MODULI SPACES OF ALGEBRAIC CURVES AND AUTOMORPHIC FORMS (60 MUNITES × 3)

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In this series of three lectures, we will review some results on arithmetic geometry of algebraic curves and their moduli space. In particular, we will explain how Schottky uniformization of Riemann surfaces is extended in arithmetic geometry, and is applied to studying Teichmüller modular forms which are defined as automorphic forms on the moduli space of curves.

In the first lecture, we consider the arithmetic Schottky uniformization theory which constructs generalized Tate curves, and show that their multiplicative periods, called universal periods, are computable integral power series. In the second lecture, using the evaluation theory on the generalized Tate curves we study arithmetic properties of Teichmüller modular forms, and apply our result to the geometry of the moduli space of curves via Mumford's isomorphism and Klein's amazing formula. In the third lecture, by Teichmüller modular forms and nonarchimedean theta functions, we consider the Schottky problem characterizing the Jacobian locus and Jacobian varieties, and give algebraic and rigid analytic versions of results of Shiota and Krichever.