Lattice Calculation of Kaon Mixing Matrix Elements from BSM Operators

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The 2nd KIAS Phenomenology Workshop, Sep. 10 – 14, 2012, KIAS

Outline

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② Lattice QCD

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Motivation & Background

Neutral Kaon System

- **Flavor eigenstates**

$$K^0 = (\bar{s}d), \qquad \bar{K}^0 = (s\bar{d})$$

- **CP eigenstates**

$$K_\pm = (K^0 \pm \bar{K}^0)/\sqrt{2}, \qquad CP|K_\pm\rangle = \pm|K_\pm\rangle$$

- **Hamiltonian eigenstates**

$$K_S = \frac{K_+ + \bar{\epsilon}K_-}{\sqrt{1+|\bar{\epsilon}|^2}}, \qquad K_L = \frac{K_- + \bar{\epsilon}K_+}{\sqrt{1+|\bar{\epsilon}|^2}}, \qquad |\bar{\epsilon}| \simeq O(10^{-3})$$

- **Preferable decays to pion states**

$$K_S \to 2\pi \text{ (via } K_+, CP \text{ even)}$$
$$K_L \to 3\pi \text{ (via } K_-, CP \text{ odd)}$$

Direct / Indirect CP Violation

$$K_L \sim K_- + \bar{\epsilon} K_+$$

$\bar{\epsilon}K_+ \xrightarrow{\text{Indirect CPV : } \epsilon} \pi\pi$

$K_- \xrightarrow{\text{Direct CPV : } \epsilon'} \pi\pi$

- **CP violating $K_L \to \pi\pi$** can occur in two ways
  - ❖ **K_-(CP odd) $\to$ $\pi\pi$(CP even)** : Direct CPV

$$\epsilon' = \frac{1}{\sqrt{2}}\left(\frac{A[K_L \to (\pi\pi)_2]}{A[K_S \to (\pi\pi)_2]} - \epsilon \frac{A[K_S \to (\pi\pi)_2]}{A[K_S \to (\pi\pi)_0]}\right)$$

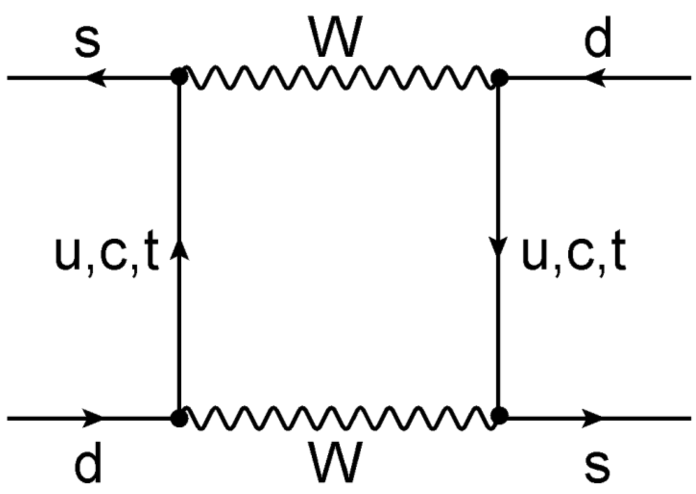
  - ❖ **$\bar{\epsilon}K_+$(CP even) $\to$ $\pi\pi$(CP even)** : Indirect CPV

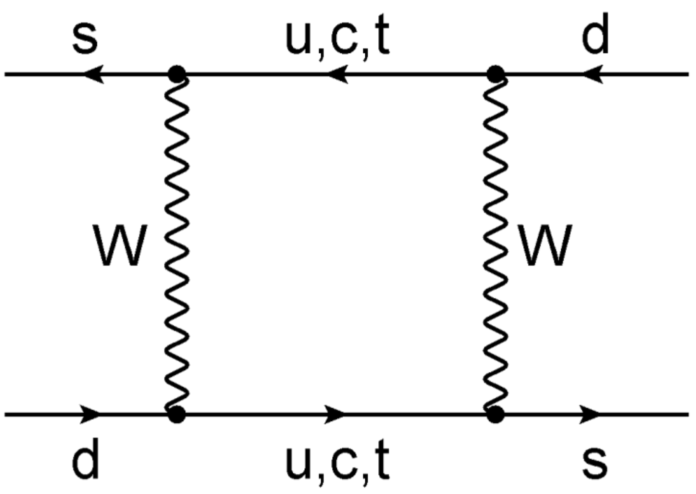
$$\epsilon = \frac{A[K_L \to (\pi\pi)_0]}{A[K_S \to (\pi\pi)_0]}$$

K_L can have small CP even component via $K^0 - \bar{K}^0$ mixing

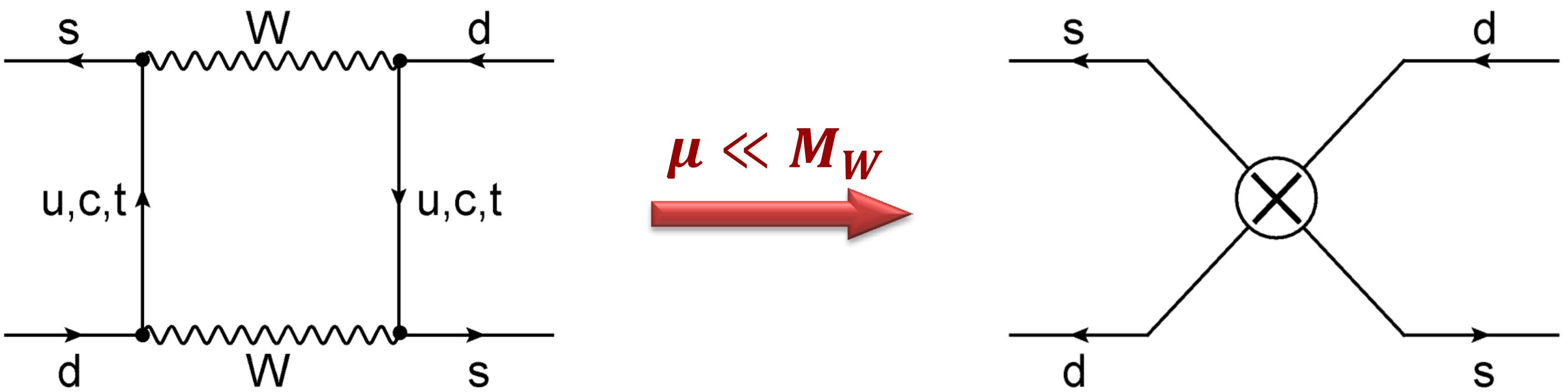
$K^0 - \bar{K}^0$ Mixing in the SM

- Arises from the $\Delta S = 2,\ s\bar{d} \to \bar{s}d$ FCNC
- Responsible for indirect CPV & $\Delta \mathrm{M_K} \equiv M_{K_L} - M_{K_S}$
- Dominated by the following box diagrams





$K^0 - \bar{K}^0$ Mixing in the SM



- Integrating out W, the box diagram can be replaced by the local, four-quark operator

$$H_{\text{eff}}^{\Delta S=2} = \frac{G_F^2 M_W^2}{16\pi^2} F^0 Q_1 + h.c.,$$

$$Q_1 = \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma_\mu(1-\gamma_5)d\,\right]$$

Kaon Bag Parameter - B_K

- In the SM, **indirect CPV** can be predicted as follows

$$\epsilon_K \sim \text{known factors} \times V_{CKM} \times \hat{B}_K$$

- $\hat{B}_K$ is the RG invariant form of B_K

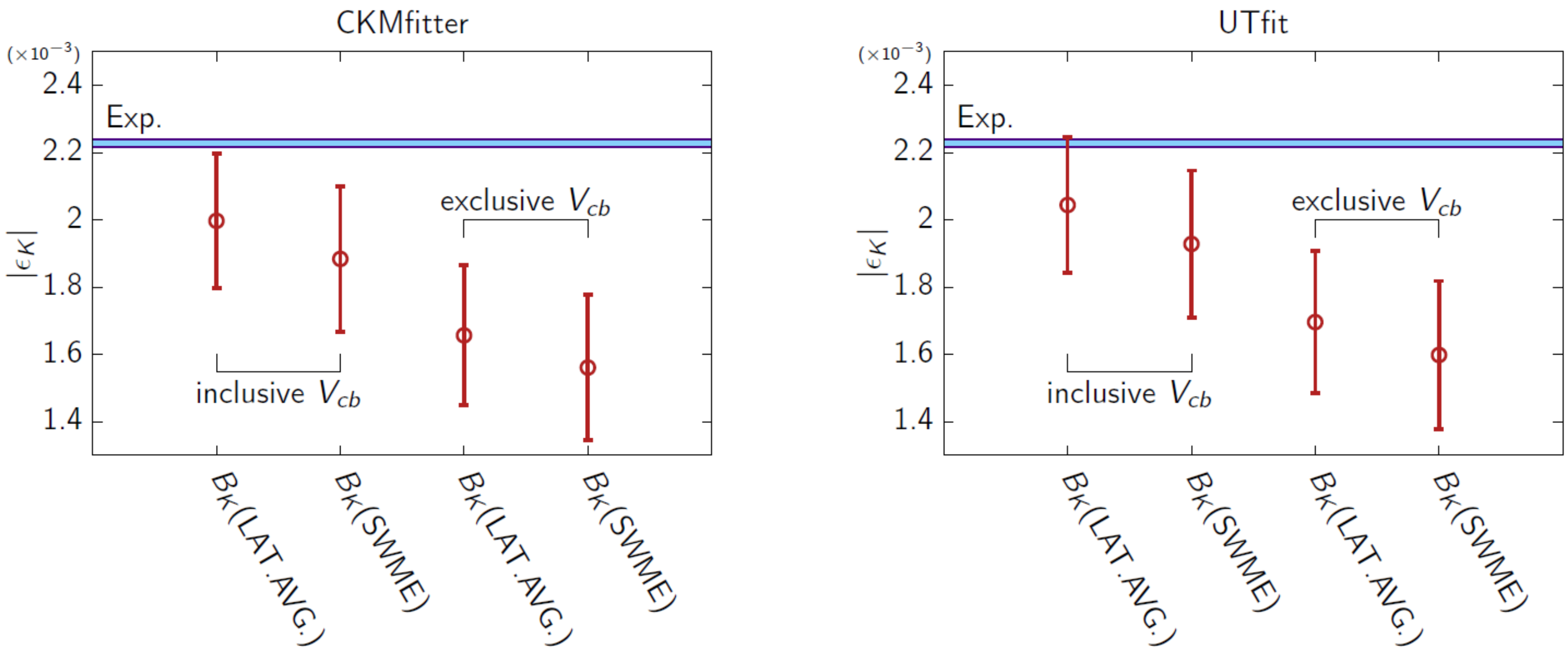
$$B_K = \frac{\langle \bar{K}^0 | [\bar{s}\gamma_\mu (1-\gamma_5) d][\bar{s}\gamma_\mu (1-\gamma_5) d] | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s}\gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu \gamma_5 d | K^0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu)$$

- $\hat{B}_K$ contains all the **non-perturbative QCD contributions for ϵ_K,** can be **calculated from lattice simulations**

Experiment vs SM Prediction on ϵ_K

(Y. Jang & W. Lee, 2012)



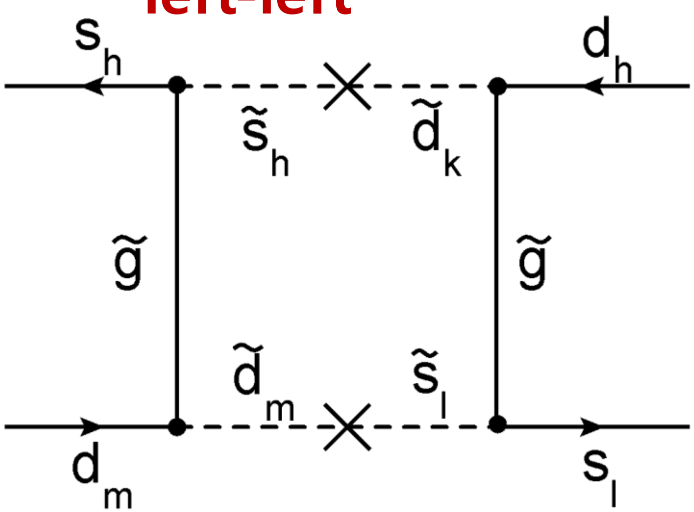
- There are two methods(exclusive, inclusive) to determine V_{cb}, whose results **do not agree each other**
- SM prediction of ϵ_K deviates from the experimental value about 3σ for exclusive V_{cb}

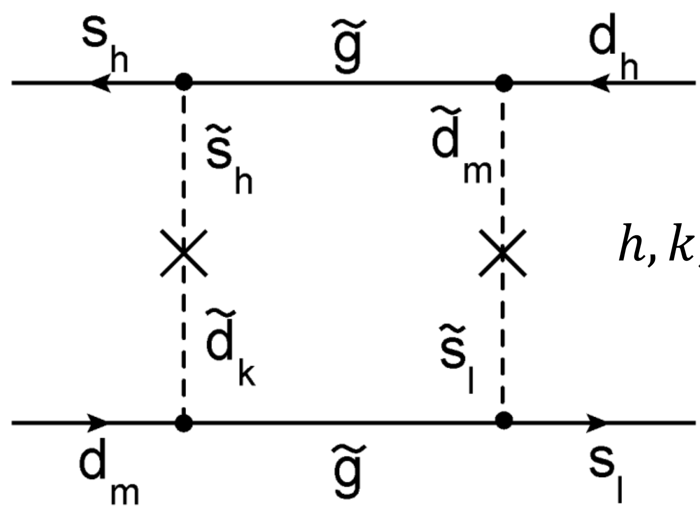
BSM Contribution to $K^0 - \bar{K}^0$ Mixing

- **In the Standard Model**, only the **"left-left" form** contribute to the $K^0 - \bar{K}^0$ **mixing** box diagram

$$\langle \bar{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K^0 \rangle$$

- Considering **BSM physics**, integrating out heavy particles (e.g. squarks & gnuinos in supersymmetric models) leads to **new operators w/ Dirac structures other than "left-left"**





$h, k, l, m \in \{L, R\}$

BSM Operators

- Considering BSM, generic effective Hamiltonian is

$$H_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i\, Q_i$$

$$Q_2 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b]$$

$$Q_3 = [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b]$$

$$Q_4 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b]$$

$$Q_5 = [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]$$

- New $\Delta S = 2$ four-fermion operators give **additional contributions to Kaon mixing** elements
- Since they are **constrained by experimental results**, calculating corresponding hadronic matrix elements

$$\langle \bar{K}^0 | Q_i | K^0 \rangle$$

can impose strong constraints on BSM physics

BSM B-parameters

- **B-parameters**

$$B_i = \frac{\langle \bar{K}^0 | Q_i | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}$$

$$Q_2 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b]$$

$$Q_3 = [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b]$$

$$Q_4 = [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b]$$

$$Q_5 = [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]$$

$$(N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3)$$

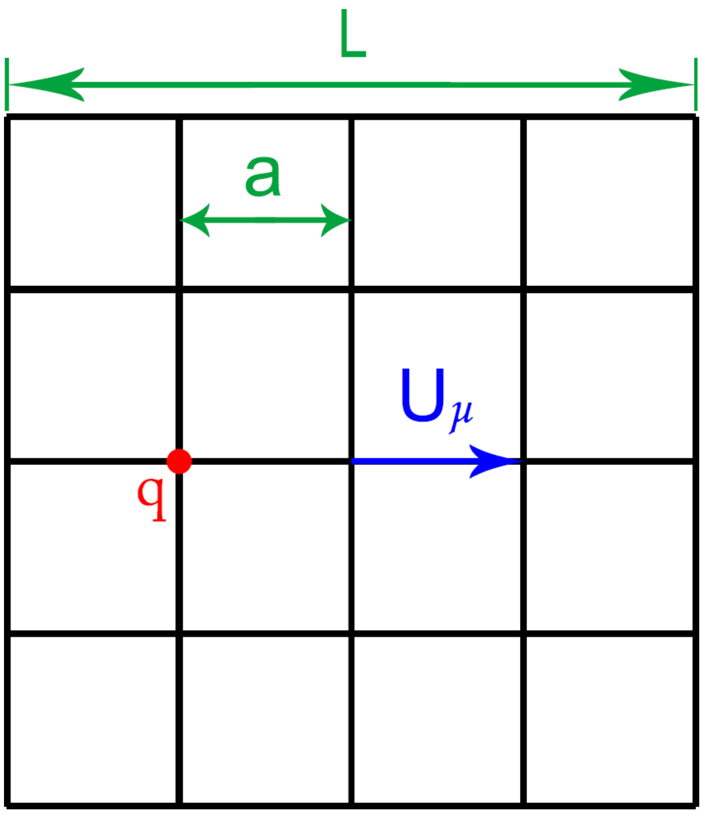
  - In lattice calculation, forming dimensionless ratio **reduces statistical and systematic error**
  - **Chiral perturbation** expression is **simpler**
  - Denominator dose not vanish in chiral limit (unlike B_K)

$$\langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle = -\left(\frac{f_K M_K^2}{m_d + m_s}\right)^2$$

Lattice QCD

Lattice QCD

- **Non-perturbative** approach to solving QCD
- Formulation of QCD on **discretized Euclidean space-time**
  - Hypercubic lattice
  - **Lattice spacing "a"**
  - Quark fields placed on sites
  - Gauge fields on the links between sites : U_μ



Lattice QCD

- Use **numerical method** (Montecarlo simulation) to calculate integral

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \mathcal{D}\Psi \mathcal{D}\bar{\Psi}\ \mathcal{O} e^{-S}$$

- "**Lattice action**" is needed to simulate in discretized space-time

$$S[U, \bar{\Psi}, \Psi] = S_G[U] + S_F[U, \bar{\Psi}, \Psi]$$

- We use "**Staggered fermion**" for the lattice fermion
  - The **fastest** lattice fermion action
  - Suffered from "**taste symmetry breaking**" but manageable

Lattice QCD

- **Expectation value**

$$\langle \mathcal{O}(U,q,\bar{q}) \rangle = \int \mathcal{D}U \mathcal{D}q \mathcal{D}\bar{q}\ \mathcal{O}(U,q,\bar{q}) \times e^{-S_G - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$= \int \mathcal{D}U\ \mathcal{O}\left(U, (D[U]+m_f)^{-1}\right) \times e^{-S_G[U]} \prod_f \det\left(D[U]+m_f\right)$$

- Integrating over the q and $\bar{q}$ gives determinant of Dirac operator and quark propagators, $\left(D[U]+m_f\right)^{-1}$
- Generate **random samples (gauge links)** according to the **probability distribution** allows us to integrate using **Montecarlo method**

Lattice QCD

$$\langle f(X)\rangle = \int dx\, f(x)\, p_X(x)$$

- **Expectation value**

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \int \mathcal{D}U\mathcal{D}q\mathcal{D}\bar{q}\; \mathcal{O}(U,q,\bar{q}) \times e^{-S_G-\sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$\boxed{= \int \mathcal{D}U\; \mathcal{O}\Big(U,(D[U]+m_f)^{-1}\Big) \times \boxed{e^{-S_G[U]} \prod_f \det\left(D[U]+m_f\right)}}$$

- Integrating over the q and $\bar{q}$ gives determinant of Dirac operator and quark propagators, $\left(D[U]+m_f\right)^{-1}$
- Generate **random samples (gauge links)** according to the **probability distribution** allows us to integrate using **Montecarlo method**

Data Analysis

Physical Results from Unphysical Simulations

- **Chiral extrapolation**
  - In the lattice simulation, the smaller quark mass requires the exponentially larger computational cost
    - ➢ Use light quark masses larger than physical d-quark mass and extrapolate to the physical down quark mass using (staggered) chiral perturbation theory
  - Tuning the strange quark mass to precise physical quark mass is not practical
    - ➢ Extrapolate to physical strange quark mass
- **Continuum extrapolation**
  - We use finite lattice spacing ($a \geq 0.45\text{fm}$)
    - ➢ Extrapolate to $a = 0$ to obtain continuum results

Data Analysis Strategy

1. **Calculate raw data**

   Calculate BSM B-parameters for different quark mass combinations, $(m_u = m_d, m_s)$

2. **Chiral fitting**

   X-fit : Fix strange quark mass and extrapolate to the light quark mass m_l to give physical down quark mass

   Y-fit : Extrapolate m_s to give physical strange quark mass

3. **RG evolution**

   Obtain results at 2GeV and 3GeV from 1/a

4. **Continuum extrapolation**

   Perform [1-3] for different lattices and extrapolate to $a = 0$

Analysis Data

a (fm)	1/a (GeV)	am_l/am_s	geometry	ens×meas
0.12	1.662	0.01/0.05	$20^3 \times 64$	671×9
0.09	2.348	0.0062/0.031	$28^3 \times 96$	995×9
0.06	3.362	0.0036/0.018	$48^3 \times 144$	749×9
0.045	4.517	0.0028/0.014	$64^3 \times 192$	747×1

- MILC 2+1 AsqTad lattice
  - Use u, d, s dynamical quarks
  - $m_u = m_d \neq m_s$
- Four different lattices

Operator Matching

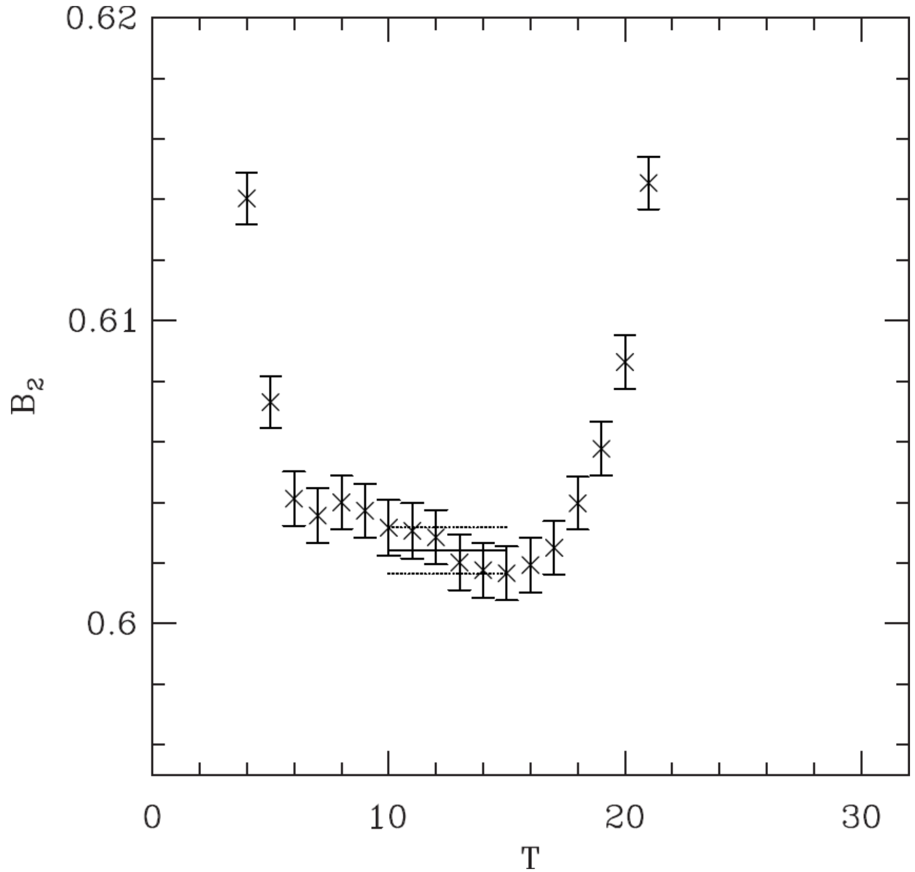
$$\mathcal{O}_i^{\mathrm{Cont}'} = \sum_{j\in(A)} z_{ij}\mathcal{O}_j^{\mathrm{Lat}} - \frac{g^2}{(4\pi)^2}\sum_{k\in(B)} d_{ik}^{\mathrm{Lat}}\mathcal{O}_k^{\mathrm{Lat}}$$

$$z_{ij} = b_{ij} + \frac{g^2}{(4\pi)^2}\Big(- \gamma_{ij}\log(\mu a) + d_{ij}^{\mathrm{Cont}} - d_{ij}^{\mathrm{Lat}} - C_F I_{MF} T_{ij}\Big)$$

- To find continuum (NDR with $\overline{\mathrm{MS}}$) results from those regularized on the lattice, "**operator matching**" is needed
- z_{ij} are the **one-loop** matching factors
  (J. Kim, W. Lee and S. Sharpe, PhysRevD.83.094503)
- We use matching scale $\mu = 1/a$

Calculation of B_j

$$B_2 = \frac{\langle \overline{K}^0 \big| [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] \big| K^0 \rangle}{(5/3)\langle \overline{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}$$



Chiral Fitting

- **Fitting functions for X-fit** (J. Bailey, et al., Phys. Rev. D85, (2012), 074507)

$$B_j(X_P) = c_1 F_0(j) + c_2 \frac{X_P}{\Lambda^2} + c_3 \frac{X_P^2}{\Lambda^4} \quad (\mathrm{S\chi PT, NNLO})$$

where X_P is the squared mass of pion, $\Lambda = 1\mathrm{GeV}$,

$$F_0(j) = 1 \pm \frac{1}{32\pi^2 f^2}\{l(X_I) + (L_I - X_I)\tilde{l}(X_I) - 2\langle l(X_B)\rangle\}$$

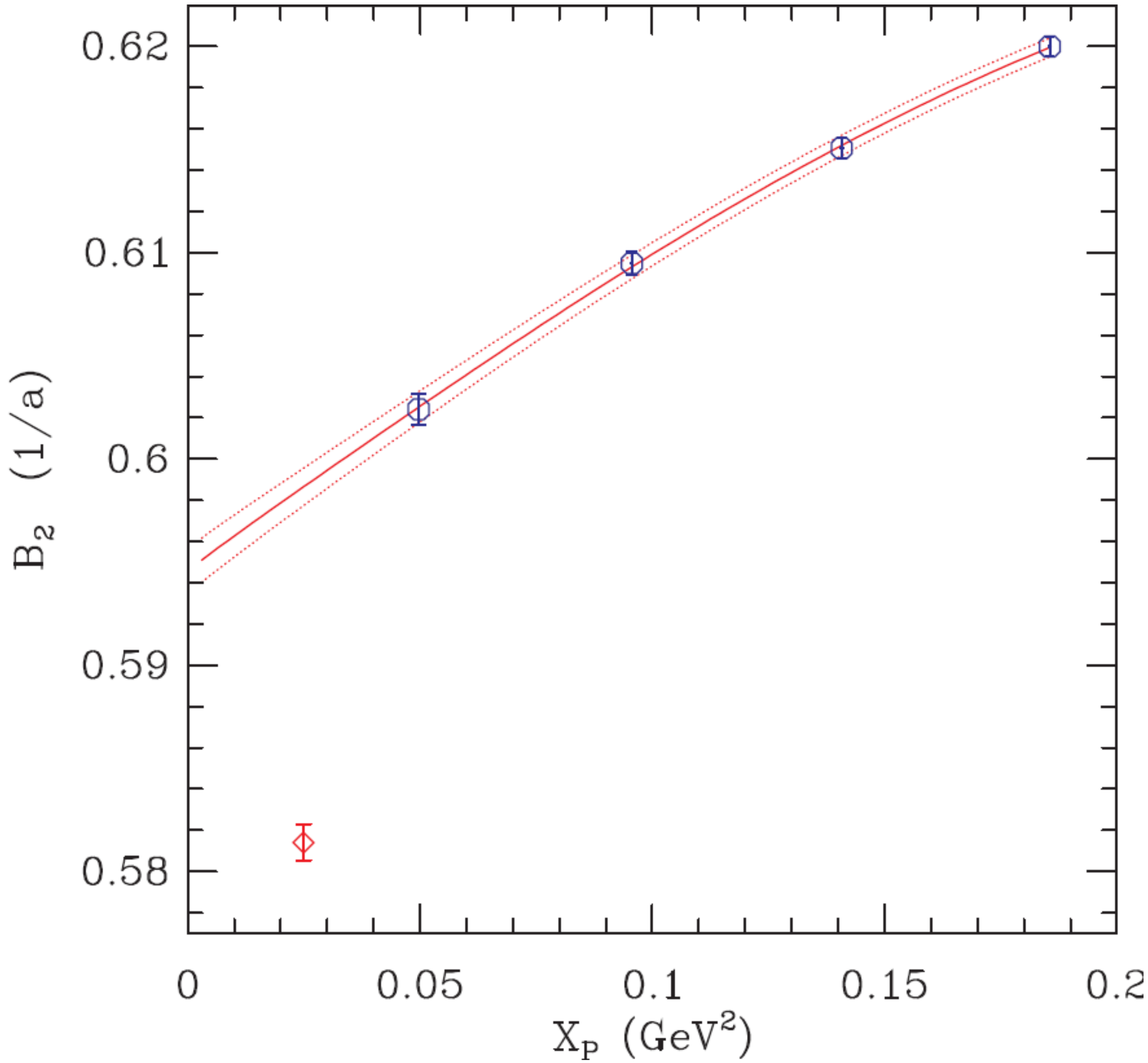
($+$ for $j = 2,3,K$, $-$ for $j = 4,5$)

- **Golden combinations**
  - Combinations that <u>cancel the leading chiral logarithms</u>

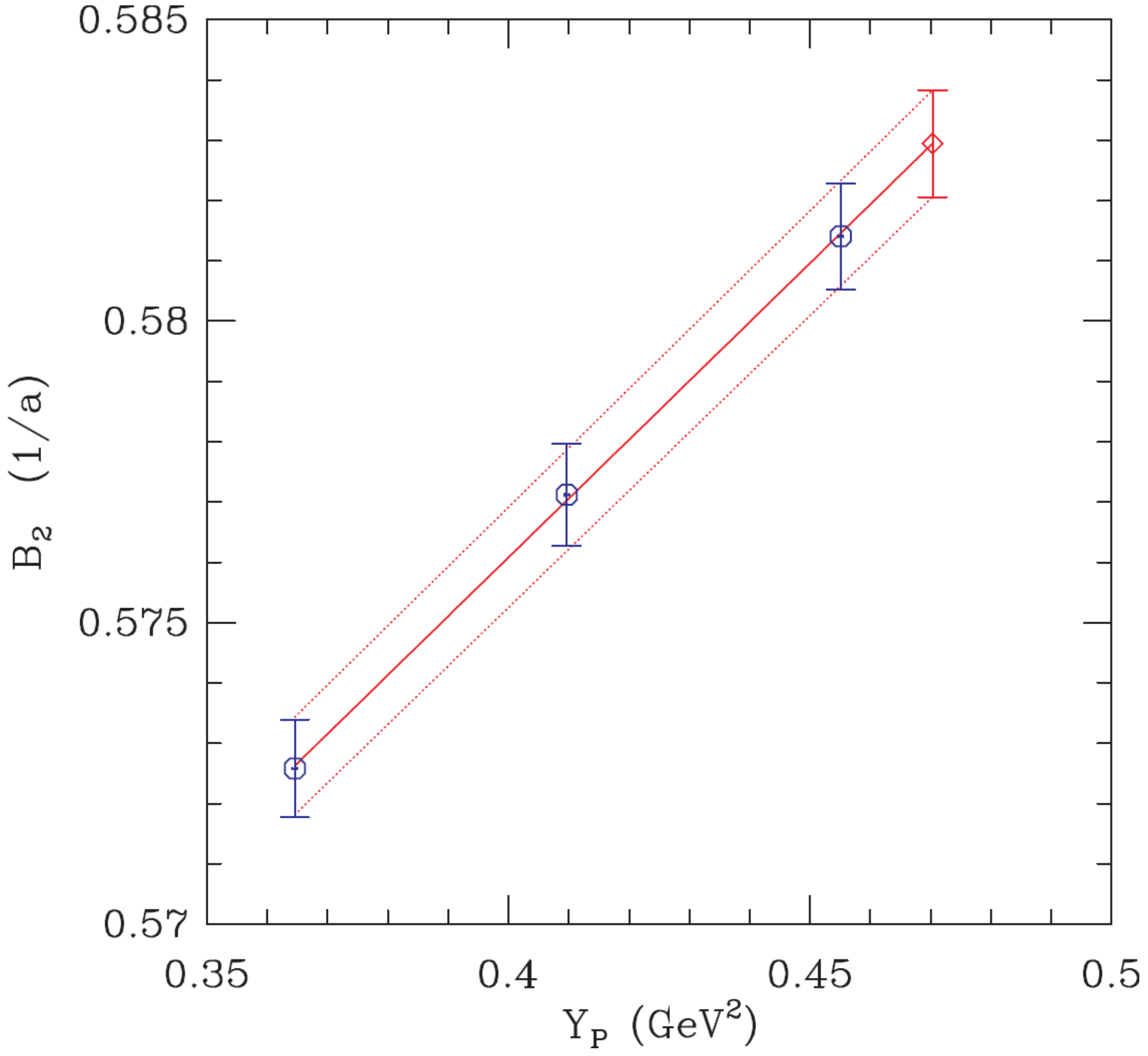
$$\left(\frac{B_2}{B_3}, \quad \frac{B_4}{B_5}, \quad B_2 \cdot B_4, \quad \frac{B_2}{B_K}\right)$$

$$R(X_P) = c_1 + c_2 \frac{X_P}{\Lambda^2} + c_3 \frac{X_P^2}{\Lambda^4} \quad (\mathrm{NNLO})$$

Chiral Fitting : X-fit



Chiral Fitting : Y-fit



RG Evolution

- Now we have B-parameter values at μ =1/a
- To perform **continuum extrapolation** with different lattices, **we need B-param. values at a common scale**
- RG running from μ_a(1/a) to μ_b(2GeV, 3GeV)

$$B_j(\mu_b) = \sum_k \frac{1}{N_j} W^R(\mu_b, \mu_a)_{jk} N_k B_k(\mu_a)$$

- Evolution kernels satisfy the RG equation

$$\frac{dW(\mu_b, \mu_a)}{d \ln \mu_b} = -\gamma(\mu_b) W(\mu_b, \mu_a), \qquad W(\mu_a, \mu_b) = 1$$

$$\gamma(\mu) = \frac{\alpha(\mu)}{4\pi}\gamma^{(0)} + \left(\frac{\alpha(\mu)}{4\pi}\right)^2 \gamma^{(1)} + \cdots$$

Continuum Extrapolation

- **Formula**
  - **Bayesian fit**

    $B_j(a^2) = c_1 + c_2^b(a\Lambda)^2 + c_3^b(a\Lambda)^2\alpha_s + c_4^b\alpha_s^2 + c_5^b(a\Lambda)^4$

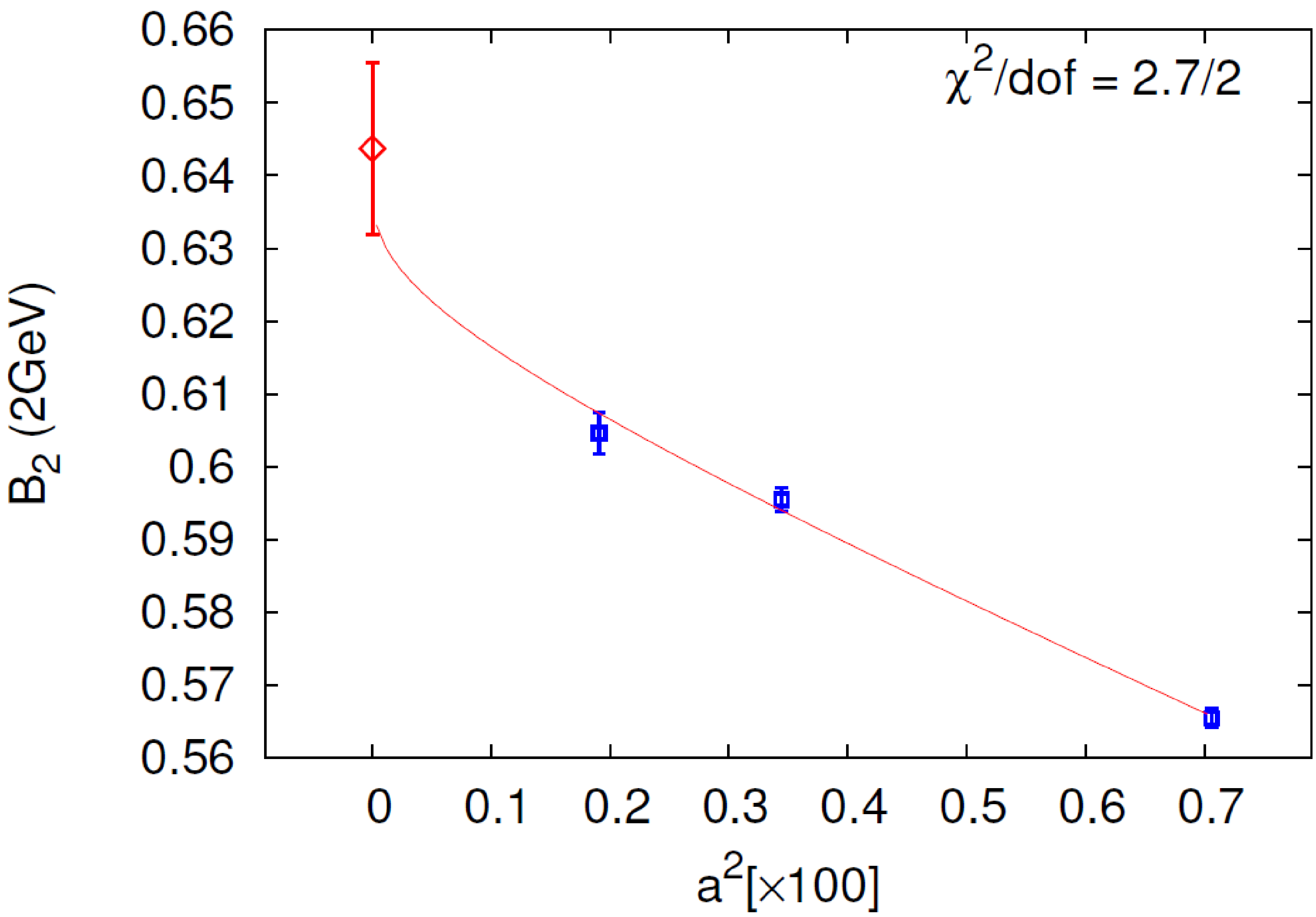
    $\Lambda = 300\mathrm{GeV}$, c_i^b are constrained by $c_i^b \approx 0 \pm 2$

  - **Linear fit**

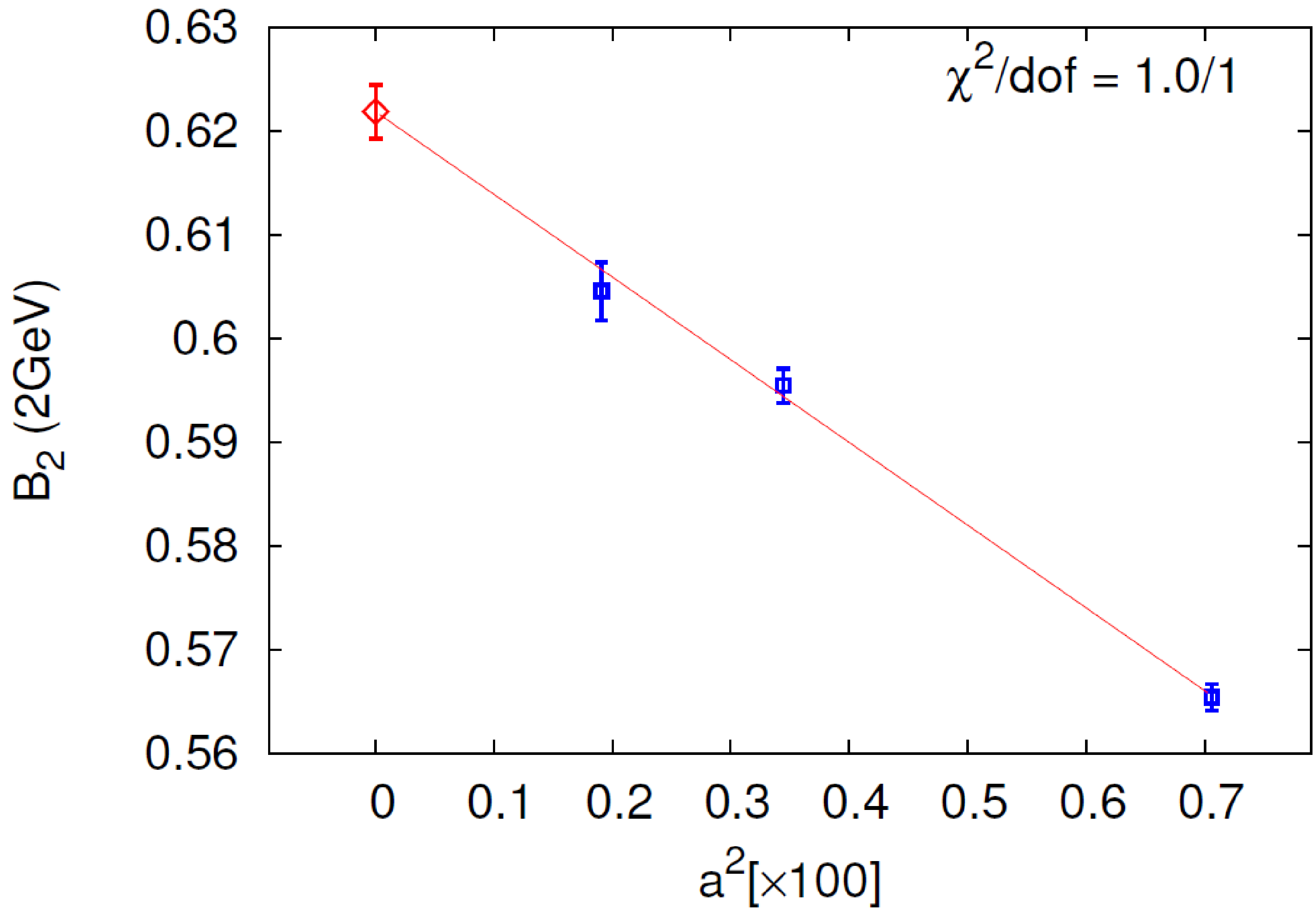
    $B_j(a^2) = c_1 + c_2 a^2$

- **Results**
  - Final results are obtained using linear fit without coarse lattice ($a \approx 0.12\mathrm{fm}$)

Continuum Extrapolation : Bayesian fit



Continuum Extrapolation : Linear fit



Results

BSM B-parameters at 2GeV and 3GeV

2GeV	Coarse	Fine	Superfine	Ultrafine	Continuum
B_K	0.5651(46)	0.5296(39)	0.5351(35)	0.5320(77)	0.5379(65)
B_2	0.5415(08)	0.5654(13)	0.5955(16)	0.6046(28)	0.6219(26)
B_3	0.3699(06)	0.4158(09)	0.4590(13)	0.4801(21)	0.5019(20)
B_4	1.0944(20)	1.1228(25)	1.0927(33)	1.0949(53)	1.0736(51)
B_5	0.9260(17)	0.9356(22)	0.8890(27)	0.8725(44)	0.8467(43)

3GeV	Coarse	Fine	Superfine	Ultrafine	Continuum
B_K	0.5459(44)	0.5115(38)	0.5169(34)	0.5139(75)	0.5195(63)
B_2	0.4798(07)	0.5009(11)	0.5275(15)	0.5355(25)	0.5509(23)
B_3	0.3169(05)	0.3511(08)	0.3843(10)	0.3998(17)	0.4167(16)
B_4	1.0456(19)	1.0726(24)	1.0438(32)	1.0457(51)	1.0255(49)
B_5	0.9124(17)	0.9250(21)	0.8834(27)	0.8711(43)	0.8473(42)

Preliminary!

Summary

Summary

- BSM physics leads to new $\Delta S = 2$ four-fermion operators that contribute to $K^0 - \bar{K}^0$ mixing
- Calculating corresponding hadronic matrix elements, $\langle \bar{K}^0 | Q_i | K^0 \rangle$, can impose strong constraints on BSM physics
- We calculate BSM B-parameters on the lattice and present preliminary results
- We are working on estimating systematic errors