# Yukawa hierarchies from spontaneous flavor symmetry breaking

Chee Sheng Fong

INFN - Laboratori Nazionali di Frascati, ITALY

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[E. Nardi, PRD84, 036008 (2011)]
 [J.R. Espinosa, CSF, E. Nardi, To appear]
 [CSF, E. Nardi, Work in progress]

- 1 Motivation: The origin of the Yukawa hierarchies ?
- Yukawa hierarchies from SSB First attempt: Via the one-loop effective potential Second attempt: Via reducible representations
- 3 Marriage between up and down
- Occursion and on-going work

#### Outline

#### 1 Motivation: The origin of the Yukawa hierarchies ?

- 2 Yukawa hierarchies from SSB First attempt: Via the one-loop effective potential Second attempt: Via reducible representations
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Isidor Rabi, "Who ordered that?"

#### Why does Nature repeat herself?



#### Why does Nature repeat herself ... Twice?

C. S. Fong (INFN, Frascati)

The family in the SM remains a puzzle. One can ask: are they actually the *same* particles?



Designer: André-Pierre Olivier http://particlequest.com/

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One can ask: are they actually the same particles?

• NO. They are different at the fundamental level but the replication is just an illusion of the low energy theory e.g. Froggatt-Nielsen mechanism [Froggatt, Nielsen (1979)], Randall-Sundrum warped extra-dimensional model (1999) [Bauer, Casagrande, Goertz, Haisch, Neubert, Pfoh, JHEP0810:094 (2008), JHEP1009:017 (2010)] and refs. therein

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- YES. They are exact replica at the fundamental level, but the flavor (continuous) symmetry (global or gauged) is broken spontaneously (SSB) at low energy.

[Mohapatra (1974)], [Mohapatra,Pati,Wolfenstein (1975)], [Wilczek, Zee (1979)], [Wilczek (1982)], [Reiss (1982)], ..., [Koide (2008), (2009)], [Koide, Nishiura (2012)], [Feldmann, Jung, Mannel (2009)], [Albrecht, Feldmann, Mannel (2010)], [Grinstein, Redi, Villadoro (2010)], [Alonso, Gavela, Merlo, Rigolin (2011)], [Nardi (2011)], [Guadagnoli, Mohapatra, Sung (2011)]

In the *Standard Model* (SM), the *q*uark and *l*epton gauge invariant kinetic terms possess the global symmetry [Chivukula, Georgi (1987)]

$$\begin{array}{lll} \mathcal{G} &=& \mathcal{G}^q \times \mathcal{G}^\ell \\ \mathcal{G}^q &=& U(3)_Q \times U(3)_u \times U(3)_d \\ \mathcal{G}^\ell &=& U(3)_\ell \times U(3)_e \end{array}$$

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Yukawa terms break explicitly the  $\mathcal{G} \rightarrow U(1)_B$  (for massive Majorana neutrinos)

$$\begin{aligned} \mathcal{G}_{\mathcal{F}} &= \mathcal{G}_{\mathcal{F}}^{q} \times \mathcal{G}_{\mathcal{F}}^{\ell} \\ \mathcal{G}_{\mathcal{F}}^{q} &= SU(3)_{Q} \times SU(3)_{u} \times SU(3)_{d} \times U(1)_{u} \times U(1)_{d} \\ \mathcal{G}_{\mathcal{F}}^{\ell} &= U(3)_{\ell} \times U(3)_{e} \end{aligned}$$

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## Yukawa hierarchies from SSB: The T, A, D invariants

Consider single sector with (global) flavor symmetry (ignore the U(1)'s)

 $\mathcal{G}_{\mathcal{F}} = SU(3)_L \times SU(3)_R$ 

with  $\psi_L(3,1), \ \psi_R(1,3), \ Y(3,\bar{3})$ 

The Yukawa interaction from dimension five operator:  $-\mathcal{L}_Y = \frac{1}{\Lambda} \overline{\psi}_L Y \psi_R H$ Writing down the characteristic equation for the eigenvalues  $\xi$  of  $YY^{\dagger}$ 

$$\det(\xi I_{3\times 3} - YY^{\dagger}) = \xi^3 - T\xi^2 + A\xi - \mathcal{D}\mathcal{D}^* = 0$$

we identify the three invariants:

 $T = \operatorname{Tr}(YY^{\dagger})$   $A = \operatorname{Tr}[\operatorname{Adj}(YY^{\dagger})] = \frac{1}{2} \left[ T^{2} - \operatorname{Tr}(YY^{\dagger}YY^{\dagger}) \right]$   $\mathcal{D} = \operatorname{det}(Y) = e^{i\delta}D$ 

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## Yukawa hierarchies from SSB: The scalar potential

The most general *renormalizable* potential for Y invariant under  $SU(3)_L \times SU(3)_R$ :

$$rac{\hat{V}_0}{\Lambda^4}\equiv V_0=V_T+V_A+V_D$$

$$V_T = \lambda \left( T - \frac{m^2}{2\lambda} \right)^2 = \lambda \left( T - v^2 \right)^2, \quad V_A = \lambda' A,$$
  
$$V_D = \tilde{\mu} \mathcal{D} + \tilde{\mu}^* \mathcal{D}^* \equiv 2\mu D \cos \left( \phi_\mu + \phi_D \right)$$

Will be not depend on the cut-off scale  $\Lambda$  or the flavor breaking scale v.

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The most general background field (remove 6 moduli, 8 phases)

$$\langle Y \rangle = \frac{1}{\sqrt{2}} \operatorname{diag}(R_{11}, R_{22}, R_{33} + iJ_{33})$$

At the minimum  $\phi_{\mu} + \phi_{D} = \pi$  such that  $V_{D}^{\min} = V_{D} = -2\mu D$ . We have degenerate but *inequivalent* vacua. As far as hierarchical spectrum is concern, we can take  $\phi_{\mu} = \pi$  and

$$\langle Y \rangle = \frac{1}{\sqrt{2}} \operatorname{diag}(R_{11}, R_{22}, R_{33}), \quad R_{ii} \ge 0$$

#### Yukawa hierarchies from SSB: The tree-level vacua

[Alonso et. al. (2011)], [Nardi (2011)]

$$T = \frac{1}{2} \left( R_{11}^2 + R_{22}^2 + R_{33}^2 \right), \quad A = \frac{1}{4} \left( R_{11}^2 R_{22}^2 + R_{11}^2 R_{33}^2 + R_{22}^2 R_{33}^2 \right), \quad D = \frac{1}{\sqrt{2}} R_{11} R_{22} R_{33}$$

At tree-level (i) For  $\lambda' < 0$ , we have  $\langle Y \rangle^s = \frac{1}{\sqrt{3}} v \operatorname{diag}(1, 1, 1)$   $\implies \langle D \rangle^{1/3} \approx \langle A \rangle^{1/4} \approx \langle T \rangle^{1/2}$  (nonhierarchical)  $\mathcal{G}_{\mathcal{F}} \rightarrow H_s = SU(3)_{L+R}$ 

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#### At tree-level (i) For $\lambda' < 0$ , we have $\langle Y \rangle^s = \frac{1}{\sqrt{3}} \nu \operatorname{diag}(1, 1, 1)$ $\implies \langle D \rangle^{1/3} \approx \langle A \rangle^{1/4} \approx \langle T \rangle^{1/2}$ (nonhierarchical) $\mathcal{G}_{\mathcal{F}} \rightarrow H_s = SU(3)_{L+R}$

(ii) For  $\lambda' > 0$ , we have  $\langle Y \rangle^h = v \operatorname{diag}(0,0,1)$  as long as [Nardi (2011)]

$$V_0(\langle Y \rangle^s) > 0 \implies \frac{\mu^2}{m^2} < 2\lambda \left[ \left( 4 + \frac{\lambda'}{\lambda} \right)^{3/2} - \left( 8 + 3\frac{\lambda'}{\lambda} \right) \right]$$

Then  $\langle T \rangle \approx 1$  and  $\langle D \rangle = \langle A \rangle = 0$ . (hierarchical)

$$\mathcal{G}_{\mathcal{F}} \to H_h = SU(2)_L \times SU(2)_R \times U(1)_{L+R}$$

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**The question**: Can the zeros be lifted?  $\langle Y \rangle^h = v \operatorname{diag}(0,0,1) \longrightarrow v \operatorname{diag}(\epsilon',\epsilon,1)$ 

• It was hypothesized that loop corrections to the effective potential could yield a structure  $\langle Y \rangle \sim v \operatorname{diag}(\epsilon', \epsilon, 1)$  i.e.  $\mathcal{G}_{\mathcal{F}} \rightarrow U(1)^2_{L+R}$  [Nardi (2011)]

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- Michel's conjecture [Michel (1979)] states that the maximal little groups  $H_s$  and  $H_h$  are the maximal stability groups of the most general <u>4-th order</u> function of the invariants. Is this true also at the loop-level?

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Including the  $SU(3)_L \times SU(3)_R$  invariant one-loop Coleman Weinberg effective potential [Coleman, Weinberg (1973)], [Jackiw (1974)] we have

$$V_{\text{eff}} = V_0 + V_1,$$
  

$$V_1 = \frac{1}{64\pi^2} \sum_i M_i^4(Y) \left[ \log \frac{M_i^2(Y)}{\Lambda^2} - \frac{3}{2} \right]$$

with  $M_i^2(Y)$  the eigenvalues of

$$\left[\mathcal{M}\right]_{ij,kl} = \frac{\partial^2 V_0}{\partial \mathcal{Y}_{ij} \partial \mathcal{Y}_{kl}} \bigg|_{\langle Y \rangle}, \qquad \mathcal{Y}_{ij} = \{\operatorname{Re}(Y_{ij}), \operatorname{Im}(Y_{ij})\}$$

(A) Brute force verification [Espinosa, CSF, Nardi, To appear]: We determined the analytical expressions for the eigenvalues (18th-order polynomial equation! But we somehow managed ...)

$$\det(M^2 \cdot I_{18 \times 18} - \mathcal{M}^2) = P^{(6)}(M^2) \times \prod_{i=1}^3 (M^2 - M_{i+1}^2)^2 (M^2 - M_{i-1}^2)^2 = 0$$

Numerically minimized the effective potential and found that vacuum structure remains separated into two:  $\langle Y \rangle^s \sim v \operatorname{diag}(1,1,1)$  and  $\langle Y \rangle^h \sim v \operatorname{diag}(0,0,1)$ 

## First attempt: Via the one-loop effective potential

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(B) A heuristic argument:



More rigorous proof in [Georgi, Pais (1977)].

C. S. Fong (INFN, Frascati)

#### Comments:

1.) <u>Cannot</u> change the tree-level vacuum through perturbative quantum effects and Michel's conjecture still holds.

2.) This result would also hold when one considers dim > 4 *nonrenormalizable* terms e.g.  $\frac{c}{\Lambda^2} \text{Tr}(YY^{\dagger}YY^{\dagger}YY^{\dagger})$  as long as they are *perturbatively* smaller the tree level terms (can be proved following Georgi-Pais approach).

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3.) In fact, we know *better* than that ... using Cayley-Hamilton theorem, it can be shown that all dim > 4 invariant terms can always be written in terms of T, A, D. Since the highest power of T, A, D is  $(YY^{\dagger})^2$ , it can be proven that at stationary points, in general, we can obtain at most two distinct eigenvalues of  $(YY^{\dagger})_c = \text{diag}(a, a, b)$  with  $a \neq b$  i.e. fully hierarchical solution is not possible with nonrenormalizable terms.

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4.) We need to break  $\mathcal{G}_{\mathcal{F}} \to U(1)^2_{L+R}$  (at least) already at tree-level.

A minimal enlargement of the scalar sector by introducing

 $Z_L(3,1), Z_R(1,3)$ 

The most general *renormalizable*  $SU(3)_L \times SU(3)_R$  invariant scalar potential:

$$\begin{split} V &= \lambda' A + \tilde{\mu} \mathcal{D} + \tilde{\mu}^* \mathcal{D}^* + V_l + V_m + V_\nu \\ V_l &= \lambda \left( T - v^2 \right)^2 + \lambda_L \left( |Z_L|^2 - v_L^2 \right)^2 + \lambda_R \left( |Z_R|^2 - v_R^2 \right)^2 \\ &+ g \left[ \left( T - v^2 \right) + \frac{g_{1L}}{g} \left( |Z_L|^2 - v_L^2 \right) + \frac{g_{1R}}{g} \left( |Z_R|^2 - v_R^2 \right) \right]^2 , \\ V_m &= g_{2L} Z_L^{\dagger} Y Y^{\dagger} Z_L + g_{2R} Z_R^{\dagger} Y^{\dagger} Y Z_R , \\ V_{\tilde{\nu}} &= \tilde{\nu} Z_l^{\dagger} Y Z_R + \tilde{\nu}^* Z_R^{\dagger} Y^{\dagger} Z_L \equiv 2\nu |Z_l^{\dagger} Y Z_R| \cos \phi_{LR} \end{split}$$

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Again as far as hierarchical spectrum is concern, we can take  $\arg \nu = \pi$  and all fields to be *real* and *positive*.

Consider only the case  $g_{2L}, g_{2R}, \lambda' > 0$ .

Since  $V_l$  fixes the 'lengths', we only have to consider

$$V_{\epsilon} = \lambda' A - 2\mu D + g_{2L} Z_L^T Y Y^T Z_L + g_{2R} Z_R^T Y^T Y Z_R - 2\nu Z_L^T Y Z_R$$

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For simplicity setting  $v = v_L = v_R = 1$  and  $g_{2L} = g_{2R} = \lambda'$ , then solving  $\frac{\partial V_{\epsilon}}{\partial \epsilon} = \dots = 0$  and by truncating to terms  $\mathcal{O}(\epsilon^2)$  we obtain a unique *global* minimum [Espinosa, CSF, Nardi, To appear]

$$\epsilon' = \frac{\lambda'\nu}{3\lambda'^2 - \mu^2}, \quad \epsilon = \frac{\mu}{\lambda'}\epsilon', \quad \epsilon_{L,R} = \epsilon'_{L,R} = 0$$

For example for  $\nu \sim \mu \sim 10^{-2} \lambda'$ , we have  $\epsilon' \sim 10^{-2}$  and  $\epsilon \sim 10^{-4}$ .

$$V = \dots - 2\mu D + 2\nu |Z_L^{\dagger} Y Z_R|$$

The vacuum

$$\begin{array}{lll} \langle Y \rangle &\simeq & v \operatorname{diag}(\epsilon, \epsilon', 1) & \epsilon' = \frac{\lambda' \nu}{3\lambda'^2 - \mu^2}, & \epsilon = \frac{\mu}{\lambda'} \epsilon' \\ \langle Z_L \rangle &\simeq & v_L \left(0, 1, 0\right) & -\mathcal{L}_Y = \frac{1}{\Lambda} \overline{\psi}_L Y \psi_R H + \frac{1}{\Lambda^2} \overline{\psi}_L Z_L Z_R^{\dagger} \psi_R H \\ \langle Z_R \rangle &\simeq & v_R \left(0, 1, 0\right) & \end{array}$$

#### Comments:

1) Small couplings terms linear in *Y*, can be achieved if *Y* charged under U(1)2) dim=6 Non-MFV, but can be forbidden assuming no *scalar doublets* in the UV complete theory.

The symmetry of the new vacua is

$$\begin{aligned} H_{LR} &= SU(2)_L \times SU(2)_R, \quad H_Y = U(1)_{L+R}^2, \\ H' &= H_{LR} \cap H_Y = U(1)_{L+R} \end{aligned}$$

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#### Marriage between up and down [CSF, Nardi, Work in progress]

Now we want to build a scalar potential invariant under  $SU(3)_Q \times SU(3)_u \times SU(3)_d$ 

The *minimal* number of fields:  $Y_u(3,\overline{3},1), Y_d(3,1,\overline{3}), Z_Q(3,1,1), Z_u(1,3,1), Z_d(1,1,3)$ 

The most general *renormalizable* potential:  $V = V_l + V_u + V_d + V_{ud}$ 

$$V_{l} = \lambda_{u} \left( T_{u} - v_{u}^{2} \right)^{2} + \lambda_{d} \left( T_{d} - v_{d}^{2} \right)^{2} + \lambda_{zQ} \left( |Z_{Q}|^{2} - w_{Q}^{2} \right)^{2} + \dots + \eta \left[ \left( T_{u} - v_{u}^{2} \right)^{2} + \frac{\eta_{d}}{\eta} \left( T_{d} - v_{d}^{2} \right)^{2} + \frac{\beta_{Q}}{\eta} \left( |Z_{Q}|^{2} - w_{Q}^{2} \right)^{2} + \dots \right] V_{u} = \lambda_{u}' A_{u} + \mu_{u} \mathcal{D}_{u} + \mu_{u}^{*} \mathcal{D}_{u}^{*} + g_{Qu} Z_{Q}^{\dagger} Y_{u} Y_{u}^{\dagger} Z_{Q} + g_{u} Z_{u}^{\dagger} Y_{u}^{\dagger} Y_{u} Z_{u} + \nu_{u} Z_{Q}^{\dagger} Y_{u} Z_{u} + \nu_{u}^{*} z_{u}^{\dagger} Y_{u}^{\dagger} Z_{Q} V_{d} = \dots V_{ud} = \lambda_{ud} T_{ud} + \gamma_{ud} Z_{u}^{\dagger} Y_{u}^{\dagger} Y_{d} Z_{d} + \gamma_{ud}^{*} Z_{d}^{\dagger} Y_{d}^{\dagger} Y_{u} Z_{u}$$

where  $T_{ud} \equiv \operatorname{Tr}\left(Y_u Y_u^{\dagger} Y_d Y_d^{\dagger}\right)$ 

#### Marriage between up and down - cont.

For the constant background fields  $Y_u$  and  $Y_d$ : 18 moduli and 18 phases With  $V_Q$ ,  $V_u$  and  $V_d$ :  $3 \times 3$  moduli and  $3 \times 5$  phases So, we end up with 9 moduli and 3 phases. We parametrize

$$Y_u = \operatorname{diag}(y_u, y_c, y_t) \operatorname{diag}(1, 1, e^{i\phi_u})$$
  

$$Y_d = V_{mix} \operatorname{diag}(y_d, y_s, y_b) \operatorname{diag}(1, 1, e^{i\phi_d})$$

where y's and  $\phi$ 's are real while

$$Z_Q^T = (z_{Q1}, z_{Q2}, z_{Q3}), \quad Z_u^T = (z_{u1}, z_{u2}, z_{u3}), \quad Z_d^T = (z_{d1}, z_{d2}, z_{d3})$$

with all z's complex.

With the standard parametrization

$$V_{mix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

with  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ . As far as mixing is concern, we can assume all fields to be *real* but in general, we should leave their *signs* free.

#### Marriage between up and down - More for Less

For complex fields, we actually have less possible vacua. The reason is at the minimum, all the complex terms will become *nonpositive* i.e. if they are zero, their phases are undetermined, otherwise their phases will become  $\pi$ .

 $\mu_u \mathcal{D}_u + \mu_u^* \mathcal{D}_u^*, \quad \nu_u Z_Q^{\dagger} Y_u Z_u + \nu_u^* z_u^{\dagger} Y_u^{\dagger} Z_Q, \quad \dots \quad , \quad \gamma_{ud} Z_u^{\dagger} Y_u^{\dagger} Y_d Z_d + \gamma_{ud}^* Z_d^{\dagger} Y_d^{\dagger} Y_u Z_u$ For example, by fixing  $\delta_{13} = \phi_u = \phi_d = 0$  and  $\arg(\mu_{u,d}) = \arg(\nu_{u,d}) = \arg(\gamma_{ud}) = \pi$ , we need to impose

$$\begin{array}{rcl} D_u &=& y_u y_c y_t \geq 0 \\ D_d &=& y_d y_s y_b \geq 0 \\ Z_Q^T Y_u^D Z_u &=& y_u z_{Q1} z_{u1} + y_c z_{Q2} z_{d2} + y_b z_{Q3} z_{d3} \geq 0 \\ Z_Q^T V_{mix} Y_d^D Z_d &=& y_d z_{Q1} z_{d1} + y_s z_{Q2} z_{d2} + y_b z_{Q3} z_{d3} + \text{mixing} \geq 0 \\ Z_u^T Y_u^{D,T} V_{mix} Y_d^D Z_d &=& y_u y_d z_{u1} z_{d1} + y_c y_s z_{u2} z_{d2} + y_t y_b z_{u3} z_{d3} + \text{mixing} \geq 0 \end{array}$$

We can choose all the fields to be real but due to the mixing terms, we should allow their freedoms of *sign*.

#### Marriage between up and down - cont.

For the <u>real</u> terms, we have the freedom to choose the signs of the couplings.

$$V_l, \quad \lambda'_u A_u, \quad g_{Qu} Z_Q^{\dagger} Y_u Y_u^{\dagger} Z_Q, \quad g_u Z_u^{\dagger} Y_u^{\dagger} Y_u Z_u, \quad \dots, \quad \lambda_{ud} T_{ud}$$

For 
$$\lambda_{ud}T_{ud} = \lambda_{ud}\operatorname{Tr} Y_u Y_u^{\dagger} Y_d Y_d^{\dagger} = \lambda_{ud} \sum_{i,j} y_i^2 y_j^2 |(V_{mix})_{ij}|^2$$
,  
(a) if  $\lambda_{ud} > 0$ , we have  $u - b, c - s, t - d$   
(b) if  $\lambda_{ud} < 0$ , we have  $u - d, c - s, t - b$   
Hence we require  $\lambda_{ud} < 0$ .

To obtain the proper mass hierarchies, all the other real couplings are chosen to be *positive*.

#### Marriage between up and down – Mixing

From the previous nonmixing solution, we know

$$Z_Q^T = (0, z_{Q2}, 0), \quad Z_u^T = (0, z_{u2}, 0), \quad Z_d^T = (0, z_{d2}, 0)$$

If we can lift the zeros of the Z's, we would be able to obtain nonzero mixing. Taking

$$Z_{Q}^{T} = (\delta_{Q}', z_{Q}, \delta_{Q}), \qquad Z_{u}^{T} = (\delta_{u}', z_{u}, \delta_{u}), \qquad Z_{d}^{T} = (\delta_{d}', z_{d}, \delta_{d})$$

we have

$$Z_{Q}^{T}V_{mix}Y_{d}^{D}Z_{d} = \left(\begin{array}{ccc}\delta_{Q}' & z_{Q} & \delta_{Q}\end{array}\right) \left(\begin{array}{ccc}V_{ud} & V_{us} & V_{ub}\\V_{cd} & V_{cs} & V_{cb}\end{array}\right) \left(\begin{array}{ccc}y_{d}\delta_{d}'\\y_{s}z_{d}\\y_{b}\delta_{d}\end{array}\right)$$
$$Z_{u}^{T}Y_{u}^{T}V_{mix}Y_{d}Z_{d} = \left(\begin{array}{ccc}y_{u}\delta_{u}' & y_{c}z_{u} & y_{t}\delta_{u}\end{array}\right) \left(\begin{array}{ccc}V_{ud} & V_{us} & V_{ub}\\V_{cd} & V_{cs} & V_{cb}\\V_{cd} & V_{cs} & V_{cb}\\V_{td} & V_{ts} & V_{tb}\end{array}\right) \left(\begin{array}{ccc}y_{d}\delta_{d}'\\y_{s}z_{d}\\y_{b}\delta_{d}\end{array}\right)$$

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we have

$$Z_Q^T V_{mix} Y_d^D Z_d = \begin{pmatrix} \delta'_Q & z_Q & \delta_Q \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} y_d \delta'_d \\ y_s z_d \\ y_b \delta_d \end{pmatrix}$$
$$Z_u^T Y_u^T V_{mix} Y_d Z_d = \begin{pmatrix} y_u \delta'_u & y_c z_u & y_t \delta_u \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} y_d \delta'_d \\ y_s z_d \\ y_b \delta_d \end{pmatrix}$$

In general, for fully hierarchical mass spectra, we can only get one nonzero mixing angle i.e.  $V_{cb}$ ,  $V_{ts} \neq 0$  (Warning: not a proof!)

#### Marriage between up and down - Extended scenario

We extend the model:

- a) We introduce a new field:  $Y_R(1,3,\overline{3})$
- b) Instead of  $Z_Q(1,3,1)$ , we would like to have  $Z_{Qu}(1,3,1)$  and  $Z_{Qd}(1,3,1)$

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To differentiate the two *Z* fields, we introduce a new symmetry  $U(1)_R$  such that  $R(Y_R) = R(Z_u) = R(Z_{Qu}) = 1$  while all other fields have R = 0. As a result, the following three terms are forbidden:

$$\gamma_{ud} Z_u^{\dagger} Y_u^{\dagger} Y_d Z_d, \quad \sigma \operatorname{Tr} Y_u Y_R Y_d^{\dagger}, \quad D_R \equiv \det(Y_R)$$

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Here we list a few relevant new terms

$$\begin{split} \lambda_{QQ} Z_{Qu}^{\dagger} Z_{Qd} Z_{Qd}^{\dagger} Z_{Qu} \\ \lambda_{R}^{\prime} A_{R}, \quad \lambda_{uR} \operatorname{Tr} Y_{u} Y_{R} Y_{R}^{\dagger} Y_{u}^{\dagger}, \quad \lambda_{dR} \operatorname{Tr} Y_{d} Y_{R}^{\dagger} Y_{R} Y_{d}^{\dagger} \\ \rho Z_{u}^{\dagger} Y_{R} Z_{d} \\ g_{uR} Z_{u}^{\dagger} Y_{R} Y_{R}^{\dagger} Z_{u}, \quad g_{dR} Z_{d}^{\dagger} Y_{R}^{\dagger} Y_{R} Z_{d} \\ \nu_{u} Z_{Qu}^{\dagger} Y_{u} Z_{u}, \quad \nu_{d} Z_{Qd}^{\dagger} Y_{d} Z_{d} \\ \zeta_{u} Z_{Qu}^{\dagger} Y_{u} Y_{R} Z_{d}, \quad \zeta_{d} Z_{Qd}^{\dagger} Y_{d} Y_{R}^{\dagger} Z_{u} \end{split}$$

Numerically use "random search" method in *Mathematica* to find a (global) minimum. Basically start with some random search points (100,200,400,...) and proceed with minimization algorithm. Then compare the values of the minima.

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b) The mixing are fixed by  $\lambda'_{R} = -2, \ \lambda_{ud} = -1.66, \ \lambda_{QQ} = 0.0236,$  $g_{uR} = g_{dR} = 0.10, \ \lambda_{uR} = \lambda_{dR} = -0.13, \ |\rho| = 0.6$ 

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A successful marriage! (Admittedly with some ad hoc parameters... minimal?)

$$Y_u^D = \text{diag}\left(-10^{-5}, -0.036, 1.47\right)$$
  

$$Y_d^D = \text{diag}\left(-10^{-3}, 0.036, -1.47\right)$$
  

$$V_{mix} = \begin{pmatrix} 0.974 & 0.225 & 0.0046 \\ -0.225 & 0.973 & 0.041 \\ 0.0046 & -0.041 & 0.999 \end{pmatrix}$$

#### The hierarchy between up and down sectors?

The full broken flavor symmetry is

$$SU(3)_Q \times SU(3)_u \times SU(3)_d \times U(1)_u \times U(1)_d$$

The effective Yukawa terms:

$$\frac{1}{\Lambda} \overline{Q} Y_{u} U \widetilde{H} + \frac{1}{\Lambda} \overline{Q} Y_{d} D H$$

$$+ \frac{1}{\Lambda^{2}} \overline{Q} Z_{Q_{u}} Z_{u}^{\dagger} U \widetilde{H} + \frac{1}{\Lambda^{2}} \overline{Q} Z_{Q_{d}} Z_{d}^{\dagger} D H + \frac{1}{\Lambda^{2}} \overline{Q} Y_{d} Y_{R}^{\dagger} U \widetilde{H} + \frac{1}{\Lambda^{2}} \overline{Q} Y_{u} Y_{R} D H$$

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Maybe we should make use of the U(1)'s ...

For example, we can assign the charges  $C(Y_u) = -C(U) = 1$  under  $U(1)_u$  $C(Y_d) = C(D) = 1$  under  $U(1)_d$ 

Then  $\frac{1}{\Lambda}\overline{Q} Y_d D\widetilde{H}$  can be naturally suppressed while  $\frac{1}{\Lambda}\overline{Q} Y_u UH$  is not.

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The bonus: The terms linear in  $Y_u$  and  $Y_d$  required to obtain hierarchical spectra will also be naturally suppressed.

#### Outline

- Motivation: The origin of the Yukawa hierarchies ?
- 2 Yukawa hierarchies from SSB First attempt: Via the one-loop effective potential Second attempt: Via reducible representations
- 3 Marriage between up and down
- Occursion and on-going work

## Conclusions and on-going work

Summary:

- Consider the most general SU(3)<sub>L</sub> × SU(3)<sub>R</sub> invariant scalar potential of Y, we have two possible tree-level vacua: ⟨Y⟩<sup>s</sup> ~ diag(1,1,1) and ⟨Y⟩<sup>h</sup> ~ diag(0,0,1).
- Yukawa hierarchies cannot be obtained through quantum corrections to  $V_0$  but the flavor symmetry has to be broken at tree-level e.g. via reducible representations with  $Y(3,\overline{3})$ ,  $Z_L(3,1)$ ,  $Z_R(1,3)$ . We obtained the vacuum solution (global minimum)  $\langle Y \rangle \sim v \operatorname{diag}(\epsilon', \epsilon, 1)$  through  $SU(3)_L \times SU(3)_R \rightarrow U(1)_{L+R}$  (could be natural).
- Couple up and down sectors with  $SU(3)_Q \times SU(3)_u \times SU(3)_q \times U(1)_R$ , we can reproduce both the **mass hierarchies** and **mixings** by considering the extended scenario with:

Three bifundamentals:  $Y_u(3,3,1)_0$ ,  $Y_d(3,1,3)_0$ ,  $Y_R(1,3,3)_1$ Four fundamentals:  $Z_{Qu}(3,1,1)_1$ ,  $Z_{Qd}(3,1,1)_0$ ,  $Z_u(1,3,1)_1$ ,  $Z_d(1,1,3)_0$ + some *ad hoc* parameters. Minimal?

### Conclusions and on-going work

#### Comments:

- We consider global flavor symmetry where the flavor breaking scale  $\boldsymbol{\nu}$  is free
- With the complete breaking of *SU*(3)<sup>3</sup>, we have 24 massless Nambu-Goldstone (NG) particles *f*.
- Consider the rare decays from [Wilczek (1982)]:

$$\Delta \mathcal{L} = \frac{1}{v} \overline{\mu} \gamma_{\rho} e \partial_{\rho} f, \quad \Delta \mathcal{L} = \frac{1}{v} \overline{K} \overleftrightarrow{\partial}_{\rho} \pi \partial_{\rho} f$$

Experimental bounds from nonobservation of rare decays: (1.)  $\mu^+ \rightarrow e^+ + f \implies v \gtrsim 10^{10} \text{ GeV}$  [Jodidio et. al. (1988)] (2.)  $K^+ \rightarrow \pi^+ + f \implies v \gtrsim 7 \times 10^{11} \text{ GeV}$  [Anisimovsky et. al. (2004)]

### Conclusions and on-going work

#### Further considerations:

- CP violation(s)
- Other interesting possibility: consider a scalar potential with additional Nambu-Goldstone massless fields in the tree-level vacuum from accidental symmetry e.g.  $V_0(\lambda', \mu \to 0) = \lambda (T v^2)^2$  (i.e.  $O(18) \longrightarrow O(17)$ ). Can additional interactions e.g. gauge be able to induce nonzero  $\langle D \rangle$ ,  $\langle A \rangle \neq 0$  at the loop-level?
- Consider lepton sector: 3 right-handed neutrinos; PMNS mixing ...
- Gauging the flavor symmetry to get rid of the massless NG bosons and lower the scales v and  $\Lambda$ . However, more fields required for anomaly cancellation... More recent work: [Albrecht, Feldmann, Mannel (2010)], [Grinstein, Redi, Villadoro (2010)], [Guadagnoli, Mohapatra, Sung (2011)]
- Consider flavor + left-right symmetry [Guadagnoli, Mohapatra, Sung (2011)]

# Thank you for your attention.

# Questions/comments?

#### "Every square matrix satisfies its own characteristic equation."

For example, the characteristic equation for the eigenvalues  $\xi$  of  $YY^{\dagger}$ 

$$\det(\xi I_{3\times 3} - YY^{\dagger}) = \xi^3 - T\xi^2 + A\xi - \mathcal{D}\mathcal{D}^* = 0$$

Then  $(YY^{\dagger})^3 - T(YY^{\dagger})^2 + A(YY^{\dagger}) - \mathcal{DD}^* = 0.$ 

"All dim > 4 invariant terms can be written in terms of T, A, D." **Proof**: The determinant det( $YY^{\dagger}...$ ) = det(Y) det(Y)\*....

 $\operatorname{Tr}(YY^{\dagger}YY^{\dagger}YY^{\dagger}) = T\operatorname{Tr}(YY^{\dagger})^{2} - AT + \mathcal{DD}^{*}$  and recall that  $2A = T^{2} - \operatorname{Tr}(YY^{\dagger})^{2}$ . Hence  $\operatorname{Tr}(YY^{\dagger}YY^{\dagger}YY^{\dagger}) = T^{3} - 3AT + \mathcal{DD}^{*}$ .

And  $\operatorname{Tr}(YY^{\dagger}YY^{\dagger}YY^{\dagger}YY^{\dagger}) = \operatorname{Tr}[T(YY^{\dagger})^{3} - A(YY^{\dagger})^{2} + \mathcal{D}\mathcal{D}^{*}(YY^{\dagger})] = \dots$ and so on...

# A function of T, A, D have 2 distinct eigenvalues at stationary point

Any function  $V(T, A, D, D^*)$ . Take the derivative w.r.t eigenvalues  $\xi_i$  of  $YY^{\dagger}$ :

$$\frac{\partial V}{\partial \xi_i} = \frac{\partial V}{\partial T} \frac{\partial T}{\partial \xi_i} + \frac{\partial V}{\partial A} \frac{\partial A}{\partial \xi_i} + \frac{\partial V}{\partial D} \frac{\partial D}{\partial \xi_i} + \frac{\partial V}{\partial D^*} \frac{\partial D^*}{\partial \xi_i} = \frac{\partial V}{\partial T} + \frac{\partial V}{\partial A} (T - \xi_i) + \frac{\partial V}{\partial D} \frac{D}{2\xi_i} + \frac{\partial V}{\partial D^*} \frac{D^*}{2\xi_i}$$

At a stationary point  $\xi_c = (x_1, x_2, x_3)$ 

$$0 = \frac{\partial V}{\partial T}\Big|_{\xi_c} + \frac{\partial V}{\partial A}\Big|_{\xi_c} (T - x_i) + \frac{\partial V}{\partial D}\Big|_{\xi_c} \frac{\mathcal{D}_c}{2x_i} + \frac{\partial V}{\partial D^*}\Big|_{\xi_c} \frac{\mathcal{D}_c^*}{2x_i}$$
$$0 = P(T_c, A_c, \mathcal{D}_0, \mathcal{D}_c^*) x_i^2 + Q(T_c, A_c, \mathcal{D}_c, \mathcal{D}_c^*) x_i + R(T_c, A_c, \mathcal{D}_c, \mathcal{D}_c^*)$$

Unless  $P(T_c, A_c, \mathcal{D}_c, \mathcal{D}_c^*) = Q(T_c, A_c, \mathcal{D}_c, \mathcal{D}_c^*) = R(T_c, A_c, \mathcal{D}_c, \mathcal{D}_c^*) = 0$ , otherwise

$$x_i = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P}$$

 $\implies$  at most *two* distinct eigenvalues.

## Yukawa hierarchies as a function of $\nu/\lambda'$

 $R = \{\epsilon', \epsilon, y\}$  and  $\mu = 0.05\lambda'$ 

Numerical Analytical R R 0.1 0.1 0.01 0.01 0.001 0.001 0.0 0.2 0.4 0.6 0.8 1.0 0.0 0.2 0.4 0.6 0.8 1.0 A successful marriage:

$$\begin{split} Y^D_u &= \text{diag} \left(-10^{-5}, -0.036, 1.47\right), \\ Y^D_d &= \text{diag} \left(-10^{-3}, 0.036, -1.47\right), \\ V_{mix} &= \begin{pmatrix} 0.974 & 0.225 & 0.0046 \\ -0.225 & 0.973 & 0.041 \\ 0.0046 & -0.041 & 0.999 \end{pmatrix}, \\ Y_R &= \begin{pmatrix} 0.571 & -2 \times 10^{-4} & 1.11 \\ -2 \times 10^{-4} & -0.521 & 0.585 \\ -2 \times 10^{-4} & -0.585 & -0.506 \end{pmatrix}, \\ Z^T_{Qu} &= & (0.0060, 0.572, -0.151), \\ Z^T_{Qu} &= & (0.124, 0.565, 0.127), \\ Z^T_d &= & (-2 \times 10^{-6}, -0.748, -0.055), \\ Z^T_d &= & (2 \times 10^{-4}, 0.748, -0.055) \end{split}$$