\frac{\sigma}{f}\right)^m \right]$$

Expected Planck sensitivity

[Kobayashi, TT 2012]

$n_{f_{\rm NL}}$ in the psuedo-Nambu-Goldstone curvaton



[Kobayashi, TT 2012]

n_{fNL} probes some aspects of models of large f_{NL}

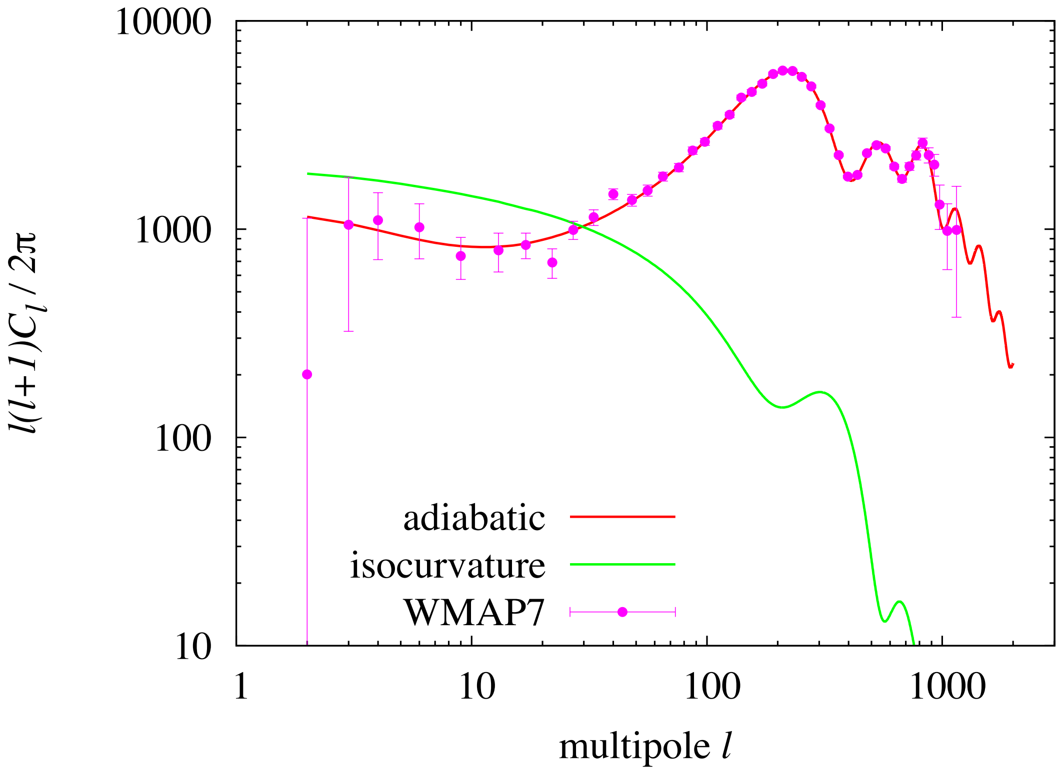
- Scale-dependence of fNL may be able to give detailed information about the model, such as the mass, potential form, ...
(if detected).

- Even if it is not detected, it can put some constraints on the model (parameters).

Non-Gaussianty in isocurvature fluctuations

Constraints on isocurvature fluctuations

- CDM isocurvature fluctuations: $\mathcal{S} \equiv \frac{\delta\rho_c}{\rho_c} - \frac{3\delta\rho_\gamma}{4\rho_\gamma}$

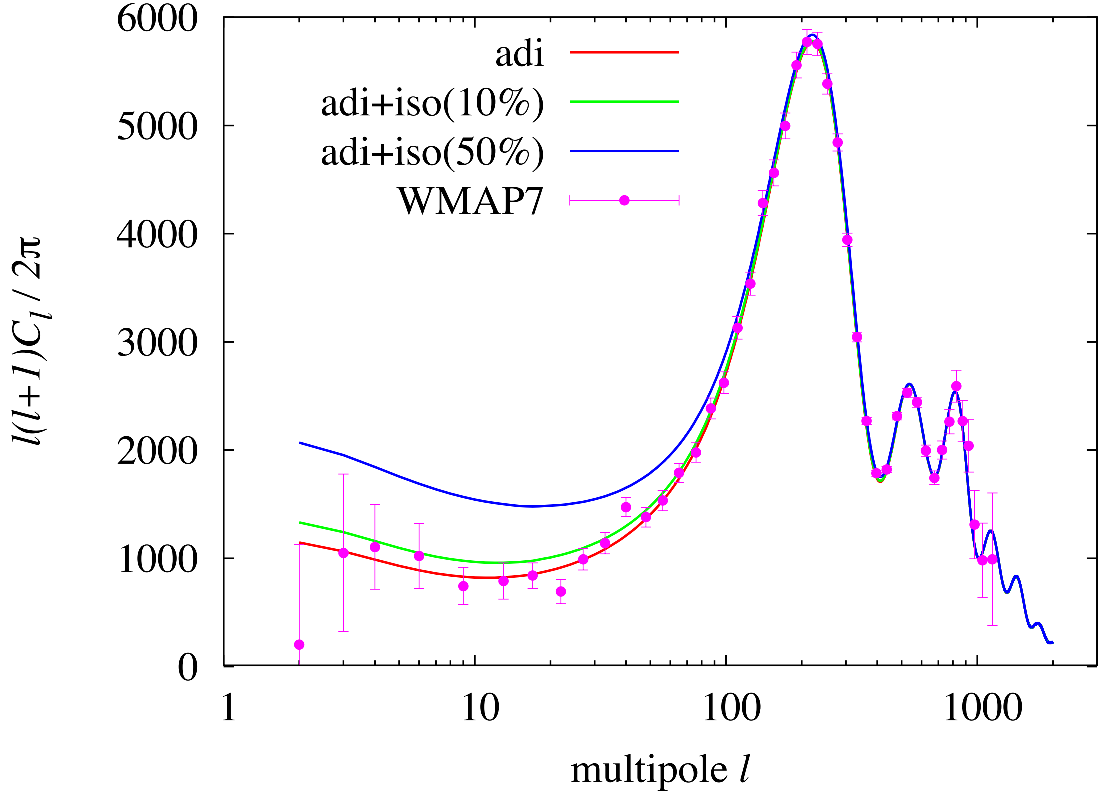


- Pure isocurvature fluctuations are excluded by the data.

Constraints on isocurvature fluctuations

- Small contribution from isocurvature is possible, although severely constrained.

$$\left(\frac{\Delta T}{T}\right)^2\Bigg|_{\rm total} = \left(\frac{\Delta T}{T}\Bigg|_{\rm adi} + \frac{\Delta T}{T}\Bigg|_{\rm iso}\right)^2$$



Constraints on isocurvature fluctuations

- Constraints for the fraction parameter: $\alpha \equiv \dfrac{P_{\mathcal{S}}(k_0)}{P_\zeta(k_0) + P_{\mathcal{S}}(k_0)} = \dfrac{P_S(k_0)}{P_{\rm total}(k_0)}$

  ($P_S \sim < S^2 >$ $\quad P_\zeta \sim < \zeta^2 >$)

  - $\alpha_0 < 0.077 \;\; (95~\%~{\rm CL})$ (for uncorrelated CDM isocurvature)
  - $\alpha_{-1} < 0.0047 \;\; (95~\%~{\rm CL})$ (for anti-correlated CDM isocurvature)

[WMAP7+BAO+SN, Komatsu et al., 2010]

■ Although iso. fluc. are severely constrained, some contaminations are still allowed.

Models with isocurvature fluctuations

- Axion model
- Affleck-Dine baryogenesis
- Curvaton model

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- Depending on when and how CDM/baryon are generated, isocurvature fluctuations can be easily produced in a light field model (such as the curvaton)

Non-Gaussianity in isocurvature fluctuations

- Non-linearity parameter for isocurvature fluctuations:

$$S(\vec{x}) = S_{\rm G}(\vec{x}) + f_{\rm NL}^{\rm (ISO)}(S_{\rm G}(\vec{x})^2 - \langle S_{\rm G}(\vec{x})^2\rangle)$$

  - Bispectrum for S

$$\langle SSS\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_{SSS}(k_1,k_2,k_3)$$

$$B_{SSS}(k_1,k_2,k_3) = 2f_{\rm NL}^{\rm (iso)}\,(P_S(k_1)P_s(k_2) + 2\text{ perm.})$$

$$B_{SSS}(k_1,k_2,k_3) \sim 2f_{\rm NL}^{\rm (iso)}\alpha^2 P_{\rm tot}^2$$

Constraint on $f_{NL}^{(\mathrm{iso})}$

- New constraint [Hikage, Kawasaki, Sekiguchi, TT 2012]

$$\alpha^2 f_{\mathrm{NL}}^{(\mathrm{iso})} = 40 \pm 66 \quad [1\sigma]$$

(from WMAP7, bispectrum)

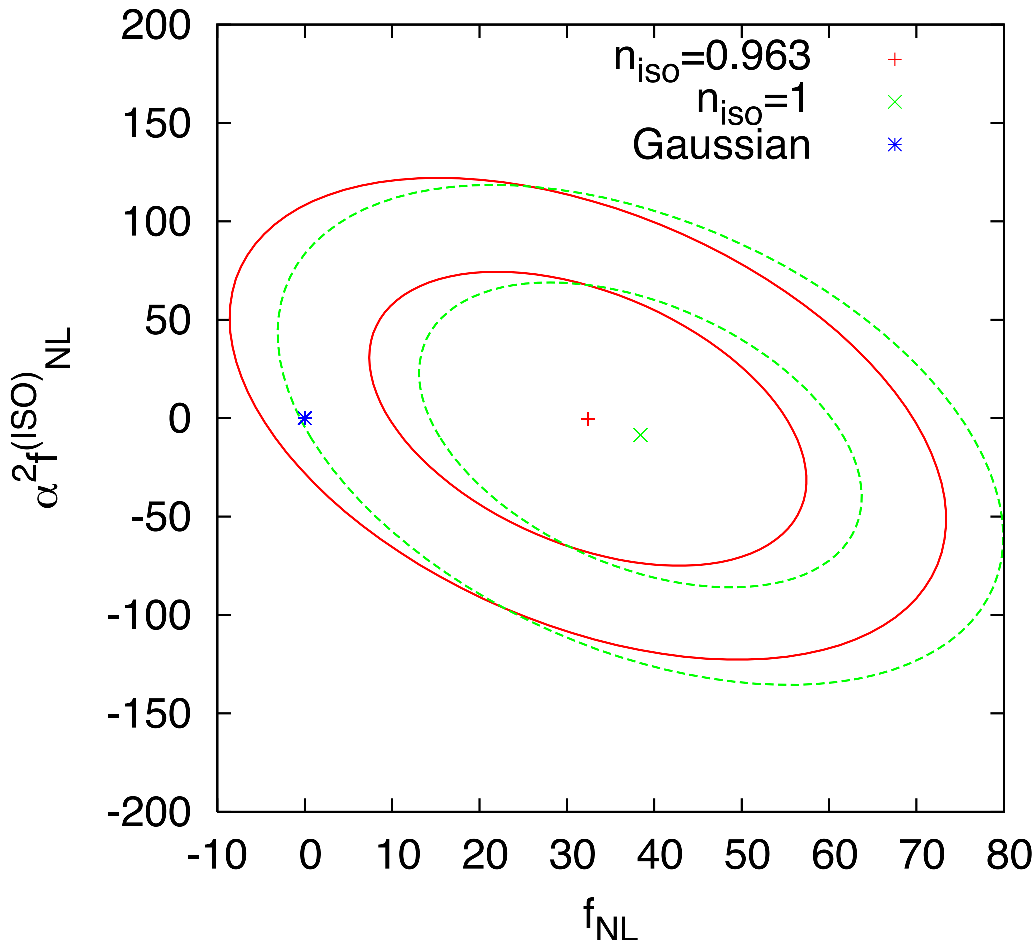
- c.f. constraint using Minkowski functional

[Hikage, Koyama, Matsubara, TT, Yamaguchi, 2008]

$$\alpha^2 f_{\mathrm{NL}}^{(\mathrm{iso})} = -15 \pm 60 \quad [1\sigma]$$

Joint constraint on f_{NL} and $f_{NL}^{(\mathrm{iso})}$

[Hikage, Kawasaki, Sekiguchi, TT 2012]

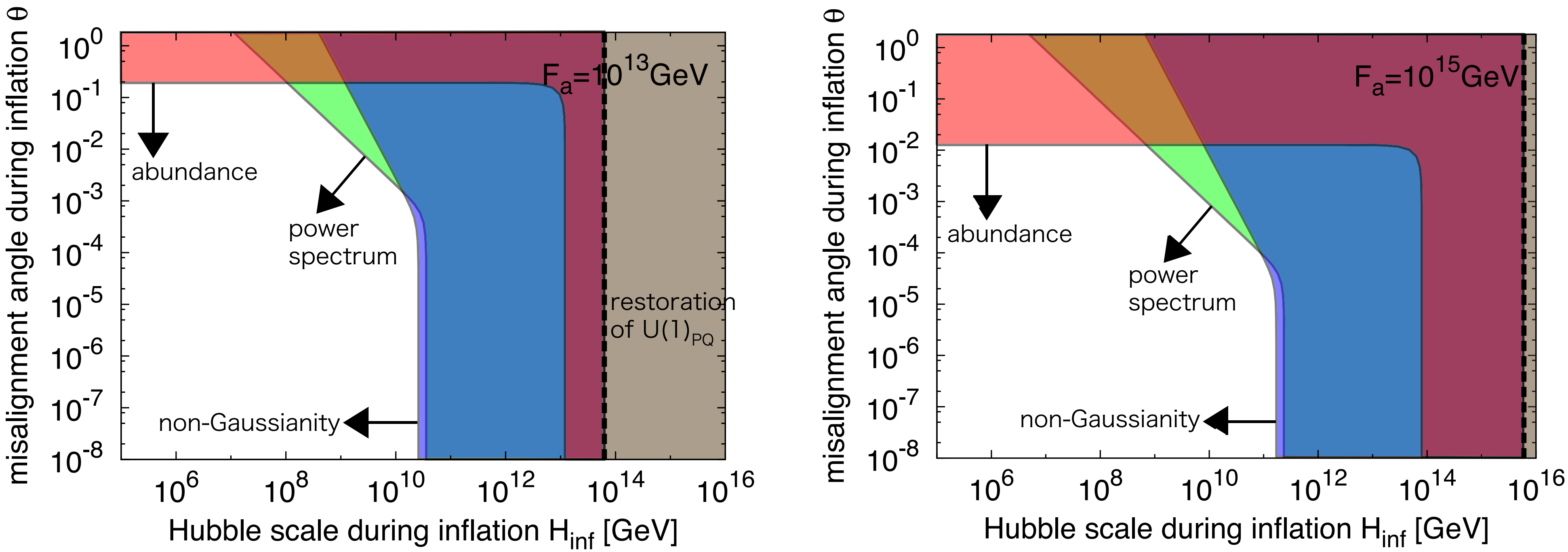


$$f_{\rm NL} = 38 \pm 24 \quad [1\sigma]$$

$$\alpha^2 f_{\rm NL}^{(\rm iso)} = -8 \pm 72 \quad [1\sigma]$$

Application to the Axion model

[Hikage, Kawasaki, Sekiguchi, TT 2012]



Summary

- Information on $f_{\rm NL}$ is NOT enough to differentiate models of primordial fluctuations.

- Relation among f_{NL}, $\tau_{\rm NL}$ and $g_{\rm NL}$ can pick up some category of models.

- Scale-dependence of non-Gaussianity ($n_{f_{\rm NL}}$) can be useful to discriminate models of large non-G.

- Isocruvature fluctuations can be a consistency check with some other aspects of cosmology.