Discrete Flavor Symmetries after RENO and Daya Bay

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based on

AFMS = G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670
AFM = G. Altarelli, F.F. and L. Merlo hep-ph/1205.5133
AFMM = Altarelli,F,Masina, Merlo 1207.0587
FHT1=F.F., C. Hagedorn, R. de A.Toroop hep-ph/1107.3486
FHT2=F.F., Hagedorn, R. de A. Toroop hep-ph/1112.1340
FHZ=F.F, Hagedorn, Ziegler in preparation

outline

- a digression:
  - some aspects of model building in quark sector
  - FCNC from New Physics at the TeV scale

- extension to the lepton sector:
  - 2012 data, models and LFV

- a special class of models: discrete flavour symmetries
  impact of RENO and DAYA Bay results

- conclusions

key question

origin of the observed hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1$$

$$|V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

goal

SM Yukawa hierarchical $\quad y_{u,d} \Longleftrightarrow Y_{u,d} \approx O(1) \quad$ fundamental O(1) parameters

paradigm: spontaneously broken $U(1)_{FN}$ [Froggatt,Nielsen 1979]

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$

$$Y_{u,d} \approx O(1)$$

$$F_X = \begin{pmatrix} \lambda^{FN(X_1)} & 0 & 0 \\ 0 & \lambda^{FN(X_2)} & 0 \\ 0 & 0 & \lambda^{FN(X_3)} \end{pmatrix} \quad (X = Q, U^c, D^c)$$

call this map Hierarchy

$FN(X_i)$ are $U(1)_{FN}$ charges [here $FN(X_i) \geq 0$]

$\lambda = \frac{\langle \vartheta \rangle}{\Lambda} \approx 0.2$ [symmetry breaking parameter]

not a mere book-keeping

take $FN(Q_1) > FN(Q_2) > FN(Q_3) \geq 0$

$$\left(V_{u,d}\right)_{ij} \approx \frac{F_{Q_i}}{F_{Q_j}} < 1 \quad (i < j) \qquad V_{CKM} = V_u^{+} V_d$$

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx O(1)$$

$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

[O.K. within a factor of 2]

independently from the specific charge choice

correct orders of magnitude of V_{ij} reproduced by e.g.

$FN(Q) = (3,2,0)$

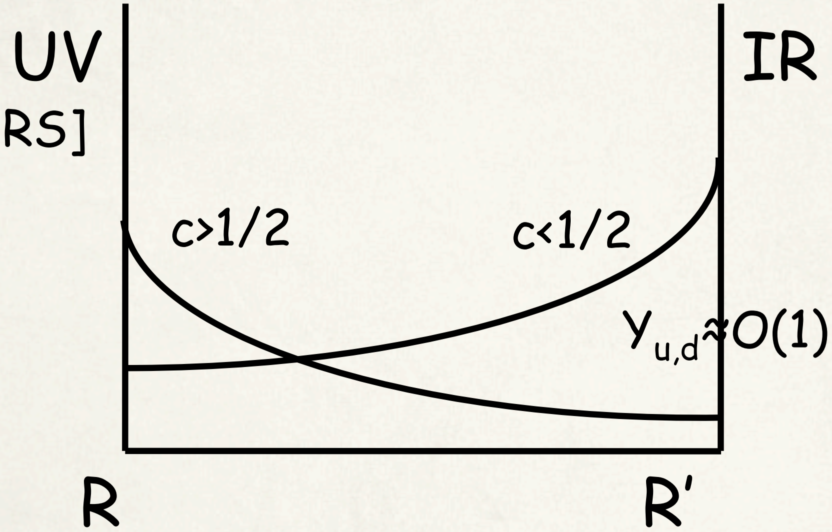
correct orders of magnitude of quark/charged lepton mass ratios [up to a couple of moderate tunings] reproduced by e.g.

$FN(U^c) = FN(E^c) = FN(Q) = (3,2,0)$
$FN(D^c) = FN(L) = (2,0,0)$

charge assignment compatible with SU(5) gauge unification

other realizations of Hierarchy

split fermions in a warped Extra Dimension [=RS]



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$M_{Pl}^{-1} \approx R \le z \le R' \approx (TeV)^{-1}$$

assign a bulk mass $c_i = M_i R$ to each fermion X_i and introduce random, O(1) Yukawa couplings $Y_{u,d}$ between bulk fermions and a Higgs localized at the IR brane

Yukawa couplings $y_{u,d}$ of zero-mode fermions as in FN [1 flavon and FN(X_i) ≥ 0]

$$y_u = F_{U^c} Y_u F_Q \qquad y_d = F_{D^c} Y_d F_Q$$

$$F_{X_i} = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}}$$

$$\approx O(1) \qquad c_i < 1/2$$

$$\approx (R/R')^{c_i - 1/2} << 1 \qquad c_i > 1/2$$

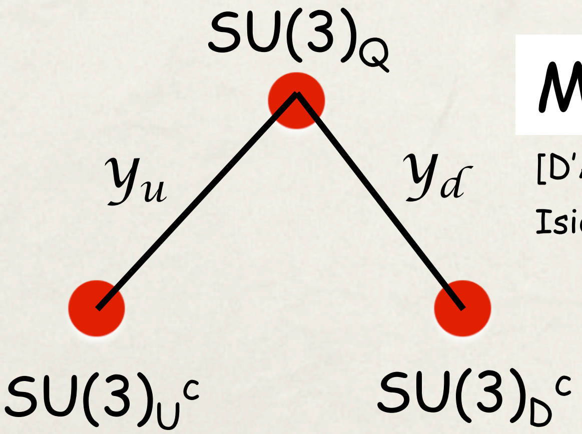
	Q	U^c	D^c
1	0.643	0.671	0.643
2	0.583	0.528	0.601
3	0.317	-0.460	0.601

[fit to c_i - Huber 0303183]

hierarchy produced by geometry

Same pattern arises when matter chiral multiplets X_i of the MSSM are coupled to a superconformal sector in some finite energy range [Nelson-Strassler 0006251]

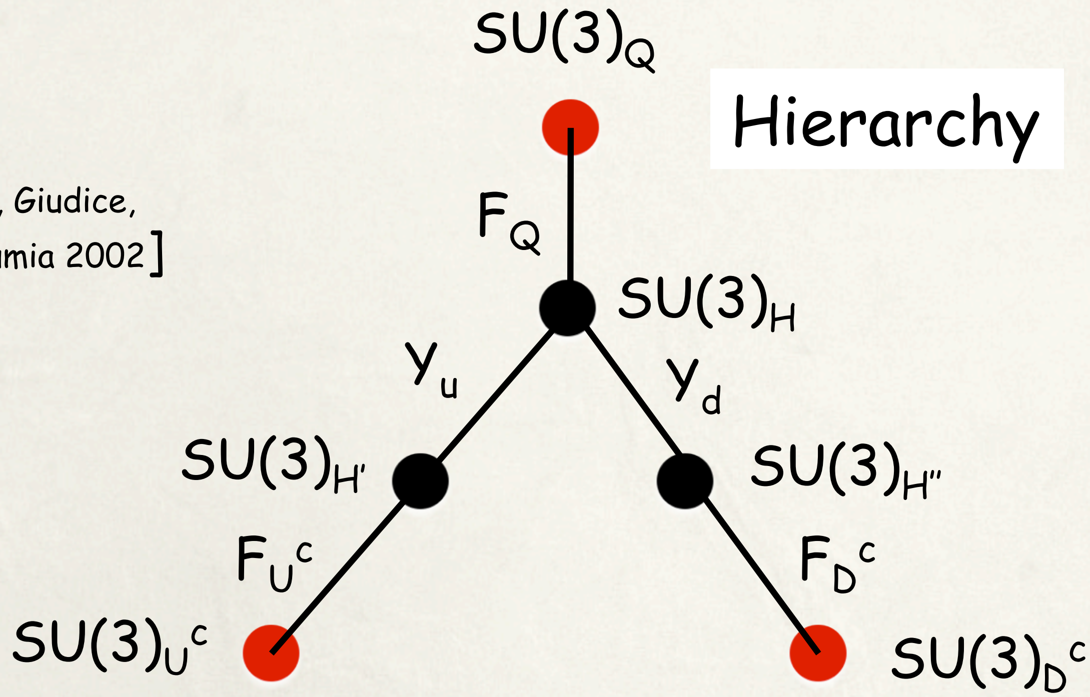
Maximal Flavour Symmetry in Hierarchy setup



MFV

[D'Ambrosio, Giudice, Isidori, Strumia 2002]

Hierarchy



Flavour group felt by quarks is the same
Hierarchy has more spurions

F_Q, F_{U^c}, F_{D^c}, Y_u, Y_d do not commute

Hierarchy MFS not enough to suppress FCNC and/or CPV at an acceptable level if there is New Physics at the TeV scale

true flavour symmetry can be even weaker, depending on the way Hierarchy is realized, as e.g. in FN models [Dudas, von Gersdorff, Parmentier, Pokorski 1007.5208]
maximal symmetry applies to RS models [RS-GIM Agashe, Perez, Soni 0408134]

dangerous FCNC

$$O_K^4 = (\bar{s}_L d_R)(\bar{s}_R d_L)$$

contributions to ε_K are both chiral and RG enhanced

arises from
$$\frac{1}{\Lambda_{NP}^2}(\bar{Q}F_Q^+\gamma_\mu F_Q Q)\ (\bar{D}^c F_{D^c}^+\gamma^\mu F_{D^c} D^c) =$$

$$= \frac{1}{\Lambda_{NP}^2} F_{Q_2} F_{Q_1} F_{D_2^c} F_{D_1^c}\ (\bar{D}_2\gamma_\mu D_1)\ (\bar{D}_2^c\gamma^\mu D_1^c) + \ldots$$

$$C_K^4 \approx \frac{1}{\Lambda_{NP}^2}\frac{1}{\langle Y_d\rangle^2}\frac{2m_d m_s}{v^2}$$

$$\mathrm{Im}(C_K^4) < (160\times 10^3\ TeV)^{-2}$$

$$\mathrm{Im}(C_K^4) \approx \mathrm{Re}(C_K^4)$$

$$\Rightarrow \quad \langle Y_d\rangle \Lambda_{NP} > 20\ TeV$$

confirmed by explicit computation in RS
O_K^4 from tree-level KK gluon exchange

[Csaki, Falkowski, Weiler 0804.1954]

$$\frac{1}{\Lambda_{NP}} = \frac{g_{s^*}}{M_{KK}} \qquad g_{s^*} = g_s(M_{KK})\times\sqrt{\log(R'/R)} \approx 6$$

$$\langle Y_d\rangle \le 3$$

$$\Rightarrow \quad M_{KK} > (22\pm 6)\ TeV$$

[also neutron EDM -> M_{KK}>O(10) TeV]

some lessons from the quark sector

- Pattern of quark masses and mixing angles well-explained by a Hierarchy map: underlying $Y_{u,d}$ are O(1)
  Hierarchy realized in several different frameworks: FN, RS, NS,….
- correct order-of-magnitude predictions
- compatible with SU(5) GUTs
- compatible with/incorporated in known solutions to the hierarchy problem
- additional ingredients probably needed to control the new sources of FC/CPV arising from New Physics at the TeV scale

  alignment
  universality
  …

  $$F_Q \quad F_{D^c} \quad F_{U^c}$$
  $$Y_d \quad Y_u$$

  some symmetry ?
- large number of independent O(1) parameters — additional constraints ?
- present precision in quark mass/mixing parameters — testable predictions beyond order-of-magnitude accuracy ?

extension to the lepton sector

2011/2012 breakthrough

- from LBL experiments searching for $\nu_\mu \to \nu_e$ conversion

T2K: muon neutrino beam produced at JPARC [Tokai] E=0.6 GeV and sent to SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \to \nu_e) = \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\sin^2\frac{\Delta m^2_{32}L}{4E} + \ldots$$

both experiments favor $\sin^2\vartheta_{13}$ ~ few %

- from SBL reactor experiments searching for anti-ν_e disappearance

Double Chooz (far detector):	$\sin^2\vartheta_{13} = 0.022 \pm 0.013$
Daya Bay (near + far detectors):	$\sin^2\vartheta_{13} = 0.024 \pm 0.004$
RENO (near + far detectors):	$\sin^2\vartheta_{13} = 0.029 \pm 0.006$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\vartheta_{13}\sin^2\frac{\Delta m^2_{32}L}{4E} + \ldots$$

- SBL reactors are sensitive to ϑ_{13} only
LBL experiments anti-correlate $\sin^2 2\vartheta_{13}$ and $\sin^2\vartheta_{23}$
also breaking the octant degeneracy $\vartheta_{23} \leftrightarrow (\pi-\vartheta_{23})$

global fit

	Lisi [Neutel2011] [0806.22517update]	Fogli et al. [1205.5254]
$\sin^2\vartheta_{12}$	$0.307^{+0.018}_{-0.016}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\vartheta_{23}$	$0.42^{+0.09}_{-0.04}$	$0.398^{+0.030}_{-0.026}$ [NO] $0.408^{+0.035}_{-0.030}$ [IO]
$\sin^2\vartheta_{13}$	$0.014^{+0.009}_{-0.008}$	$0.0245^{+0.0034}_{-0.0031}$ [NO] $0.0246^{+0.0034}_{-0.0031}$ [IO]
$\Delta m^2_{sol}\ (eV^2)$	$(7.54^{+0.25}_{-0.22})\times 10^{-5}$	$(7.54^{+0.26}_{-0.22})\times 10^{-5}$
$\lvert\Delta m^2_{atm}\ (eV^2)\rvert$	$(2.36^{+0.12}_{-0.10})\times 10^{-3}$	$(2.43^{+0.07}_{-0.09})\times 10^{-3}$ [NO] $(2.42^{+0.07}_{-0.10})\times 10^{-3}$ [IO]

hint for non maximal ϑ_{23} ?

7σ away from 0

$$\vartheta_{13} = (9.0 \pm 0.6)^0$$

open questions
- is L violated or not?
- mass ordering: Normal or Inverted?
- δ_{CP}

- no evidence for big hierarchies in neutrino mixing angles [ϑ_{13} ?]
clear hierarchy only in the charged lepton masses

$$\Rightarrow \quad \begin{array}{l} F_{E_1^c} << F_{E_2^c} << F_{E_3^c} \\ F_{L_1} \approx F_{L_2} \approx F_{L_3} \end{array}$$

[independently on whether neutrinos are Majorana or Dirac]

several possibilities [here focus on Majorana neutrinos]:

- $F_{L_1} = F_{L_2} = F_{L_3}$

all mixing angles expected to be large

Anarchy

ϑ_{12} O.K. $\quad\quad$ ϑ_{13} ?

$\Delta m^2_{12} \approx \Delta m^2_{13}$?

fit to present data in a non-negligible portion of parameter space, better with see-saw
[Hall, Murayama, Weiner 1999]

- $F_{L_1} < F_{L_2} = F_{L_3}$

$\vartheta_{13} \approx F_{L_1} / F_{L_3}$ small

[next slide]

$\vartheta_{12} \approx \vartheta_{13} \approx F_{L_1} / F_{L_3}$

$\Delta m^2_{12} \approx \Delta m^2_{13}$

[bad]

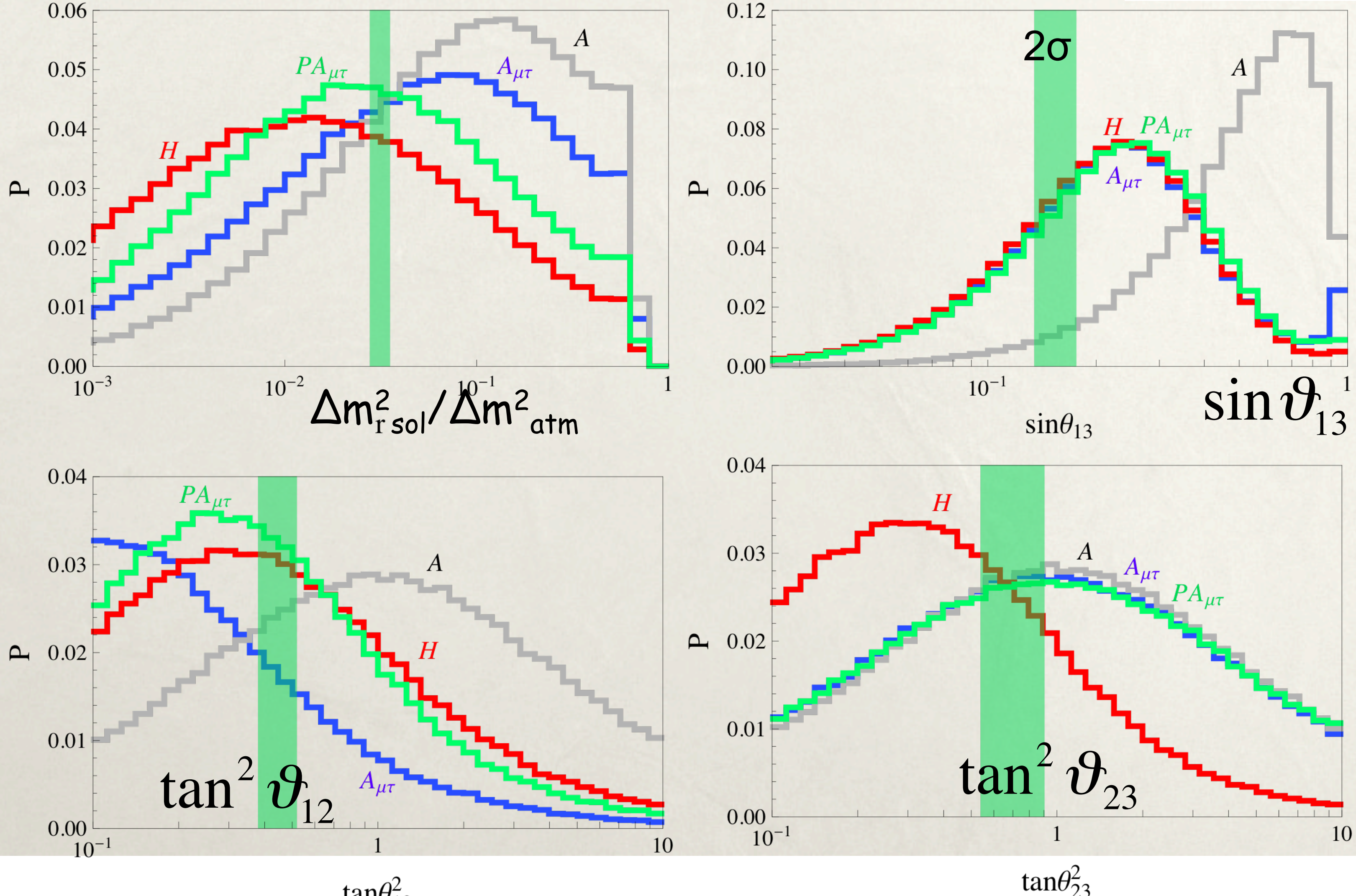
[both fixed if det(23) small;
may occur accidentally or
by allowing FN charges of both signs or
via dominance of a single N^c exchange [King 1998]
$\vartheta_{13} \approx \sqrt{\Delta m^2_{sol} / \Delta m^2_{atm}}$ normal mass ordering]

- viable $U(1)_{FN}$ models, quarks and leptons treated on equal foot
- compatible with SU(5) unification
- difficult to go beyond order-of-magnitude predictions

	$FN(L)$	λ
A	(0,0,0)	
$A_{\mu\tau}$	(1,0,0)	0.25
$PA_{\mu\tau}$	(2,0,0)	0.35
H	(2,1,0)	0.45

$$F(L_i) = \lambda^{FN(L_i)}$$

[Altarelli,F,Masina, Merlo 1207.0587]



constraints from lepton flavour violation

take the limit $m_\nu = 0$

if MFV applied, we would expect no LFV [y_e diagonal]

in our setup, in general F_{E^c}, F_L, Y_e do not commute [not even when F_L is universal] LFV expected at some level

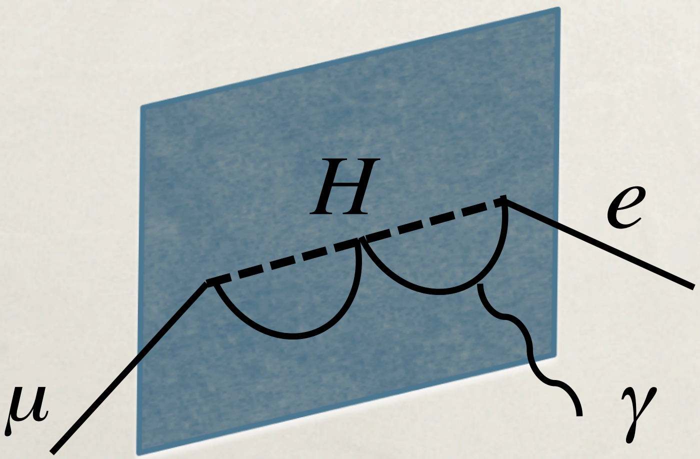
dominant LFV dipole operator

$$L_{dip} = \frac{e}{\Lambda_{NP}^2} E^c (\sigma_{\mu\nu} F^{\mu\nu}) \underbrace{(F_{E^c} Y_e Y_e^+ Y_e F_L)}_{\text{not diagonal}} (H^+ L)$$

when $y_e = F_{E^c} Y_e F_L$ diagonal

Explicit computation in RS

[Agashe, Blechman, Petriello 0606021
Csaki, Grossman, Tanedo, Tsai 1004.2037]



$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12} \qquad \left(\frac{M_{KK}}{3\,TeV}\right)\frac{2}{\langle Y_e \rangle} > 15$$

$$BR(\mu \to e)_{Ti} < 6.1 \times 10^{-13} \qquad \left(\frac{M_{KK}}{3\,TeV}\right)^2 \langle Y_e \rangle > 4$$

$$M_{KK} > O(10)\ TeV$$

additional sources of LFV when neutrino couplings are turned on

F_L universality is not enough

a sufficient condition for the absence of LFV: F_{E^c}, Y_e, F_L diagonal in the same basis

for instance:

$$F_L \propto 1 \qquad F_{E^c} \propto Y_e Y_e^+$$

[M.C. Chen and Yu, 08042503
Perez, Randall 0805.4652]

are there models of lepton masses that already include such conditions ?

- further constraints on the parameters F_L,F_{E^c},Y_e,... of the lepton sector are required if we believe that some feature of the data deserves a special explanation

$$\vartheta_{13} << \vartheta_{12}, \vartheta_{23}$$
$$\vartheta_{23} \approx \text{maximal}$$

less sharp after the 2012 data

$$\vartheta_{12} + O(\lambda_C) \approx \pi/4$$
$$\Delta m^2_{sol} << \left|\Delta m^2_{atm}\right|$$

accidental features
mixing angles and mass ratios are O(1)
no special pattern beyond the data:
Anarchy

"Evidence" for some property of the fundamental theory

$U_{PMNS} = U^0_{PMNS}$ + corrections

the new data have strengthened the case for Anarchy

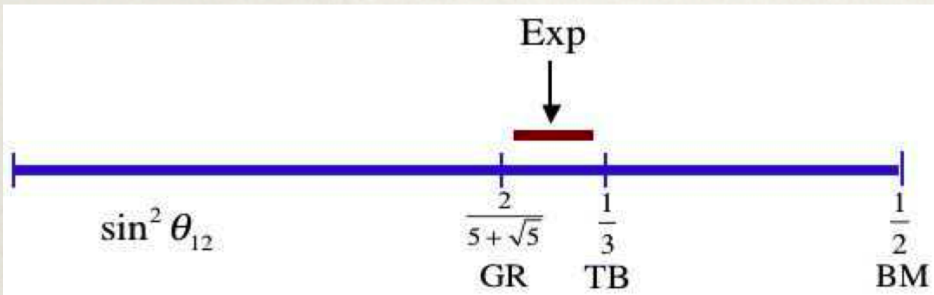
Mixing patterns U^0_{PMNS} (an incomplete list)

	U^0_{PMNS}	$\sin^2\vartheta^0_{23}$	$\sin^2\vartheta^0_{13}$	$\sin^2\vartheta^0_{12}$
TB	$\begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$	$1/2$	0	$1/3$
GR	$\begin{pmatrix} c & s & 0 \\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ $s/c = 1/\varphi$	$1/2$	0	$1/(\sqrt{5}\varphi)$ ≈ 0.276
BM	$\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & -1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$	$1/2$	0	$1/2$
	3σ range [NO]	$(0.330 \div 0.638)$	$(0.0149 \div 0.0344)$	$(0.259 \div 0.359)$

[TB <->Harrison, Perkins and Scott]

[GR<-> Kajiyama, Raidal, Strumia 2007]

$\varphi = \frac{(1+\sqrt{5})}{2}$ Golden Ratio



mixing patterns U^0_{PMNS} from discrete symmetries

[in some limit: u->0]

G_f

[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

G_e — residual symmetry of $(m_e^+ m_e)$

G_ν — residual symmetry of m_ν

$(m_e^+ m_e)$ and m_ν misaligned because G_e and G_ν do not commute

[non degenerate mass spectrum: G_e and G_ν abelian]

assign l to a 3-dim irrep ρ(g) of G_f

$$U_\nu^+ \rho(g_\nu) U_\nu = \rho(g_\nu)_{diag}$$
$$U_e^+ \rho(g_e) U_e = \rho(g_e)_{diag}$$

$$\Rightarrow \quad U_{PMNS} = U_e^+ U_\nu$$

- the most general group leaving $\nu^T m_\nu \nu$ invariant, and m_i unconstrained

$$G_\nu = Z_2 \times Z_2$$

Majorana neutrinos imply G_ν discrete!

- G_e can be continuous but the simplest choice is G_e discrete

$$G_e = \begin{cases} Z_2 \times Z_2 \\ Z_n \quad n \geq 3 \end{cases}$$

LO result gets corrected in the full theory $\quad \vartheta_{ij} = \vartheta_{ij}^0 + O(u)$

- invariance under a single Z_2 parity in $G_\nu=Z_2\times Z_2$ determines two (combinations of) mixing angles
for instance the invariance under

$$P_{23}=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

μ-τ or 2-3 exchange symmetry
[in the basis where the elements of G_e are diagonal]

[in A_4, P_{23} arises as an accidental symmetry]

$\Rightarrow$ $\vartheta_{13}=0$ and $\vartheta_{23}=\pi/4$

[Mohapatra, Nasri, Yu;Koide; Kubo et al. Kaneko et al Caravaglios et al. Morisi; Picariello; Grimus, Lavoura....]

- the second Z_2 parity determines the third angle and a phase

- neutrino masses unconstrained: fitted, not predicted

- empirical mixing patterns arise from small groups

G_f	G_e	G_ν	U^0_{PMNS}
A_4	Z_3	$Z_2\times Z_2$	U_{TB}
S_4	Z_3	$Z_2\times Z_2$	U_{TB}
	$Z_4, Z_2\times Z_2$	$Z_2\times Z_2$	U_{BM}
A_5	Z_5	$Z_2\times Z_2$	U_{GR}

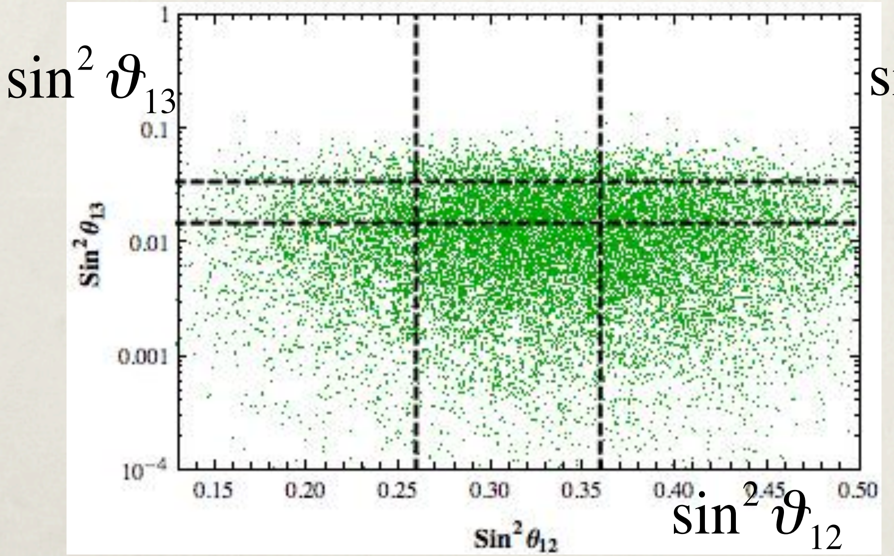
[in A_4, one of the two parities arises as an accidental symmetry] general feature

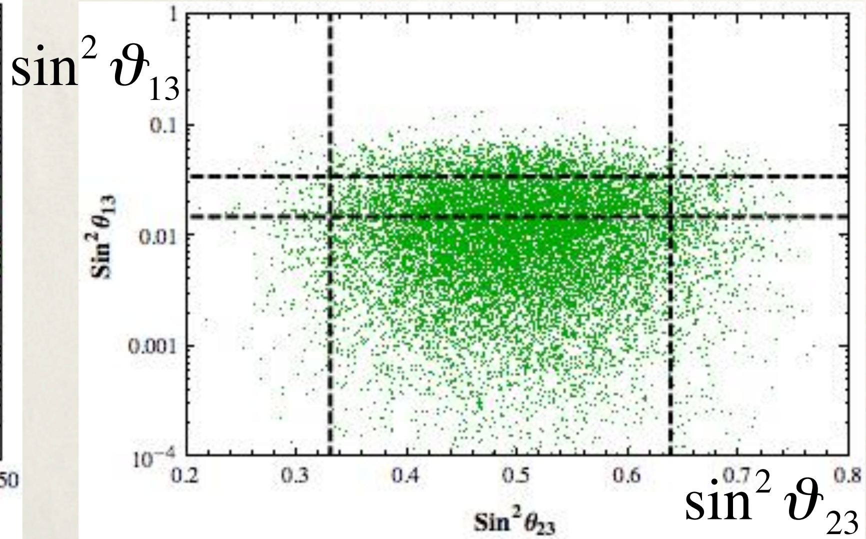
$$U_{PMNS} = U^0_{PMNS} + O(u) \qquad u \equiv \frac{\langle \varphi \rangle}{\Lambda} < 1 \quad \Rightarrow \quad \vartheta_{ij} = \vartheta^0_{ij} + O(u)$$

a challenge for models such as A_4 leading to $U^0 = U_{TB}$
is to generate $\vartheta_{13} \approx 0.1$ while keeping ϑ_{12} almost unchanged

A_4 model with typical O(0.1) corrections

[size of the corrections – 0.08 - optimized to maximize the success rate] [AFMS 2012]





lack of predictability: $\sin^2 \vartheta_{12}$ ranges from 0.2 up to 0.45 now
success rate (about 13%) indicates the need of tuning

A_4 models with special corrections: $G_\nu=Z_2$

group theoretical origin of TB mixing suggests how to get $\vartheta_{13}\approx 0.1$ while keeping ϑ_{12} almost unchanged

assume $G_e=Z_3$ as before and remove Z_2 generated by P_{23} from $G_\nu=Z_2\times Z_2$

-- natural in the context of A_4 that does not contain P_{23} [Hernandez,Smirnov 1204.0445]
-- explicit constructions proposed before T2K,... [Lin 2009]
-- starting from the full $G_\nu=Z_2\times Z_2$, the parity P_{23} can be broken at a high scale

$$U^0 = U_{TB}\times\begin{pmatrix}\cos\alpha & 0 & e^{i\delta}\sin\alpha\\ 0 & 1 & 0\\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha\end{pmatrix}$$

$$0\le\alpha\le\pi/2$$
$$0<\delta\le 2\pi$$

$$\sin\vartheta_{13}=\sqrt{2/3}\,\alpha+\ldots$$
$$\sin^2\vartheta_{12}=1/3+2/9\,\alpha^2+\ldots$$
$$\sin^2\vartheta_{23}=1/2+\alpha/\sqrt{3}\cos\delta+\ldots$$
$$\delta_{CP}=\delta$$

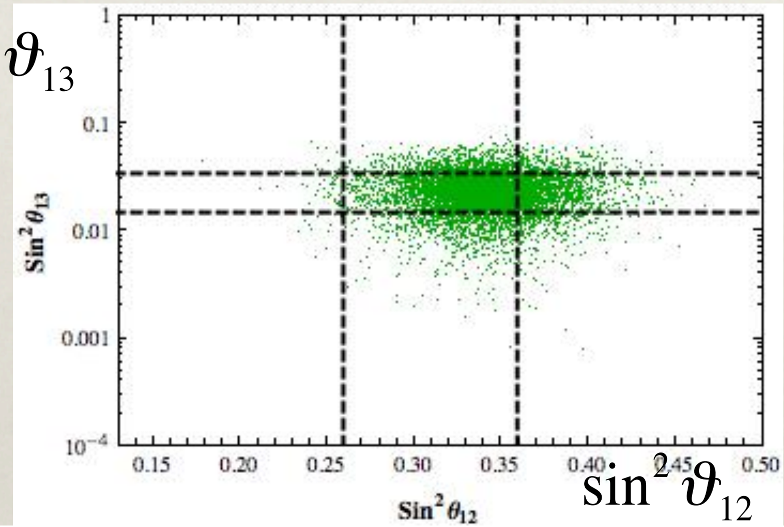
[assuming α=0.1 and expanding in powers of α]

Trimaximal mixing

gives back TB when α=0

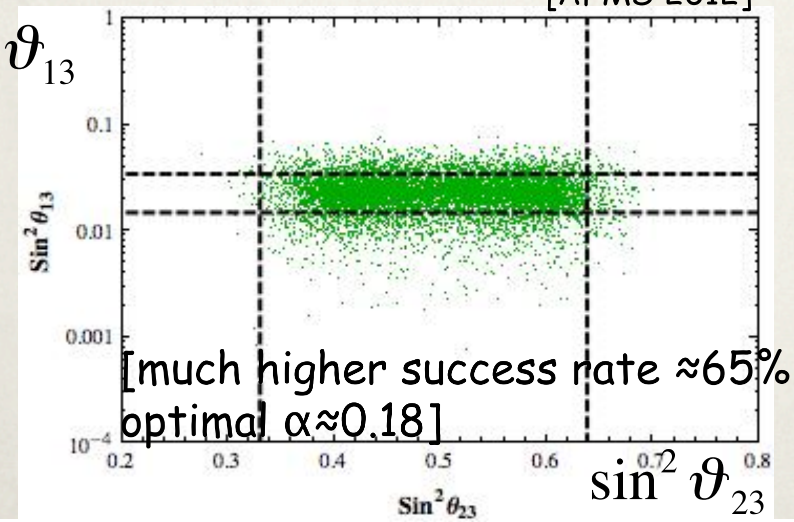
[AFMS 2012]

$\sin^2\vartheta_{13}$



$\sin^2\vartheta_{12}$

$\sin^2\vartheta_{13}$

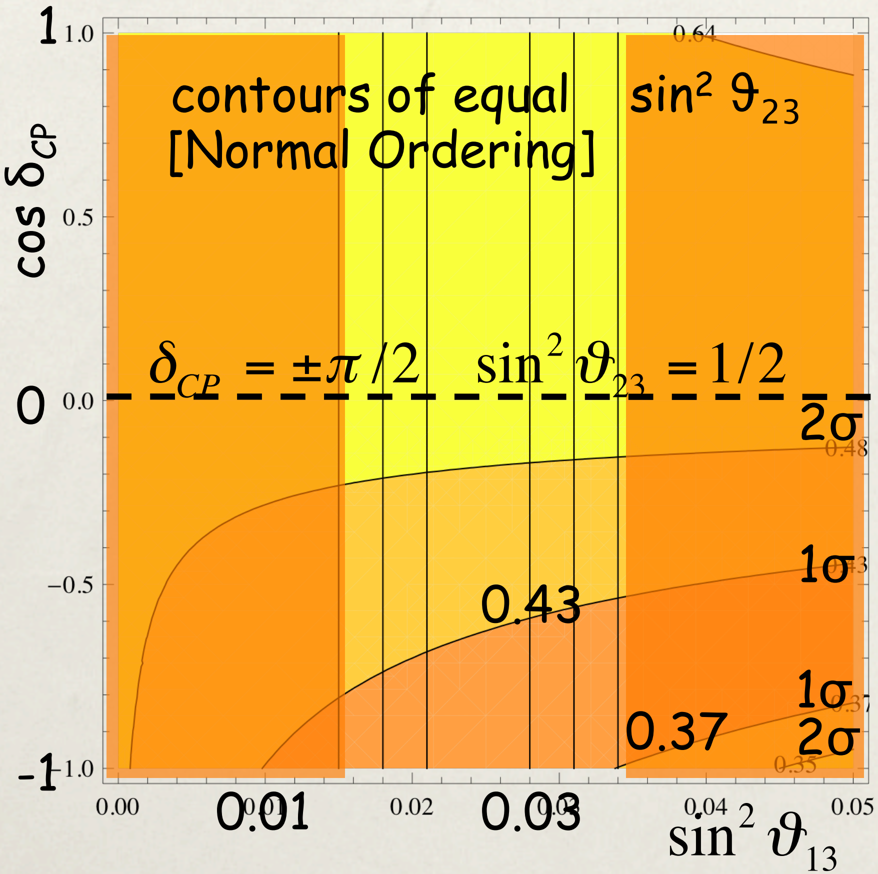


[much higher success rate ≈65% optimal α≈0.18]

$\sin^2\vartheta_{23}$

from the previous relations

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$



indication for $\sin^2\vartheta_{23} \approx 0.4$ would favor $-1 < \cos\delta_{CP} < -0.5$

can be tested by measuring δ_{CP} and improving on $\sin^2\vartheta_{23}$

Trimaximal ansatz proposed with different motivations by many authors

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011]

[similar tests can be realized in S_4 (TM,BM) and A_5 (GR)]

δ_{CP} determined if residual symmetry is $G_\nu = Z_2 \times CP$

consider a non-trivial action of CP in flavour space

$$\begin{array}{lccc} G_f: & \varphi & \to & \rho(g)\varphi \\ CP: & \varphi & \to & X\,\varphi^* \end{array}$$

X act in flavour space
consistency condition between X and $\rho(g)$ required

example: $G_f = S_4$ $G_e = Z_3$ [as for TB mixing]

$G_\nu = Z_2 \times CP$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- here CP acts as a 23 reflection symmetry
  [Harrison, Scott 0210197
  Grimus, Lavoura 0305309
  Mohapatra, Nishi 1208.2875]
- other posibilities are allowed
- full classification in
  F, Hagedorn, Ziegler, to appear

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & \pm i\sin\alpha \\ 0 & 1 & 0 \\ \pm i\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

$0 \le \alpha \le \pi/2$

Trimaximal mixing

$$\begin{aligned} \sin\vartheta_{13} &= \sqrt{2/3}\,\alpha + \ldots \\ \sin^2\vartheta_{12} &= 1/3 + 2/9\,\alpha^2 + \ldots \\ \sin^2\vartheta_{23} &= 1/2 \\ \delta_{CP} &= \pm\pi/2 \end{aligned}$$

[assuming α=0.1 and expanding in powers of α]

LFV - signatures of discrete symmetries

discrete symmetries are weaker than continuous ones such as MFV, SO(3)... and allow for G_f-invariant and LFV operators

in all models: $l \sim 3$ of G_f

	A_4	S_4	A_5
$\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{\mu}ee+...)$	Yes	Yes	Yes
$\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{e}\mu\mu+...)$	Yes	No	No
$\frac{1}{\Lambda_{NP}^2}(\overline{\mu}\overline{e}\tau\tau+...)$	Yes	No	No

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = 0, \pm 2$

$\tau^- \to \mu^+ e^- e^-$ in A_4, S_4, A_5

$\tau^- \to e^+ \mu^- \mu^-$ in A_4

$BR(\tau^- \to \mu^+ e^- e^-) < 2.0 \times 10^{-8}$

$BR(\tau^- \to e^+ \mu^- \mu^-) < 2.3 \times 10^{-8}$

$\Lambda_{NP} > 10$ TeV

$m_{NP} > 500$ GeV $\quad (m_{NP} = g\Lambda_{NP}/4\pi)$

- in simplest realizations of the above groups these operators are not generated at the LO

$$\frac{BR(\tau^- \to \mu^+ e^- e^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^4) \qquad \frac{BR(\tau^- \to e^+ \mu^- \mu^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^2 \frac{m_\mu}{m_\tau})$$

LFV - radiative decays $l_i \to l_j\gamma$

from loops of SUSY particles

$G_f = A_4 \times$ SUSY...

- allowing for the most general slepton mass matrix compatible with pattern of flavour symmetry breaking

$$\hat{m}^2_{LL} = \begin{pmatrix} n & n_{12}\,u^2 & n_{13}\,u^2 \\ n_{12}\,u^2 & n & n_{23}\,u^2 \\ n_{13}\,u^2 & n_{23}\,u^2 & n \end{pmatrix} m^2_{SUSY} + \ldots$$

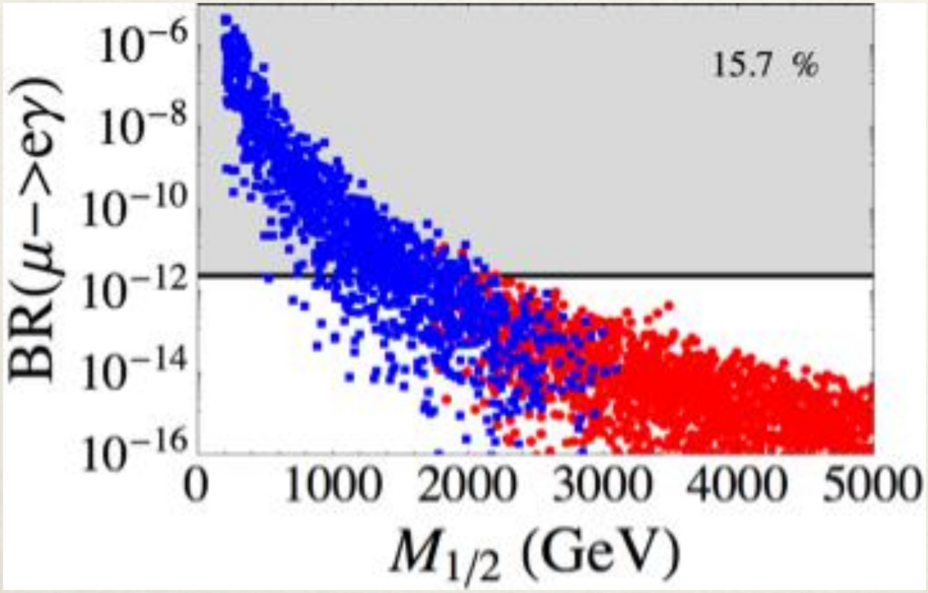
m^2_{XY} (X,Y=L,R) are almost diagonal
off-diagonal terms $(\delta_{ij})_{XY}$
proportional to SB parameter $u = \langle\varphi\rangle/\Lambda$

[in super-"CKM" basis]

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} = \frac{6 m_W^4 \alpha_{em}}{\pi m^4_{SUSY}}\left[\left|w^{(1)}_{ij}u^2\right|^2 + \frac{m_j^2}{m_i^2}\left|w^{(2)}_{ij}u\right|^2\right]$$

[$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters]

- R_{ij} vanishes as u->0
  but the expansion parameter u is now of order 0.1 and the predicted rates can be too large



$\tan\beta = 15 \quad m_0 = 200\ GeV$

$\vartheta_{13} > 0$ from $G_\nu = Z_2 \times Z_2$ at the LO ?

[de Adelhart Toorop, F, Hagedorn 1107.3486 and 1112.1340]

how to "deform" A_4/A_5 or S_4? no continuous parameter

abstract definition
in terms of generators
and relations

$$S^2 = (ST)^3 = T^n = 1$$

$n = 3 \quad A_4$

$n = 4 \quad S_4$

$n = 5 \quad A_5$

all subgroups of the (infinite) modular group Γ

$$S^2 = (ST)^3 = 1$$

look for other subgroups of Γ, the so-called finite modular groups Γ_N
an infinite series, but there are only six of them admitting (independent) 3-dimensional irreducible representations [Nobs, 1976]

N	3	4	5	7	8	16
Γ_N	A_4	S_4	A_5	$PSL(2,Z_7)$	Γ_8	Γ_{16}

new interesting patterns in N=8,16 choosing $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$

$$\Gamma_8 \supset \Delta(96): \qquad S^2 = (ST)^3 = T^8 = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

$$\Gamma_{16} \supset \Delta(384): \qquad S^2 = (ST)^3 = T^{16} = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

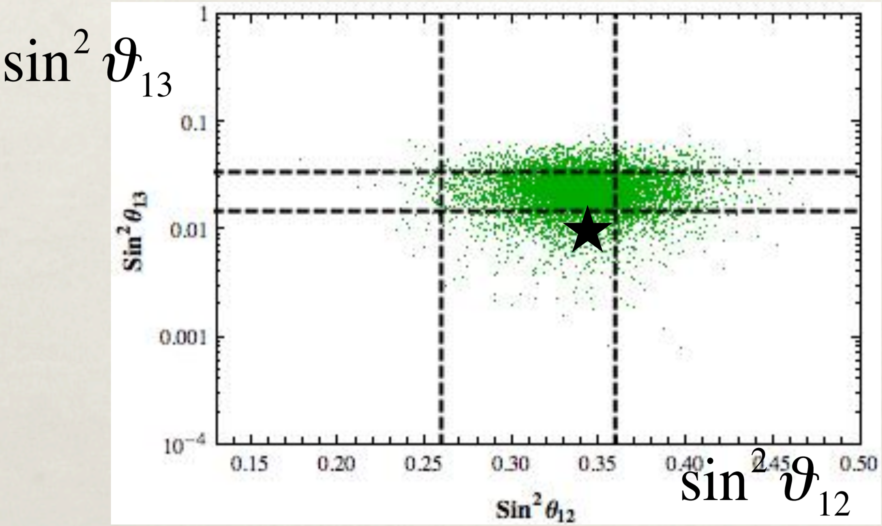
new mixing patterns are special forms of Trimaximal mixing

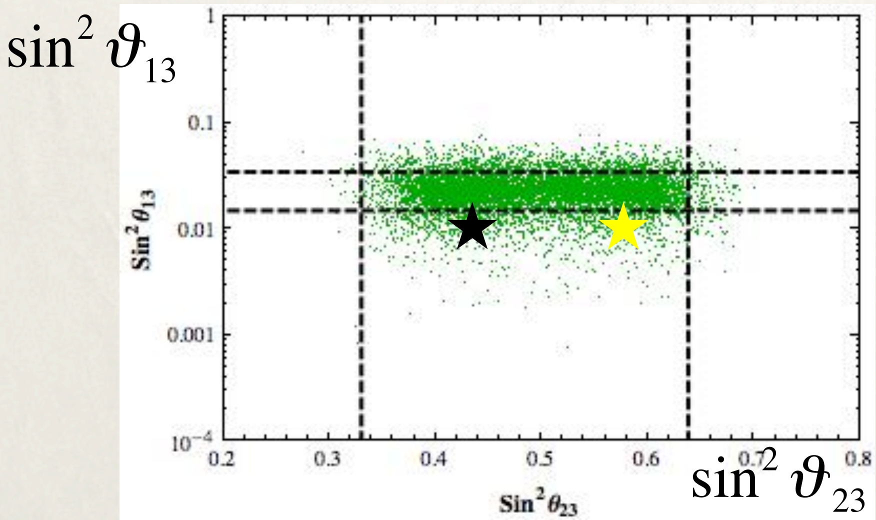
$$U^0_{PMNS} = U_{TB} U_{13}(\alpha) \qquad U_{13}(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

but δ_{CP} =0,π (no CP violation) and
the angle α is not a free parameter:
it is "quantized" by group theory

G_f	Γ_8	Γ_{16}
α	$\pm\pi/12$	$\pm\pi/24$

patterns from Γ_{16} (compared to A_4 with "special" corrections)





recent scan of all finite subgroups of SU(3) up to order 511 finds only one group accommodating ϑ_{13} and ϑ_{23} : Δ(150). [Lam, 1208.5527]

Conclusions

- the measurement of ϑ_{13} has had a great impact on neutrino model building
  constructions leading to small deviations from TB mixing are no longer viable

- models based on Anarchy or variant of Anarchy provide a decent description of the data

- do the data still suggest a first approximation to lepton mixing angles? Is this relevant for model building ? Less clear after RENO and DAY Bay results

- if so, it is rather different from $V_{CKM} \approx 1$
  lepton mixing angles look independent from neutrino masses
  special values can be understood in terms of a broken flavour symmetry

- non-abelian discrete groups like A_4, S_4, A_5,... can provide the basis for a realistic model of neutrino masses (CP can also play a role)

- there are breaking patterns giving rise to realistic deformations of TB mixing, with interesting correlation between ϑ_{13}, ϑ_{23} and δ_{CP}

- when New Physics at the TeV scale is present, it is difficult to keep LFV below the present bounds

Perspectives/open questions

- advantages for quarks from a discrete flavour symmetry?
- quarks can be accommodated in this framework [even in GUTs];
  several existence proofs
- not clear the role of the discrete group in the quark sector
  [large hierarchies and small angles seem not require finite groups]
- it might be easier to reconcile u 0.1 with Cabibbo angle 0.2

- perhaps we should still find the correct embedding of quarks back up slides

there are many possibilities [for a review, see: G. Altarelli and F.F arXiv:1002.0211]

for instance TB mixing can be realized

- with larger discrete groups such as T', S_4,...
  [F,Hagedorn, Lin, Merlo 2007; Chen, Mahanthappa 2007, Frampton, Kehpard 2007; Lam; Bazzocchi, Merlo, Morisi 2009; Hagedorn, King, Luhn 2010, Luhn, Nasri, Ramond 2007, King, Luhn 2009,....]
- continuous groups such as SO(3) and SU(3) and their finite subgroups
  [Varzielas, King, Ross 2007; Luhn, Nashri, Ramond 2007,...]
- discrete symmetries coupled to Sequential Dominance and Form Dominance
  [King 2005, Varzielas, King, Ross 2007,...]

also many version with the same flavour symmetry
for instance there are many A_4 versions

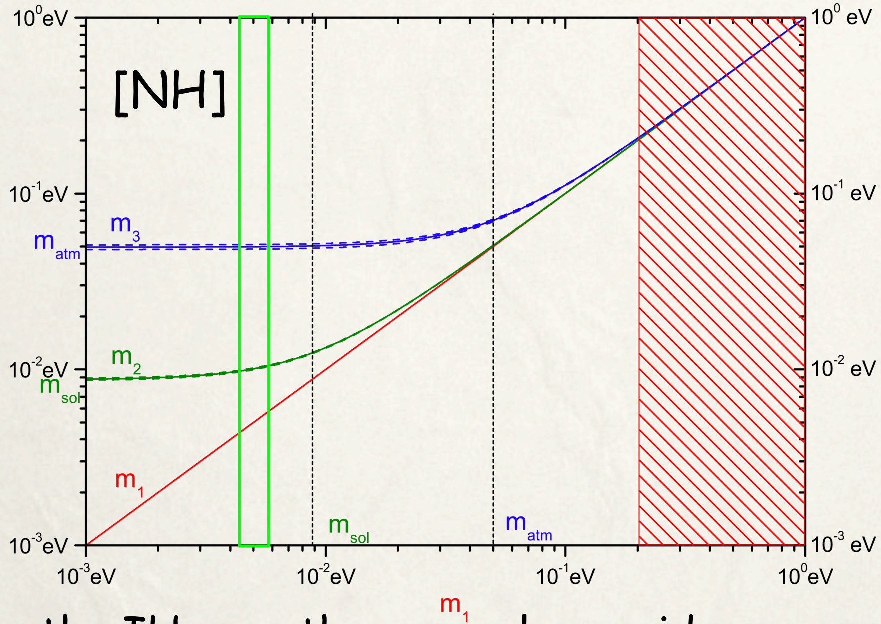
- m_ν from dimension 5 operator or from see-saw
- SUSY or non-SUSY
- 4-dimensional or with ED [Altarelli,F,Lin 2007; Csaki, Delaunay, Grojean, Grossman 2008; Kadosh, Pallante 2010,...]
- charged lepton hierarchy with or without $U(1)_{FN}$ [Lin 2008, Altarelli, Meloni 2009,...]
- extension to quarks without GUT
- GUT extension [Altarelli, F, Hagedorn 2008, Bazzocchi et al. 2009, Antush, King, Spinrath 2010,...]

predictions based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SEE-SAW] [Altarelli, F 2005]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
at the LO neutrino mass spectrum depends on two complex parameters
there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

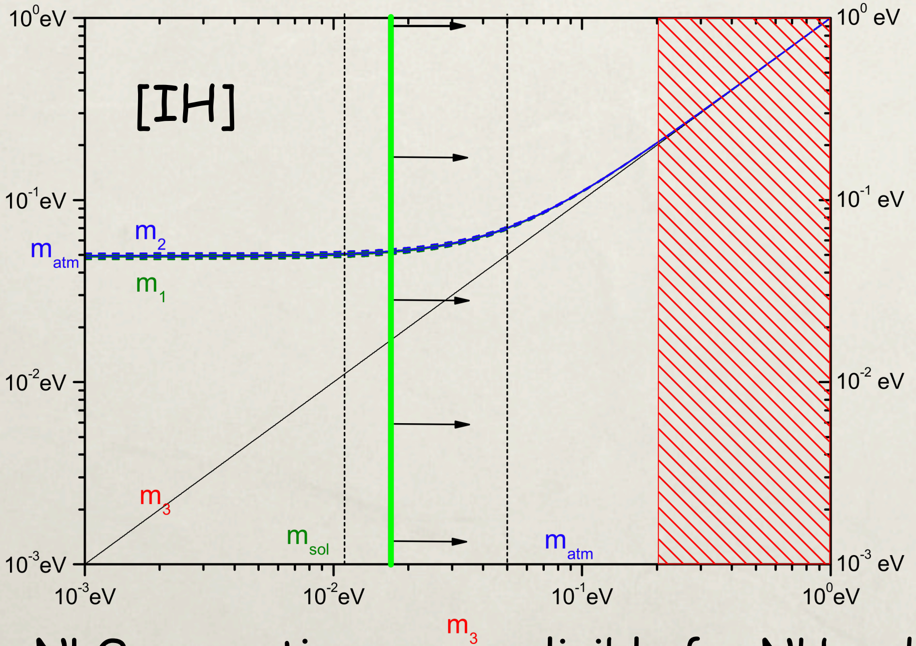
both normal [NH] and inverted [IH] hierarchy are allowed



in the NH case the sum rule completely determines the spectrum

$$m_1 \approx 0.005\ eV \quad m_2 \approx 0.01\ eV \quad m_3 \approx 0.05\ eV$$

$$|m_{ee}| \approx 0.007\ eV$$

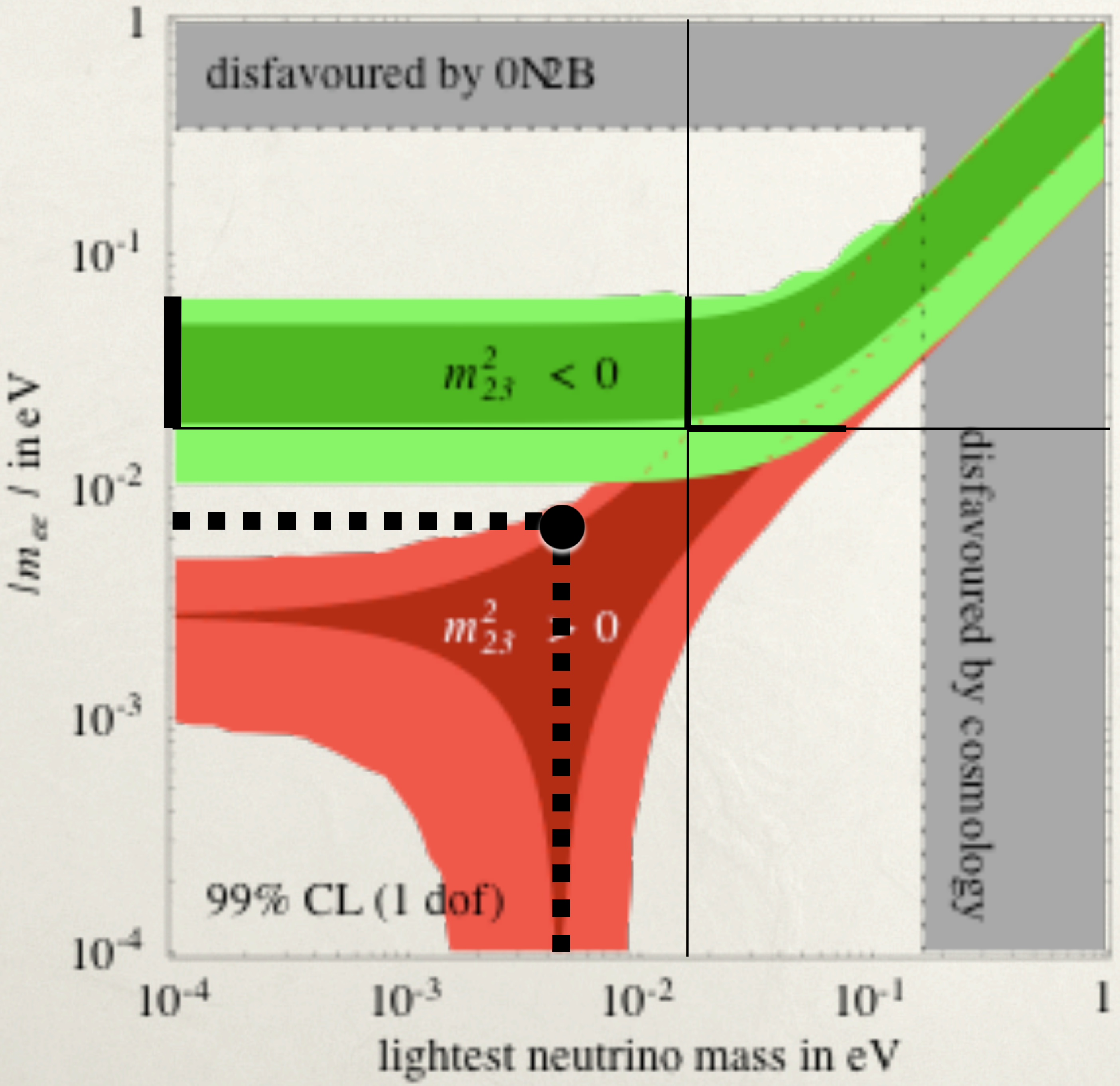


in the IH case the sum rule provides a lower bound on m_3

$$m_3 \geq 0.017\ eV$$

$$|m_{ee}| \geq 0.017\ eV$$

NLO corrections are negligible for NH and for IH close to the lower bound



disfavoured by 0N2B
$m^2_{23} < 0$
$m^2_{23} > 0$
disfavoured by cosmology
99% CL (1 dof)
$|m_{ee}|$ in eV
lightest neutrino mass in eV
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Additional tests: LFV from 1-loop SUSY particle exchange

in a class of SUSY realizations [F, Hagedorn, Lin, Merlo, 2008-2009]

$$\frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} = \frac{6 m_W^4 \alpha_{em}}{\pi m_{SUSY}^4} \left[\left| w_{ij}^{(1)} u^2 \right|^2 + \frac{m_j^2}{m_i^2} \left| w_{ij}^{(2)} u \right|^2 \right]$$

$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters

$$BR(\mu \to e\gamma) \approx BR(\tau \to \mu\gamma) \approx BR(\tau \to e\gamma)$$

independently from $u \approx \vartheta_{13}$

cfr. MFV [Cirigliano, Grinstein, Isidori, Wise 2005]

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left| \frac{2}{3} r \pm \sqrt{2} \sin\vartheta_{13} e^{i\delta} \right|^2 < 1$$

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

present (expected) sensitivity to m_{SUSY}

Assuming $w^{(1,2)}_{ij} = 1$

$BR(\mu\to e\gamma) < 1.2\times10^{-11}$ (10^{-13})	
$m_{SUSY} >$ 255 (820) GeV	u=0.005
$m_{SUSY} >$ 0.7 (2.5) TeV	u=0.05

$BR(\mu\to eee) < 10^{-12}$ (10^{-13})	
$m_{SUSY} >$ 140 (225) GeV	u=0.005
$m_{SUSY} >$ 400 (700) GeV	u=0.05

[F.F. and A. Paris 1005.5526]

$CR^{Ti}(\mu\to e) <$ (10^{-18})	
$m_{SUSY} >$ (2.3) TeV	u=0.005
$m_{SUSY} >$ (6.6) TeV	u=0.05

m_{susy} in the region of interest for LHC

[also Hagedorn, Molinaro, Petcov 0911.3605]

Leptogenesis

if ν^c_i transform in a 3-dim irreducible representation of G_f then ε_i=0 in the exact symmetry limit u=0.

$\Rightarrow$ ε_i = 0 at the LO

$\varepsilon_i \neq 0$ from the NLO corrections

$$\varepsilon_i \approx \frac{u^2}{16\pi} \qquad [\mathrm{NH}]$$

$$\varepsilon_i \approx \frac{u^2}{16\pi r} \qquad [\mathrm{IH}] \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{30}$$

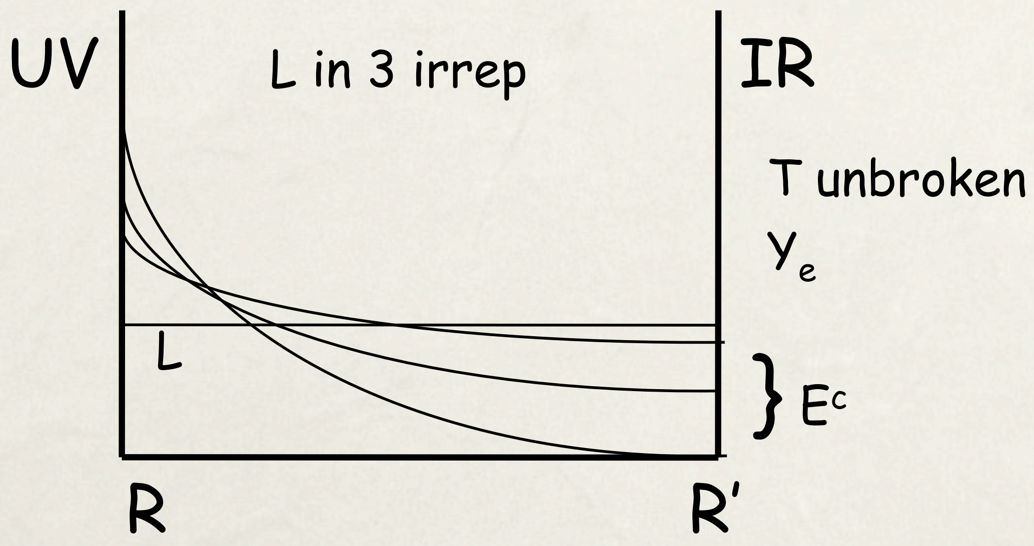
$\varepsilon_i \geq 10^{-6}$ to produce an acceptable baryon asymmetry

$$u \geq \begin{cases} 0.01 & [\mathrm{NH}] \\ 0.002 & [\mathrm{IH}] \end{cases}$$

in agreement with expected range of u

[Jenkins, Manohar 0807.4176
Bertuzzo, Di Bari, FF, Nardi 0908.0161
Hagedorn, Molinari, Petcov 0908.0240]

A_4 in RS setup [Csaki, Delaunay, Grojean, Grossman 0806.0356]



$$F_{E^c}, Y_e, F_L$$

diagonal in the same basis

- no tree level FCNC
  no μ−>eγ contribution from Y_e alone
- dominant contribution to μ−>eγ from (KK) W and (KK) N^c exchange
  BR≈10^{-13} for M_{KK}≈3 TeV
- more natural solution to vacuum alignment:
  G_f->T on IR brane and G_f->S on UV brane

solutions

(I) alignment

[Fitzpatrick, Perez, Randall 0710.1869
Csaki, Perez, Surnjon, Weiler 0907.0479]

5D quark mass terms are aligned with Yukawas $Y_{u,d}$

$$C_{U^c} \propto Y_u Y_u^+ \qquad C_{D^c} \propto Y_d Y_d^+ \qquad C_Q \propto Y_d^+ Y_d + r Y_u^+ Y_u$$

no FC in down sector in the limit r=0 -> requires r << 1

(II) universality

[Santiago 0806.1230
Csaki, Falkowski, 0806.3757]

$SU(3)_{D^c}$ flavour symmetry
unbroken by bulk: $C_{D^c}, F_{D^c} \propto 1$

indeed, from Huber's fit

$$c_{D_1^c} = 0.643 \quad c_{D_2^c} = 0.601 \quad c_{D_3^c} = 0.601$$

when C_{D^c} is proportional to the identity, the couplings of the 1st KK gluon to the D^c sector are flavour blind and $C_K^4=0$

(III) New Physics at the TeV completely flavour blind

e.g. low-energy SUSY in gauge mediation

Quark masses - grand unification

quarks assigned to the same A_4 representations used for leptons?

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- fermion masses from dim ≥ 5 operators, e.g. $\frac{\tau^c \varphi_T l H_d}{\Lambda}$
  good for leptons, but not for the top quark
- naïve extension to quarks leads diagonal quark mass matrices and to $V_{CKM}=1$
  departure from this approximation is problematic
  [expansion parameter (VEV/Λ) too small]

possible solution within T′, the double covering of A_4

[FHLM1]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1′ 1″ 3 2 2′ 2″

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$(t \quad b)$	t^c	b^c	η	ξ''
T'	2''	2''	2''	1	1	1	2'	1''

[older T′ models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators) at the leading order

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243$ $\quad$ 0.2257 ± 0.0021

$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

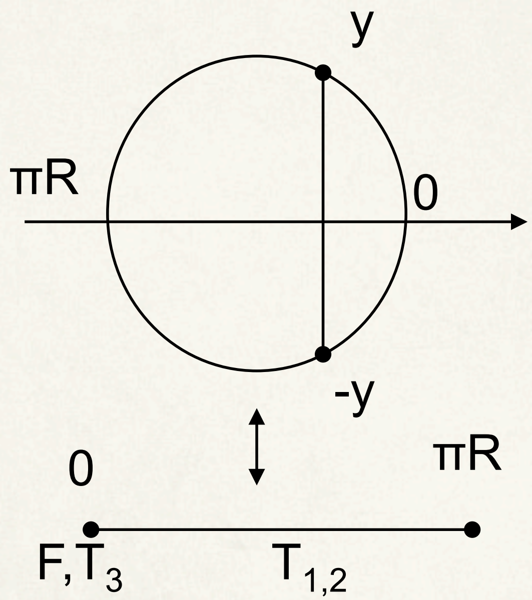
$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

other option:

[AFH]

SUSY SU(5) in 5D=M_4x(S^1x Z_2)
+
flavour symmetry A_4x$U(1)$



DT splitting problem solved
via SU(5) breaking induced by compactification

dim 5 B-violating operators forbidden!
p-decay dominated by gauge boson exchange (dim 6)

unwanted minimal SU(5) mass relation $m_e=m_d^T$ avoided by assigning $T_{1,2}$ to the bulk

the construction is compatible with A_4!

	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$
$SU(5)$	1	$\bar{5}$	10	10	10	5	$\bar{5}$
A_4	3	3	1''	1'	1	1	1'

reshuffling of singlet reps.

unsuppressed top Yukawa coupling T_3T_3

realistic quark mass matrices
by an additional U(1) acting on $T_{1,2}$

neutrino masses from see-saw
compatible with both normal and
inverted hierarchy

TB mixing + small corrections

A_4 as a leftover of Poincare symmetry in D>4

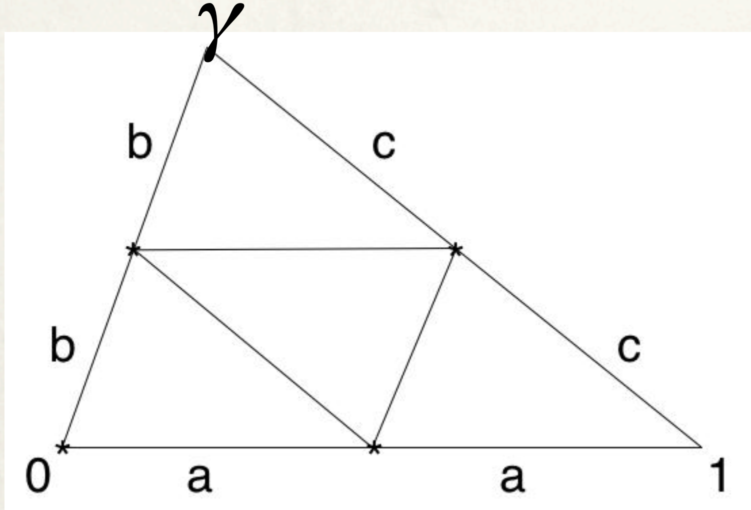
[AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

$\Longleftrightarrow$

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x ...

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \to z + \frac{1}{2} \qquad \text{[translation]}$$

$$T: \quad z \to \gamma^2 z \qquad \text{[rotation by } 120^0\text{]}$$

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)] the four fixed points (z_1,z_2,z_3,z_4) are permuted under the action of S and T

$$S: \qquad (z_1,z_2,z_3,z_4) \rightarrow (z_4,z_3,z_2,z_1)$$
$$T: \qquad (z_1,z_2,z_3,z_4) \rightarrow (z_2,z_3,z_1,z_4)$$

S and T satisfy $$\boxed{S^2 = T^3 = (ST)^3 = 1}$$

the compact space is invariant under a remnant of 2-translations x SO(2) isomorphic to the A_4 group

Field Theory

brane fields $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$, $\varphi_4(x)$ transform as 3 + (a singlet) under A_4

The previous model can be reproduced by choosing l, e^c, μ^c, τ^c, $H_{u,d}$ as brane fields and φ_T, φ_S and ξ as bulk fields.

other realizations of Anarchy (II)

Nelson-Strassler [0006251 "Suppressing Flavor Anarchy"]

Anarchy can arise when matter chiral supermultiplets X_i of the MSSM are coupled to a superconformal sector in some finite energy range

e.g.

$\Lambda_c = M_{GUT}$ $\qquad\qquad$ $\Lambda = M_{Pl}$

large positive anomalous dimensions for X_i: $\qquad \frac{\gamma_i}{2} \equiv d(X_i) - 1 > 0$

$$K = \sum_i Z_i X_i^+ X_i + \dots \qquad\qquad Z_i(\Lambda_c) = \underbrace{Z_i(\Lambda)}_{1}\left(\frac{\Lambda_c}{\Lambda}\right)^{-\gamma_i}$$

Anarchy through wave function renormalization: $X_i \to F_{X_i} X_i$

$$w = Y_{ij} X_i X_j H + \dots \to (F_{X_i} Y_{ij} F_{X_j}) X_i X_j H + \dots$$

$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{\gamma_i}{2}} < 1$$

[as in FN with a single flavon and positive FN charges]

no underlying flavour symmetry

[an anomaly free R symmetry is generated dynamically at the IR stable fixed point: $\dim(X_i) = 2/3\, R(X_i)$]

anomalous dimensions γ_i calculable when gauge group and field content are known

[Polland, Simmons-Duffin 0910.4585]

Minimal Flavor Violation [MFV]

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]

$$G_f = SU(3)_l \times SU(3)_{e^c} \times \ldots$$

the largest G_f

$$l = (\bar{3},1) \qquad e^c = (1,3)$$

$$\varphi \equiv \begin{cases} y_e = (3,\bar{3}) \\ Y = (6,1) \end{cases}$$

G_f broken only by the Yukawa coupling of L_{SM} and L_5

y_e and Y can be expressed in terms of lepton masses and mixing angles

$$y_e = \sqrt{2}\frac{m_e^{diag}}{v} \qquad\qquad Y = \frac{\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$ are of the same size as in A_4x…
similar lower bounds on the scale M

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij} = \beta\,(y_e Y^+ Y)_{ij} + \dots$$

$$= \sqrt{2}\beta \frac{(m_l)_{ii}}{v} \frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3}\right] + \dots$$

+ for normal hierarchy
- for inverted hierarchy

a positive signal at MEG $10^{-11} < R_{\mu e} < 10^{-13} \div 10^{-14}$ always accommodated [but for a small interval around $\vartheta_{13} \approx 0.02$ where $R_{\mu e} = 0$]

non-observation of R_{ij} can be accommodated by lowering Λ_L

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]

both $\mu \to e\gamma$ and $\tau \to \mu\gamma$ could be above future sensitivity

$R_{\mu e} < 1.2 \times 10^{-11}$ implies $R_{\tau\mu} < 10^{-9}$

0.02

ϑ_{13}

0 0.1 0.2

here $\mu \to e\gamma$ vanishes

MFV [scale M can be of order 1 TeV]

both
$\mu \to e\gamma$ and $\tau \to \mu\gamma$
could be above future sensitivity

$$R_{\mu e} < 1.2 \times 10^{-11}$$

implies

$$R_{\tau\mu} < 10^{-9}$$

0.02 here $\mu \to e\gamma$ vanishes

ϑ_{13}

0 0.05 0.1 0.2

only
$\mu \to e\gamma$
can be above experimental sensitivity

disfavoured by A_4

SUSYxA_4 [scale M can be of order 1 TeV]

String Theory [heterotic string compactified on orbifolds]

in string theory the discrete flavour symmetry is in general bigger than the isometry of the compact space. [Kobayashi, Nilles, Ploger, Raby, Ratz 2006]

orbifolds are defined by the identification

$$(\vartheta\, x) \approx x + l \qquad \begin{cases} l = n_a e_a & \text{translation in a lattice} \\ \vartheta & \text{twist} \end{cases}$$

group generated by (ϑ, l) is called space group

fixed points: special points x_F satisfying

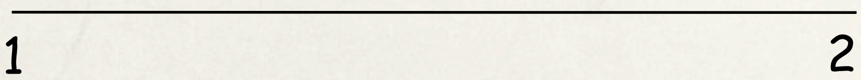
$$x_F \equiv (\vartheta_F^K\, x_F) + l_F \qquad \text{for some} \qquad (\vartheta_F^K, l_F)$$

twisted states living at the fixed point $x_F = (\vartheta_F^K, l_F)$ have couplings satisfying space group selection rules [SGSR]. Non-vanishing couplings allowed for

$$\prod_F (\vartheta_F^K, l_F) \equiv (1,0)$$

G_f is the group generated by the orbifold isometry and the SGSR

Example: S^1/Z_2



Isometry group = S_2 generated by σ^1 in the basis {|1>,|2>}

SGSR = $Z_2 \times Z_2$ generated by $(\sigma^3,-1)$

[allowed couplings when number n_1 of twisted states at |1> and the number n_2 of twisted states at |2> are even]

G_f = semidirect product of S_2 and $(Z_2 \times Z_2) \equiv D_4$

group leaving invariant a square

relation between A_4 and the modular group

[AF2]

modular group PSL(2,Z): linear fractional transformation

complex variable

$$z \to \frac{a\,z+b}{c\,z+d}$$

$$a,b,c,d \in Z$$
$$ad-bc=1$$

discrete, infinite group generated by two elements

$$\underbrace{z \to -\frac{1}{z}}_{S}$$

$$\underbrace{z \to z+1}_{T}$$

obeying

$$S^2 = (ST)^3 = 1$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$A_4 = \frac{PSL(2,Z)}{H}$$

$\Rightarrow$

representations of A_4 are representations of PSL(2,Z)

infinite discrete normal subgroup of PSL(2,Z)

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \rightarrow \nu_\mu$ disappearance channel

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

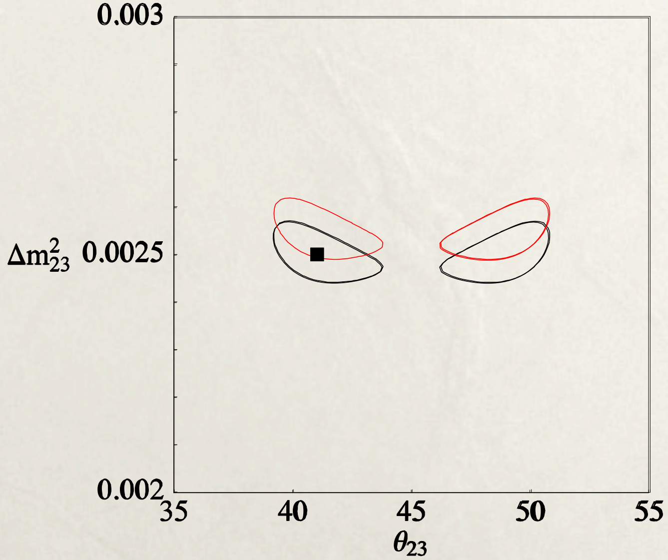
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\text{rad} \leftrightarrow 2.9^0$$

improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]