Singular Modular Forms on the Exceptional Domain

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Abstract

A modular form f with the Fourier expansion

$$f(Z) = \sum_{T \geq 0} a(T) e^{2\pi i (T,Z)}$$

is called a singular modular form if $a(T) = 0$ when T is of full rank.

According to a general theorem by E. Freitag, all singular modular forms on the Siegel upper half plane

$$\mathcal{H}_n = \left\{ Z = X + iY \mid {}^tX = X \in M_n(\mathbb{R}), {}^tY = Y \in M_n(\mathbb{R}), Y > 0 \right\},$$

are nothing but finite linear combinations of theta series of the form

$$\vartheta(S, Z) = \sum_{G \in \mathbb{Z}^{m \times n}} e^{\pi i (S[G], Z)} \quad (m < n)$$

where S is a $m \times m$ positive definite integral symmetric matrix and $S[G] = {}^tGXG$.

Such an assertion is not quite true on the exceptional domain of 27 dimensions. Indeed, it is a long-standing open problem to construct possible theta series on the exceptional domain since Baily initiated the theory of modular forms on such domain in 1970.