A Storm in A "T" Cup

(Guide to transverse projections and mass-constraining variables)

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In collaboration with: A. Barr, T. Khoo, P. Konar, C. Lester, K. Matchev, M. Park
arXiv:1105.2977 [hep-ph], to appear in PRD

KIAS Phenomenology Workshop
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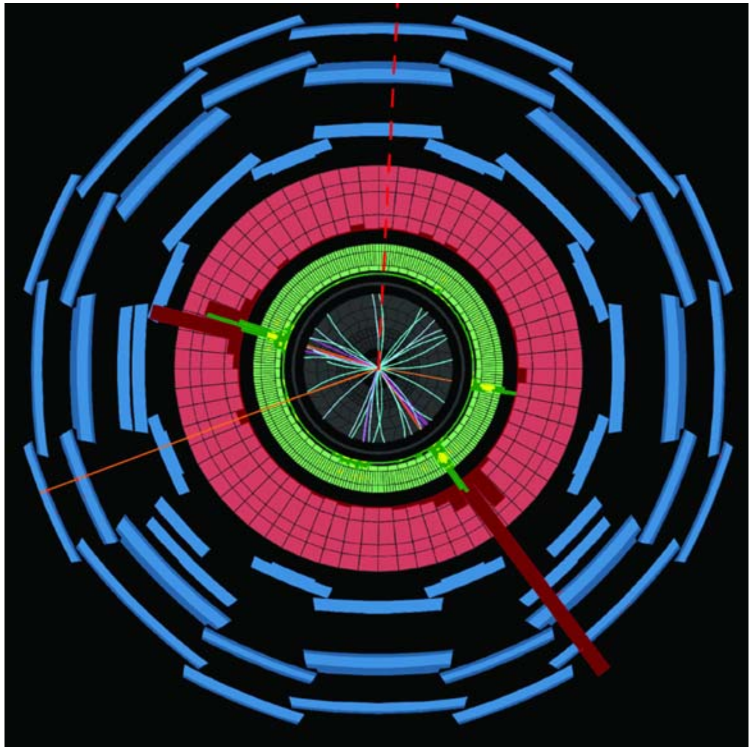
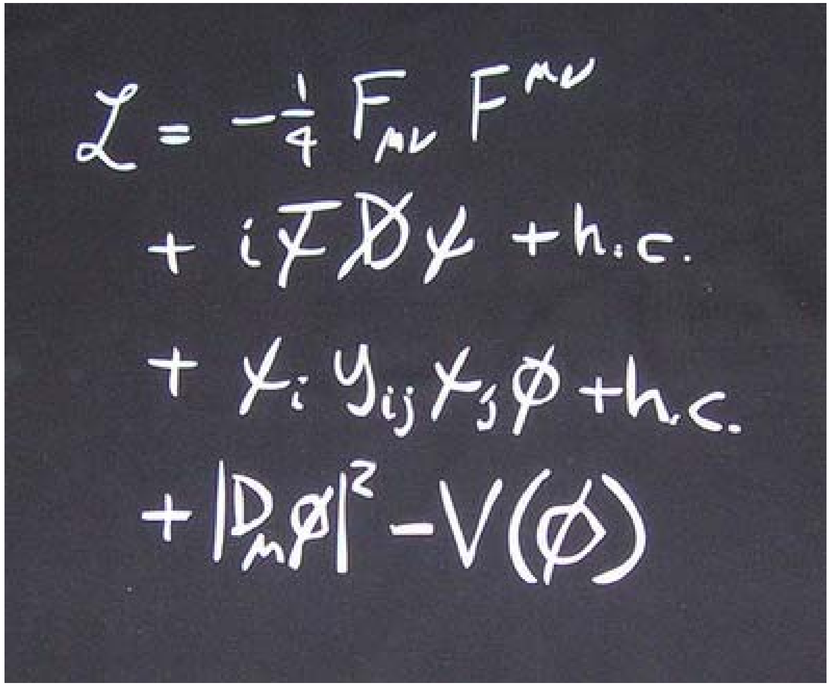
The game



40 M / second over 10 years



$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi + h.c. \\ & + \psi_i y_{ij} \psi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

+ more terms…?

At some point, 5000 people will shout:

"We've found a …

[long pause]

… SOMETHING!"

A large collider of hadrons …
… not a collider of large hadrons

What is that *something*?

How hard is it to identify what was found?

“mass measurement methods”

... short for ...

“parameter estimation and discovery techniques”

Do we care about masses?

- Common Parameters in the Lagrangian
- Interpretation
  - SUSY breaking mechanism, geometry of ED
- Prediction of new things
  - Mass of W,Z -> indirect top quark mass "measurement"
  - Masses of W/Z/t -> indirect measurement of the Higgs mass
- Expedites discovery - optimal selection

Some methods ...(no way to include all)

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	$M_{eff,} M_{est}, H_T$	Wedgebox	
Approximate	$S_{min,}$ M_{Tgen}	M_{T2}, M_{2C}, $M_{3C,}$ $M_{CT,}$ $M_{T2}(n,p,c)$	As usual (MAOS)
Exact	?	Polynomial method	As usual

pessimism

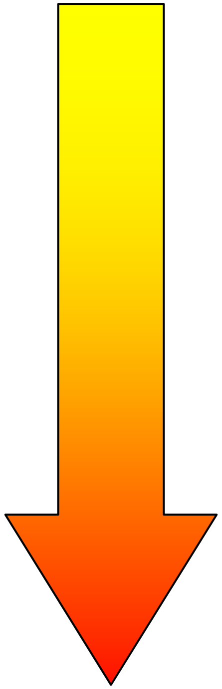
optimism

pessimism **optimism**

Types of Technique

Few

assumptions



Many

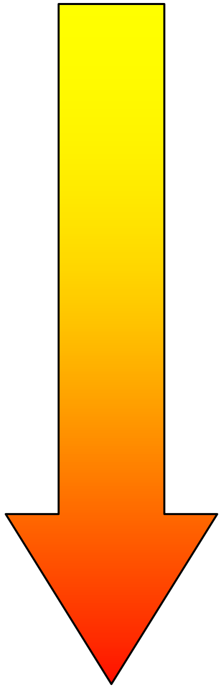
assumptions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



Specific

conclusions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
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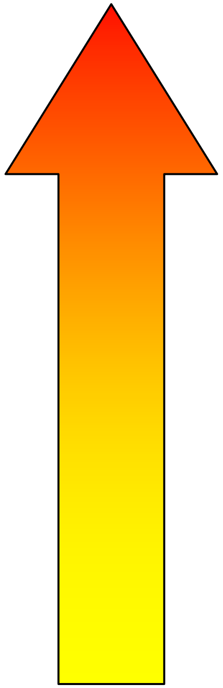
- Max Likelihood / Matrix Element

Types of Technique

Robust



Fragile

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
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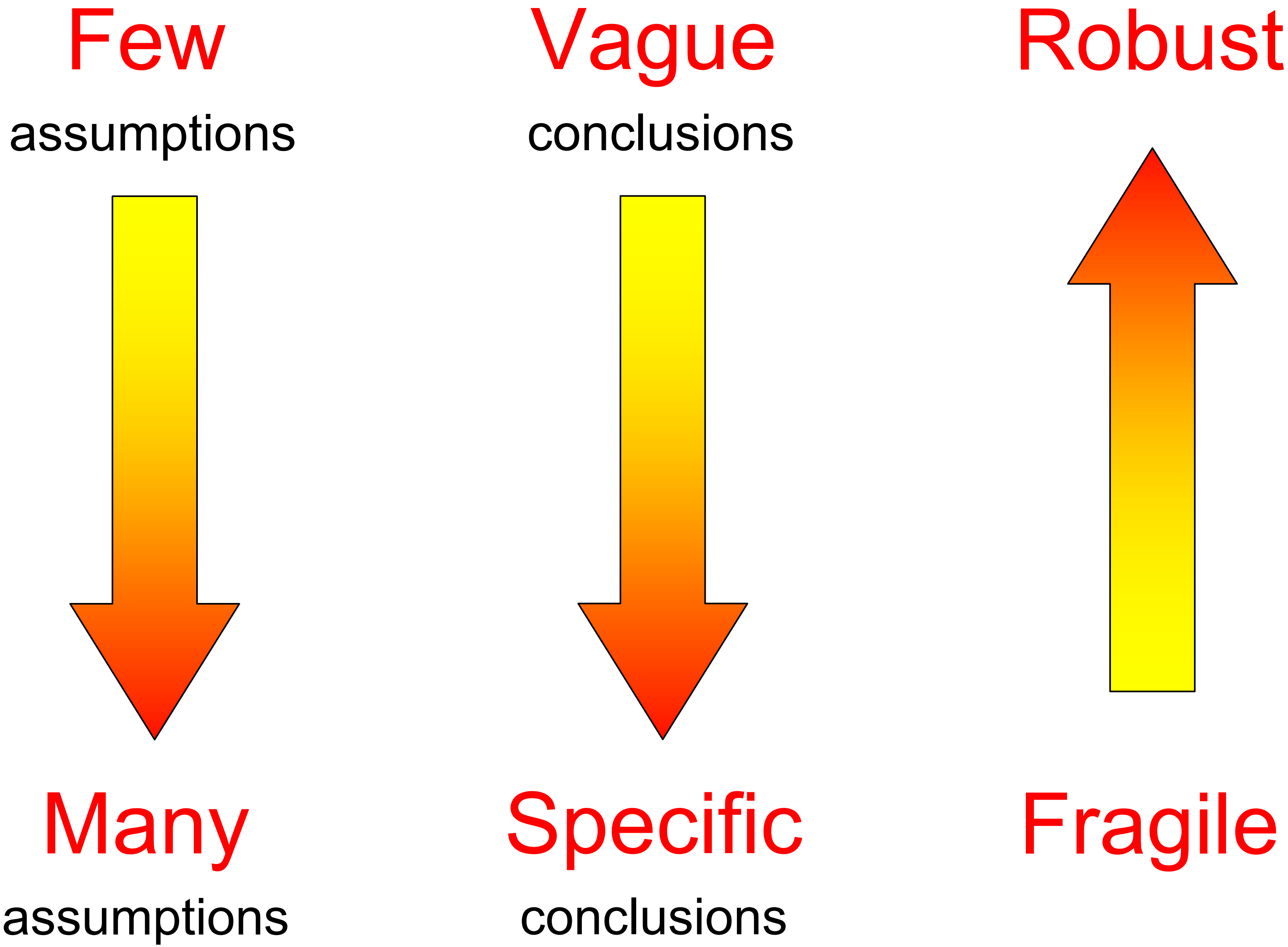
Interpretation : the balance of benefits



Few

assumptions

Vague

conclusions

Robust

Many

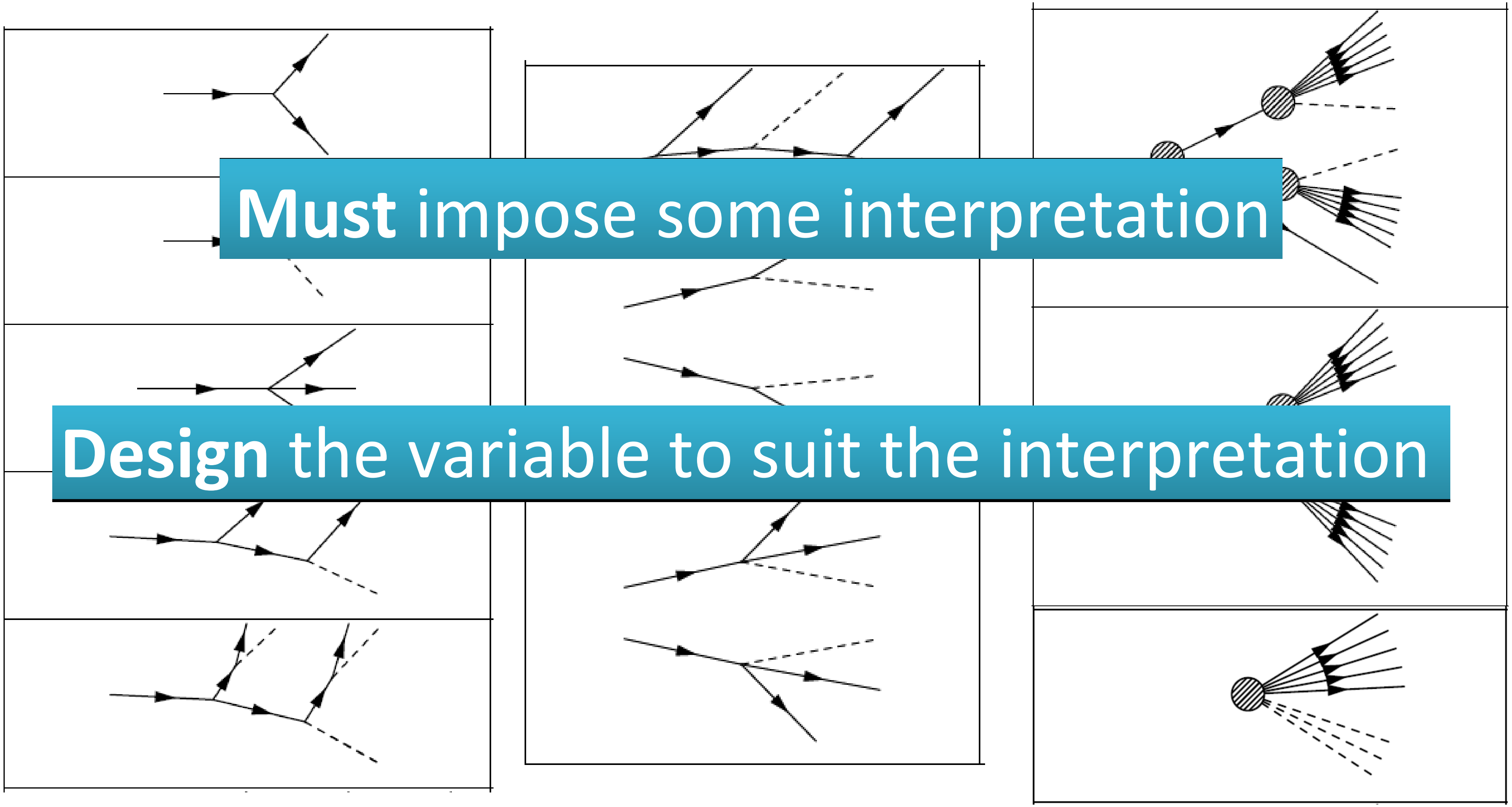
assumptions

Specific

conclusions

Fragile

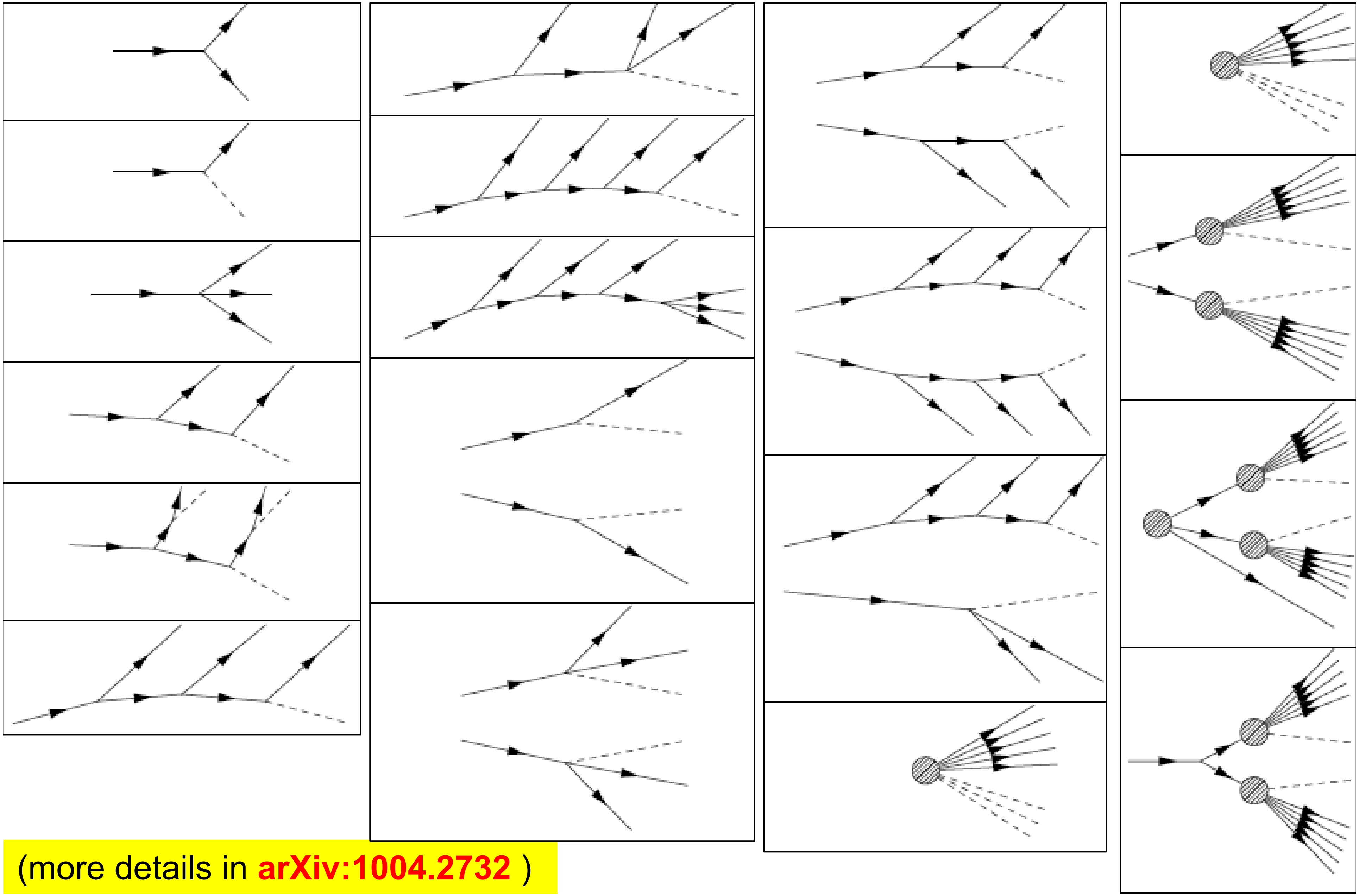
Topology / hypothesis



Must impose some interpretation

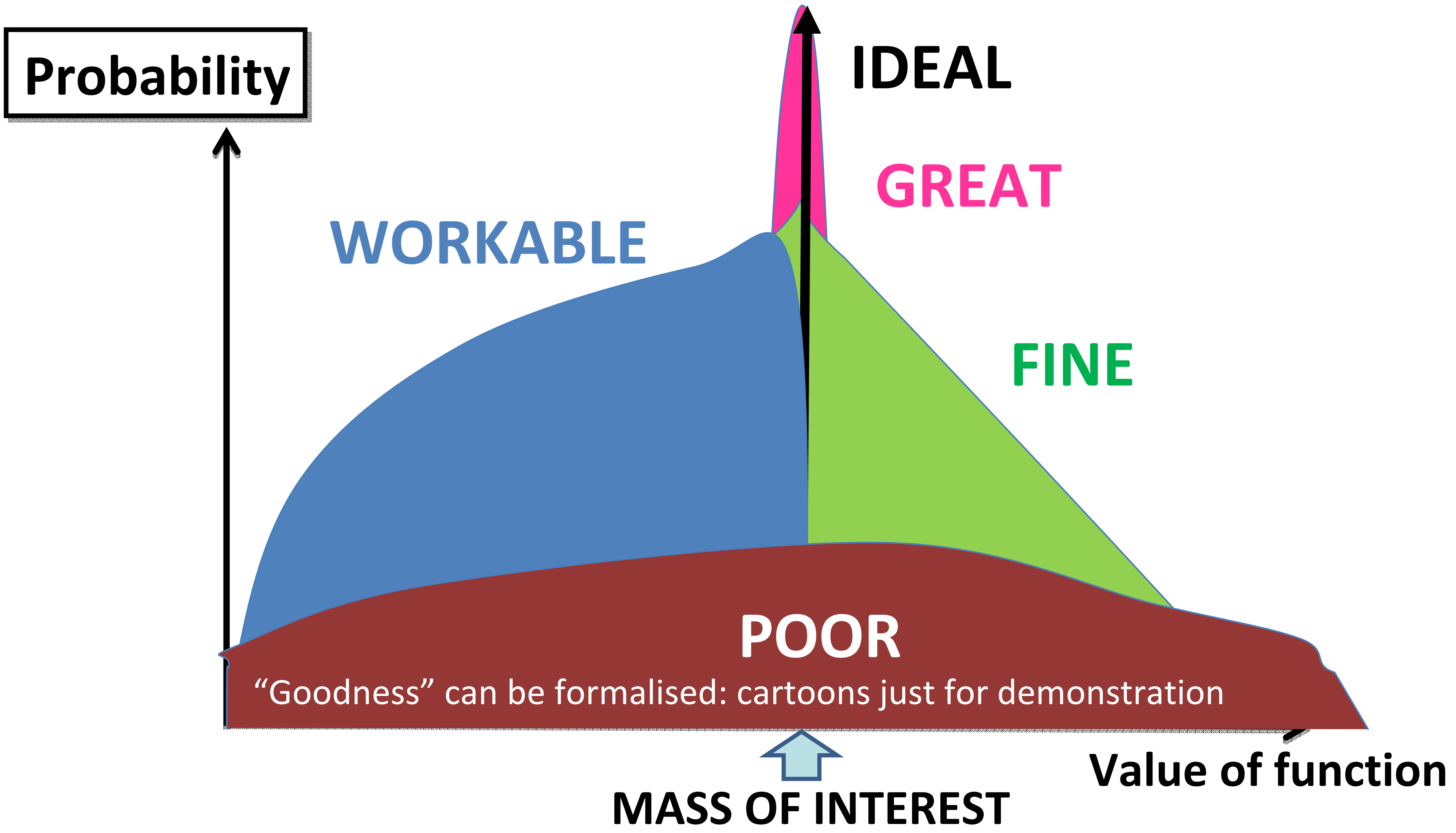
Design the variable to suit the interpretation

Not all proposed new-physics chains are short!

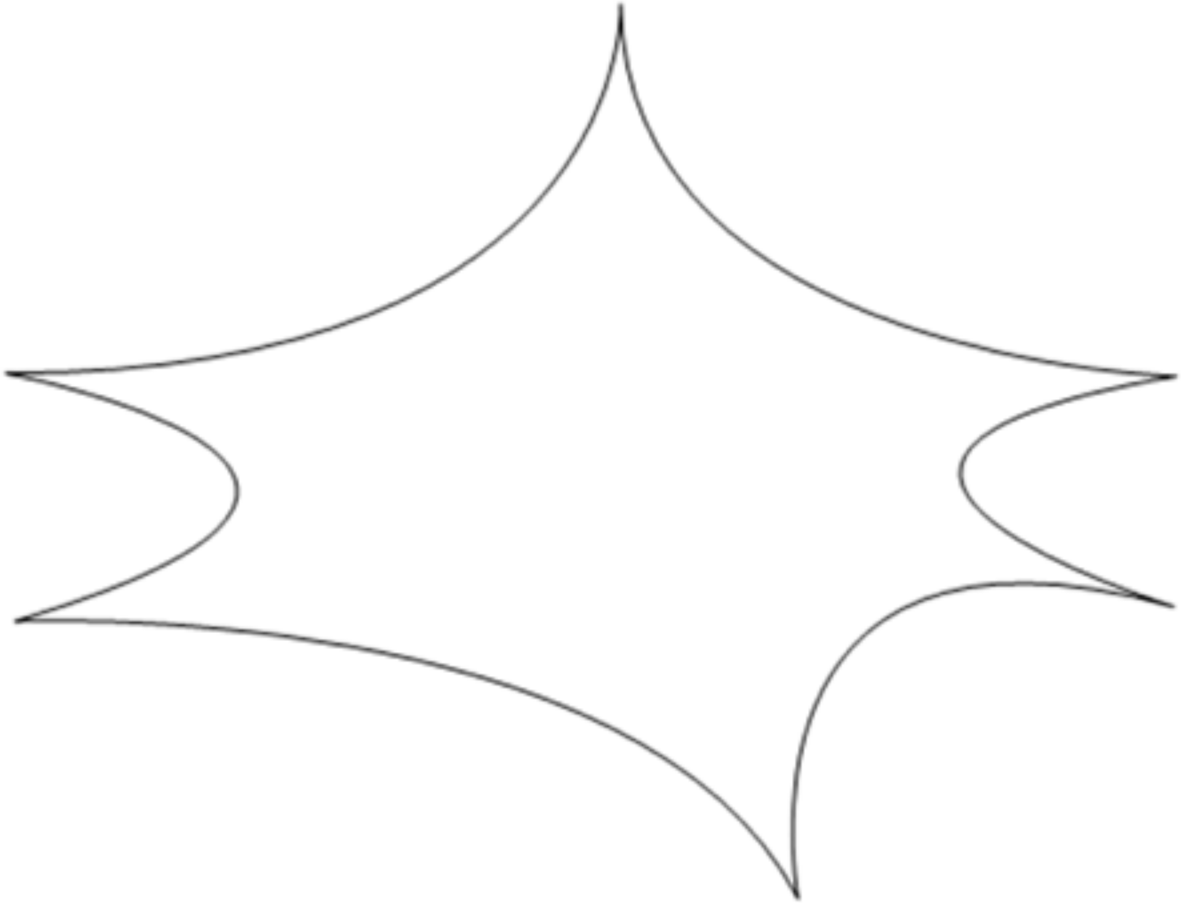


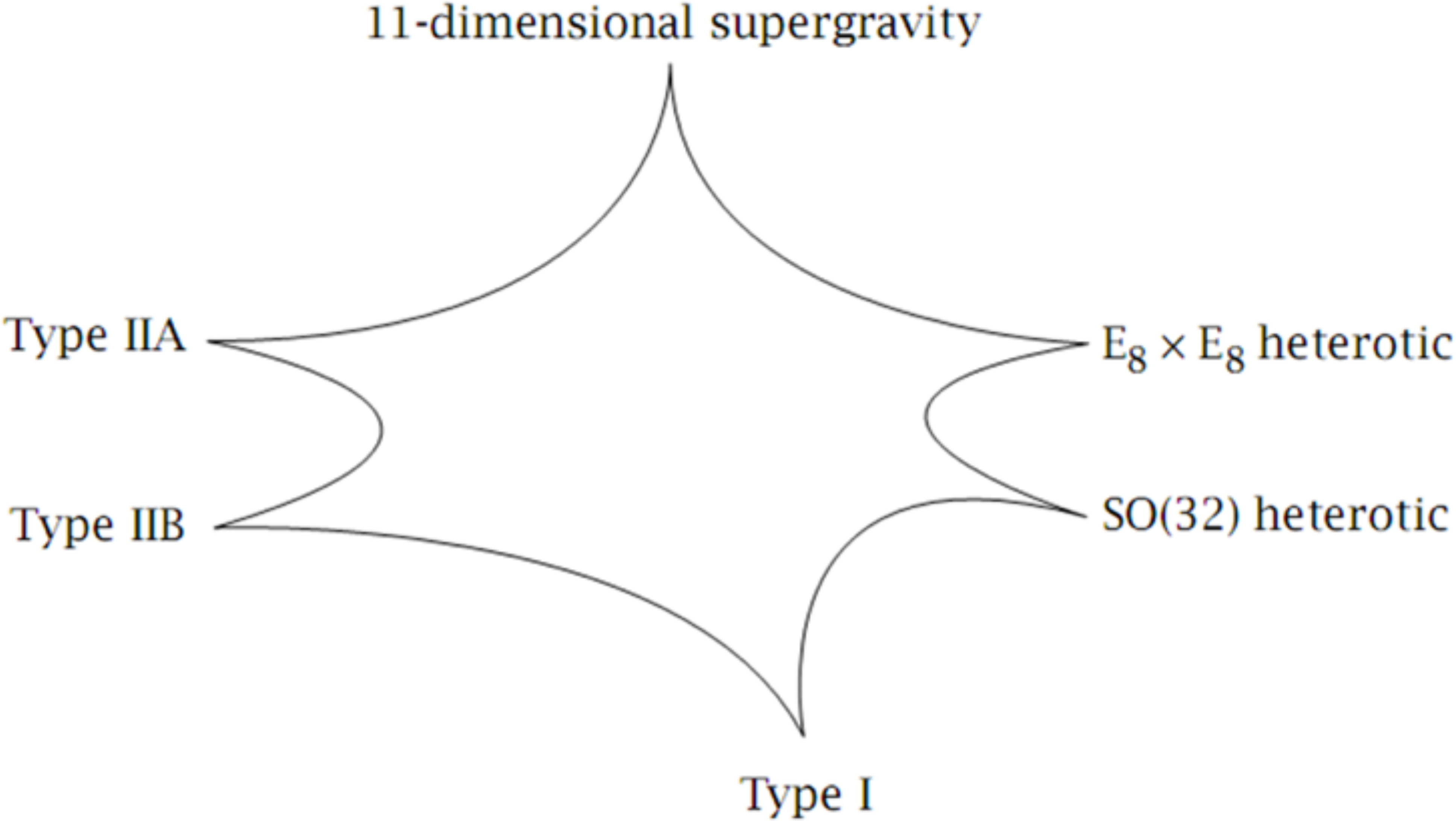
(more details in **arXiv:1004.2732**)

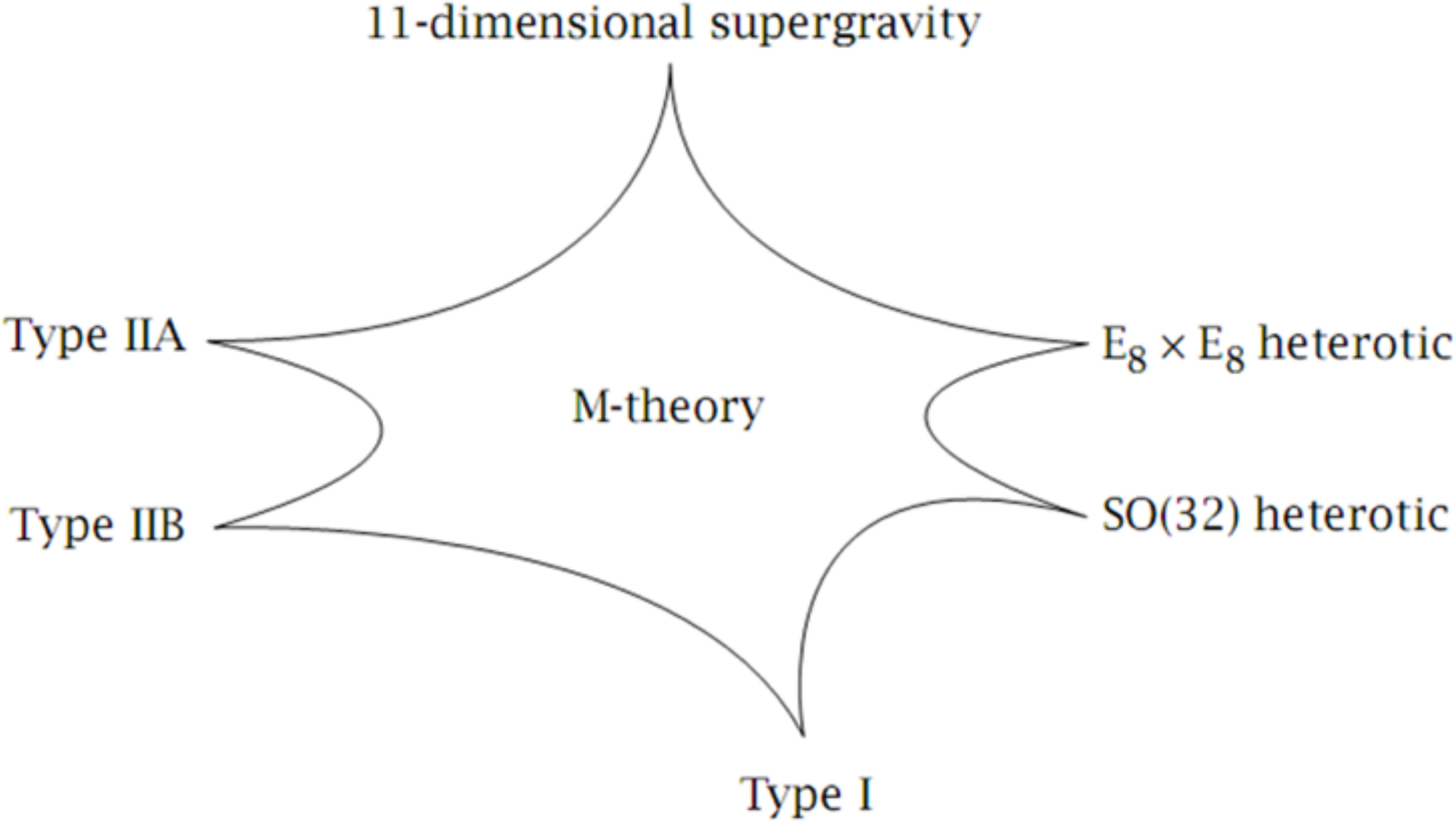
Good vs poor variables

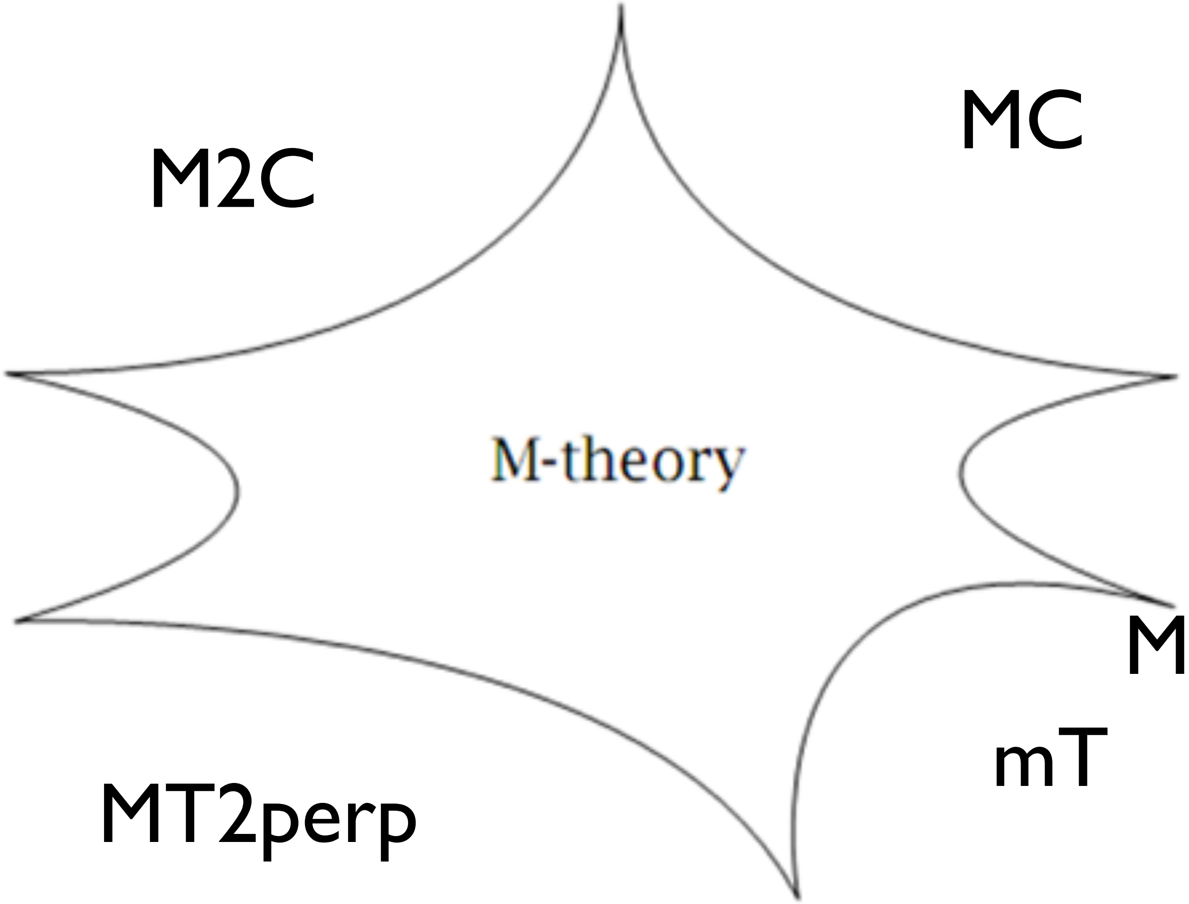


“Goodness” can be formalised: cartoons just for demonstration




11-dimensional supergravity
Type IIA
$E_8 \times E_8$ heterotic
Type IIB
SO(32) heterotic
Type I


11-dimensional supergravity
Type IIA
$E_8 \times E_8$ heterotic
M-theory
Type IIB
SO(32) heterotic
Type I



MT

M2C

MC

MT2

Smin

MET

Meff, HT

mT

MT2perp

MTGEN

MT

MC

M2C

MT2

Smin

- M-theory at the LHC
- M(ass) theory at the LHC
- M(other) theory ...

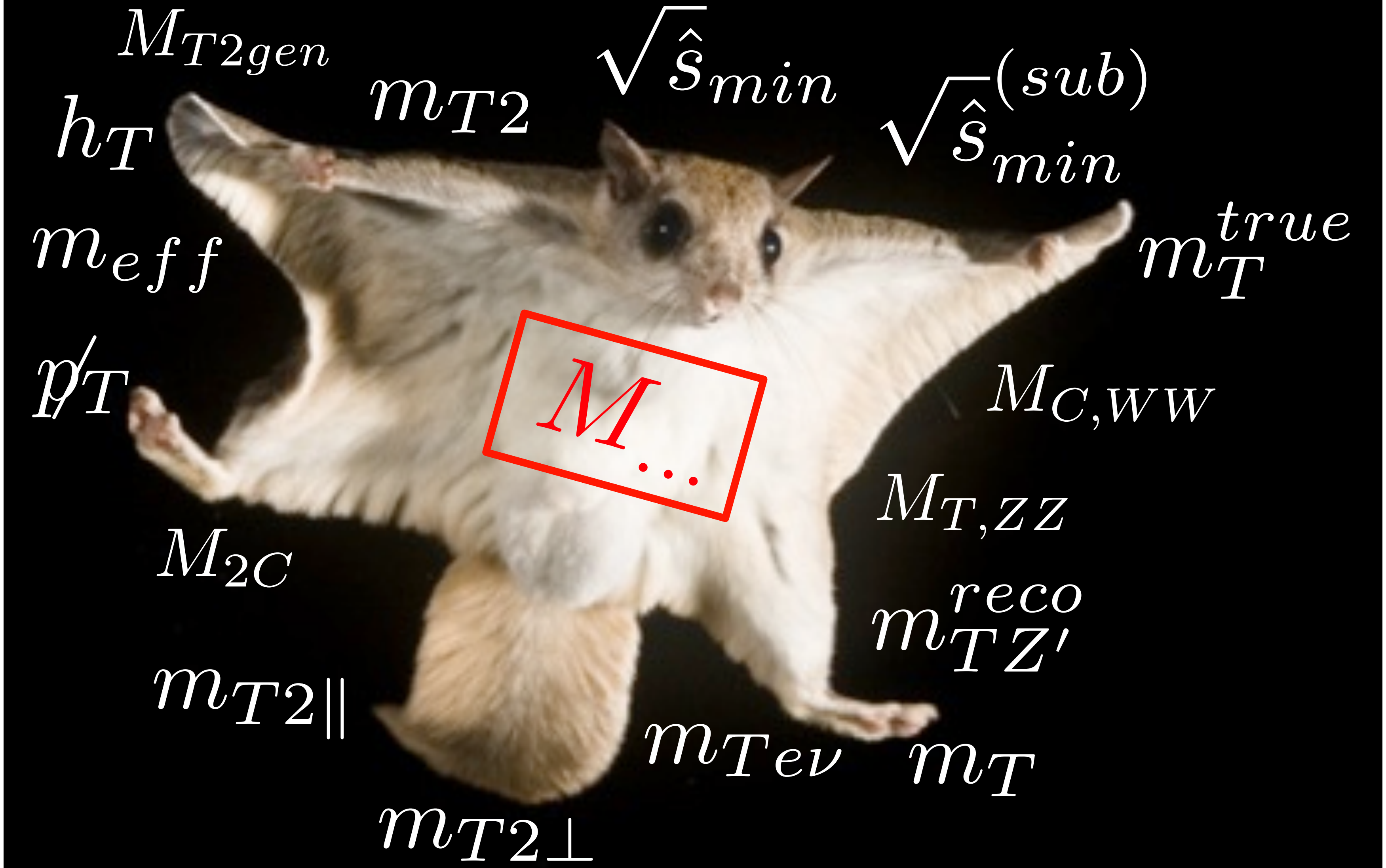
MET

Meff, HT

mT

MT2perp

MTGEN



W. Lamb (1955): “The finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine”

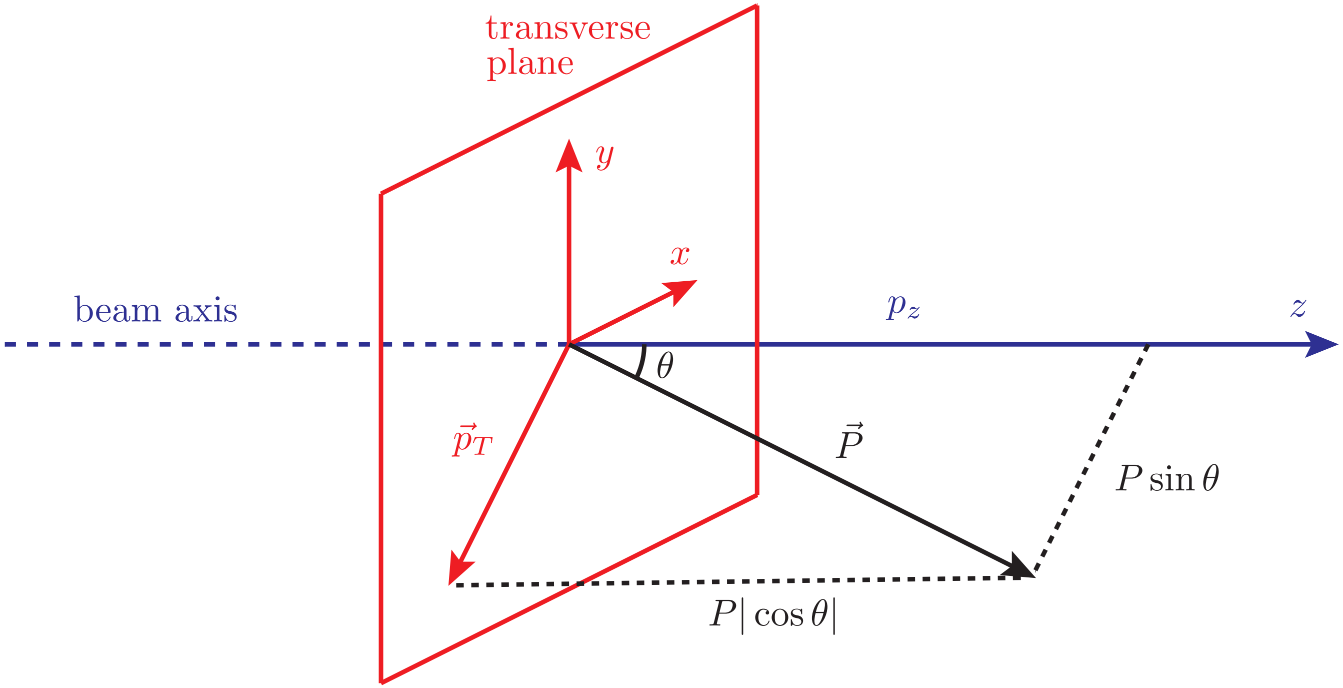
© Joe McDonald

Outline

- Transversification
  - how do we project particle momenta?
- Agglomeration
  - how do we add transverse momenta?
- Interpretation
  - how do we categorize reconstructed objects?
- Generalization
  - how do we define the most general mass-bound variables?
- Specialization
  - how do we recover the existing variables?
  - illustration: dilepton tt-bar and h->WW examples.

Transversification of 3-vectors

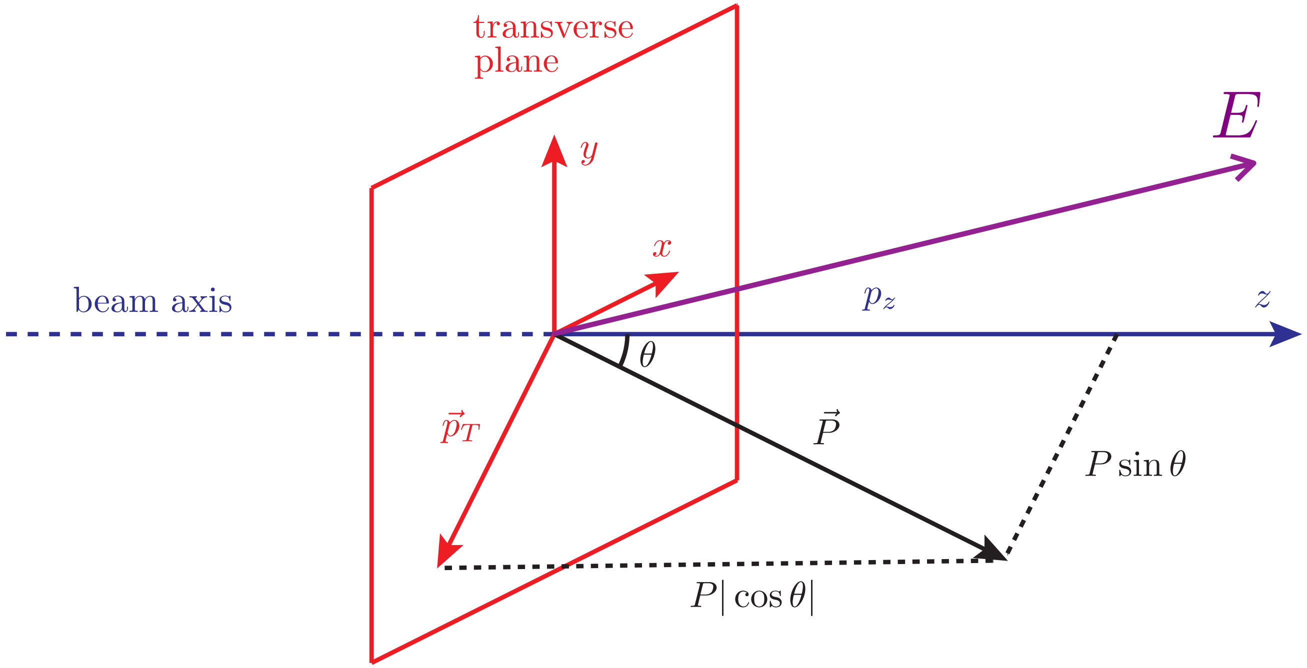
- Warm-up exercise: geometrical projection



$$p_T = P\sin\theta$$

Transversification of 1+3-vectors

- What to do with the energy (time-like) component?



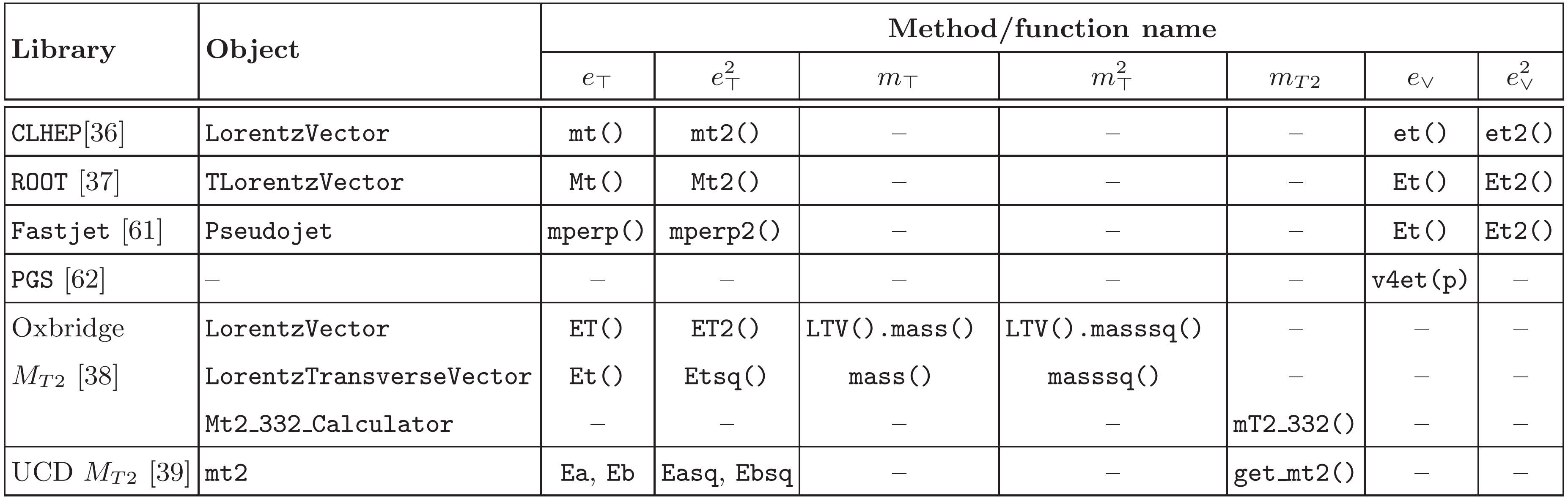
- Well, isn’t it obvious? Not really: there are at least three different options for the “transverse” energy: “T”, “V” and “0”.

Summary of transverse projections

Quantity	**Transverse projection method**		
	Mass-preserving ‘$\top$’	**Speed-preserving ‘$\vee$’**	**Massless ‘$\circ$’**
Original (4)-momentum	$P^\mu = (E, \vec{p}_T, p_z)$		
(1+3)-mass invariant	$M = \sqrt{E^2 - \vec{p}_T^{\,2} - p_z^2}$		
Transverse momentum	$\vec{p}_T \equiv (p_x, p_y)$		
(1+2)-vectors	$p_\top^\alpha \equiv (e_\top, \vec{p}_\top)$	$p_\vee^\alpha \equiv (e_\vee, \vec{p}_\vee)$	$p_\circ^\alpha \equiv (e_\circ, \vec{p}_\circ)$
Transverse momentum under the projection	$\vec{p}_\top \equiv \vec{p}_T$	$\vec{p}_\vee \equiv \vec{p}_T$	$\vec{p}_\circ \equiv \vec{p}_T$
Transverse energy under the projection	$e_\top \equiv \sqrt{M^2 + \vec{p}_T^{\,2}}$	$e_\vee \equiv E\,\lvert\sin\theta\rvert = \lvert\vec{p}_T\rvert/V$	$e_\circ \equiv \lvert\vec{p}_T\rvert$
Transverse mass under the projection	$m_\top^2 = e_\top^2 - \vec{p}_\top^{\,2}$	$m_\vee^2 \equiv e_\vee^2 - \vec{p}_\vee^{\,2}$	$m_\circ^2 \equiv e_\circ^2 - \vec{p}_\circ^{\,2} = 0$
Relationship between transverse quantity and its (1+3) analogue	$m_\top = M$	$m_\vee = M\,\lvert\sin\theta\rvert$	$m_\circ = 0$
	$\frac{1}{v_\top} = \frac{1}{V}\sqrt{1 + (1 - V^2)\frac{p_z^2}{p_T^2}}$	$v_\vee = V$	$v_\circ = 1$
Equivalence classes under $(1+3) \xmapsto{\text{proj}} (1+2)$	All P^μ with the same p_x, p_y and M	All P^μ with the same p_x, p_y and V	All P^μ with the same p_x and p_y

A guide to existing computer codes

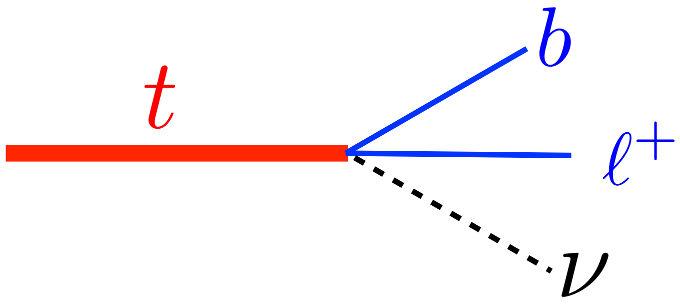
- Both “T” and “V” projections appear to be used in the existing computer libraries and codes



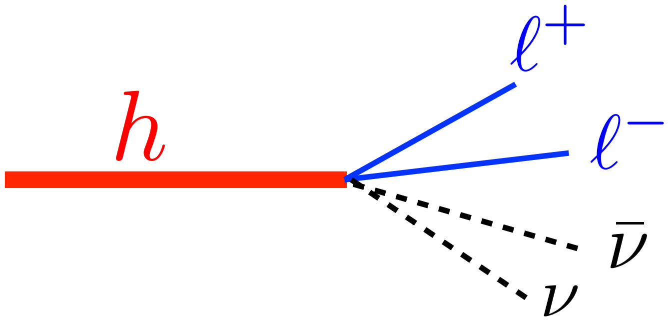
Library	**Object**	**Method/function name**						
		$e_\top$	$e_\top^2$	$m_\top$	$m_\top^2$	m_{T2}	$e_\vee$	$e_\vee^2$
CLHEP[36]	LorentzVector	mt()	mt2()	–	–	–	et()	et2()
ROOT [37]	TLorentzVector	Mt()	Mt2()	–	–	–	Et()	Et2()
Fastjet [61]	Pseudojet	mperp()	mperp2()	–	–	–	Et()	Et2()
PGS [62]	–	–	–	–	–	–	v4et(p)	–
Oxbridge M_{T2} [38]	LorentzVector	ET()	ET2()	LTV().mass()	LTV().masssq()	–	–	–
	LorentzTransverseVector	Et()	Etsq()	mass()	masssq()	–	–	–
	Mt2_332_Calculator	–	–	–	–	mT2_332()	–	–
UCD M_{T2} [39]	mt2	Ea, Eb	Easq, Ebsq	–	–	get_mt2()	–	–

Agglomeration

- Heavy, promptly, semi-invisibly decaying resonances are reconstructed by agglomerating their daughter particles



$$t \to b\ell^+\nu$$



$$h \to W^+W^- \to \ell^+\ell^-\nu\bar{\nu}$$

- Transverse quantities are constructed by transverse projections
- Which should come first: the projection or the agglomeration? The results are different!

“Early” versus “late” projections

- The order of the operations makes a big difference for the time-like components

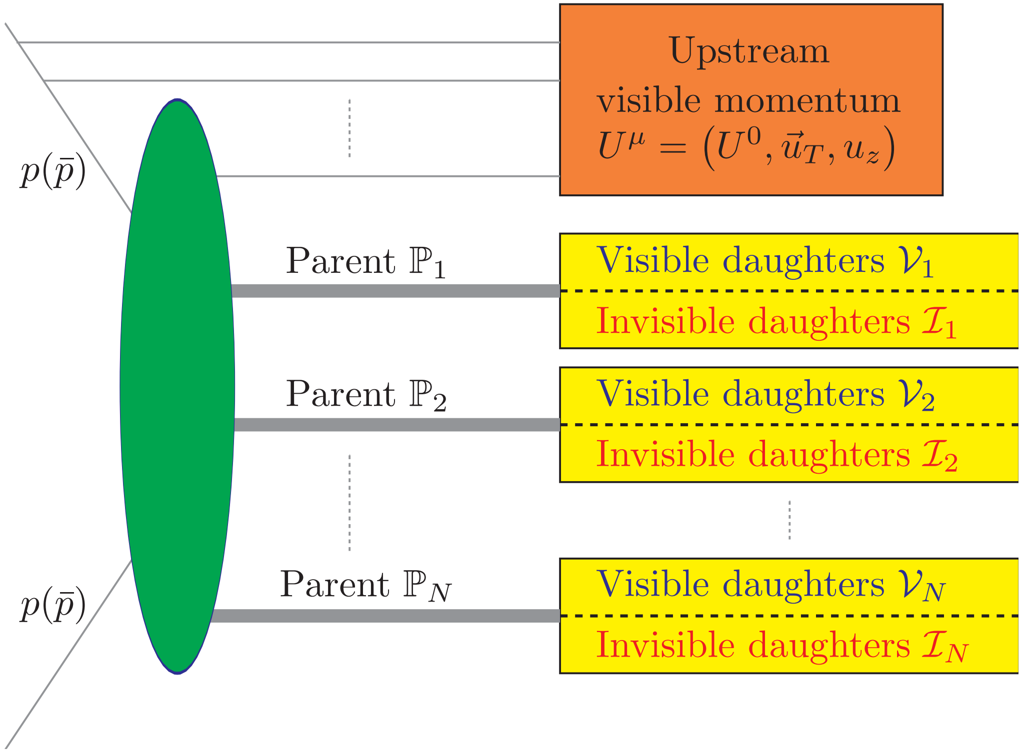
$$\sum_i \vec{p}_{i\top} = \left(\sum_i \vec{P}_i\right)_\top \qquad \sum_i e_{i\top} \neq \left(\sum_i E_i\right)_\top ,$$

$$\sum_i \vec{p}_{i\vee} = \left(\sum_i \vec{P}_i\right)_\vee \qquad \sum_i e_{i\vee} \neq \left(\sum_i E_i\right)_\vee ,$$

$$\sum_i \vec{p}_{i\circ} = \left(\sum_i \vec{P}_i\right)_\circ \qquad \sum_i e_{i\circ} \neq \left(\sum_i E_i\right)_\circ .$$

- Our convention: the order of indices (from left to right) denotes the order of operations, e.g.
  - – add first, project later: $p^\alpha_{aT} \equiv (e_{aT}, \vec{p}_{aT})$
  - – project first, add later: $p^\alpha_{Ta} \equiv (e_{Ta}, \vec{p}_{Ta})$

Interpretation (of an event)



- N “parents”. For each:
  - Visible daughters
  - Invisible daughters
- Upstream momentum
- Missing p_T

$$\not{\vec{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}$$

- Notation for particle momenta:
  - “P” (“p”) for visible daughters
  - “Q” (“q”) for invisible daughters

How to form mass-bound variables

- Goal: find a lower bound on the mass of the heaviest (next-heaviest, etc.) parent
- There are various possibilities:
  - – 1 unprojected $\mathcal{M}_a \equiv \sqrt{g_{\mu\nu}\,(\mathbf{P}_a^\mu + \mathbf{Q}_a^\mu)(\mathbf{P}_a^\nu + \mathbf{Q}_a^\nu)}$
  - – 3 late-projected $\mathcal{M}_{aT} \equiv \sqrt{g_{\alpha\beta}\,(\mathbf{p}_{aT}^\alpha + \mathbf{q}_{aT}^\alpha)(\mathbf{p}_{aT}^\beta + \mathbf{q}_{aT}^\beta)}$
  - – 3 early-projected $\mathcal{M}_{Ta} \equiv \sqrt{g_{\alpha\beta}\,(\mathbf{p}_{Ta}^\alpha + \mathbf{q}_{Ta}^\alpha)(\mathbf{p}_{Ta}^\beta + \mathbf{q}_{Ta}^\beta)}$
- Then minimize over the momenta of the invisible particles:

$$
\begin{aligned}
M_N &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_a\right]\right], \\
M_{NT} &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{aT}\right]\right], \\
M_{TN} &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{Ta}\right]\right],
\end{aligned}
$$

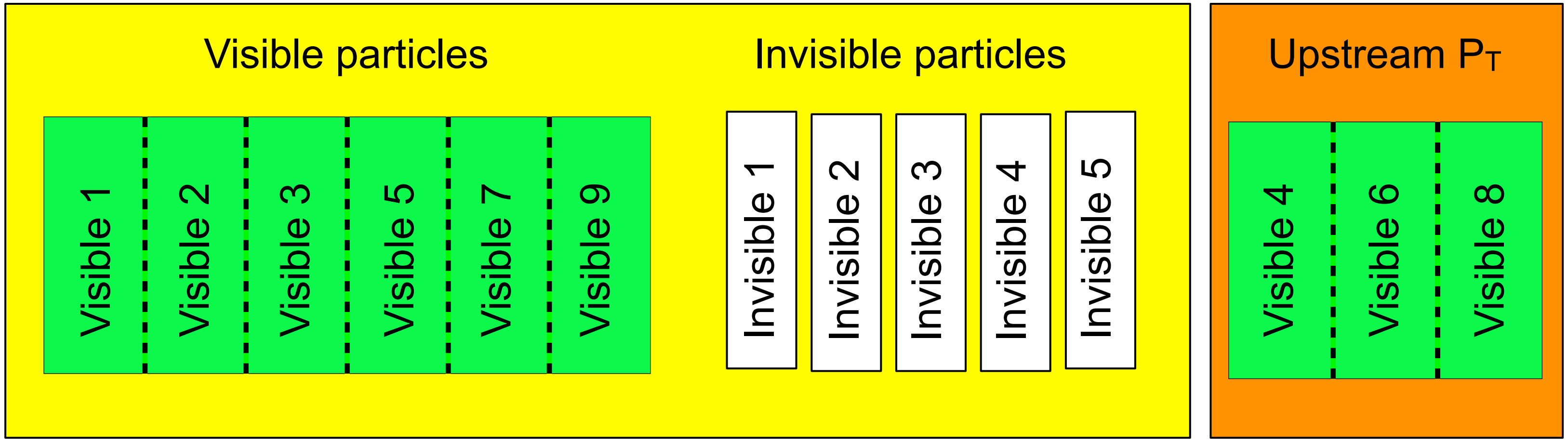
The 7 basic mass bound variables

Type of variables	Operations			Notation
	First	Second	Third	
Unprojected	Partitioning	Minimization	—	M_N √
Early partitioned (late projected) M_{NT}	Partitioning	$T = \top$ projection	Minimization	$M_{N\top}$ √
	Partitioning	$T = \vee$ projection	Minimization	$M_{N\vee}$
	Partitioning	$T = \circ$ projection	Minimization	$M_{N\circ}$ √
Late partitioned (early projected) M_{TN}	$T = \top$ projection	Partitioning	Minimization	$M_{\top N}$ √
	$T = \vee$ projection	Partitioning	Minimization	$M_{\vee N}$
	$T = \circ$ projection	Partitioning	Minimization	$M_{\circ N}$ √

- Can you recognize which one is the Cambridge M_{T2}?

Example: The unprojected M_1

- This is the minimum total invariant mass of the single-parent subsystem



Visible particles

Visible 1

Visible 2

Visible 3

Visible 5

Visible 7

Visible 9

Invisible particles

Invisible 1

Invisible 2

Invisible 3

Invisible 4

Invisible 5

Upstream P_T

Visible 4

Visible 6

Visible 8

$$M_1^2(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2+\mathbf{p}_{1T}^2}+\sqrt{\not{\mathbf{M}}_1^2+\not{p}_T^2}\right)^2 - u_T^2 \equiv \hat{s}_{min}^{(sub)}$$

Total visible mass: $\mathbf{M}_1 \equiv \sqrt{\mathbf{E}_1^2 - \vec{\mathbf{p}}_{1T}^2 - \mathbf{p}_{1z}^2},$

Konar, Kong, Matchev, Park 2010

Total invisible mass: $\not{\mathbf{M}}_1 \equiv \sum_{i=1}^{N_{\mathcal{I}}} \tilde{M}_i.$

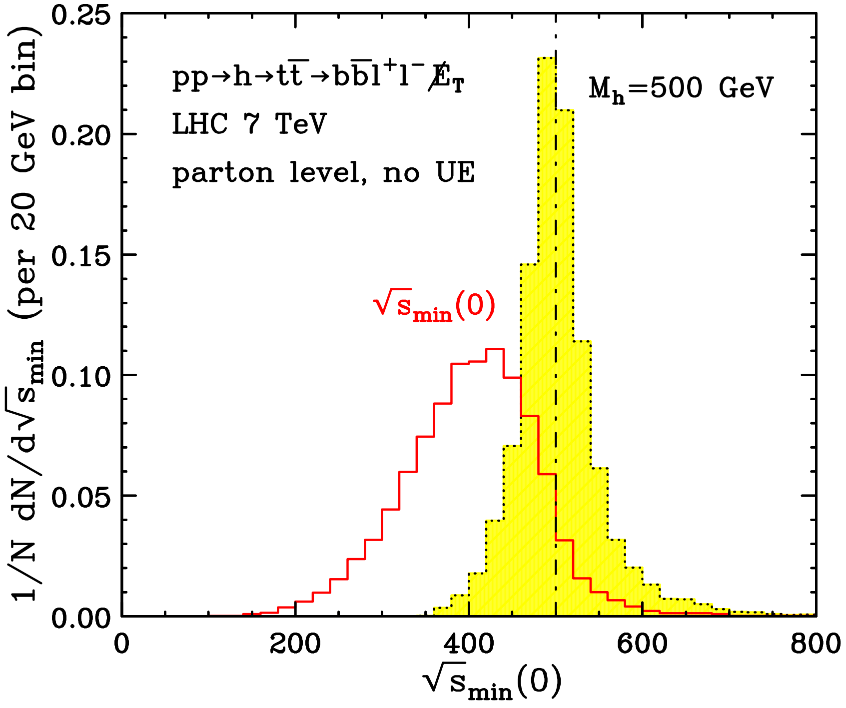
Applications of $\boxed{\sqrt{s_{min}}}$

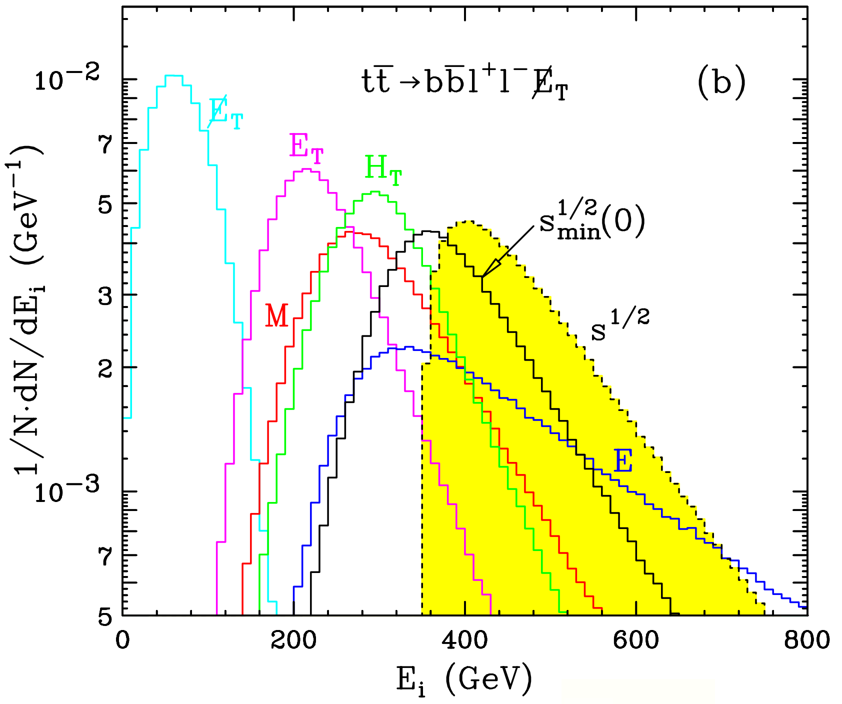
Konar, Kong, Matchev 2008

Konar, Kong, Matchev, Park 2010

- N=1: Single semi-invisibly decaying particle
  - SM Higgs to tt-bar
  - endpoint at the parent mass

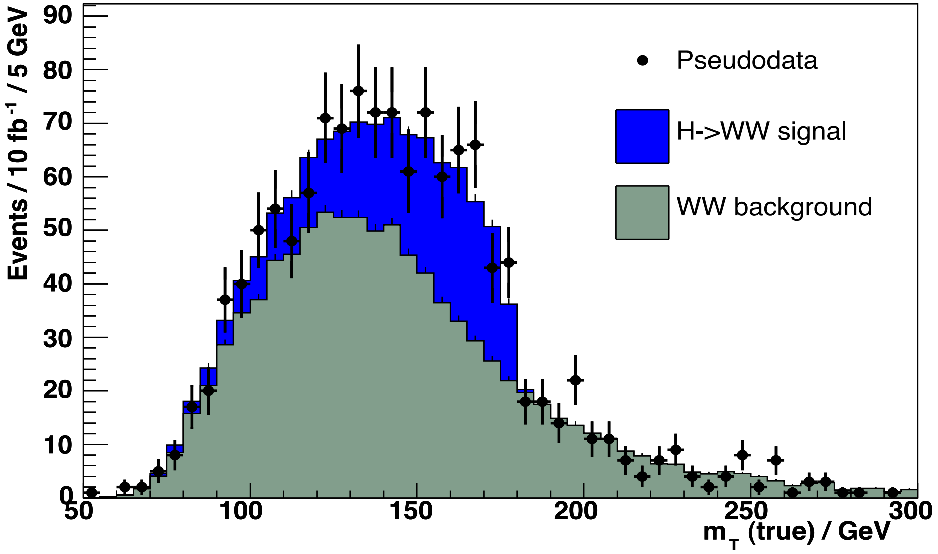
- N=2: A pair of semi-invisibly decaying particles
  - direct tt-bar production
  - peak at the total parent mass

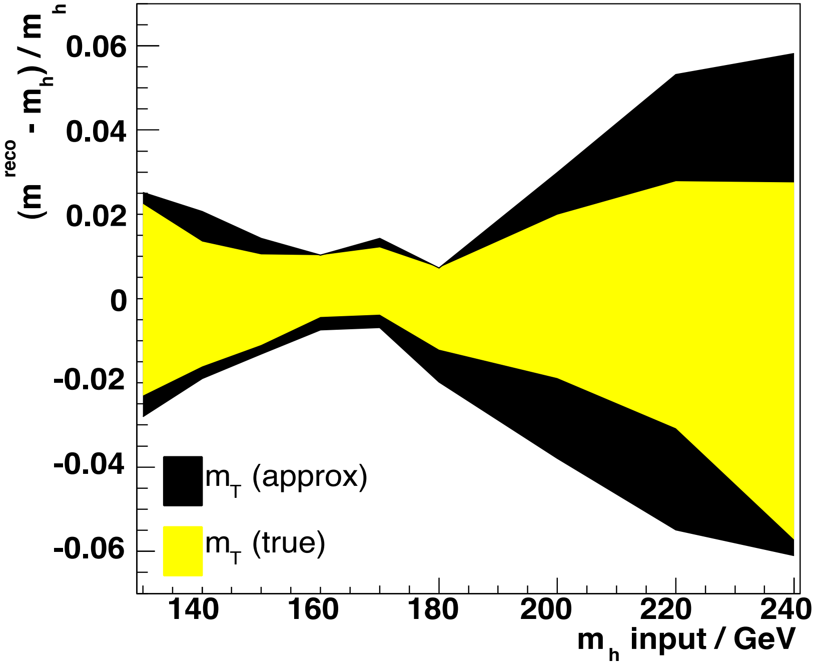


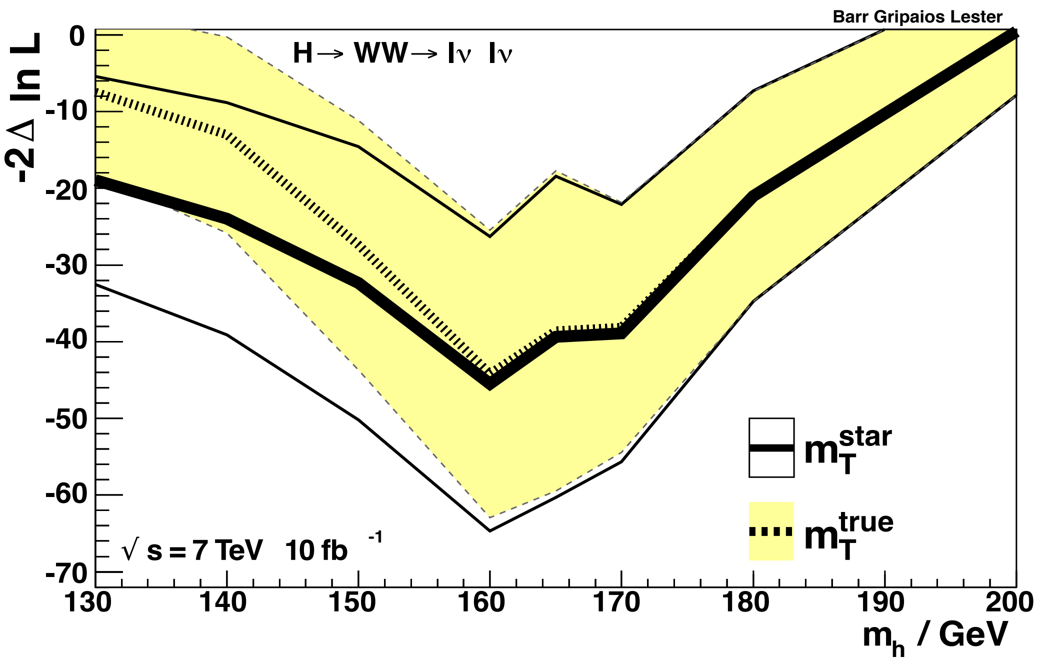


Barr, Gripaios, Lester 2009, 2011

$$h \to WW^{(*)} \to \ell^+\ell^-\nu\bar{\nu}$$







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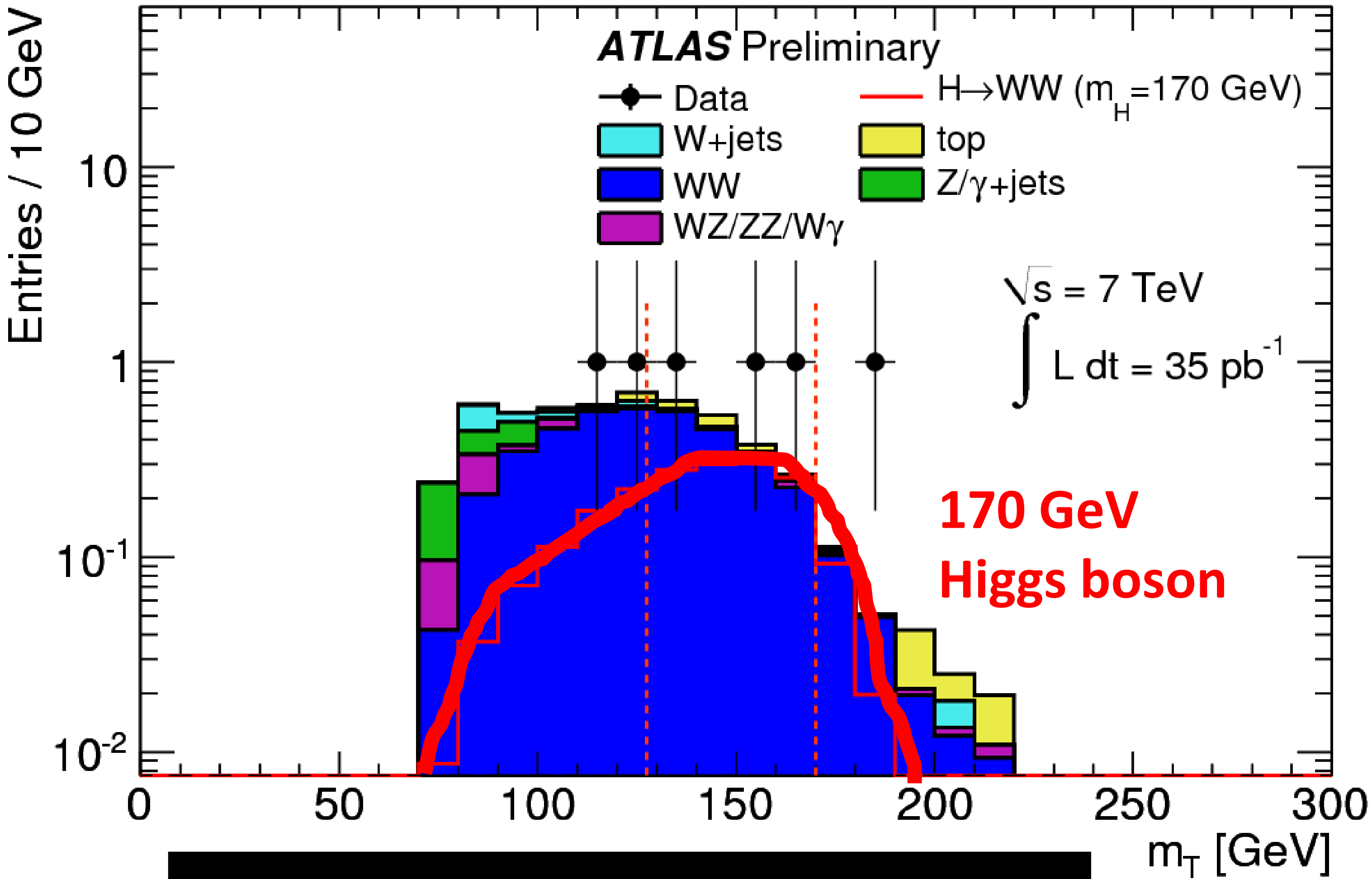
Against the 2010 LHC data…



170 GeV Higgs boson

Big improvement in LHC Higgs Search

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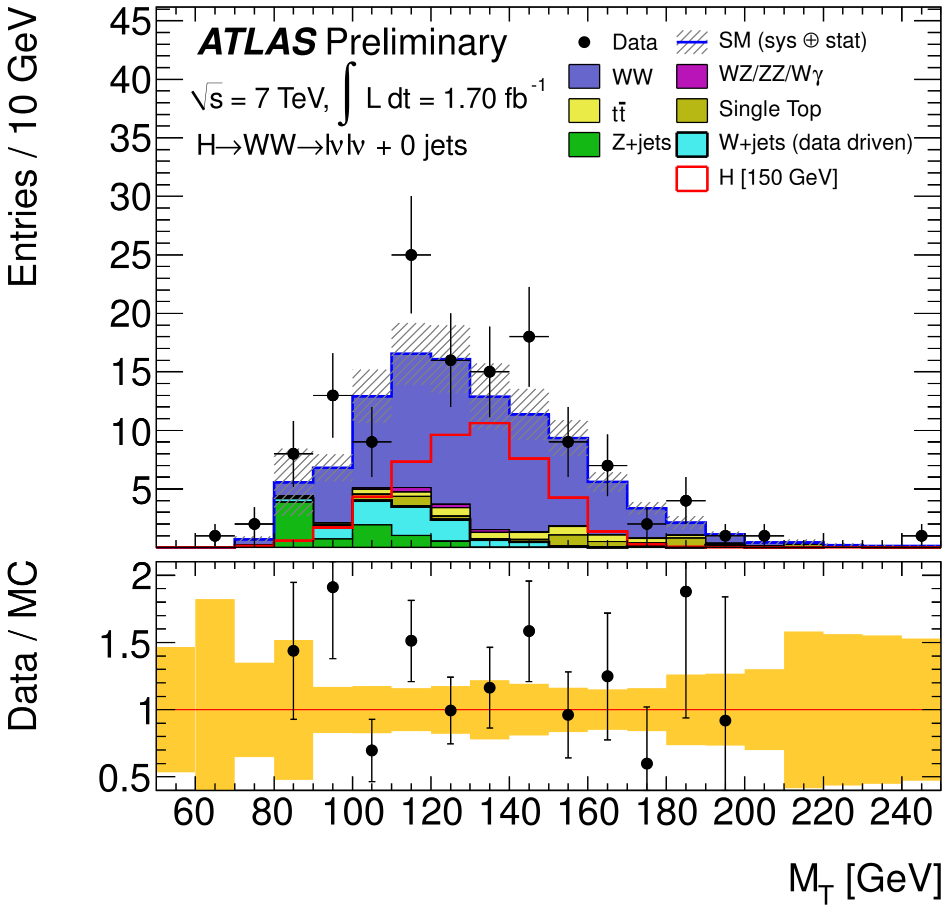
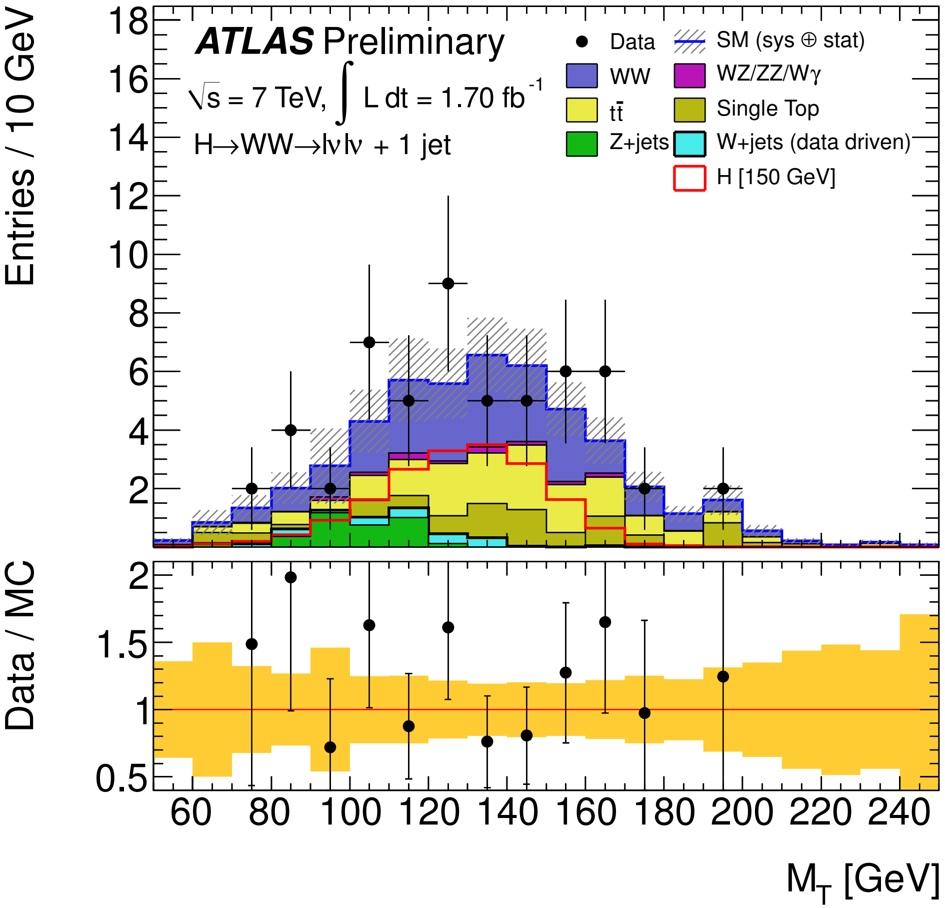




History repeats but we learn more and understand better

$$M_T = 2\sqrt{p_T^2(ll) + m^2(ll)},$$

$$M_C = \sqrt{p_T^2(ll) + m^2(ll)} + \not{E}_T$$

Han, Zhang 1998,1999

$$\begin{aligned}\sqrt{s}_{min}^{(sub)}(\not{M}) &= \left\{\left(\sqrt{E_{(sub)}^2 - P_{z(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2}\right)^2 - P_{T(up)}^2\right\}^{\frac{1}{2}} \\ &= \left\{\left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2}\right)^2 - P_{T(up)}^2\right\}^{\frac{1}{2}} \\ &= \left\{\left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2}\right)^2 - (\vec{P}_{T(sub)} + \vec{\not{P}}_T)^2\right\}^{\frac{1}{2}} \\ &= ||p_{T(sub)} + \not{p}_T||\,,\end{aligned}$$

$$p_{T(sub)} \equiv \left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2}\,,\ \vec{P}_{T(sub)}\right)$$

$$\not{p}_T \equiv \left(\sqrt{\not{M}^2 + \not{P}_T^2}\,,\ \vec{\not{P}}_T\right)\,.$$

Konar, Kong, Matchev, Park, 2008 2010

$$(m_T^{\mathrm{true}})^2 \equiv m_T^2(m_i = 0) = m_v^2 + 2(e_v|\mathbf{p}_i| - \mathbf{p}_v \cdot \mathbf{p}_i)$$

Barr, Gripaios, Lester 2009, 2011

$$\boxed{M_1^2(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2 + \mathbf{p}_{1T}^2} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2}\right)^2 - u_T^2}$$

Barr, Khoo, Konar, Kong,
Lester, Matchev, Park 2011

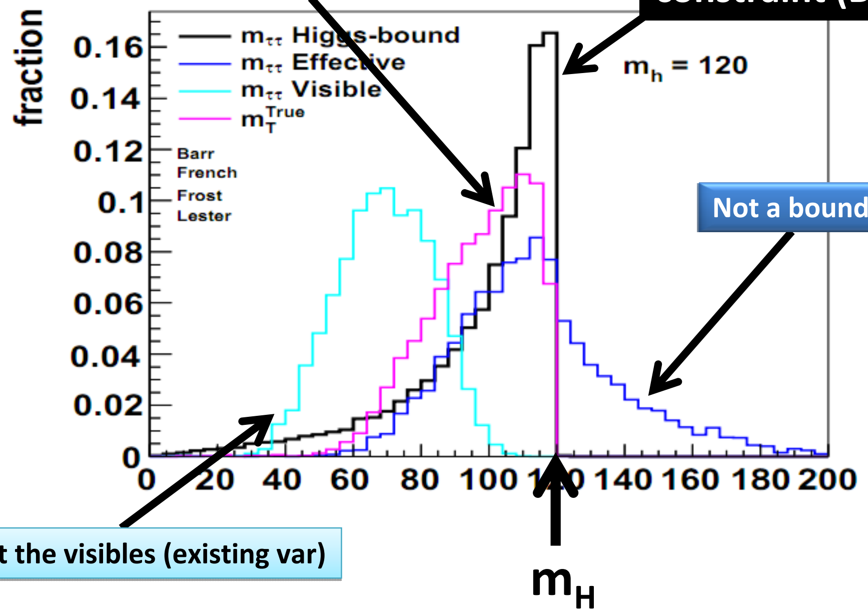
Barr, French, Frost, Lester 2011

Application in Higgs to a tau-pair

Parent mas bound (no intermediate constraint) = M1T

Result

Including the intermediate constraint (BEST)



Not a bound (existing var)

Just the visibles (existing var)

Dramatic difference to Higgs observability?

$$h \to \tau\tau$$

$$m_{\tau\tau}^{\text{Higgs}-\text{bound}} = \min_{\{Q_1^\mu, Q_2^\mu | \aleph\}} \sqrt{H^\mu H_\mu}$$

$$H^\mu = P_1^\mu + Q_1^\mu + P_2^\mu + Q_2^\mu$$

$$\begin{aligned} Q_1^\mu Q_{1\mu} &= 0, \\ Q_2^\mu Q_{2\mu} &= 0, \\ (Q_1^\mu + P_1^\mu)(Q_{1\mu} + P_{1\mu}) &= m_\tau^2, \\ (Q_2^\mu + P_2^\mu)(Q_{2\mu} + P_{2\mu}) &= m_\tau^2, \end{aligned}$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \vec{\not{p}}_{\mathrm{T}}.$$

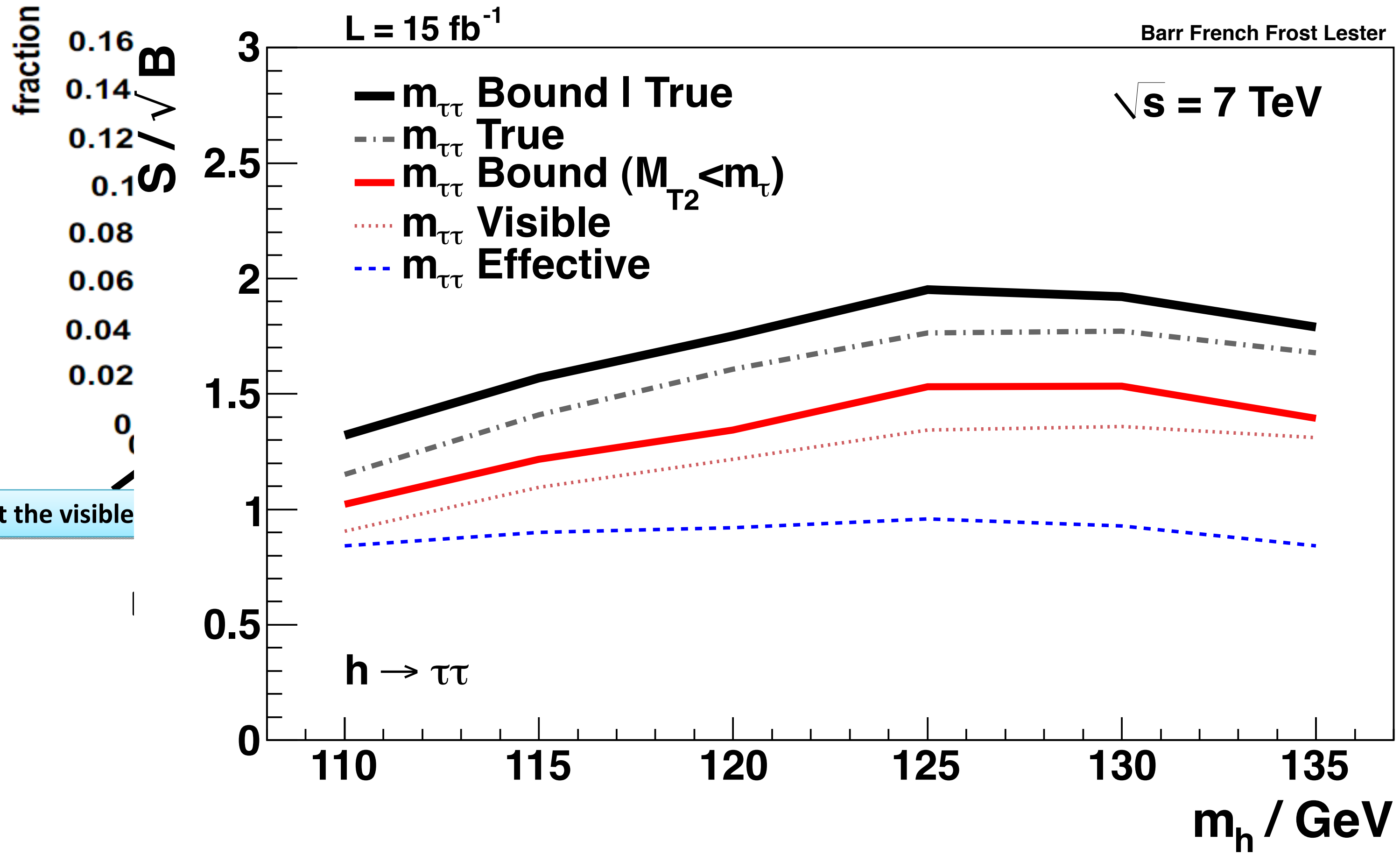
Barr, French, Frost, Lester 2011

Application in Higgs to a tau-pair

Parent mas bound (no intermediate constraint) = M1T

Result

Including the intermediate



Just the visible

$$\min_{{}^{\mu}_{1}, Q^{\mu}_{2} | \aleph\}} \sqrt{H^\mu H_\mu}$$

$$- P^\mu_2 + Q^\mu_2$$

$$Q_{1\mu} = 0,$$

$$Q_{2\mu} = 0,$$

$$P_{1\mu}) = m_\tau^2,$$

$$(Q_2 + P_2)(Q_{2\mu} + P_{2\mu}) = m_\tau^2,$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \not{\vec{p}}_{\mathrm{T}}.$$

The late “T”-projected variable M$_{NT}$

- The order is: agglomerate, “T”-project, then minimize over q$_{iT}$ and q$_{iz}$. First form each parent mass

$$\mathcal{M}^2_{a\top}(\mathbf{p}^\alpha_{a\top}, \mathbf{q}^\alpha_{a\top}, \tilde{\mu}_a) \equiv (\mathbf{p}_{a\top}+\mathbf{q}_{a\top})^2 \equiv (\mathbf{e}_{a\top}+\tilde{\mathbf{e}}_{a\top})^2 - (\vec{\mathbf{p}}_{aT}+\vec{\mathbf{q}}_{aT})^2$$

- Then minimize the largest one:

$$M_{N\top}(\not{\mathbf{M}}) \equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{a\top}(\mathbf{p}^\alpha_{a\top}, \mathbf{q}^\alpha_{a\top}, \tilde{\mu}_a)\right]\right]$$

- For N=1 the result is

$$M^2_{1\top}(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2 + \mathbf{p}^2_{1T}} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2}\right)^2 - u_T^2 \equiv \hat{s}^{(sub)}_{min}$$

- In general one finds the identity

$$\boxed{M_{N\top} = M_N}$$

The early“T”-projected variable M_{TN}

- The order is: “T”-project, agglomerate, then minimize over q_{iT} (there is no q_{iz} dependence).

$$\mathcal{M}^2_{\top a}(\mathbf{p}^\alpha_{\top a}, \mathbf{q}^\alpha_{\top a}, \tilde{\mu}_a) \equiv (\mathbf{p}_{\top a} + \mathbf{q}_{\top a})^2 \equiv (\mathbf{e}_{\top a} + \tilde{\mathbf{e}}_{\top a})^2 - (\vec{\mathbf{p}}_{aT} + \vec{\mathbf{q}}_{aT})^2$$

- Then minimize the largest one:

$$M_{\top N}(\not{\mathbf{M}}) \equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{\top a}(\mathbf{p}^\alpha_{\top a}, \mathbf{q}^\alpha_{\top a}, \tilde{\mu}_a) \right] \right]$$

- For N=1 the result is

$$M^2_{\top 1}(\not{\mathbf{M}}_1) = \left(\sum_{i=1}^{N_{\mathcal{V}}} \sqrt{M_i^2 + \vec{p}^{\,2}_{iT}} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2$$

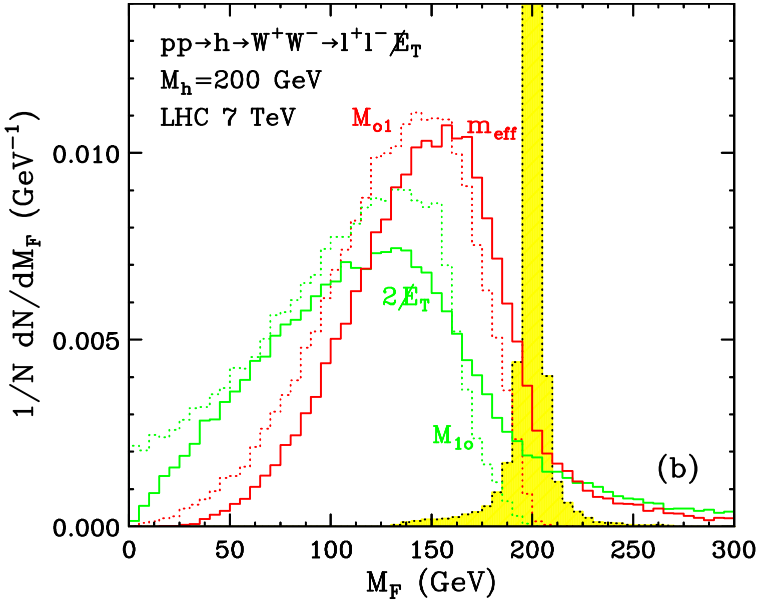
- For massless visible particles (leptons or jets)

$$\boxed{\lim_{M_i \to 0} M^2_{\top 1}(\not{\mathbf{M}}_1) = \left(h_T + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2}$$

The early “0”-projected variable M_{0N}

$$M_{\circ N} \equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_{a} \left[\mathcal{M}_{\circ a}(\mathbf{p}^{\alpha}_{\circ a}, \mathbf{q}^{\alpha}_{\circ a}) \right] \right]$$

$$\begin{aligned}
M^2_{\circ 1} &= \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\left(\sum_{i=1}^{N_{\mathcal{V}}} e_{i\circ} + \sum_{i=1}^{N_{\mathcal{I}}} \tilde{e}_{i\circ} \right)^2 - u_T^2 \right] \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} e_{i\circ} + \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\sum_{i=1}^{N_{\mathcal{I}}} \tilde{e}_{i\circ} \right] \right)^2 - u_T^2 \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} p_{iT} + \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\sum_{i=1}^{N_{\mathcal{I}}} q_{iT} \right] \right)^2 - u_T^2. \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} p_{iT} + \not{p}_T \right)^2 - u_T^2 \\
&= \left(h_T + \not{p}_T \right)^2 - u_T^2, \\
&= m^2_{\text{eff}} - u_T^2.
\end{aligned}$$



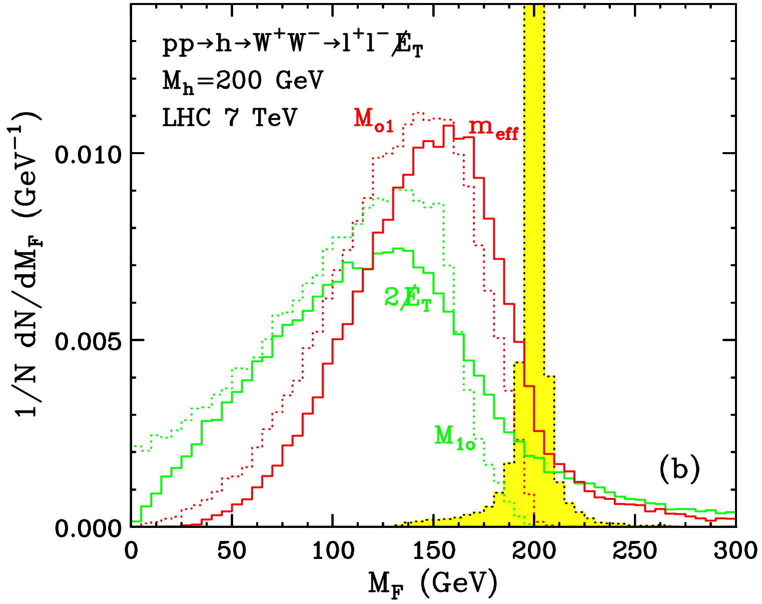
$$h_T \equiv \sum_{i=1}^{N_{\mathcal{V}}} p_{iT}$$

$$m_{\text{eff}} \equiv h_T + \not{p}_T$$

The late “0”-projected variable M_{N0}

$$M_{N\circ} \equiv \min_{\sum \vec{q}_{iT} = \vec{\not{p}}_T} \left[\max_a \left[\mathcal{M}_{a\circ}(\mathbf{p}^\alpha_{a\circ}, \mathbf{q}^\alpha_{a\circ}) \right] \right]$$

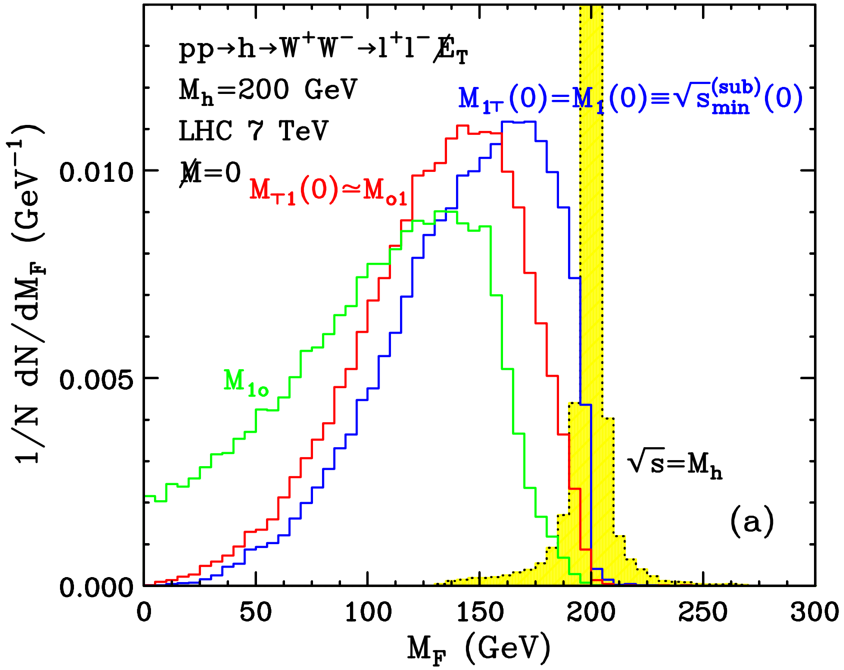
$$\begin{aligned}
M^2_{1\circ} &= \min_{\sum \vec{q}_{iT} = \vec{\not{p}}_T} \left[(\mathbf{e}_{1\circ} + \tilde{\mathbf{e}}_{1\circ})^2 - u_T^2 \right] \\
&= \left(\mathbf{e}_{1\circ} + \min_{\sum \vec{q}_{iT} = \vec{\not{p}}_T} [\tilde{\mathbf{e}}_{1\circ}] \right)^2 - u_T^2 \\
&= \left(\mathbf{e}_{1\circ} + \min_{\sum \vec{q}_{iT} = \vec{\not{p}}_T} \left[\left| \sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT} \right| \right] \right)^2 - u_T^2. \\
&= \left(\left| \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT} \right| + \not{p}_T \right)^2 - u_T^2 \\
&= \left(|\vec{\not{p}}_T + \vec{u}_T| + \not{p}_T \right)^2 - u_T^2 \\
&= 2\left(\vec{\not{p}}_T \cdot (\vec{\not{p}}_T + \vec{u}_T) + \not{p}_T \, |\vec{\not{p}}_T + \vec{u}_T| \right)
\end{aligned}$$

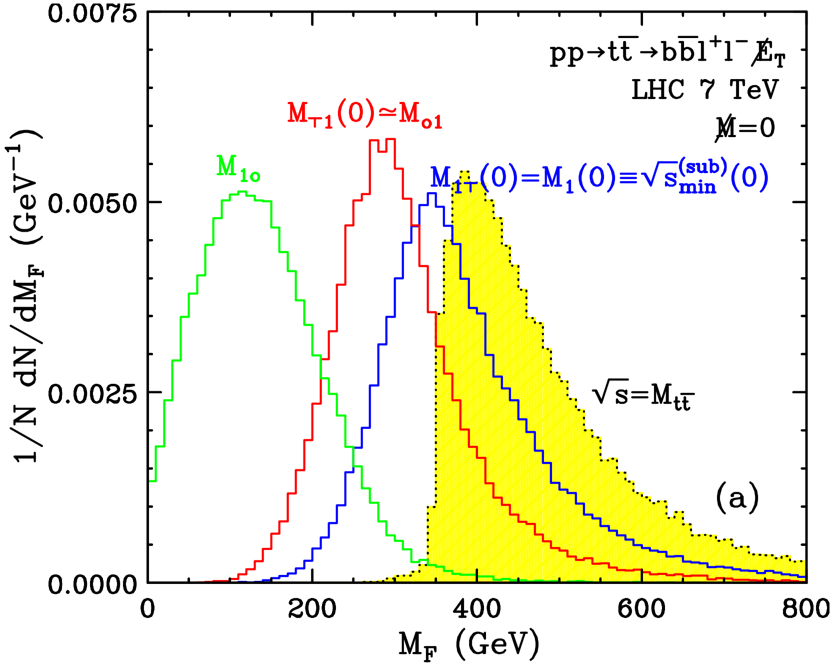


$$\boxed{\lim_{u_T \to 0} M^2_{1\circ} = 4\not{p}_T^2}$$

Which variable is best?

$$\boxed{M_N = M_{N\top} \geq M_{\top N} \geq M_{\circ N} \geq M_{N\circ}}$$



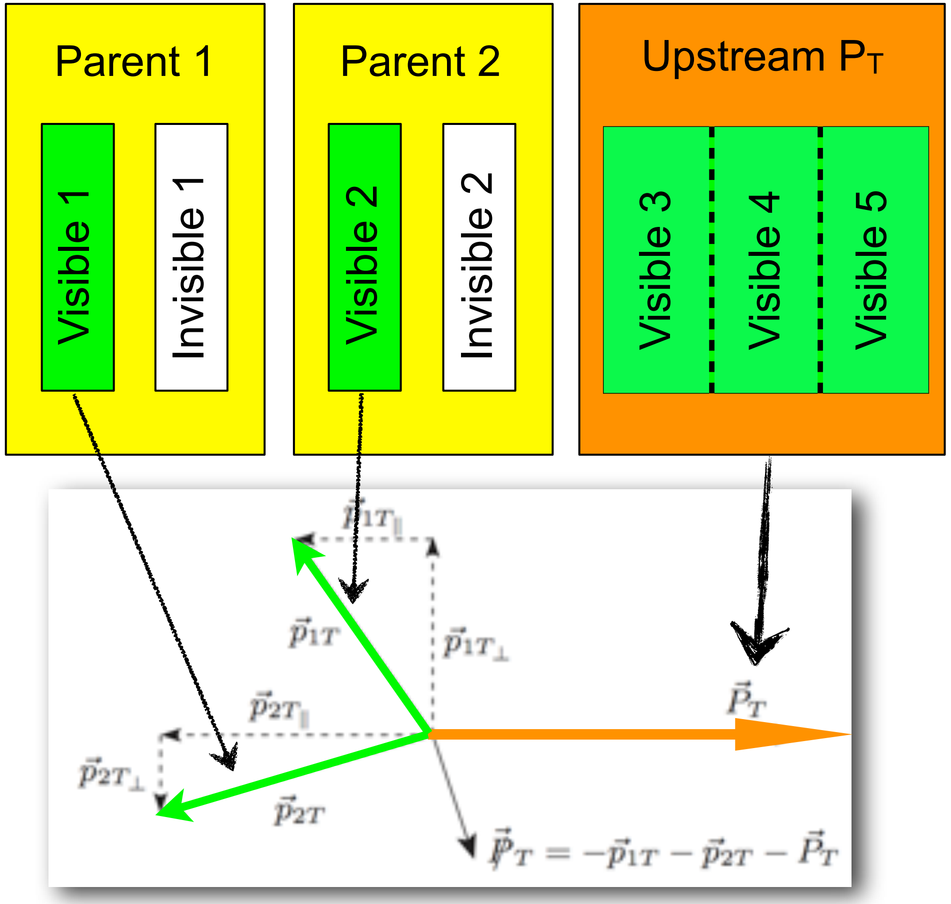


- Late (or no) projection gives a better endpoint structure
- Early projection less sensitive to forward hadronic activity

Transversification (twice)

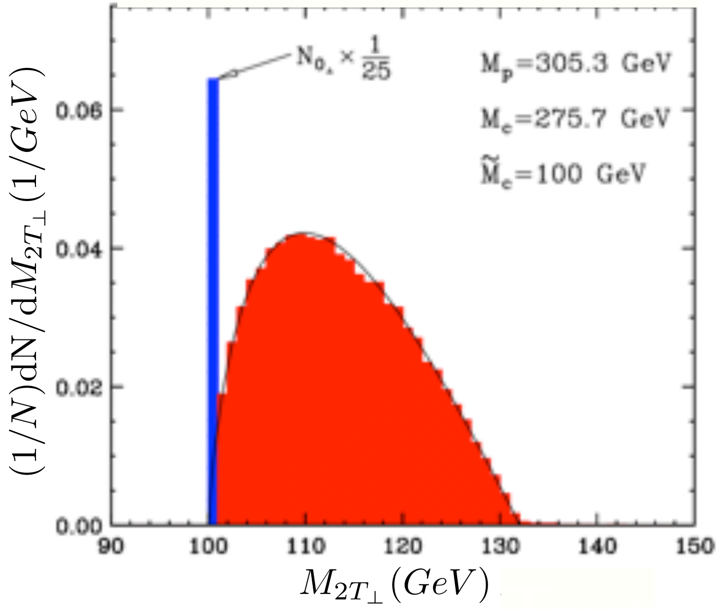
Matchev, Park 2009

- Having projected on the transverse plane, one can additionally project on the direction of Upstream P_T:

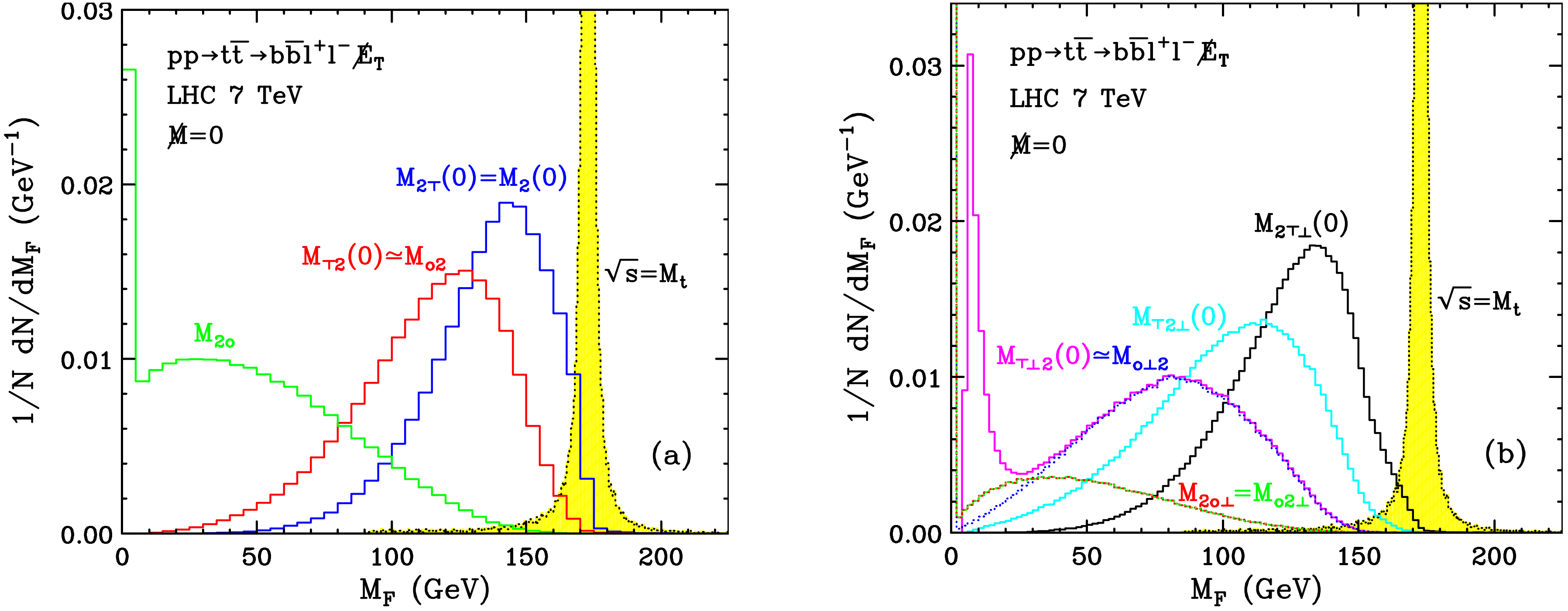


- The endpoints of “perp” distributions are stable against P_T variations

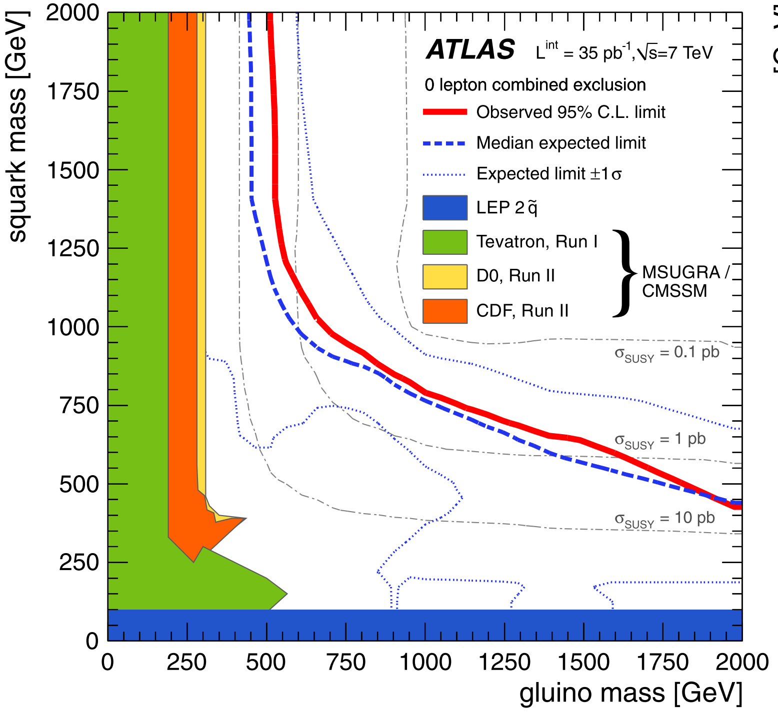
Konar, Kong, Matchev, Park 2009

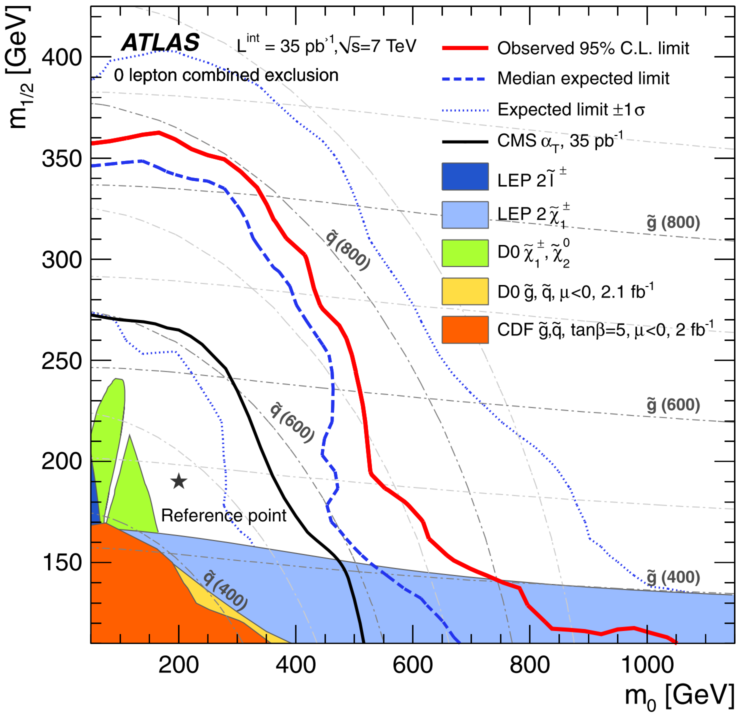


Cambridge M_{T2}-type variables



- The “2” in M_{T2} referred to the number of invisibles
- The “2” here refers to the number of parents





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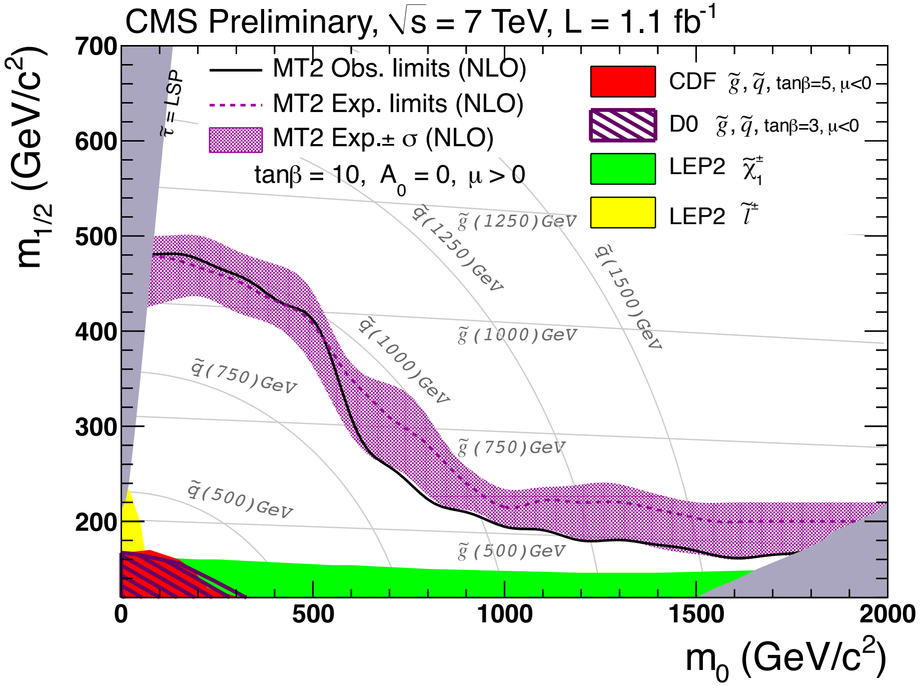

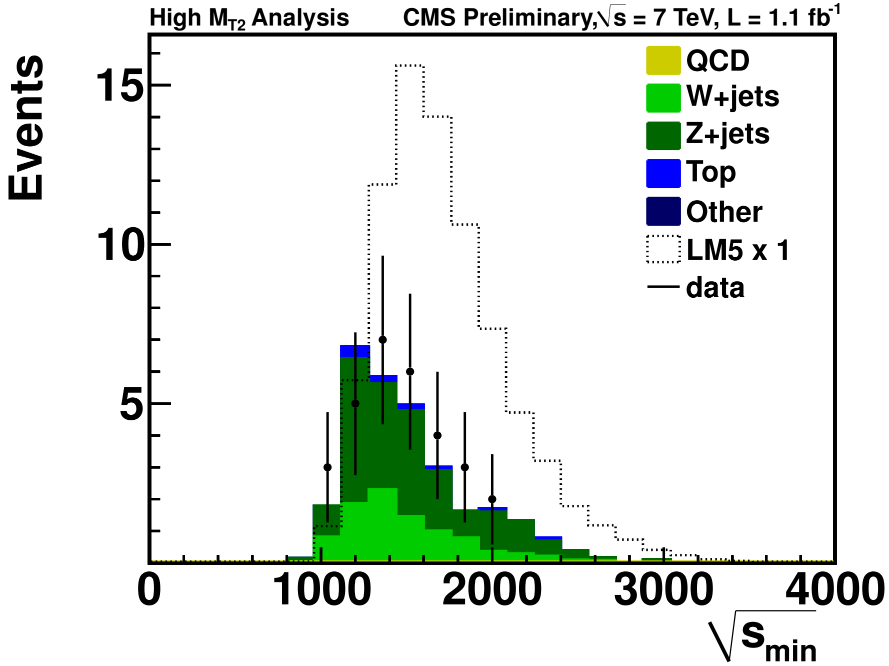

A common framework

	Mass-bound variable					
Existing variable	$N = 1$				$N = 2$	
	$M_1(\not{\mathbf{M}}_1) = M_{1\top}(\not{\mathbf{M}}_1)$	$M_{\top 1}(\not{\mathbf{M}}_1)$	$M_{\circ 1}$	$M_{1 \circ}$	$M_2(\not{\mathbf{M}}) = M_{2\top}(\not{\mathbf{M}})$	$M_{2\top\perp}(\not{\mathbf{M}})$
$2\not{p}_T = 2\not{E}_T$				$u_T \to 0$		
$m_{\rm eff}$		$\not{\mathbf{M}}_1 \to 0, u_T \to 0$	$u_T \to 0$			
$\sqrt{\hat{s}}_{\rm min}^{(\rm sub)}(\not{\mathbf{M}}_1)$	✓					
$\sqrt{\hat{s}}_{\rm min}(\not{\mathbf{M}}_1)$	$u_T \to 0$					
$m_{Te\nu}(M_e, M_\nu)$	✓	✓	$M_e, M_\nu \to 0$	$M_e, M_\nu \to 0$		
$M_{T,ZZ}(M_Z)$	✓	✓				
$M_{C,WW}$	$\not{\mathbf{M}}_1 \to 0$					
$m_T^{\rm true}$	$\not{\mathbf{M}}_1 \to 0$					
$m_{TZ'}^{reco}(M_Z)$	$u_T \to 0$	$u_T \to 0$				
$m_{T2}(\not{\mathbf{M}})$					✓	
$m_{T2\perp}(\not{\mathbf{M}})$						✓

All previous variables are just specializations to a specific event topology, massless invisibles or uT=0

Take home lessons

- There are different ways to project on the transverse plane
- Be mindful of the way in which composite particles are agglomerated (before or after T)
- Always think which of the 61 variables is most suited for the particular case at hand
- The early-agglomerated (late-projected) “transverse” variables are “secretly” 1+3 dimensional

$$\boxed{M_{N\top}(\not{\mathbf{M}}) = M_N(\not{\mathbf{M}})}$$

- The dependence on the unknown masses is only through the N summed-mass parameters

$$\boxed{\not{\mathbf{M}}_a \equiv \sum_{i\in\mathcal{I}_a} \tilde{M}_i .}$$