Radiatively Enhanced Higgs Mass with the MSSM Singlets

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@ KIAS

CMS, ATLAS reported the excesses of events
for $\gamma\gamma$, ZZ^*, $WW^* \longrightarrow$ 4 leptons
around 125 -126 GeV at 5σ level.

They seemingly imply the discovery of
the Higgs boson with 125 GeV mass.

Higgs mass in the MSSM

- at tree level

$$m_h^2 < M_Z^2 \cos^2 2\beta$$

- LEP bound: $m_h^2 > 114$ GeV.
- By including the radiative corrections,
  the Higgs mass can be raised above 100 GeV.

Radiative Corr. to m_h^2 in the MSSM

$$\Delta m_h^2 = (3/4\pi^2)\,(y_t M_t)^2 \sin^2\beta\ \log(\,M_t^2 + m_t^2/M_t^2)$$

(y_t: top quark Yukawa coupling, M_t : top quark mass, m_t : S-top mass)

For a large radiative correction to the Higgs mass,

the S-top should be quite heavy.

(a few TeV at 2-loop for m_h=125 GeV)

Radiative Corr. to m_h^2 in the MSSM

$\Delta V_{CW}''$ contributes to the renormalization of m_{2h}^2.

One of the extremum conditions with the MSSM Higgs pot. reads

$$m_2^2 + |\mu|^2 \approx m_3^2 \cot\beta + (M_Z^2/2)\cos 2\beta - \Delta m_2^2$$

where $$\Delta m_2^2 \approx (3y_t^2/8\pi^2)\, m_t^2 \log(m_t^2/M_G^2)$$

A large m_t (> a few TeV) requires a fine-tuning (< 0.1%) among the soft parameters to fit M_Z .

"Little Hierarchy Problem"

Radiative Corr. to m_h^2 in the MSSM (mixing eff.)

Large mixing between the L- and R-hnd. S-tops through the "A-term" is helpful for raising m_h:

$$\Delta m_h^2 \approx (3/4\pi^2)\,(y_t M_t)^2 \sin^2\beta \left[\, \log(m_t^2/M_t^2) + (X_t/m_t)^2 \left\{1 - (X_t/m_t)^2/12\right\} \right]$$

$$X_t = A_t - \mu \cot\beta$$

The maximal mixing [$(X_t/m_t)^2 = 6$]
can push m_h up to 135 GeV. But, ...

4^{th} family, extra vec.-like matter

- By introducing extra order one Yukawa coupling of extra unknown matter, 125 GeV Higgs mass can be explained by Rad. Corr. without a serious fine-tuning.

- But introduction of new colored particles with order one Yukawa couplings would exceedingly affect

1. the production rate of $gg \to h$ and also
2. the decay rate of $h \to \gamma\gamma$ at the LHC.

$\longrightarrow$ immoderate **deviation** from the **SM prediction**

Higgs mass in the NMSSM

- promote the MSSM μ term to $\lambda S H_u H_d$ in the superpot., introducing a singlet S.
- The Higgs mass can be raised to

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \lambda^2 v_H^2 \sin^2 2\beta + \Delta m_h^2$$

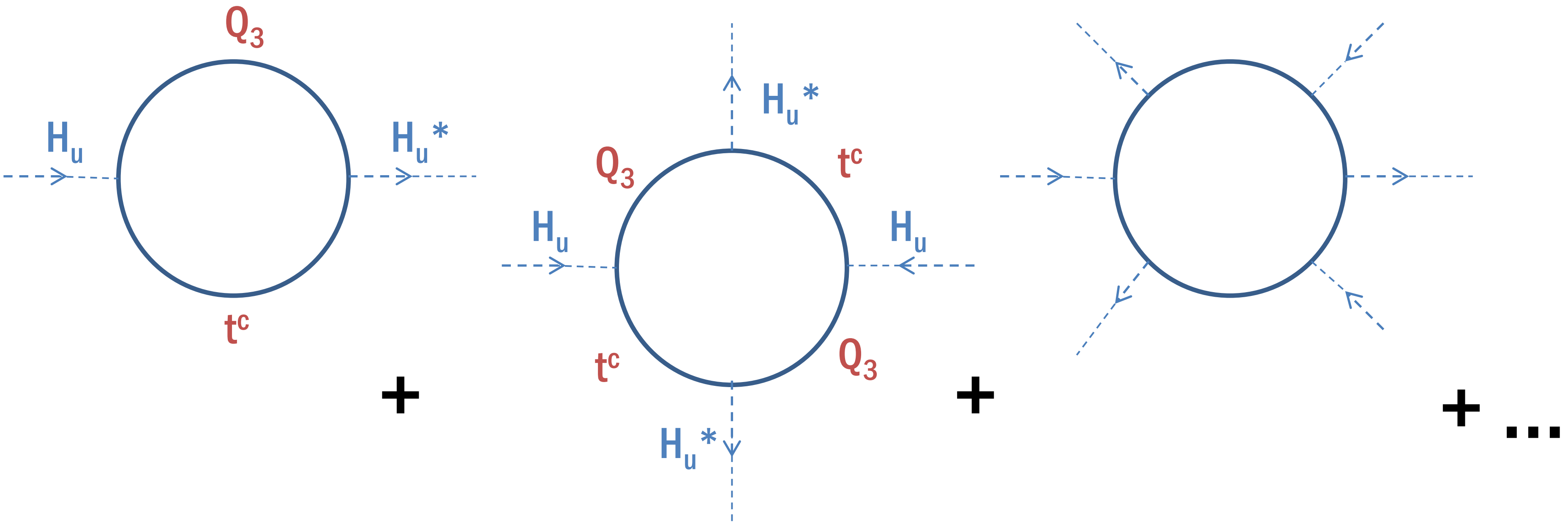
- But λ is restricted by the Landau pole constraint:

$$\lambda < 0.7$$

- For $m_t \ll 1$ TeV, $0.5 < \lambda < 0.7$, $1 < \tan\beta < 3$.

Can the Radiative Correction be enhanced by MSSM singlets?

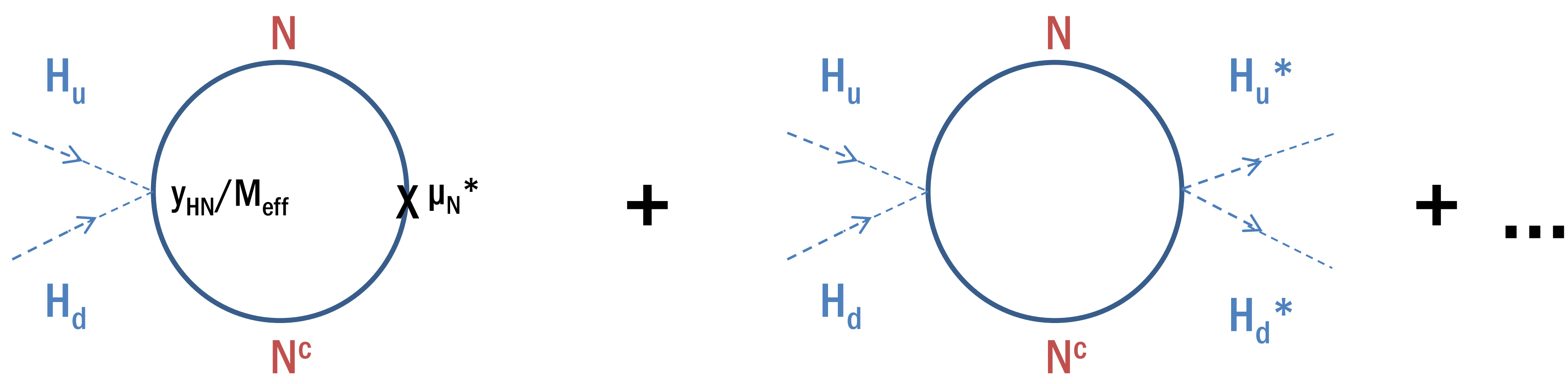
1-loop Effective Potential in the MSSM



+ bosonic loops

$$y_t\, H_u Q_3 t^c$$

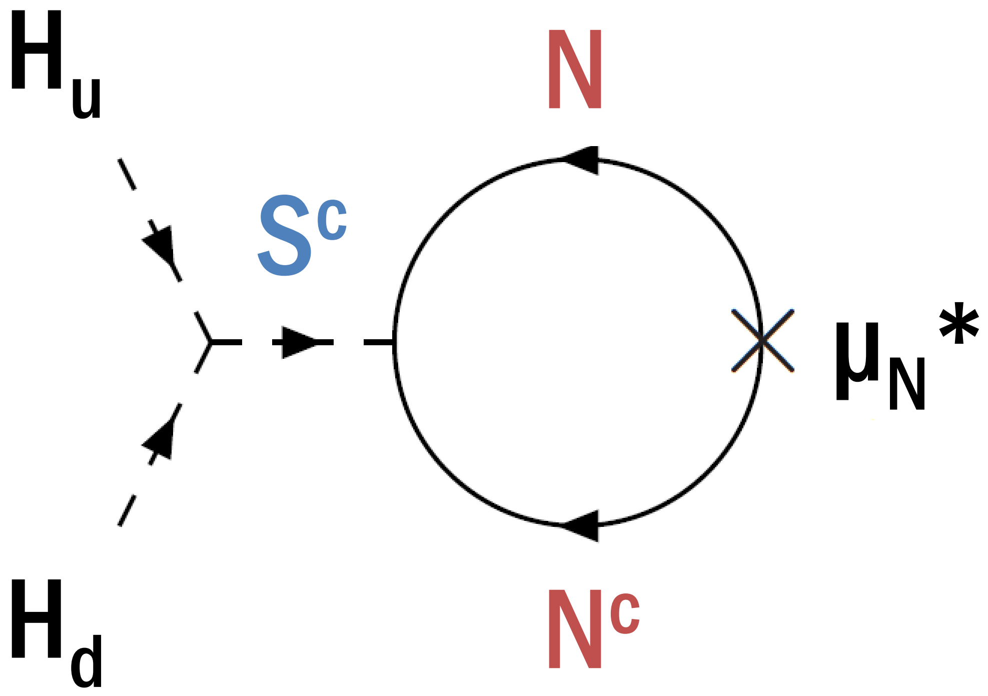
1-loop Effective Potential by SM Singlets



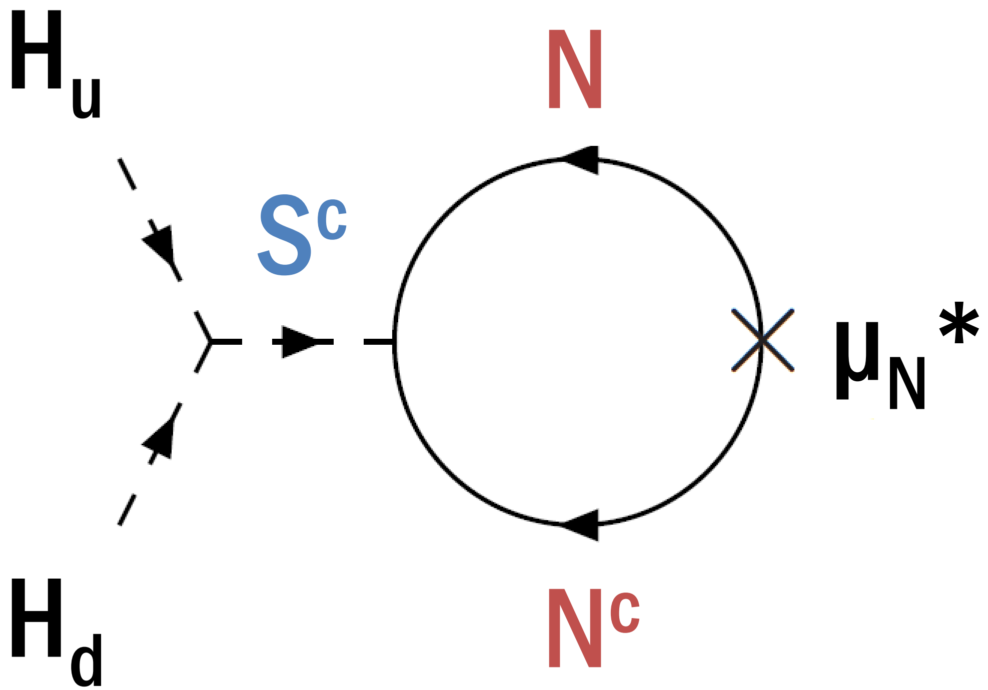
+ bosonic loops

$$(y_{HN}/M_{eff})\, H_u H_d N N^c + \mu_N N N^c$$

1-loop Effective Potential by SM Singlets



1-loop Effective Potential by SM Singlets

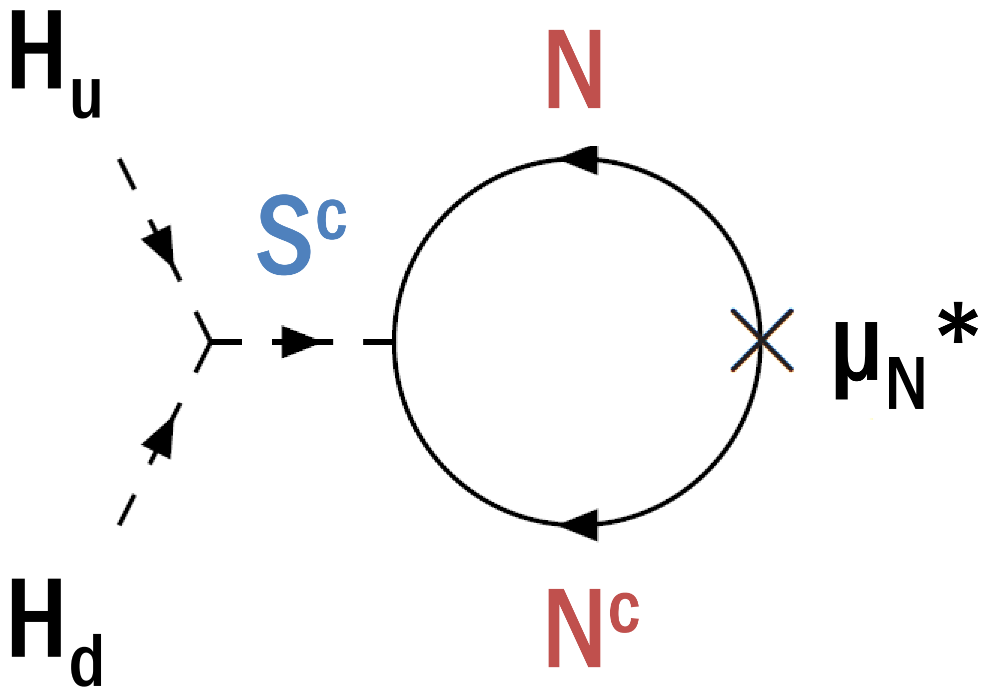


Higgs sec. (visible sec.)

Mediation sec. (messenger sec.)

Mass generation sec. (hidden sec.)

1-loop Effective Potential by SM Singlets



$$W_{eff} = \mu H_u H_d + y_H S H_u H_d + \mu_S S S^c + y_N S^c N N^c + \mu_N N N^c$$

Higgs sec. (visible sec.)

Mediation sec. (messenger sec.)

Mass generation sec. (hidden sec.)

A singlet extension of the MSSM

Introducing neutral fields under SM, $\{S, S^c\}$, $\{N, N^c\}$,
where $\{N, N^c\}$ are n-dim. Rep. of a (large) Hidden gauge group.

Visible sec. Messenger sec. Hidden sec.

$$W = (\mu + y_H S) H_u H_d + \mu_S S S^c + (\mu_N + y_N S^c) N N^c$$

$y_H < O(1)$, $y_N \sim O(1)$

y_N does NOT blow up at higher energies.

A singlet extension of the MSSM

Introducing neutral fields under SM, $\{S, S^c\}$, $\{N, N^c\}$,
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Visible sec. Messenger sec. Hidden sec.

$$W = (\mu + y_H S) H_u H_d + \mu_S S S^c + (\mu_N + y_N S^c) N N^c$$

$y_H < O(1)$, $y_N \sim O(1)$ $\mu_S , \mu_N \ll 1$ TeV , e.g. by G.-M. mech.

A singlet extension of the MSSM

Introducing neutral fields under SM, $\{S, S^c\}$, $\{N, N^c\}$,
where $\{N, N^c\}$ are n-dim. Rep. of a (large) Hidden gauge group.

Visible sec. Messenger sec. Hidden sec.

$$W = (\mu + y_H S) H_u H_d + \mu_S S S^c + (\mu_N + y_N S^c) N N^c$$

$$m_{S^c}^2 \ll \mu^2 \ll m_{3/2}^2 , \mu_S^2 \ll m_S^2 \ll \mu_N^2 , m_N^2 , m_{N^c}^2$$

1-loop Effective Higgs Potential

$$\Delta V(h) = \Delta V_{(00)} + \Delta V_{(1,1)} h_u h_d + \Delta V_{(2,2)} (h_u h_d)^2 / (2!2!) + \dots$$

$$\Delta V_{(00)} = \text{const.}$$

$$\Delta V_{(11)} = (n/8\pi^2)\,(y_H y_N \mu_N/\mu_S)\,[\,(\mu_N{}^2 + m_N{}^2)\,\{\log(\mu_N{}^2 + m_N{}^2/\Lambda^2) - 1\} - \mu_N{}^2\,\{\log(\mu_N{}^2/\Lambda^2) - 1\}\,]$$

$$\Delta V_{(22)} = (n/4\pi^2)\,(y_H y_N \mu_N/\mu_S)^2 \log(\,\mu_N{}^2 + m_N{}^2/\mu_N{}^2)$$

1-loop Effective Higgs Potential

$$\Delta V(h) = \Delta V_{(00)} + \Delta V_{(1,1)} h_u h_d + \Delta V_{(2,2)} (h_u h_d)^2 / (2!2!) + \ldots$$

$\Delta V_{(00)}$: vacuum energy

$\Delta V_{(11)}$: renormalize the $B\mu$ term, asso. with tuning

$\Delta V_{(22)}$: contribute to the Higgs mass

1-loop Effective Higgs Potential

$\Delta V(h)$ is valid below the messenger scale ($\approx \mu_S$).
(It can be a local op. below μ_S.)

Above the μ_N scale, one should return to
the original renormalizable ops. given in the model,
in which y_N can be of order unity,
for discussing the consistency.

Radiative Corr. to m_h^2 by the Singlets

$$\Delta m_h^2 = (n/4\pi^2)\,(y_H y_N \mu_N/\mu_S)^2\,(v_H^2 \sin^2 2\beta)\,\log(\mu_N^2 + m_N^2/\mu_N^2)$$

Δm_h^2 can be enlarged by n, $(y_H y_N \mu_N/\mu_S)^2$, etc.

Compared with the case of the MSSM:

$$\Delta m_h^2|_{top} = (3/4\pi^2)\,(y_t M_t)^2 \sin^2\beta\,\log(M_t^2 + m_t^2/M_t^2)$$

(y_t: top quark Yukawa coupling, M_t : top quark mass, m_t : S-top mass)

Radiative Corr. to m_h^2 by the Singlets

$$\Delta m_h^2 = (n/4\pi^2)\,(y_H y_N \mu_N/\mu_S)^2\,(v_H^2 \sin^2 2\beta)\,\log(\mu_N^2 + m_N^2/\mu_N^2)$$

Ex) $\mu_N \sim 600$ GeV , $\mu_S \sim 300$ GeV

Note:

- Above μ_S, $\Delta V(h)$ can NOT be a local op. any longer.

  (μ_S=300 – 500 GeV, quite low messenger scale)

- y_N ($\sim$ O(1)) does NOT blow up at higher energies.

Radiative Corr. to m_h^2 by the Singlets

$\Delta V_{CW}''$ contributes to the renormalization of m_3^2 ($=B\mu$).

One of the extremum conditions becomes

$$-2m_3^2 = (m_{1h}^2 - m_{2h}^2)\tan 2\beta + M_Z^2 \sin 2\beta$$
$$-(n/4\pi^2)(y_H y_N \mu_N/\mu_S)\left[(\mu_N^2 + m_N^2)\{\log(\mu_N^2 + m_N^2/\Lambda^2) - 1\} - \mu_N^2\{\log(\mu_N^2/\Lambda^2) - 1\}\right]$$

μ_N, $m_N \ll$ 1 TeV to avoid a serious fine-tuning

Radiative Corr. to m_h^2 by the Singlets

ΔV_{CW}'' contributes to the renormalization of m_3^2 (=$B\mu$).

One of the extremum conditions becomes

$$-2m_3^2 = (m_{1h}^2 - m_{2h}^2)\tan 2\beta + M_Z^2 \sin 2\beta$$
$$-(n/4\pi^2)\,(y_H y_N \mu_N/\mu_S)\left[(\mu_N^2 + m_N^2)\{\log(\mu_N^2 + m_N^2/\Lambda^2) - 1\} - \mu_N^2\{\log(\mu_N^2/\Lambda^2) - 1\} \right]$$

Below μ_S, the RG running frozen. So $\Lambda = \mu_S \sim 300$ GeV.

Radiative Corr. to m_h^2 by the Singlets

$$\Delta m_h^2 = (n/4\pi^2)(y_H y_N M_N/M_S)^2 (v_H^2 \sin^2 2\beta) \log(M_N^2 + m_N^2/M_N^2)$$

$$-2m_3^2 = (m_{1h}^2 - m_{2h}^2) \tan 2\beta + M_Z^2 \sin 2\beta$$
$$-(n/4\pi^2)(y_H y_N M_N/M_S)\left[m_N^2\{ \log(m_N^2/\mu_S^2)-1\} - M_N^2\{ \log(M_N^2/\mu_S^2)-1\} \right]$$

Compared with the MSSM/4th family scenario,

$$\Delta m_h^2 = (3/4\pi^2)\, (y_t M_t)^2 \sin^2\beta \;\; \log(M_t^2+m_t^2/M_t^2)$$

$$m_{2h}^2 = m_3^2 \cot\beta + (M_Z^2/2)\cos 2\beta$$
$$-(3y_t^2/8\pi^2)[\, m_t^2\{ \log(m_t^2/\Lambda^2)-1\} - M_t^2\{ \log(M_t^2/\Lambda^2)-1\} \,]$$

$\Lambda = M_{GUT}$

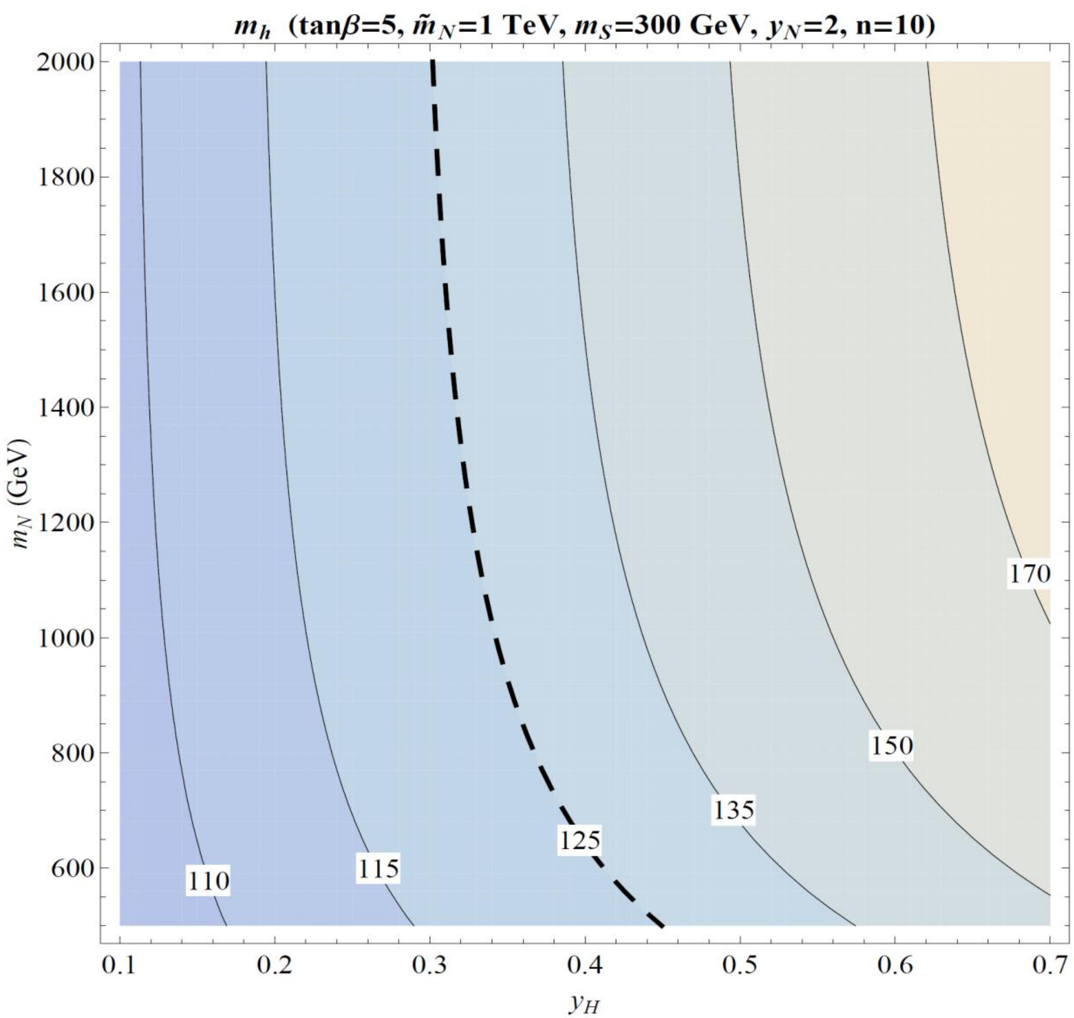

FIG. 2: Contour plots for the lightest Higgs mass m_h in the $y_H - m_N$ plane. Here we set $\Delta m_h|^2_{\rm top} = (66\ \text{GeV})^2$, which corresponds to $\widetilde{m}_t \approx 500$ GeV at two-loop level, but turn off the mixing effect. The tree level contribution from the NMSSM is ignored. We fix the other parameters as shown in the figure. The thick dashed line corresponds to $m_h = 125$ GeV.

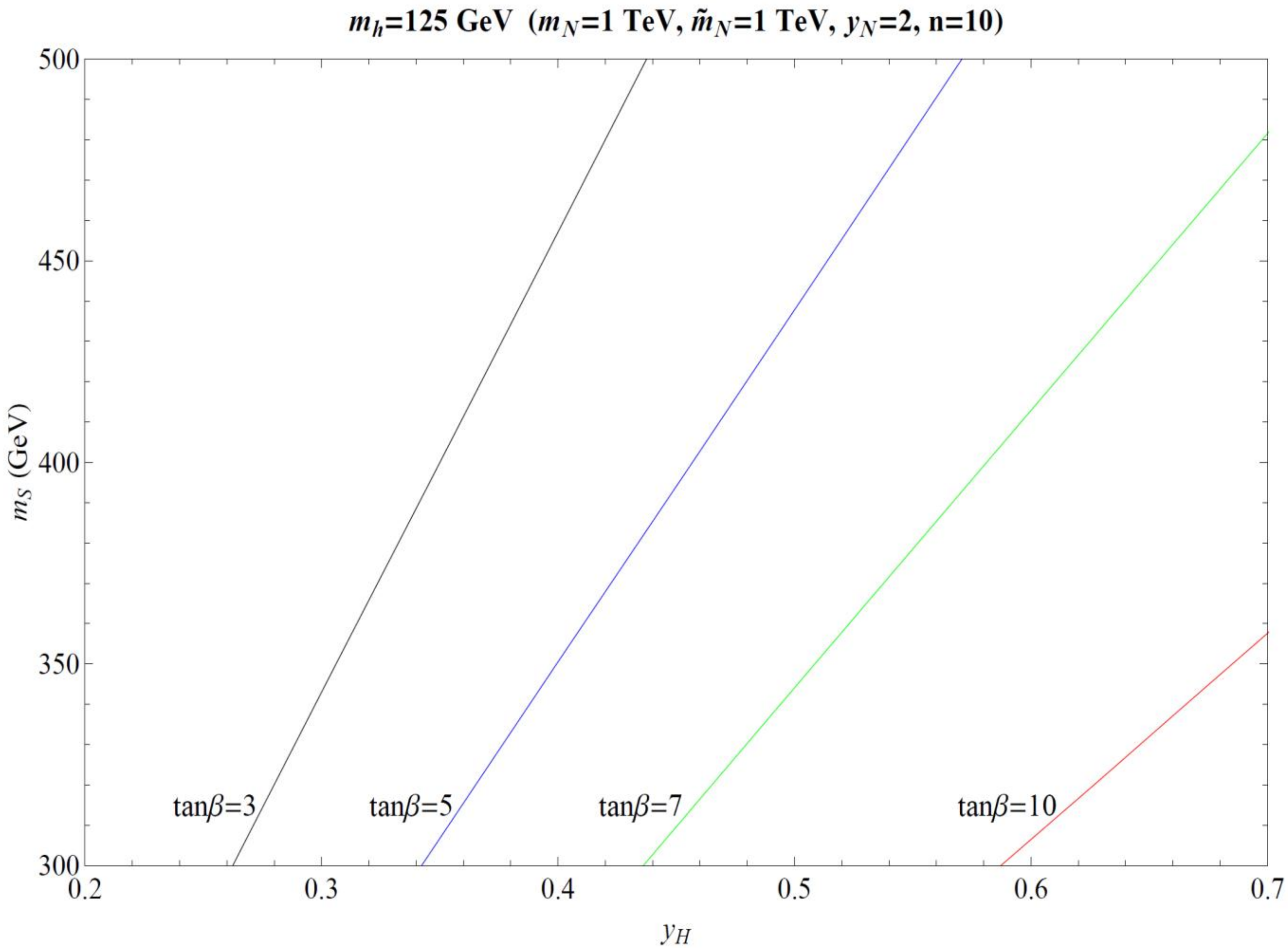

FIG. 3: Lightest Higgs mass $m_h = 125$ GeV lines for various $\tan\beta$s in the $y_H - m_S$ plane. Here we set $\Delta m_h|^2_{\rm top} = (66 \text{ GeV})^2$, which corresponds to $\widetilde{m}_t \approx 500$ GeV at two-loop level, but turn off the mixing effect. The tree level contribution from the NMSSM is ignored. The other parameters are fixed as shown in the figure.

The least tuning condition

$$F^2 \equiv \frac{\Delta m_h^2}{f^2 v_H^2} = R^2 \log\left(1 + r^2\right),$$

$$G \equiv \frac{2\Delta m_3^2}{g\mu_S^2} = R^3 \left[\left(1 + r^2\right)\left\{\log(1 + r^2) + \log R^2 - 1\right\} - \left\{\log R^2 - 1\right\}\right]$$

where R, r, f^2 and g are defined as

$$R \equiv \frac{\mu_N}{\mu_S}\,, \quad r \equiv \frac{m_N}{\mu_N}\,, \quad \text{and}$$
$$f^2 \equiv \frac{n}{4\pi^2} y_H^2 y_N^2 \sin^2 2\beta\,, \quad g \equiv \frac{n}{4\pi^2} y_H y_N.$$

The least tuning condition

Inserting F into G,

$$G = R^3 \left[e^{\frac{F^2}{R^2}} \left(\frac{F^2}{R^2} + \log R^2 - 1 \right) - \left(\log R^2 - 1 \right) \right]$$

If F is fixed, G is minimized at $R \approx F/(1+\varepsilon)$, $\varepsilon \ll 1$.
When G is minimized,

$$r^2 \approx 1.72 + 5.44\,\varepsilon\,,$$

$$G \approx F^3\,[\,(1.72 + 0.28\,\varepsilon)\log F^2 + (1 - \varepsilon)\,]$$

The least tuning condition

$0.3 < r < 1.8$ for $-0.3 < \varepsilon < 0.3$.

So μ_N and m_N need to be comparable to each other to minimize G.

But F and G is Not much sensitive to r.

G could be further minimized with a small F.

Since $\Delta m_h^2 \approx m_h^2 - M_Z^2\cos^2 2\beta - \Delta m_h^2|_{\mathrm{MSSM}}$,

F^2 is minimized when $\sin^2 2\beta = 1$ (or $\tan\beta = 1$):

$$F^2 \approx \frac{m_h^2 - M_Z^2 - \Delta m_h^2|_{\mathrm{MSSM}} + M_Z^2 \sin^2 2\beta}{\frac{n}{4\pi^2}(y_H y_N)^2 v_H^2 \sin^2 2\beta} \geq \frac{m_h^2 - \Delta m_h^2|_{\mathrm{MSSM}}}{\frac{n}{4\pi^2}(y_H y_N)^2 v_H^2}$$

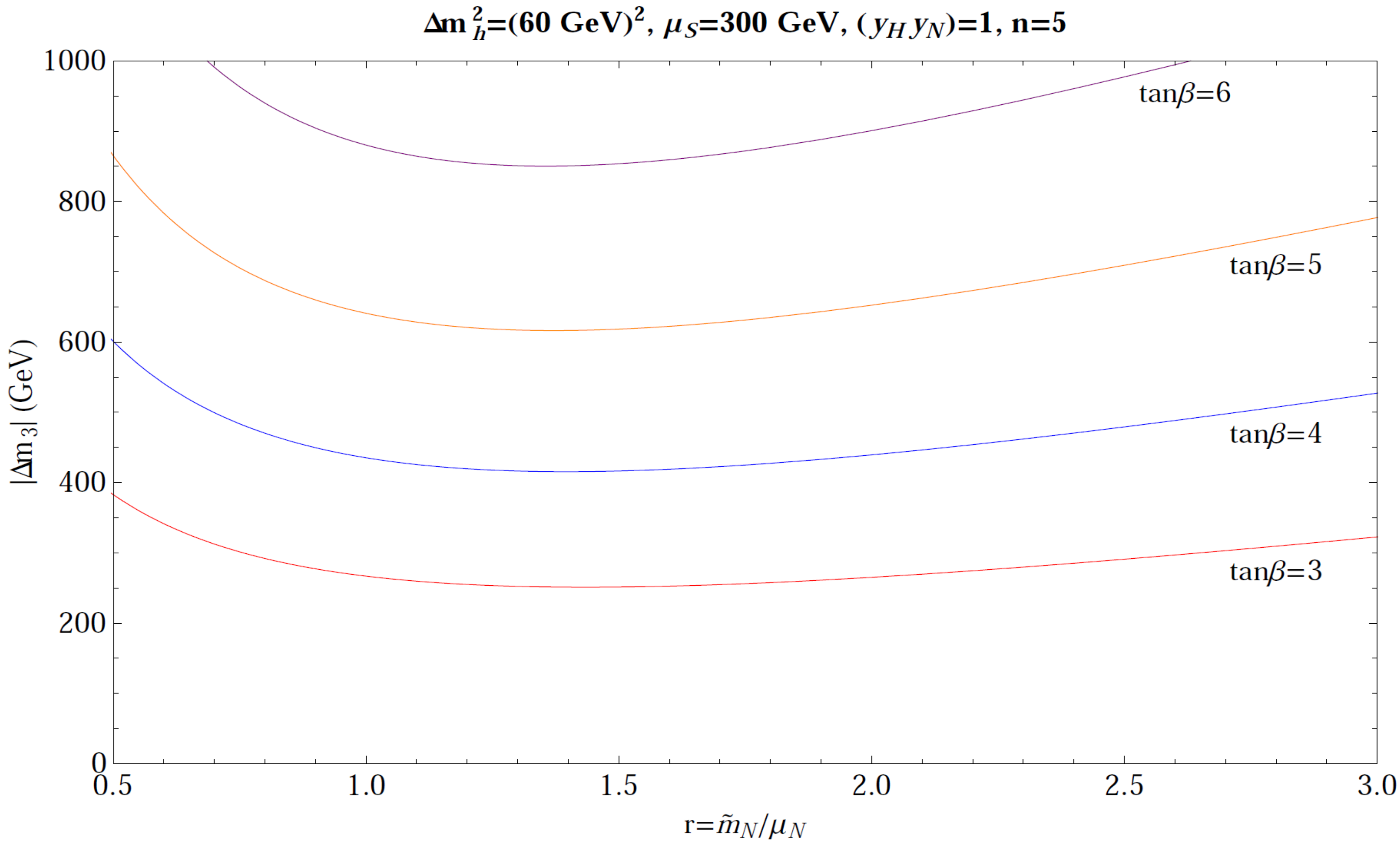

FIG. 3: Radiative correction $|\Delta m_3|$ ($\equiv \sqrt{B_\mu \mu}$) vs. $\widetilde{m}_N/\mu_N$ for various values of tanβ. The radiative correction to the Higgs mass Δm_h^2 is set to (60 GeV)^2. Thus, $|\Delta m_h|_{\rm MSSM} \approx (68, 70, 75, 82)$ GeV for tan$\beta = (6, 5, 4, 3)$ are assumed to be supplemented from the (s-) top's contributions for the 125 GeV Higgs mass. They correspond to $\widetilde{m}_t \approx (530, 590, 780, 1300)$ GeV at two-loop level, when turning off the mixing effect of $(\tilde{t}_L, \tilde{t}_R)$. We fix the other parameters as shown in the figure.

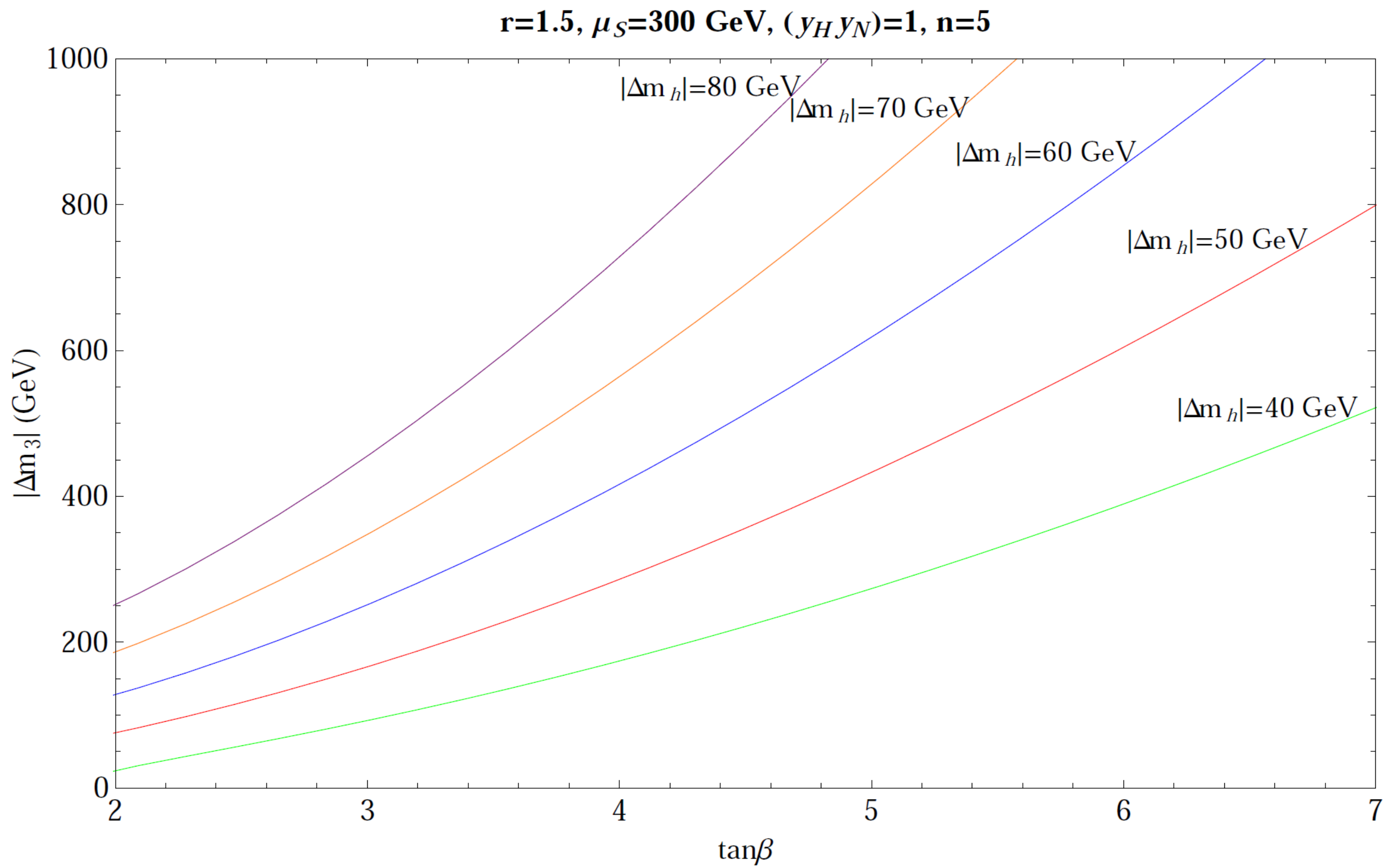

FIG. 4: Radiative correction $|\Delta m_3|$ ($\equiv \sqrt{B_\mu \mu}$) vs. $\tan\beta$ for various values of Δm_h^2 around the least tuning points ($\widetilde{m}_N/\mu_N \approx 1.5$). $|\Delta m_h| = (80, 70, 60, 50, 40)$ GeV for $\tan\beta = 5$ require the supplements of $|\Delta m_h|_{\rm MSSM} \approx (47, 61, 70, 78, 83)$ GeV, respectively, by the (s-) top's contributions. They correspond to $\widetilde{m}_t \approx (230, 390, 590, 940, 1400)$ GeV at two-loop level, when turning off the mixing effect of $(\tilde{t}_L, \tilde{t}_R)$. The other parameters are fixed as shown in the figure.

The solution of the naturalness problem and the gauge coupling unification, which are the great achievements of the MSSM, can still be valid.

Diphoton Decay Enhancement

$$\frac{\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)}{[\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)]_{\mathrm{SM}}} \sim 1.5 - 2,$$

$$\frac{\sigma(gg \to h) \times \mathrm{Br}(h \to VV)}{[\sigma(gg \to h) \times \mathrm{Br}(h \to VV)]_{\mathrm{SM}}} \sim 1,$$

If the large excess persists even after further more precise analyses with more data, one must seriously consider the possibility of the presence of new charged particles at low energies.

By assigning the EM charges to N, N^c,
the diphoton excess can be explained.

Diphoton Decay Enhancement

After integrating out S^c,

$$-L_{eff} = (y_H y_N / \mu_S)\, H_u H_d N N^c + \text{h.c.}$$

$$M_N \approx \mu_N - v_u v_d / \mu_S$$

$$R_{\gamma\gamma} \approx \left| 1 - \frac{y_H y_N}{\sqrt{2}} \frac{v_H^2 \sin 2\beta}{\mu_S M_N} \frac{n Q_N^2 \left\{ A_{1/2}(x_N) + \mathcal{O}\left(\frac{m_h}{\tilde{m}_N}\right) \right\}}{A_1(x_W) + 3\left(\frac{2}{3}\right)^2 A_{1/2}(x_t)} \right|^2$$

where $x_i \equiv 4m_i^2/m_h^2$

$$A_1(x) = -x^2\left[2x^{-2} + 3x^{-1} + 3\left(2x^{-1} - 1\right) f(x^{-1})\right],$$

$$A_{1/2}(x) = 2x^2\left[x^{-1} + \left(x^{-1} - 1\right) f(x^{-1})\right],$$

where $f(x^{-1}) \equiv \arcsin^2 x^{-1/2}$

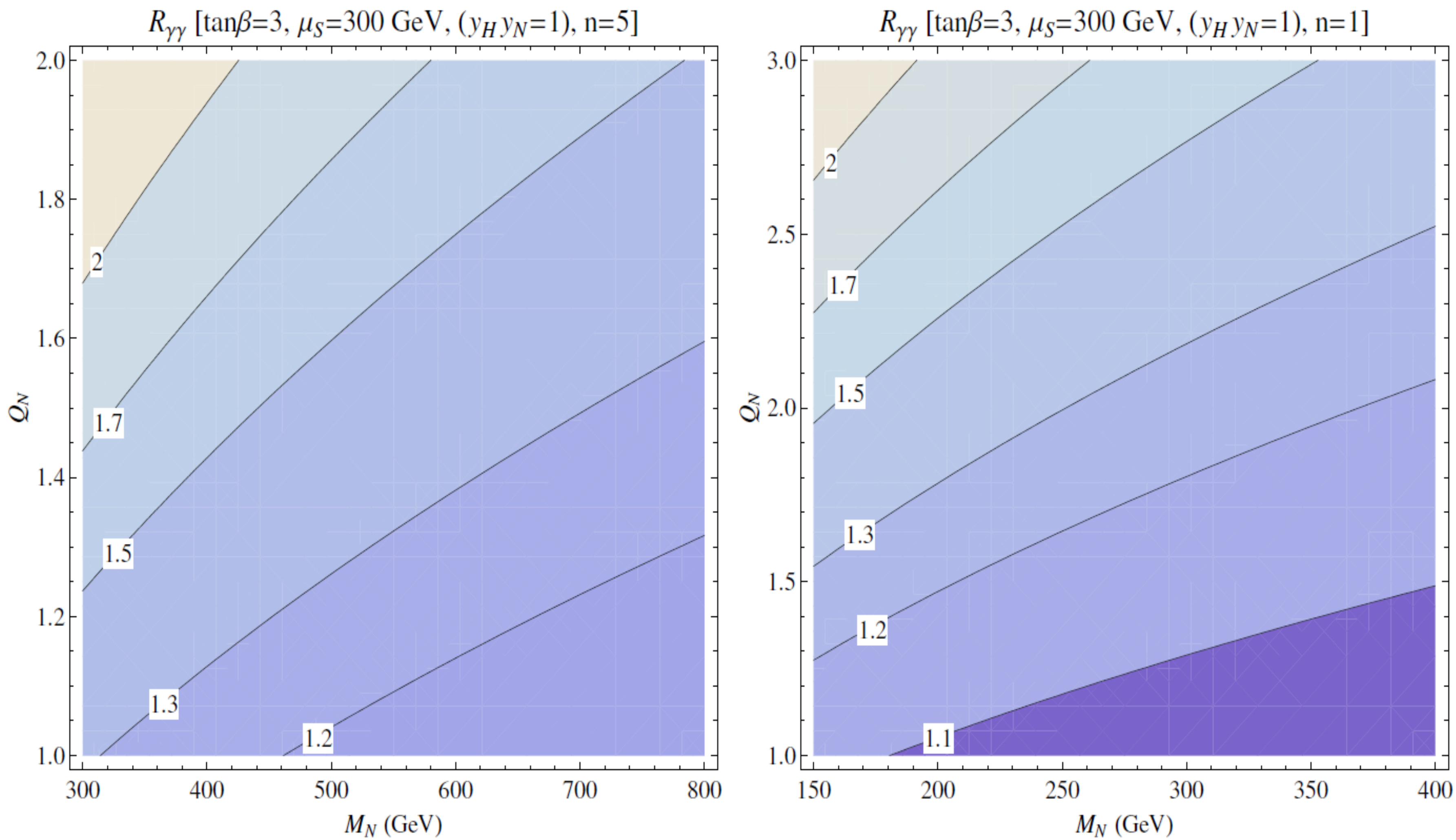

FIG. 5: Contour plots for the enhancement factor over the SM diphoton width in the M_N–Q_N plane. We fix the other parameters as shown in each figure.

The mechanism of raising the Higgs mass and
mitigating the fine-tuning
can be closely associated with
the excess of the diphoton decay rate of the Higgs
in this framework.

The Model

$$W = (\mu + y_H S) H_u H_d + \mu_S S S^c + (\mu_N + y_N S^c) N N^c$$

$$W_{\mathrm{UV}} = y_H S H_u H_d + y_N S^c N N^c$$

$$+ \frac{f_1}{M_P} \Sigma_1^2 H_u H_d + \frac{f_2}{M_P} \Sigma_2^2 N N^c + \frac{f_3}{M_P} \Sigma_3^2 S S^c$$

$$+ \frac{g_1}{M_P} \Sigma_3 \Sigma_1 \overline{\Sigma}_1^2 + \frac{g_2}{M_P} \Sigma_3 \Sigma_2 \overline{\Sigma}_2^2 + \frac{g_3}{M_P} \Sigma_3^2 \overline{\Sigma}_3^2$$

The Model

Superfields	H_u	H_d	N	N^c	S	S^c	Σ_1	Σ_2	Σ_3	$\overline{\Sigma}_1$	$\overline{\Sigma}_2$	$\overline{\Sigma}_3$
$\mathrm{U(1)_R}$	0	0	0	0	2	2	1	1	-1	1	1	2
$\mathrm{U(1)_{PQ}}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{4}$

TABLE I: R and Pecci-Quinn charges of the superfields. The MSSM *matter* superfields carry the unit R charges, and also the PQ charges of $1/8$. N and N^c are assumed to be proper n-dimensional vector-like representations of a hidden gauge group, under which all the MSSM fields are neutral. Σs and $\overline{\Sigma}$s carry some Z_2 charges.

The Model

The "A-terms" corresponding to the g_1, g_2, g_3 terms
and
the soft mass terms admit the VEVs,

$$< \Sigma_{1,2,3} > \sim < \Sigma^c_{1,2,3} > \sim (m_{3/2} M_P)^{1/2}$$

Then,
$f_i \sum_i^2 / M_P \sim m_{3/2}$. So μ, M_S, M_N are of EW scale.

The Model

The "A-terms" corresponding to the g_1, g_2, g_3 terms
and
the soft mass terms admit the VEVs,

$$<\Sigma_{1,2,3}> \ \sim \ <\Sigma^c_{1,2,3}> \ \sim \ (m_{3/2} M_P)^{1/2}$$

The domain wall problem can be avoided,
if $T_r < 10^9$ GeV.

Conclusion

- SUSY Higgs mass can increase through the radiative correction by Hidden sector fields, which is transmitted to the Higgs via a messenger field with 300 GeV mass.

- Since the Higgs mass is raised by the superpot. para., lifting the Higgs mass is quite efficient as in the extra matt. scenario.

- But our model is free from the constraint by LHC exp. on extra colored particles with order one Yukawa coupling with the Higgs boson.

Conclusion

- The fine-tuning problem asso. with the light Higgs mass can be remarkably mitigated by taking low enough messenger (～300 GeV) and mass para. (< 500 GeV) scale, explaining the 125 GeV Higgs mass.

- The mechanism of raising the Higgs mass and mitigating the fine-tuning can be closely associated with the excess of the diphoton decay rate of the Higgs in this framework.

4^{th} family, extra vec.-like matter

$W = M_Q Q Q^c + y Q H_u u^c$ if one pair of extra $\{Q, Q^c\}$ introduced,

$$\Delta V = (3/16\pi^2) \left[(M^2 + m^2)^2 \{ \log(M^2 + m^2/\Lambda^2) - 3/2 \} - M^4 \{ \log(M^2/\Lambda^2) - 3/2 \} \right] + \text{const.}$$

where $M^2 = M_Q{}^2 + y^2 |H_u|^2$

Λ : renorm. scale

(All the soft mass squareds are set to be m^2 for simplicity.)

$$
\begin{aligned}
V_{\mathrm{HS}} = &\left(m_2^2 + |\mu + y_H \widetilde{S}|^2\right)|H_u|^2 + \left(m_1^2 + |\mu + y_H \widetilde{S}|^2\right)|H_d|^2 \\
&+ \left(\widetilde{m}_{S^c}^2 + \mu_S^2\right)|\widetilde{S}^c|^2 + \left(\widetilde{m}_S^2 + \mu_S^2\right)|\widetilde{S}|^2 + y_H^2 |H_u H_d|^2 \\
&+ \left[\left(y_H \mu_S \widetilde{S}^{c*} + B_\mu \mu + y_H A_S \widetilde{S}\right) H_u H_d + B_S \mu_S \widetilde{S}\widetilde{S}^c + \mathrm{h.c.}\right] \\
&+ \frac{1}{8}(g^2 + g'^2)\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 + \Delta V(\widetilde{S}^c) \ ,
\end{aligned}
$$

1-loop Effective Potential: integrating out $\{\delta N, \delta N^c\}$

$$V = V_{tree} + \Delta V(S^c)$$

$$\Delta V(S^c) = (n/16\pi^2)\,[\,(M_N^2 + m_N^2)^2\,\{\log(M_N^2 + m_N^2/\Lambda^2) - 3/2\,\} - M_N^4\,\{\log(M_N^2/\Lambda^2) - 3/2\,\}\,]$$

where $\quad M_N^2 = |\mu_N + y_N S^c|^2$

Extrm. Cond. (e.o.m): $\partial_{Nc} V = \partial_N V = 0$. The solns. are inserted in V.

1-loop Effective Potential: integrating out $\{S, S^c\}$

$$V = V_{tree} + \Delta V(S^c)$$

Extrm. Cond. (e.o.m) : $\partial_{Sc} V = \partial_S V = 0,$

$$S^c \approx (-1/\mu_S^2)\,[\, y_H \mu_S H_u H_d + \partial_{Sc^*} \Delta V \,]\,,$$

$$S \approx (-1/m_S^2 + \mu_S^2)\,[\, y_H (A_S^* - B_S^*) H_u H_d - (B_S^*/\mu_S) \partial_{Sc^*} \Delta V \,]\,,$$

Plugging them into V,

1-loop Effective Higgs Potential

$$V_H \approx V_{MSSM} - |A_S - B_S|^2/(m_S{}^2+\mu_S{}^2)\ y_H{}^2 |H_u H_d|^2 + \Delta V(H)\ ,$$

$$M_N{}^2 \approx \mu_N{}^2 - (y_H y_N\, \mu_N/\mu_S)\, h_u h_d$$

$\mathrm{Re}\, H_{u,d} = h_{u,d} / \sqrt{2}$