Charm semileptonic form factors from lattice QCD with 2+1 flavors of light staggered fermions

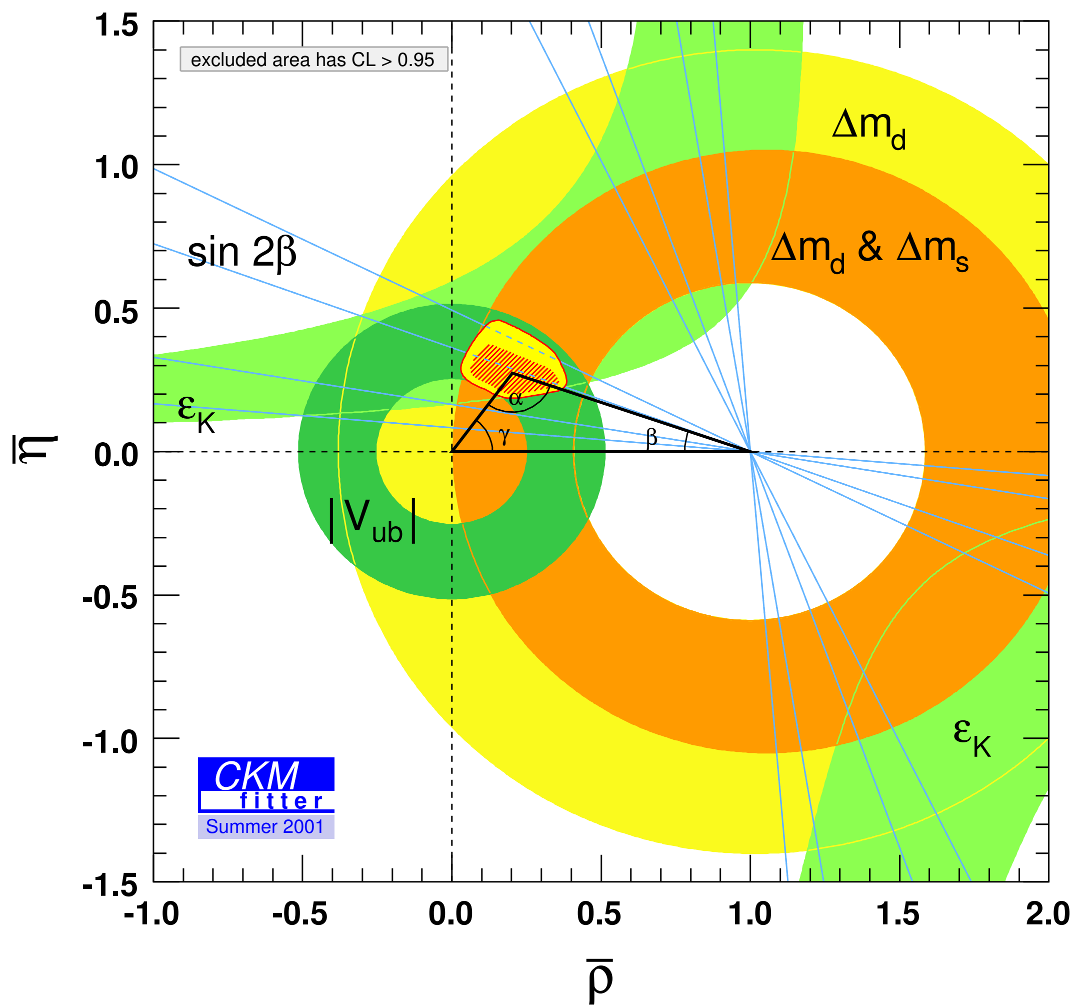
Jon A. Bailey
Seoul National University

Fermilab Lattice and MILC Collaborations

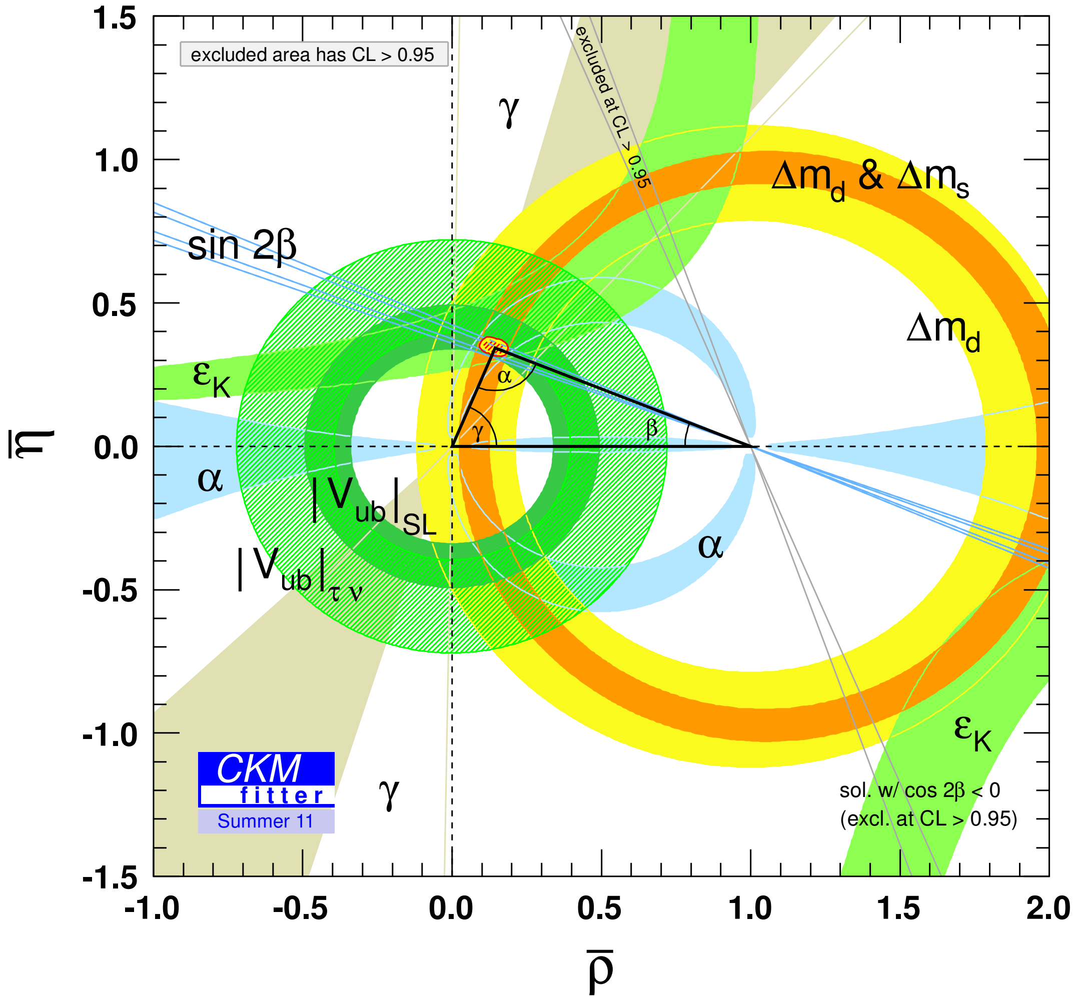
Weonjong Lee, Hyung-Jin Kim (SNU)

KIAS Phenomenology Workshop, Nov. 17-19, 2011

UT analysis, Summer 2001



UT analysis, Summer 2011



$B \to \pi \ell \nu$

- Tension btwn. constraints, excl. vs. incl.
  (Lunghi, Lattice 2011; Laiho et al., CKM 2010)

$$|V_{ub}| \quad \leftrightarrow \quad \sin 2\beta$$

- Uncertainty from lattice QCD dominates exclusive determination (FNAL-MILC, PRD 2009)
  – LQCD + BABAR

$$\Rightarrow |V_{ub}| = (3.38 \times 10^{-3})(1 \pm 10.7\%)$$

- Decreasing unc. to less than few percent could increase tension significantly

$B \to D^{(*)}\ell\nu$

- Interplay btwn. constraints, tension with incl.

$$\epsilon_K \sim |V_{cb}|^4 \quad \leftrightarrow \quad |V_{ub}|$$

- Uncertainty from lattice QCD approx. experimental error for exclusive determinations
(FNAL-MILC, CKM 2010, Lattice 2011)

$$\Rightarrow |V_{cb}| = (3.97 \times 10^{-2})(1 \pm 2.5\%)$$

- Decreasing unc. could lead to interesting tensions

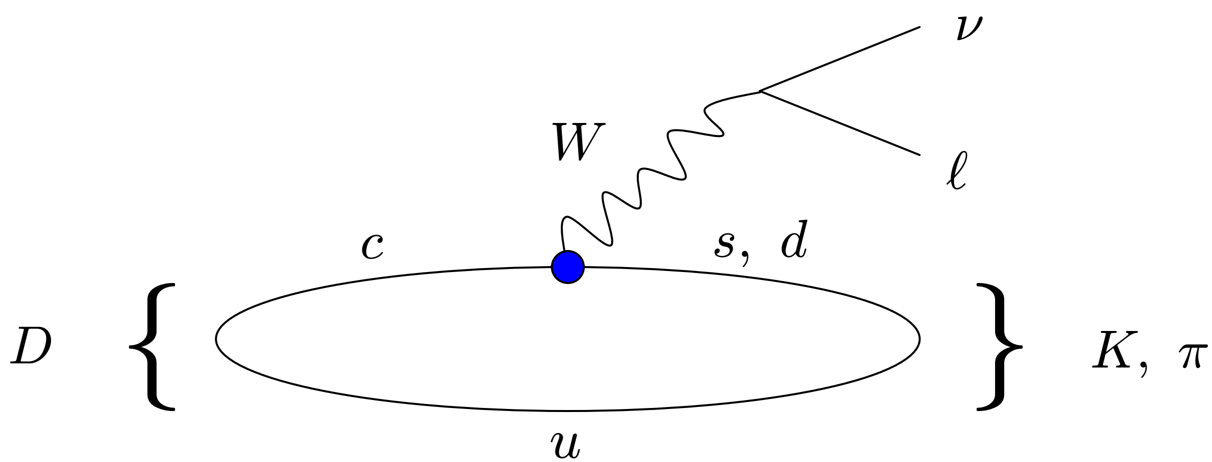
Gold-plated processes for LQCD

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & \end{pmatrix}$$

$$\text{and } B \to K\ell^+\ell^-,\ B_s \to D_s\ell\nu,\ \ldots$$

Charm semileptonic decays

$$D \to K\ell\nu \quad \text{and} \quad D \to \pi\ell\nu$$



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{cs(d)}|^2|\mathbf{p}_{K(\pi)}|^3|f_+^{D\to K(\pi)}(q^2)|^2$$

Looking for new physics

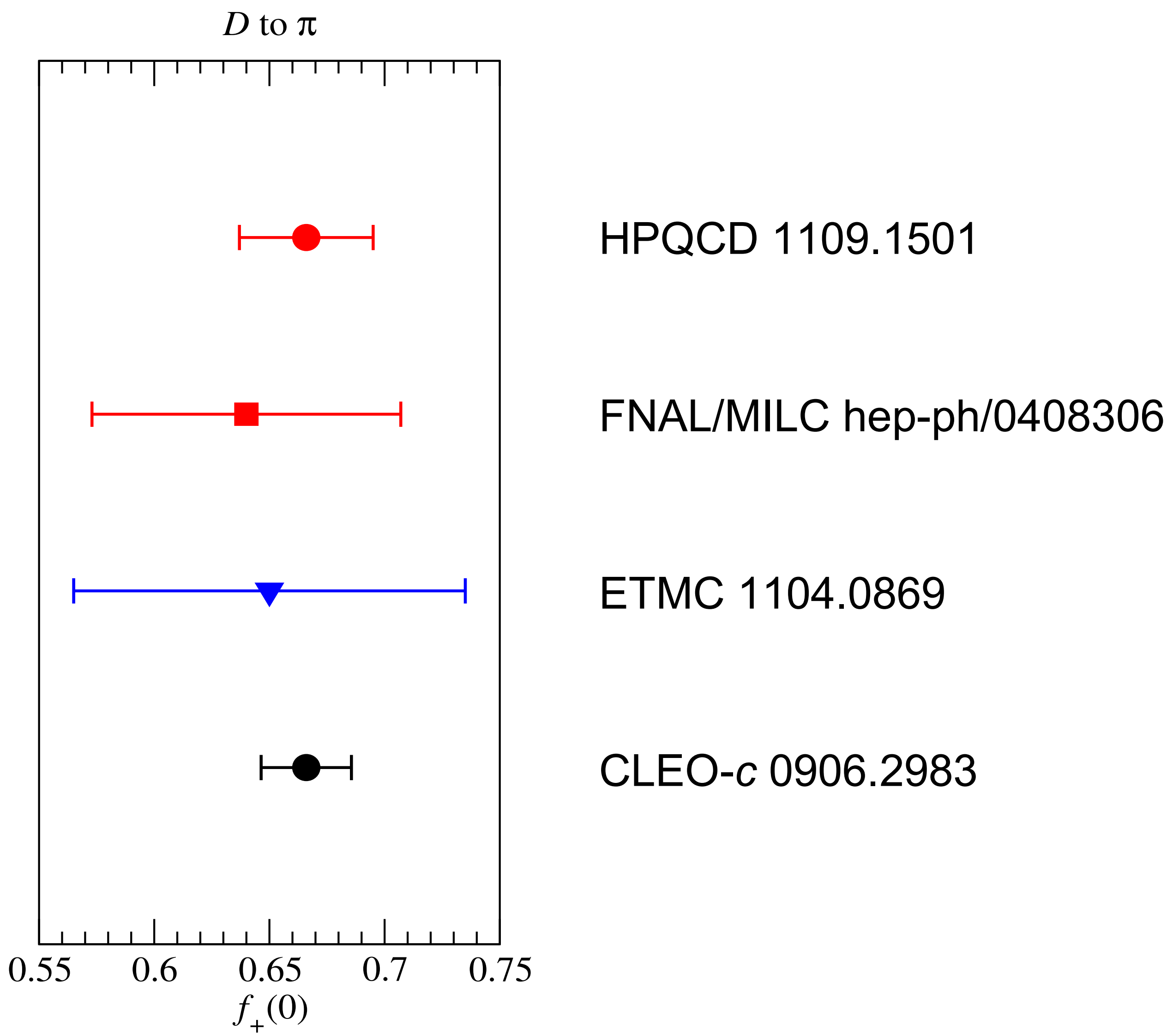
- Tests of CKM unitarity
  - LQCD form factors + experiment $\rightarrow |V_{cs}|$
  - 2nd row and column unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1?$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1?$$

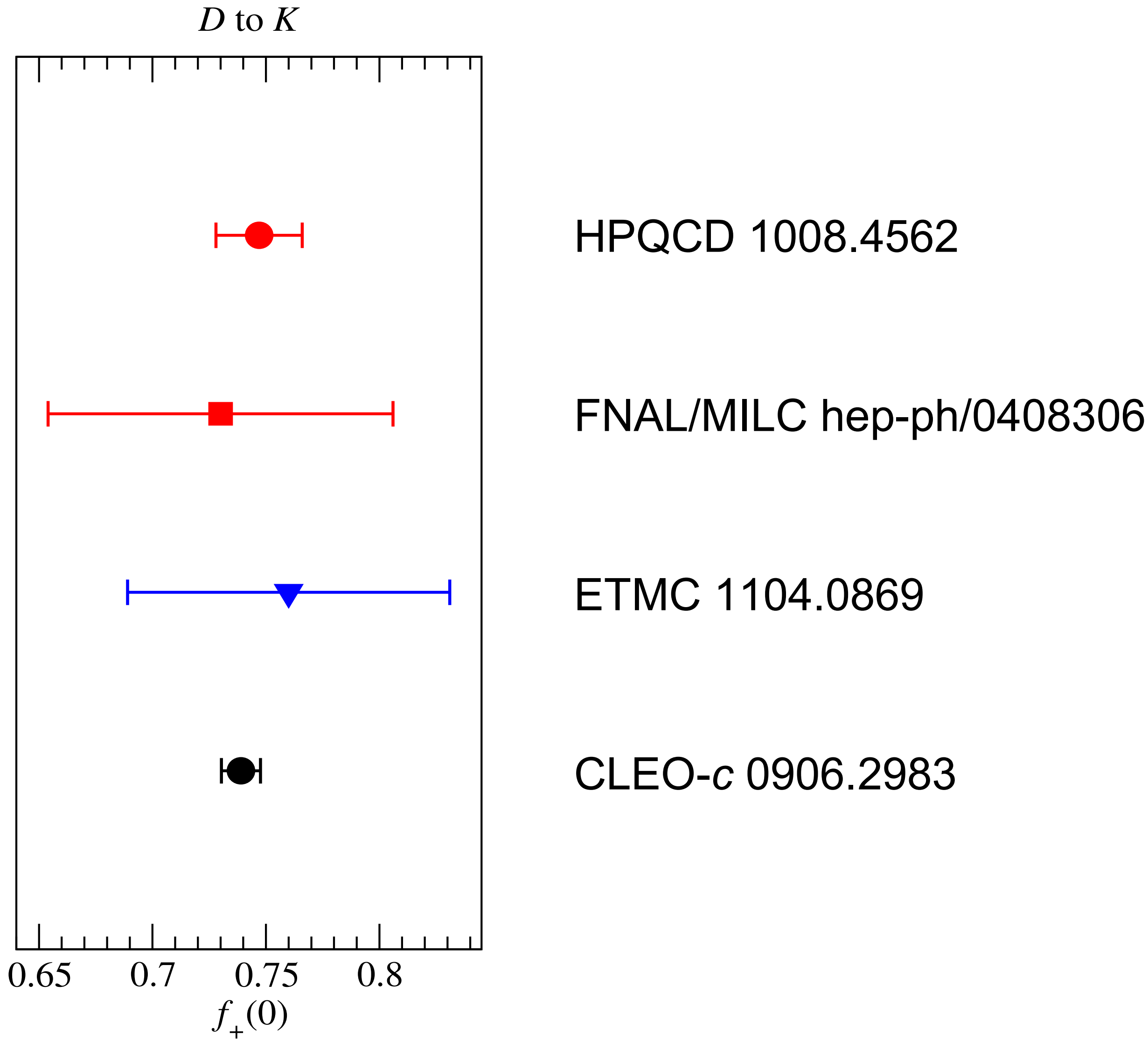
- Tests of lattice QCD
  - Global fit in SM + experiment $\rightarrow f_+^{D\rightarrow K(\pi)}(q^2)$
  - LQCD form factors?

$$\Rightarrow f_+^{B\rightarrow\pi}(q^2),\ f_+^{B\rightarrow K}(q^2),\ f_T^{B\rightarrow K}(q^2),\ \ldots$$

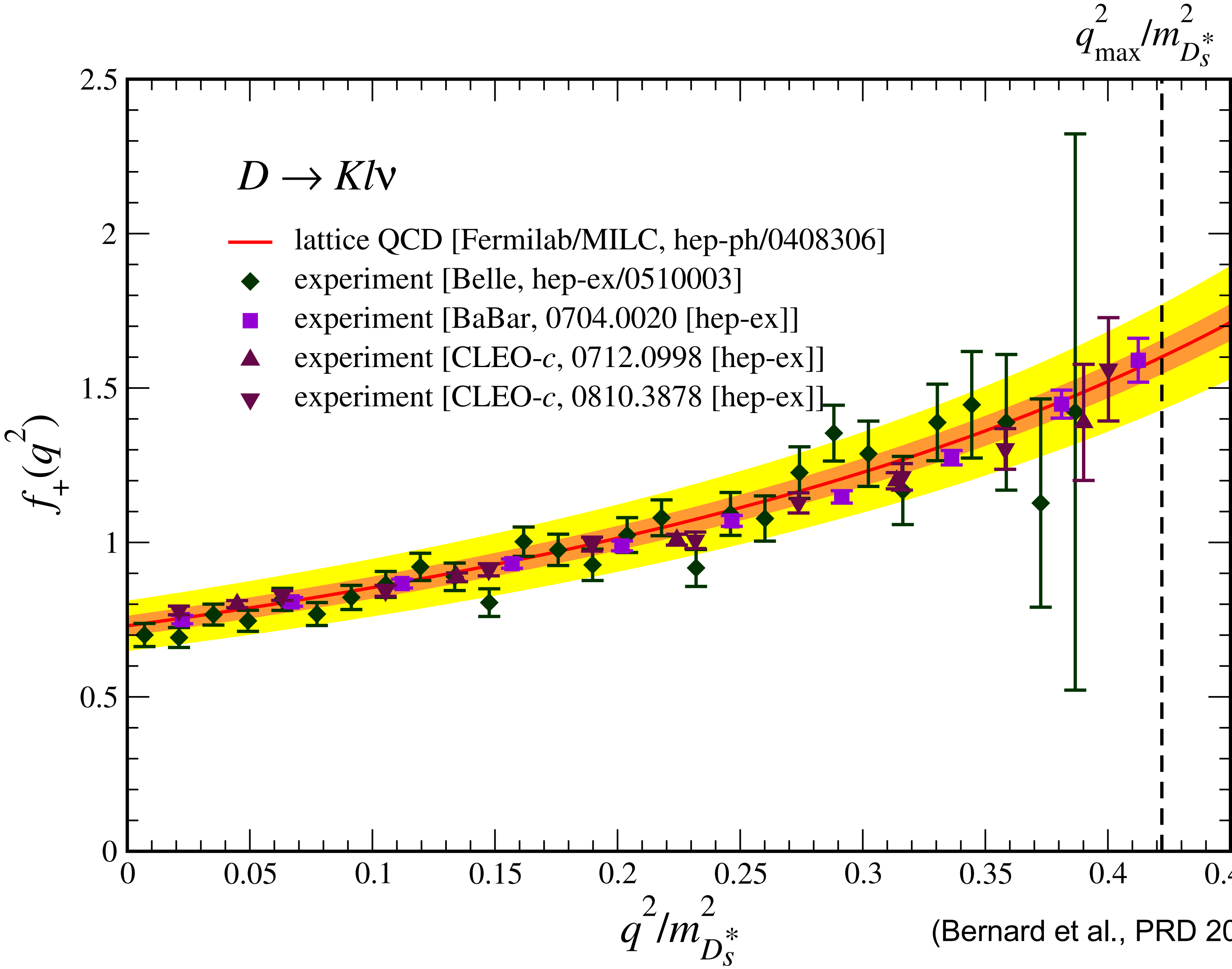
Results for $f_+^{D\to\pi}(q^2 = 0)$



Results for $f_+^{D\to K}(q^2=0)$



FNAL/MILC, 2009: Visualization of correlations



(Bernard et al., PRD 2009)

Testing lattice QCD

- Fermilab method, RHQ (Tsukuba, Columbia)
  - Control lattice artifacts for arbitrary masses
  - Apply to charm and bottom

$$(B \text{ form factor calcs.}) \equiv (D \text{ form factor calcs.})$$

- Consistency with SM for charm decays

$\longrightarrow$ direct validation

- Staggered gauge ensembles for sea employ fourth root to remove doublers

Challenges ~ Uncertainties

- Discretization effects
  - Light quarks and gluons
  - Heavy quarks
- Large *u*, *d* quark masses
- Finite-volume effects
- Inputs
  - Scale
  - Light, heavy quark mass tunings
- Operator matching
  - Perturbative
  - Nonperturbative
- Fitting ansatz
- Electromagnetic effects
- Quenching
  - Charm
  - Strange
- Statistics

Form factors from correlators

- Form factors defined ~ matrix elements

$$\langle K|i\bar{s}\gamma_\mu c|D\rangle = f_+^{D\to K}(q^2)\left(p_D + p_K - \frac{m_D^2 - m_K^2}{q^2}q\right)_\mu + f_0^{D\to K}(q^2)\frac{m_D^2 - m_K^2}{q^2}q_\mu$$

$$= \sqrt{2m_D}\left[v_\mu f_\parallel^{D\to K}(E_K) + p_{\perp\mu} f_\perp^{D\to K}(E_K)\right]$$

$$f_\parallel^{D\to K}(E_K) = \frac{\langle K|i\bar{s}\gamma_4 c|D\rangle}{\sqrt{2m_D}}, \quad f_\perp^{D\to K}(E_K) = \frac{\langle K|i\bar{s}\gamma_k c|D\rangle}{\sqrt{2m_D}}\frac{1}{p_K^k}$$

- Matrix elements from Green's functions ~ correlators

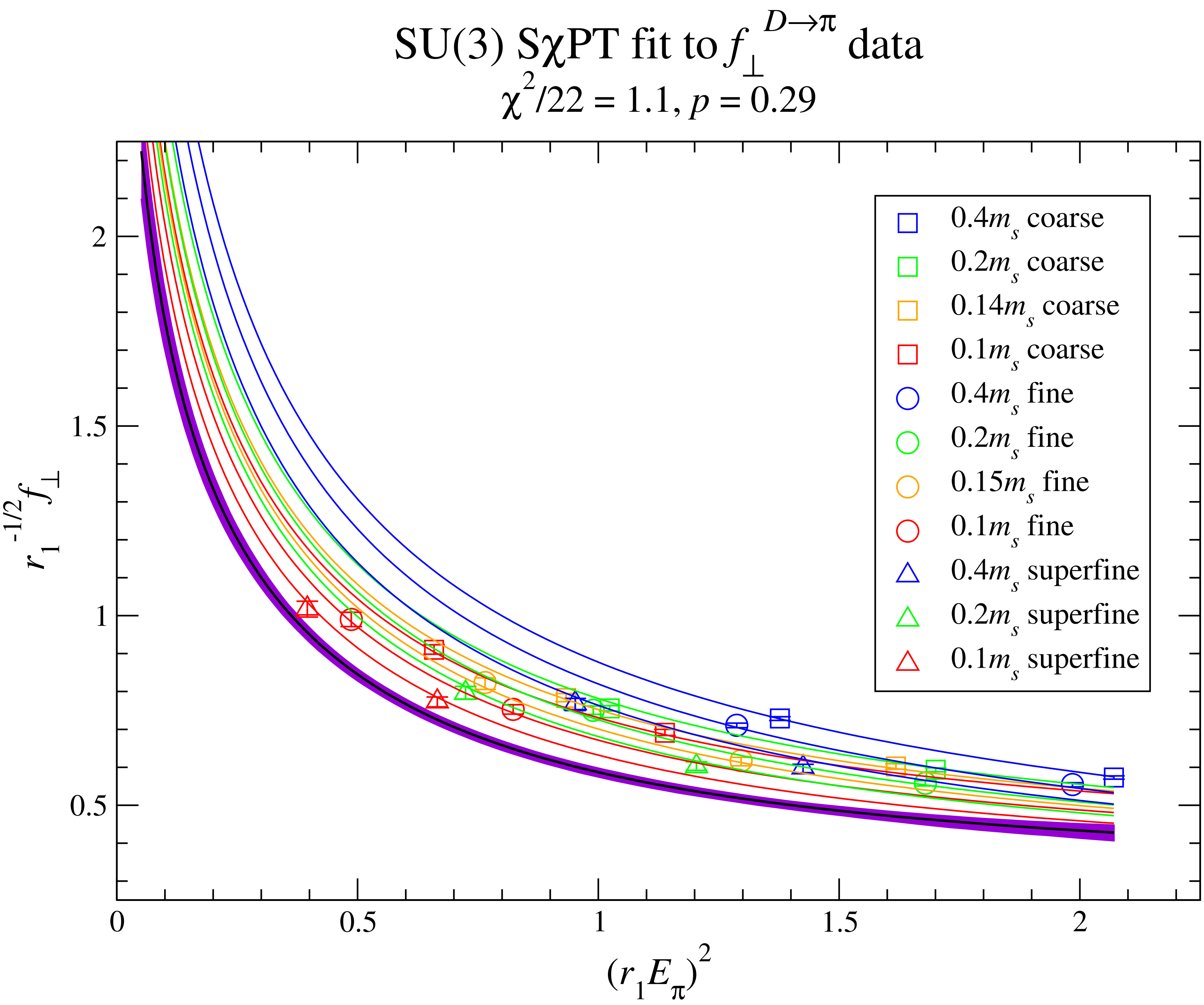
$$f_{\parallel,\perp}^{\text{lat}}(E) \sim \frac{C_{3,\mu}(t,T;\mathbf{p})}{C_2^{K(\pi)}(t;\mathbf{p})C_2^D(T-t)}, \quad t_{\text{src}} \ll t \ll t_{\text{src}} + T$$

Correlators

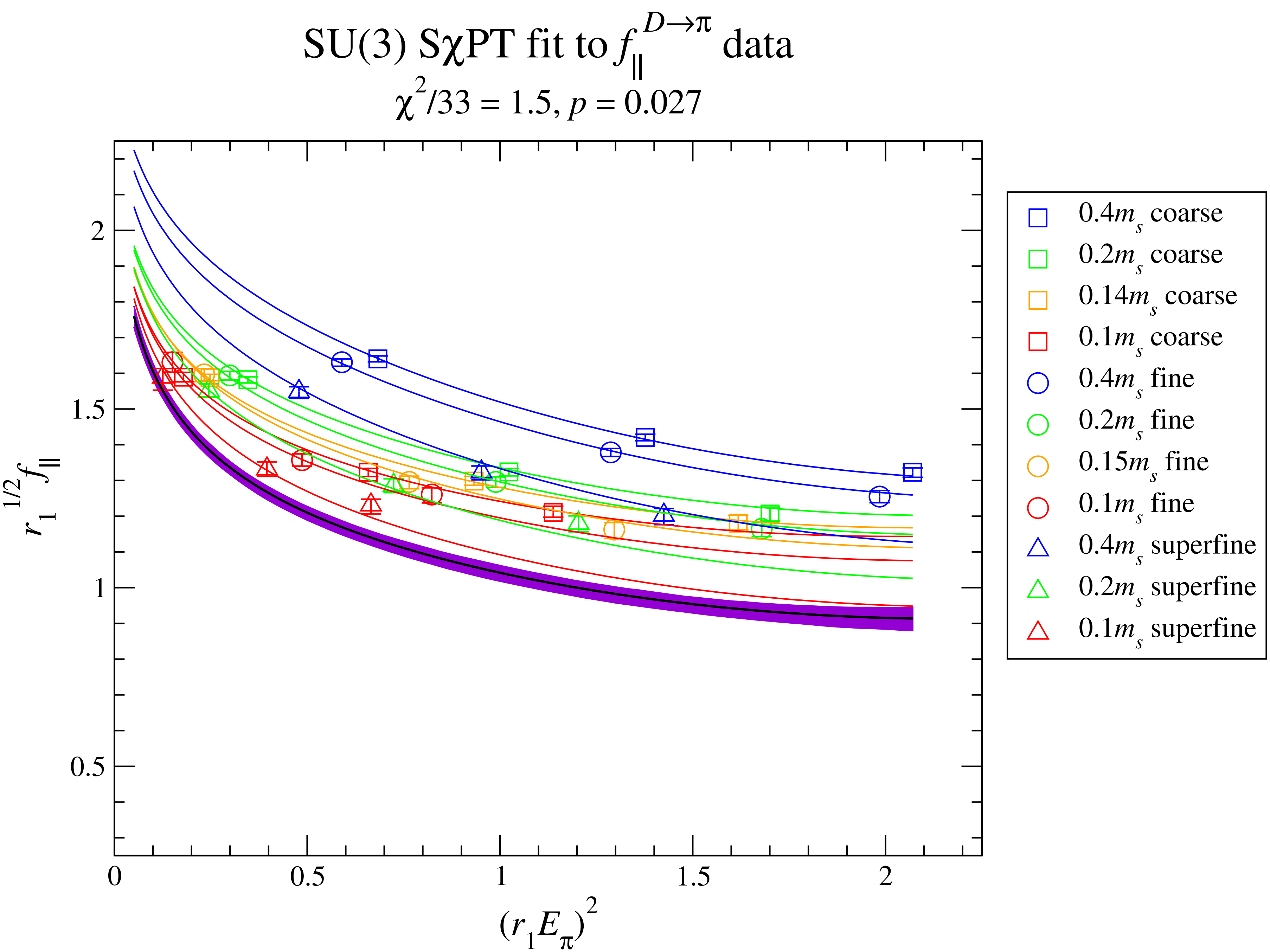
- Four lattice spacings ~ 0.045 fm to 0.12 fm
- Multiple *u*, *d* masses ~ $0.05m_s$ to $0.4m_s$
- Five outgoing meson momenta
- Gauge ensembles of ~ 600 to 2000 lats
- Four source times
- Four source-sink separations
  - Two physical separations
  - Two for constructing ratio
- Three masses near charm, one 0.12 fm ensemble

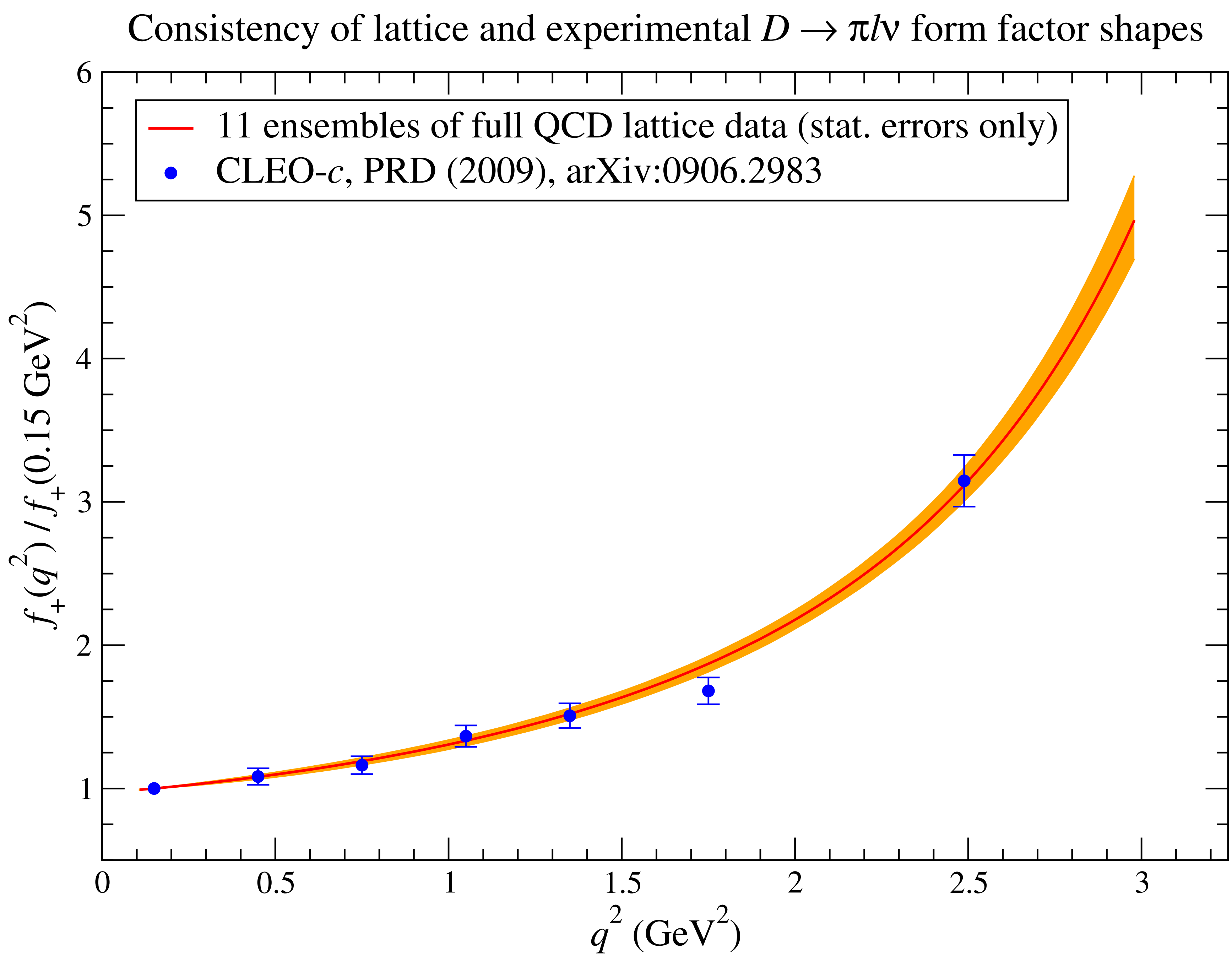
Chiral-continuum-*E* extra-interpolation

- EFT: Heavy-meson rooted staggered chiral perturbation theory (Aubin and Bernard, PRD 2007)
  - Advantages
    - Dependence on quark masses, energy of outgoing meson, lattice spacing
    - Leading light-quark and gluon discretization errors, taste violations, fit and removed
    - Include heavy-quark discretization errors for more systematic estimate than power counting
    - Model-independent for light quark masses, small energies
  - Limitations
    - Model-independent only for light masses, small energies

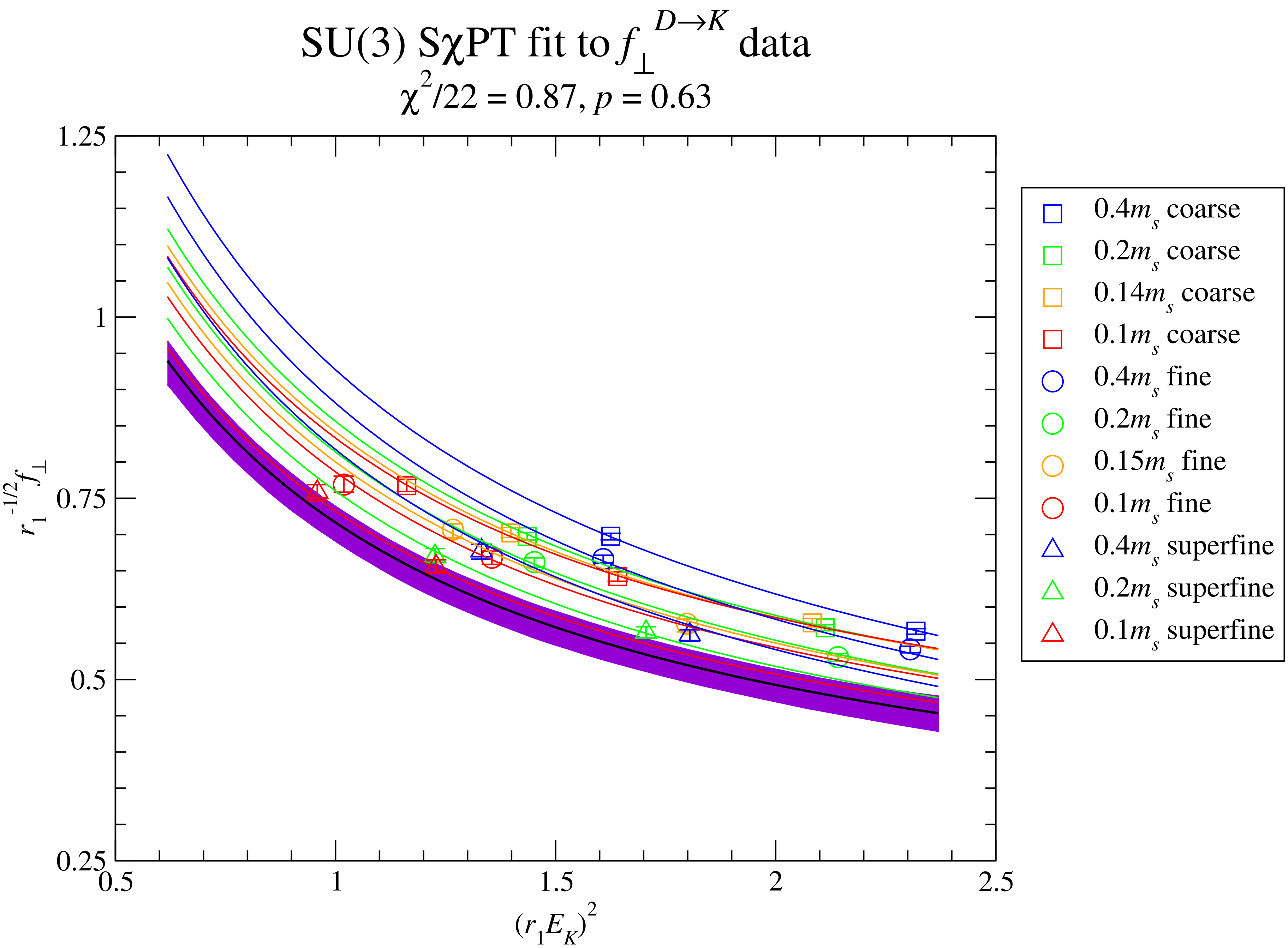


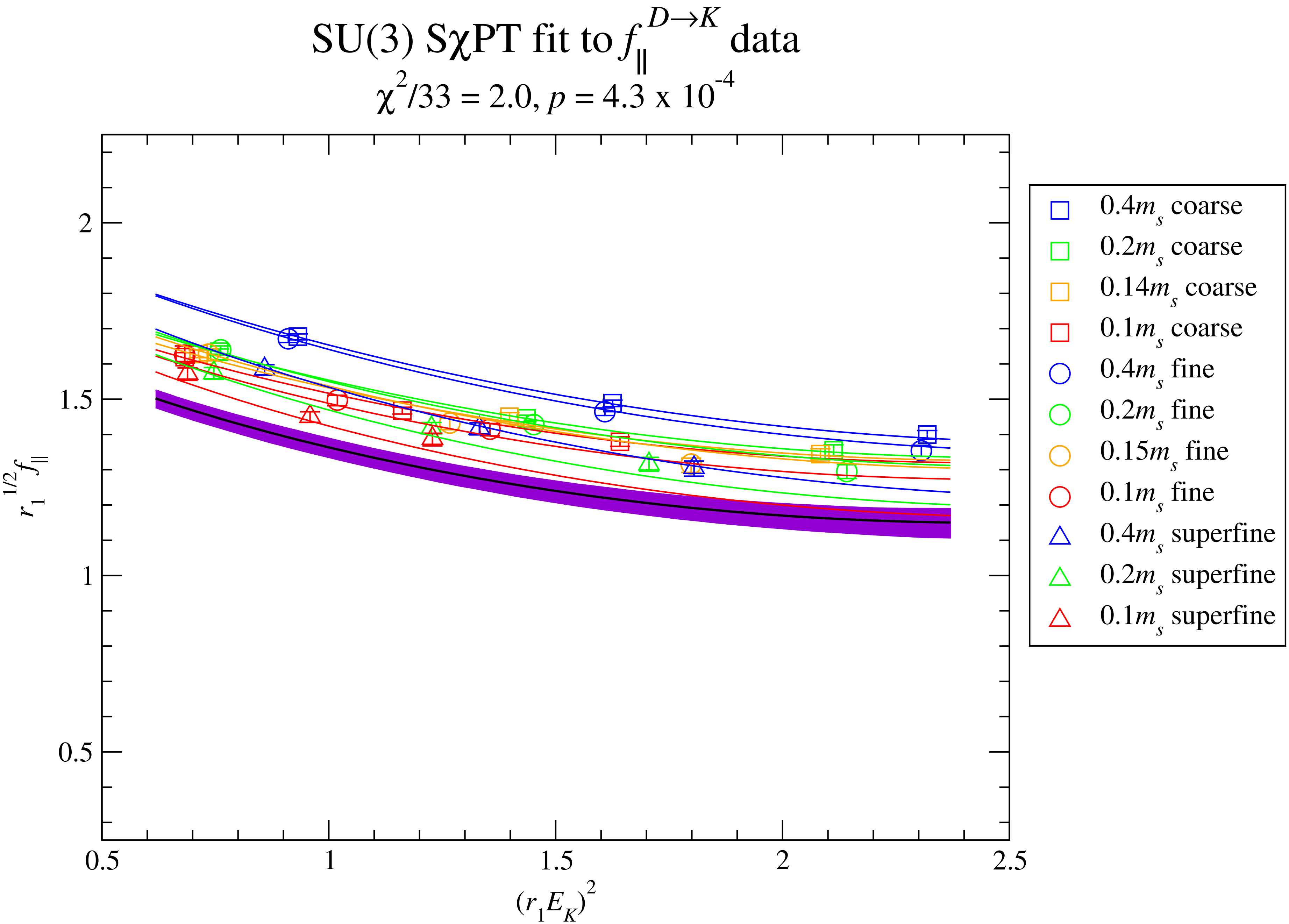
SU(3) SχPT fit to $f_\perp^{D\to\pi}$ data
$\chi^2/22 = 1.1$, $p = 0.29$
$r_1^{-1/2} f_\perp$
$(r_1 E_\pi)^2$
0.4m_s coarse
0.2m_s coarse
0.14m_s coarse
0.1m_s coarse
0.4m_s fine
0.2m_s fine
0.15m_s fine
0.1m_s fine
0.4m_s superfine
0.2m_s superfine
0.1m_s superfine
0
0.5
1
1.5
2


SU(3) SχPT fit to $f_{\parallel}^{D\to\pi}$ data
$\chi^2/33 = 1.5$, $p = 0.027$
$r_1^{1/2} f_{\parallel}$
$(r_1 E_\pi)^2$
0
0.5
1
1.5
2
0.5
1
1.5
2
0.4m_s coarse
0.2m_s coarse
0.14m_s coarse
0.1m_s coarse
0.4m_s fine
0.2m_s fine
0.15m_s fine
0.1m_s fine
0.4m_s superfine
0.2m_s superfine
0.1m_s superfine

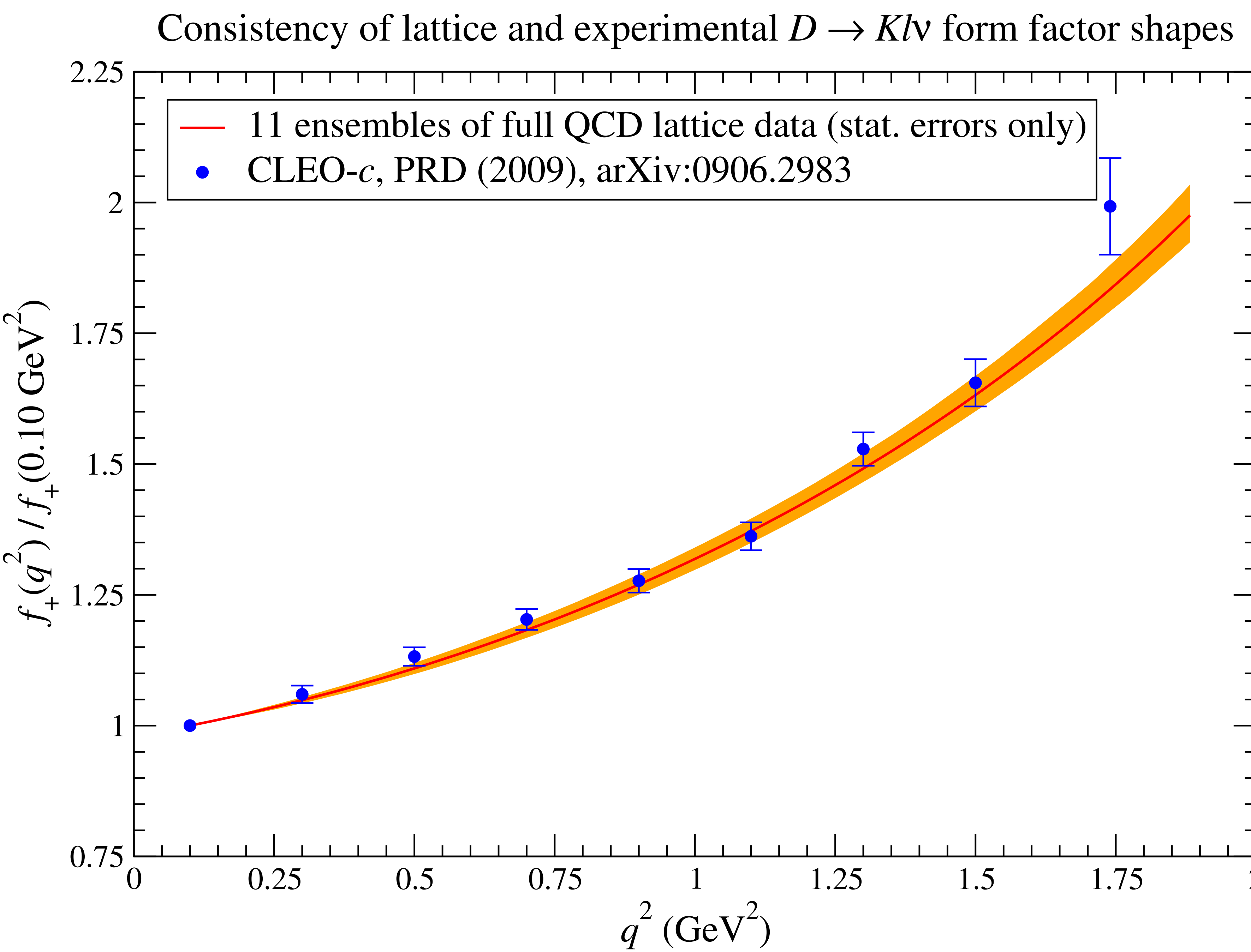


Consistency of lattice and experimental $D \to \pi l\nu$ form factor shapes
11 ensembles of full QCD lattice data (stat. errors only)
CLEO-c, PRD (2009), arXiv:0906.2983
$f_+(q^2) / f_+(0.15\ \mathrm{GeV}^2)$
q^2 (GeV^2)
0
0.5
1
1.5
2
2.5
3
1
2
3
4
5
6


SU(3) SχPT fit to $f_\perp^{D\to K}$ data
$\chi^2/22 = 0.87$, $p = 0.63$
$r_1^{-1/2} f_\perp$
$(r_1 E_K)^2$
1.25
1
0.75
0.5
0.25
0.5
1
1.5
2
2.5
$0.4m_s$ coarse
$0.2m_s$ coarse
$0.14m_s$ coarse
$0.1m_s$ coarse
$0.4m_s$ fine
$0.2m_s$ fine
$0.15m_s$ fine
$0.1m_s$ fine
$0.4m_s$ superfine
$0.2m_s$ superfine
$0.1m_s$ superfine



SU(3) SχPT fit to $f_{\parallel}^{D\to K}$ data
$\chi^2/33 = 2.0$, $p = 4.3 \times 10^{-4}$
$r_1^{1/2} f_{\parallel}$
$(r_1 E_K)^2$
0.5
1
1.5
2
2.5
0.4m_s coarse
0.2m_s coarse
0.14m_s coarse
0.1m_s coarse
0.4m_s fine
0.2m_s fine
0.15m_s fine
0.1m_s fine
0.4m_s superfine
0.2m_s superfine
0.1m_s superfine



Consistency of lattice and experimental $D \to Kl\nu$ form factor shapes
11 ensembles of full QCD lattice data (stat. errors only)
CLEO-c, PRD (2009), arXiv:0906.2983
$f_+(q^2) / f_+(0.10\ \mathrm{GeV}^2)$
q^2 (GeV^2)
0.75
1
1.25
1.5
1.75
2
2.25
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2

Projected error budget for $f_+^{D\to K(\pi)}(0)$

stat. + χPT (%)	2.0	$\leftarrow$ Increased statistics, new ensembles, fit method
g_π	2.9	
r_1	1.4	$\leftarrow f_\pi$
$\hat{m}$	0.3	$\leftarrow m_\pi,\ m_K$ (with m_s)
m_s	1.3	
m_c	0.2	$\leftarrow$ updated tuning to $\overline{M}_{D_s}$
HQ disc.	2.5	$\leftarrow$ power counting
nonpert. Z_V	0.6	$\leftarrow$ statistics dominated
pert. ρ	0.5	$\leftarrow$ power counting
finite vol.	0.5	$\leftarrow$ χPT
total syst. (%)	4.4	
total (%)	4.8	

Goldstone bosons on the lattice

- Staggered fermions have an exact chiral symmetry at nonzero lattice spacing

- PGBs exist that are massless in chiral limit at nonzero lattice spacing

- Chiral symmetry protects against additive mass renormalization, operator mixing, . . .

Non-Goldstone bosons

- Remnant doublers of staggered fermions show up as pseudoflavors, “tastes”
- Four tastes for each physical flavor
  - Fourth root (if correct) means one per flavor in sea
  - Valence sector retains four-fold degeneracy in continuum limit ~ partial quenching

$$M = \begin{pmatrix} \hat{m} I_4 & 0 & 0 \\ 0 & \hat{m} I_4 & 0 \\ 0 & 0 & m_s I_4 \end{pmatrix}$$

  - Enlarged flavor symmetry, PGB sector

$$SU(3)_F \to SU(12)_f$$
$$\mathbf{8} \to \mathbf{143}$$

Strategy

- Calculate masses of taste non-Goldstone pions and kaons in staggered chiral perturbation theory through NLO (1 loop)
- Fit correlators in corresponding taste channels
  - Fix low-energy couplings, Gasser-Leutwyler parameters
  - Extract light quark masses

Summary

- Encouraging agreement between form factor shapes from lattice + experiment
  - Preliminary, work in progress
  - Matching factors estimated, omitted
  - Modeling energy dependence?
- SU(2) SChPT fits in progress, alternatives
- Masses of taste non-Goldstone PGBs calculated through NLO in SChPT