Thermal Production of Axino Dark Matter

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Outline

- Motivation and introduction
- Effective interactions of the axion supermultiplet
- Thermal production of axino in the early Universe
- Conclusion

Motivation and Introduction

- Gauge hierarchy problem

  ➔ Weak scale SUSY

- Strong CP problem

  ➔ Anomalous global $U(1)_{PQ}$ symmetry (Peccei and Quinn)
  spontaneously broken at 10^9 GeV < F_a < 10^{12} GeV

  ➔ Axion

It is then natural to combine these two ideas together.

➔ Supersymmetric axion model

In SUSY axion model, the MSSM mu-parameter can be generated as

$\mu \sim \frac{F_a^2}{M_{Planck}}$, which would solve the mu-problem. (Kim and Nilles)

Supersymmetric axion model necessarily contains the fermionic superpartner of axion, the axino, which can have a variety of cosmological implications.

- Axino can be a good DM candidate

  Bonometto,Gabbiani, Masiero; Rajagopal,Turner,Wilczek; Covi,Kim,Roszkowski

- Even when axino is not the LSP, so not DM, it can severely affect the cosmological feature of the model, e.g. DM (neutralino or gravitino) can be produced by decaying axinos.

These issues depend crucially on the cosmological production of axinos in the early Universe.

Thermal production of axinos in the early Universe ($T < F_a$)

Covi, Kim, Kim, Roszkowski; Brandenburg, Steffen; Strumia

Previous works on thermal axino production are mostly based on

$$\mathcal{L}_{\rm eff} = \frac{1}{32\pi^2 F_a}\int A W^i W^i + {\rm h.c} \quad (A = s + ia + \theta\tilde{a} + F^A)$$
$$= \frac{1}{32\pi^2 F_a}\left[a G^i_{\mu\nu}\tilde{G}^{i\mu\nu} + i\tilde{a}\sigma^{\mu\nu}\gamma_5\tilde{\lambda}^i G^i_{\mu\nu} - \tilde{a}\tilde{\lambda}^i D^i + \ldots \right]$$

However, analysis using this effective interaction alone can lead to a highly overestimated axino production rate, in particular for the DFSZ-type models.

For correct result, we need more careful analysis incorporating other effective interactions of the axion supermultiplet, and then the resulting relic axino density can be very different from the previous result.

Effective interactions of axion supermultiplet

Axion effective interactions (Energy < F_a):

$$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2}\frac{a}{F_a}G^i_{\mu\nu}\tilde{G}^{i\mu\nu} + y_Q\frac{\partial_\mu a}{2F_a}\bar{Q}\gamma^\mu\gamma_5 Q - m_Q\bar{Q}\left[\exp\left(ix_Q\gamma_5\frac{a}{F_a}\right)\right]Q$$

U(1)$_{PQ}$: $a \rightarrow a + \alpha F_a, \quad Q \rightarrow \exp\left(-\frac{i\alpha x_Q\gamma_5}{2}\right)Q$

$$J^\mu_{PQ} = F_a\partial^\mu a + x_Q\bar{Q}\gamma^\mu\gamma_5 Q$$

$$\partial_\mu J^\mu_{PQ} = \frac{C_{PQ}}{32\pi^2}G^i_{\mu\nu}\tilde{G}^{i\mu\nu} \qquad (C_{PQ} = C_W + x_Q)$$

Change of field basis: $Q \rightarrow \exp\left(-iz_Q\gamma_5\frac{a}{2F_a}\right)Q$

➔ Reparameterization of the Wilsonian axion couplings

$$C_W \rightarrow C_W + z_Q, \qquad y_Q \rightarrow y_Q + z_Q, \qquad x_Q \rightarrow x_Q - z_Q$$

For low energy effective lagrangian of axion, one often chooses the field basis for which all fields except the axion are invariant under U(1)$_{PQ}$, and the PQ anomaly is encoded entirely in C_W : Georgi, Kaplan, Randall

$$x_Q = 0 , \qquad C_{PQ} = C_W$$

$$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2}\frac{a}{F_a}G^i_{\mu\nu}\tilde{G}^{i\mu\nu} + y_Q\frac{\partial_\mu a}{2F_a}\bar{Q}\gamma^\mu\gamma_5 Q - m_Q\bar{Q}\left[\exp\left(ix_Q\gamma_5\frac{a}{F_a}\right)\right]Q$$

All observables derived from L_{eff} should be invariant under

$$C_W \to C_W + z_Q, \qquad y_Q \to y_Q + z_Q, \qquad x_Q \to x_Q - z_Q$$

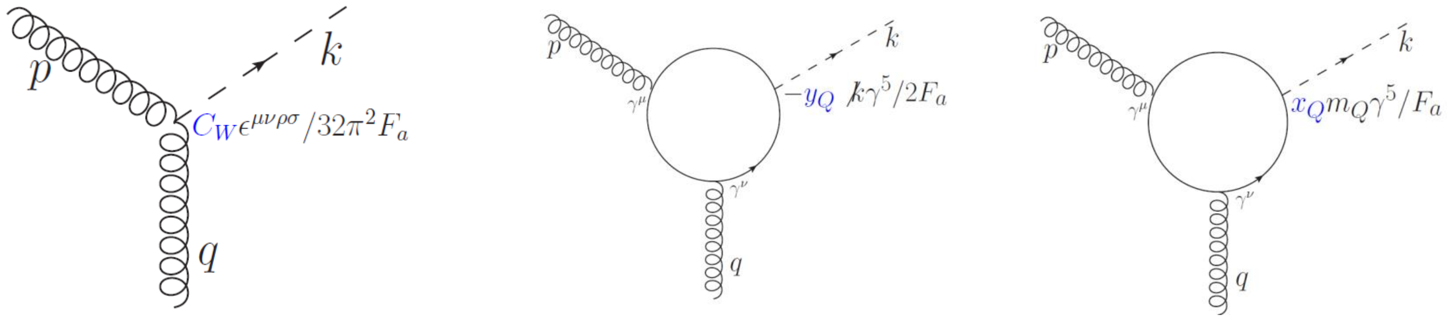
- PQ-breaking axion potential:

$$V_{\rm axion} = -f_\pi^2 m_\pi^2\sqrt{\frac{m_u^2 + m_d^2 + 2m_u m_d\cos(C_{PQ}a/F_a + \bar{\theta})}{(m_u + m_d)^2}}$$

Nonzero PQ anomaly coefficient $C_{PQ} = C_W + x_Q$ for QCD is essential for the strong CP problem to be solved by the axion VEV.

- PQ-invariant 1PI axion-gluon-gluon amplitude:

$$\mathcal{A}_{1PI}[a(k), g(p), g(q)] = \frac{C_{1PI}}{32\pi^2 F_a}\delta^4(k+p+q)\epsilon_{\mu\nu\rho\sigma}k^\mu p^\nu \epsilon_1^\rho \epsilon_2^\sigma$$



C_W y_Q x_Q

$$C_{1PI} = (C_W - y_Q) + (x_Q + y_Q)\, L(p, q; m_Q)$$

$$L(p,q;m_Q) = \int_0^1 dx \int_0^{1-x} dy \frac{2m_Q^2}{m_Q^2 - [p^2 x(1-x) + q^2 y(1-y) + 2(p\cdot q)xy]}$$

$$C_{1PI} = \begin{cases} C_W - y_Q + \mathcal{O}\left(m_Q^2/p^2\right) & \text{for} \quad |p^2| \gg m_Q^2, \quad k^2 = q^2 = 0 \\ C_W + x_Q + \mathcal{O}\left(p^2/m_Q^2\right) & \text{for} \quad |p^2| \ll m_Q^2, \quad k^2 = q^2 = 0 \end{cases}$$

(Recall $C_{PQ} = C_W + x_Q$ at every energy scales.)

The Wilsonian axion coupling **C_W**, the PQ anomaly coefficient **C_{PQ}** for QCD, and the 1PI axion-gluon-gluon amplitude **C_{1PI}** look similar to each other, but they can have very different values.

- **C_W** **:** field-basis-dependent lagrangian parameter

- **$C_{PQ} = C_W + x_Q$** **:**

  1) field-basis-independent constant which is a true measure of the explicit breaking of $U(1)_{PQ}$ by the QCD anomaly and instantons.

  2) has a common value at every energy scales

  3) exactly determined at 1-loop order

- **$C_{1PI} = C_W - y_Q + (x_Q + y_Q)L(p,q;m_Q)$** **:**

  1) field-basis-independent, and generically contains non-local piece

  2) can have different values at different energy scales

  3) receives higher order corrections

  4) determines the axion production by gluons at each energy scale and is suppressed by m_Q^2/p^2 in the limit $|p| \gg m_Q$

◆ UV completion with linearly realized $U(1)_{PQ}$:

Axion models with linearly realized $U(1)_{PQ}$ include

i) PQ-charged scalar field X whose VEV sets the axion scale: $\langle X \rangle = F_a$

ii) PQ-charged and gauge-charged fermions Q_L+Q_R which become massive as a consequence of spontaneous $U(1)_{PQ}$ breaking.

$$\mathcal{L}_{UV} = \partial_\mu X \partial^\mu X^* - \frac{1}{4g^2} G^{i\mu\nu} G^i_{\mu\nu} + \frac{1}{32\pi^2} \bar{\theta}\, G^{i\mu\nu} \tilde{G}^i_{\mu\nu}$$
$$+ i\bar{Q}\gamma^\mu D_\mu Q - \left(\kappa \frac{X^n}{M_{Planck}^{n-1}} \bar{Q}_L Q_R + \text{h.c} \right) - V(|X|) + \ldots$$

The Yukawa couplings between PQ-breaking scalar fields and Q_L+Q_R make $U(1)_{PQ}$ anomalous, and provide connection between the axion sector and the visible sector.

$$m_Q = \kappa \frac{F_a^n}{M_{Planck}^{n-1}}$$

➔ In the limit $m_Q \to 0$, axion becomes a Goldstone boson of anomaly-free U(1) and is decoupled from the visible sector.

$$i\bar{Q}\gamma^\mu D_\mu Q - \left(\kappa \frac{X^n}{M_{Planck}^{n-1}} \bar{Q}_L Q_R + h.c\right)$$

Q = the heaviest PQ-charged and gauge-charged fermion which becomes massive as a consequence of spontaneous PQ- breaking

Two distinct class of (SUSY) axion models:

* SUSY **KSVZ** (Kim, Shifman, Vainshtein, Zakharov) model :

  Q = exotic quark, n = 1,

  m_Q can take any value between the weak scale and the axion scale

* SUSY **DFSZ** (Dine, Fischler, Srednicki, Zhitnitsky) model :

  $\bar{Q}_L = \tilde{H}_u, \; Q_R = \tilde{H}_d$, n = 2 , m_Q = Higgs mu-parameter

➔ In SUSY DFSZ model, m_Q = weak scale << F_a.

It is also possible that m_Q << F_a in KSVZ model.

The decoupling of axion from the visible sector in the limit m_Q ➔ 0 should reveal in physical 1PI amplitudes at energy scales above m_Q.

$$\mathcal{L}_{\rm UV} = \partial_\mu X \partial^\mu X^* - \frac{1}{4g^2} G^{i\mu\nu} G^i_{\mu\nu} + \frac{1}{32\pi^2} \bar{\theta}\, G^{i\mu\nu} \tilde{G}^i_{\mu\nu}$$
$$+ i\bar{Q}\gamma^\mu D_\mu Q - \left(\kappa \frac{X^n}{M_{\rm Planck}^{n-1}} \bar{Q}_L Q_R + {\rm h.c} \right) - V(|X|) + \ldots$$

➔ $$\mathcal{L}_{\rm eff} = \frac{C_W}{32\pi^2} \frac{a}{F_a} G^i_{\mu\nu} \tilde{G}^{i\mu\nu} + y_Q \frac{\partial_\mu a}{2F_a} \bar{Q}\gamma^\mu \gamma_5 Q - m_Q \bar{Q} \left[\exp\left(i x_Q \gamma_5 \frac{a}{F_a} \right) \right] Q$$

with $C_W = z_Q, \quad y_Q = z_Q, \quad x_Q = n - z_Q$

$$C_{1PI} = \begin{cases} \mathcal{O}\left(m_Q^2/p^2\right) & \text{for} \quad m_Q < p < F_a \\ n + \mathcal{O}\left(p^2/m_Q^2\right) & \text{for} \quad p < m_Q \end{cases}$$

The same observation applies also to the axino effective interactions:

$$\mathcal{L}_{\rm UV} = \int d^4\theta \sum_{\rm I} \Phi_{\rm I}^* \Phi_{\rm I} + \int d^2\theta \left(\kappa \frac{X^n}{M_{\rm Planck}^{n-1}} Q Q^c + \ldots \right)$$

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2} \frac{A}{F_a} W^{i\alpha} W_\alpha^i + \exp\left(\frac{(x_Q + x_{Q^c}) A}{F_a} \right) m_Q Q Q^c \right] + \int d^4\theta \left(\frac{A + A^*}{F_a} \right) (y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c) \qquad (A = s + ia + \theta \tilde{a} + F^A)$$

$Q + Q^c$ = the heaviest PQ-charged and gauge-charged matter field whose mass can be far below F_a

1PI axino-gluino-gluon amplitude showing the decoupling in the limit $m_Q \rightarrow 0$

$$\mathcal{A}_{\rm 1PI}[\tilde{a}(k), g(p), \tilde{g}(q)] = \frac{C_{\rm 1PI}}{32\pi^2 F_a} \delta^4(k+p+q) \bar{\tilde{a}}(k) \sigma_{\mu\nu} \gamma_5 \tilde{g}(q) \epsilon^\mu p^\nu$$

$$C_{\rm 1PI} = \begin{cases} \mathcal{O}\left(m_Q^2/p^2\right) & \text{for} \quad m_Q < p < F_a \\ n + \mathcal{O}\left(p^2/m_Q^2\right) & \text{for} \quad p < m_Q \end{cases}$$

Thermal production of axinos

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2}\frac{A}{F_a}W^{i\alpha}W^i_\alpha + \exp\left(\frac{(x_Q + x_{Q^c})A}{F_a}\right) m_Q Q Q^c\right]$$
$$+ \int d^4\theta \left(\frac{A + A^*}{F_a}\right)(y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c)$$

The AWW-coupling does not involve any suppression by m_Q, while the physical 1PI axino-gluino-gluon amplitude C_{1PI} is suppressed by m_Q^2/p^2 at energy scale $p >> m_Q$.

➔ The analysis using the AWW-coupling alone for the axino production at $T >> m_Q$ gives a highly overestimated production rate.

We need more careful analysis including other effective interactions together.

Axino production per unit spacetime volume :

$$\mathcal{L}_{\rm eff} = \int d^2\theta \left[\frac{C_W}{32\pi^2}\frac{A}{F_a} W^{i\alpha}W^i_\alpha + \exp\left(\frac{(x_Q + x_{Q^c})A}{F_a}\right) m_Q Q Q^c\right] + \int d^4\theta \left(\frac{A + A^*}{F_a}\right)(y_Q Q^* Q + y_{Q^c} Q^{c*} Q^c)$$

$$\mathcal{A}_{1PI}[\tilde{a}(k), g(p), \tilde{g}(q)] = \frac{C_{1PI}}{32\pi^2 F_a}\delta^4(k+p+q)\bar{\tilde{a}}(k)\sigma_{\mu\nu}\gamma_5\tilde{g}(q)\epsilon^\mu p^\nu$$

* Previous result for the axino production by the gluon multiplet using the effective AWW-coupling alone :

$$\Gamma_{\tilde{a}}(C_W) \sim \frac{C_W^2 g^6 T^6}{(32\pi^2 F_a)^2} \quad \text{for all} \quad T < F_a$$

* Correct production rate using the 1PI amplitude C_{1PI} :

$$\Gamma_{\tilde{a}}(C_{1PI}) \sim \begin{cases} g^6 m_Q^4 T^2 \ln^4(T^2/m_Q^2)/(32\pi^2 F_a)^2 & \text{for} \quad m_Q < T < F_a \\ g^6 T^6/(32\pi^2 F_a)^2 & \text{for} \quad T \ll m_Q \end{cases}$$

* Production by the matter multiplet Q :

$$\Gamma_{\tilde{a}}(y_Q, x_Q) \sim \frac{(x_Q + y_Q)^2 g^2 m_Q^2 T^4}{F_a^2} \quad \text{for} \quad m_Q < T < F_a$$

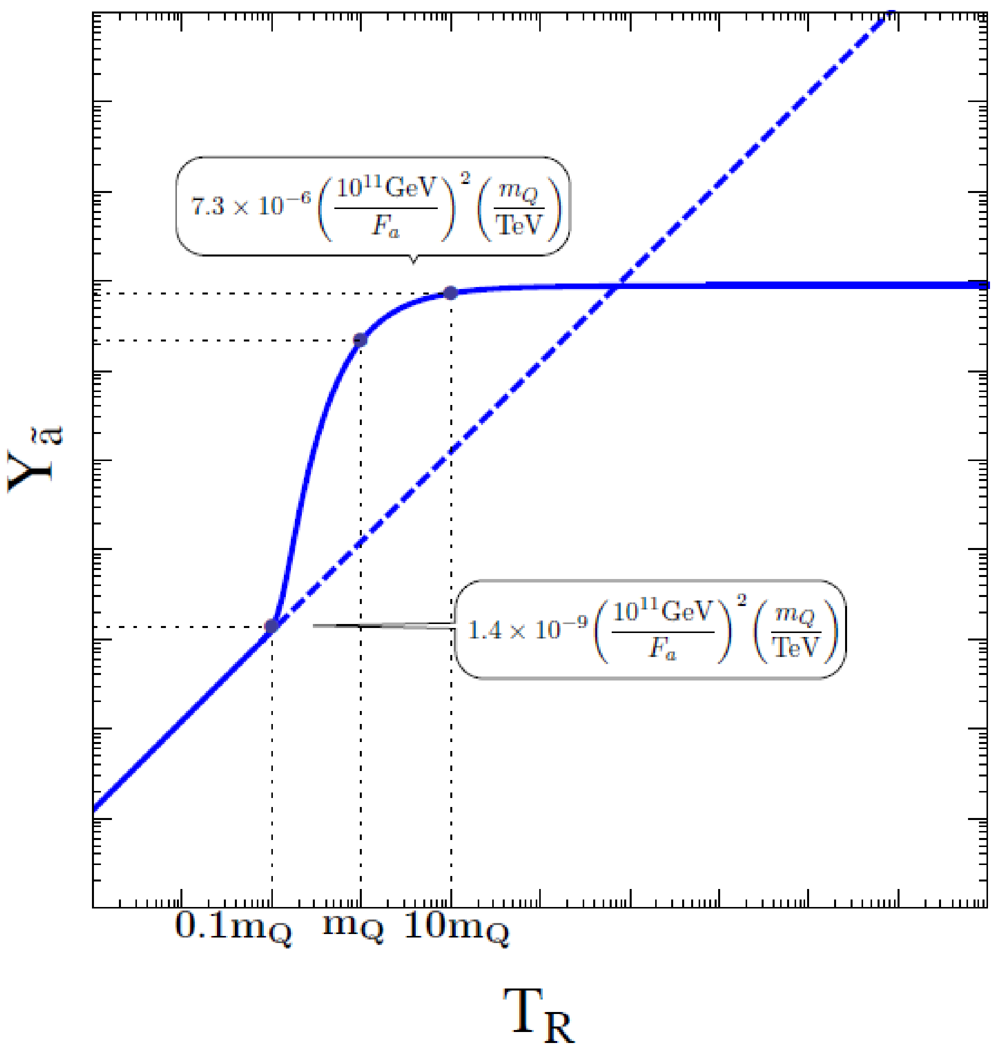
Axino production processes

	Process	$\lvert\mathcal{M}\rvert^2(m_Q < T \ll F_a)$	$\lvert\mathcal{M}\rvert^2(T \ll m_Q)$
A	$g+g\to\tilde{a}+\tilde{g}$	negligible	$4\mathcal{C}_2(s+2t+2t^2/s)$
B	$g+\tilde{g}\to\tilde{a}+g$	negligible	$-4\mathcal{C}_2(t+2s+2s^2/t)$
C	$\tilde{Q}+g\to\tilde{a}+Q$	$-\mathcal{C}_1(1+\frac{s-m_Q^2}{t-m_Q^2})$	$2s\mathcal{C}_3$
D	$Q+g\to\tilde{a}+\tilde{Q}$	$\mathcal{C}_1(1+\frac{t-m_Q^2}{s-m_Q^2})$	$-2t\mathcal{C}_3$
E	$\bar{\tilde{Q}}+Q\to\tilde{a}+g$	$-\mathcal{C}_1\frac{s-m_Q^2}{t-m_Q^2}$	$-2t\mathcal{C}_3$
F	$\tilde{g}+\tilde{g}\to\tilde{a}+\tilde{g}$	negligible	$-8\mathcal{C}_2(s^2+t^2+u^2)^2/stu$
G	$Q+\tilde{g}\to\tilde{a}+Q$	$\mathcal{C}_1(4+\frac{2m_Q^2}{s-m_Q^2}+\frac{2m_Q^2}{t-m_Q^2})$	$-4\mathcal{C}_3(s+s^2/t)$
H	$\tilde{Q}+\tilde{g}\to\tilde{a}+\tilde{Q}$	$\mathcal{C}_1(2-\frac{t-3m_Q^2}{s-m_Q^2}-\frac{s-3m_Q^2}{t-m_Q^2})$	$-2\mathcal{C}_3(t+2s+2s^2/t)$
I	$Q+\bar{Q}\to\tilde{a}+\tilde{g}$	$\mathcal{C}_1(4+\frac{2m_Q^2}{u-m_Q^2}+\frac{2m_Q^2}{t-m_Q^2})$	$-4\mathcal{C}_3(t+t^2/s)$
J	$\tilde{Q}+\bar{\tilde{Q}}\to\tilde{a}+\tilde{g}$	$\mathcal{C}_1(2-\frac{t-3m_Q^2}{u-m_Q^2}-\frac{u-3m_Q^2}{t-m_Q^2})$	$2\mathcal{C}_3(s+2t+2t^2/s)$

$\mathcal{C}_1 = 8g^2m_Q^2\delta^{ab}\delta_{ab}/F_a^2$ $\quad\mathcal{C}_2 = g^6\lvert f^{abc}\rvert^2/128\pi^4F_a^2$ $\quad\mathcal{C}_3 = g^6\sum_q\lvert T_{ij}^a\rvert^2/128\pi^4F_a^2$

Relic axino number density :

$$Y_{\tilde{a}} = \frac{N_{\tilde{a}}}{\text{entropy}} = \int_{T_0}^{T_R} \frac{dT}{T} \frac{\Gamma_{\tilde{a}}}{s(T)H(T)} \propto T_R^N \qquad N = \begin{cases} 0 & \text{for} \quad T_R \gg m_Q \\ 1 & \text{for} \quad T_R \ll m_Q \end{cases}$$

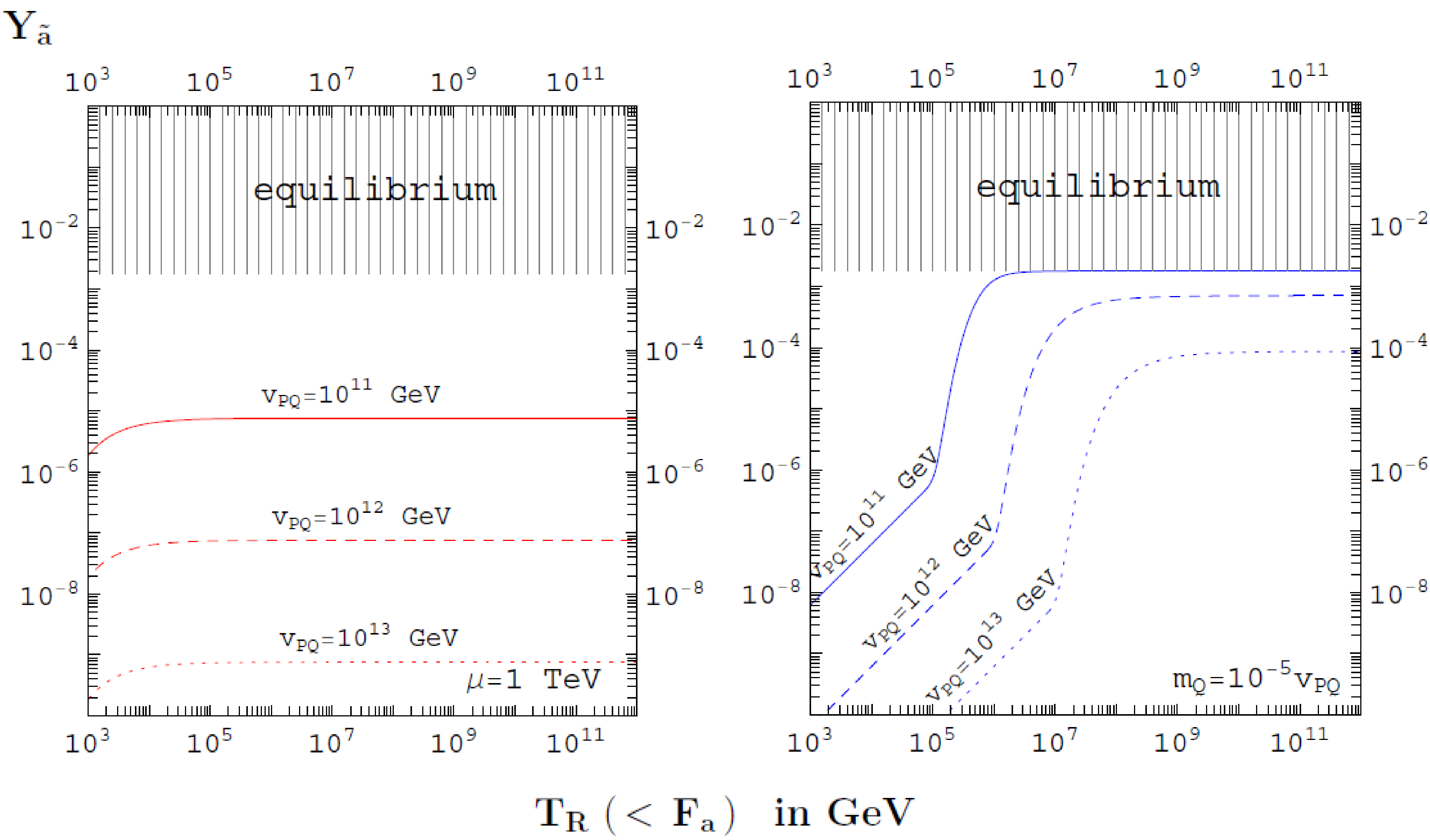


T_R = reheat temperature

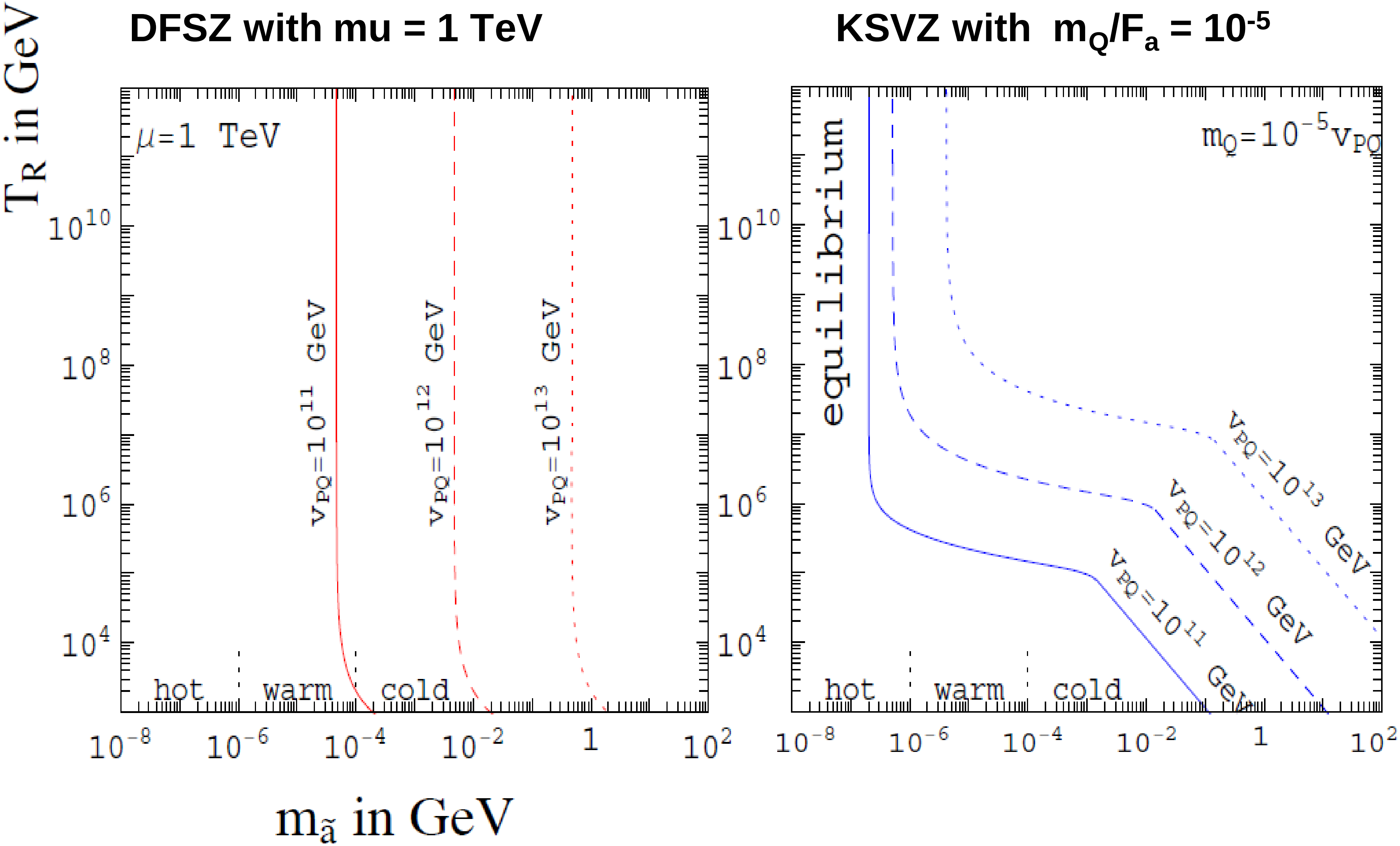
m_Q = Higgsino mass in DFSZ model (exotic quark mass in KSVZ model)

Relic axino number density

DFSZ with mu = 1 TeV **KSVZ with $m_Q/F_a = 10^{-5}$**



Axino mass vs Reheat temperature for $\Omega_{\tilde{a}} h^2 = 0.11$



High T_R can be allowed for wider range of the axino mass.

Conclusion

* Combining SUSY with axion is a very compelling idea, which might solve the gauge hierarchy problem, the strong CP problem, and the mu-problem altogether in an elegant manner.

* The fermionic superpartner of axion, the axino, can have a variety of cosmological implications, in particular it can be a good DM candidate.

* Thermal production of axinos requires more careful analysis incorporating all relevant effective interactions together, which correctly reveals the decoupling in the limit $m_Q/T \rightarrow 0$, which is a generic feature of axion models which have a UV completion with linearly realized PQ symmetry.

* Compared to the previous analysis using only one particular type of effective interaction, the correct analysis can give very different axino production rates at high temperature, particularly for the DFSZ model solving the mu-problem with PQ symmetry.