Schedule of Lectures

August, 24 (Wednesday)
14:30 - 15:50: Minking Eie (National Chung Cheng University),
- Singular modular forms on the exceptional domain
16:10 - 17:30: Takahiro Shiota (Kyoto University),
- Soliton equations and the Riemann-Schottky problem I

August, 25 (Thursday)
09:30 - 10:30: Valery Gritsenko (Lille University of Science and Technology),
- Jacobi modular forms and Borcherds automorphic products I
10:50 - 11:50: Chang Heon Kim (Hanyang university),
- Application of two geometric theta lifts to traces of singular moduli
11:50 - 14:30: Lunch Time
14:30 - 15:50: Minking Eie (National Chung Cheng University),
- Shuffle product formulae of multiple zeta values
16:10 - 17:30: Takahiro Shiota (Kyoto University),
- Soliton equations and the Riemann-Schottky problem II

August, 26 (Friday)
09:30 - 10:30: Valery Gritsenko (Lille University of Science and Technology),
- Jacobi modular forms and Borcherds automorphic products II
10:50 - 12:10: Minking Eie (National Chung Cheng University),
- The sum formula, the restricted sum formula and their generalizations
12:10 - 14:30: Lunch Time
14:30 - 15:30: Valery Gritsenko (Lille University of Science and Technology),
- Jacobi modular forms and Borcherds automorphic products III
15:50 - 17:10: Riccardo Salvati Manni (Sapienza Universita di Roma)
- Modular forms, hyperelliptic loci and applications

August, 29 (Monday)
09:30 - 10:30: Takakshi Ichikawa (Saga University),
- Moduli spaces of algebraic curves and automorphic forms I
10:50 - 11:50: Subong Lim (Postech),
- Modular forms and L-functions
11:50 - 14:30: Lunch Time
14:30 - 15:30: Takakshi Ichikawa (Saga University),
- Moduli spaces of algebraic curves and automorphic forms II
15:30 - 17:10: Riccardo Salvati Manni (Sapienza Universita di Roma)
- Siegel modular varieties with a Calabi-Yau model

August, 30 (Tuesday)
09:30 -10:30: Takakshi Ichikawa (Saga University),
- Moduli spaces of algebraic curves and automorphic forms III
10:50 - 11:50: Dohoon Choi (Korea Aerospace University),
- T. B. A.

GALOIS REPREPRESENATIONS OVER FINITE FIELDS (60 MINUTES)

DOHOON CHOI

This is a survey talk on Serre's modularity conjecture and its generalization.

Singular Modular Forms on the Exceptional Domain

Minking Eie

Department of Mathematics
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June 28, 2011

Abstract

A modular form f with the Fourier expansion

$$f(Z) = \sum_{T \geq 0} a(T) e^{2\pi i (T,Z)}$$

is called a singular modular form if $a(T) = 0$ when T is of full rank.

According to a general theorem by E. Freitag, all singular modular forms on the Siegel upper half plane

$$\mathcal{H}_n = \left\{ Z = X + iY \mid {}^tX = X \in M_n(\mathbb{R}), {}^tY = Y \in M_n(\mathbb{R}), Y > 0 \right\},$$

are nothing but finite linear combinations of theta series of the form

$$\vartheta(S, Z) = \sum_{G \in \mathbb{Z}^{m \times n}} e^{\pi i (S[G], Z)} \quad (m < n)$$

where S is a $m \times m$ positive definite integral symmetric matrix and $S[G] = {}^tGXG$.

Such an assertion is not quite true on the exceptional domain of 27 dimensions. Indeed, it is a long-standing open problem to construct possible theta series on the exceptional domain since Baily initiated the theory of modular forms on such domain in 1970.

Shuffle Product Formula of Multiple Zeta Values

Minking Eie
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National Chung Cheng University
Taiwan, Republic of China

Abstract

A multiple zeta values or r-fold Euler sum defined by

$$\zeta(\alpha_1, \alpha_2, \cdots, \alpha_r) = \sum_{1 \le n_1 < n_2 < \cdots < n_r} n_1^{-\alpha_1} n_2^{-\alpha_2} \cdots n_r^{-\alpha_r}$$

with $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_r)$ an r-tuple of positive integers and $\alpha_r \ge 2$, is a natural generalization of the classical Euler sum

$$S_{p,q} = \sum_{k=1}^{\infty} \frac{1}{k^q} \sum_{j=1}^{k} \frac{1}{j^p}.$$

It is a problem proposed by Goldbach to Euler in an attempt to evaluate $S_{p,q}$ in terms of the special values at positive integers of the Riemann zeta function defined by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad Res > 1.$$

The shuffle product formula of two multiple zeta values then express the product of two multiple zeta values as a linear combination of multiple zeta values with integral coefficients.

The Sum Formula, the Restricted Sum Formula, Their Generalizations and Applications

Minking Eie
Department of Mathematics
National Chung Cheng University
Taiwan, Republic of China

Abstract

The sum formula

$$\sum_{\substack{|\boldsymbol{\alpha}|=m+r+1\\ \alpha_j\geq 1}} \zeta(\alpha_0,\alpha_1,\ldots,\alpha_r+1)=\zeta(m+r+2)$$

asserted that the sum of all multiple zeta values of the same depth $r+1$ and weight $m+r+2$ is equal to a single zeta values $\zeta(m+r+2)$. In particular, when $m=0$, it implies that

$$\zeta(\{1\}^r,2)=\zeta(r+2)$$

which is a special case of duality. It was originally conjectured by C.Moen and M.Schmidt independently around 1994. It was proved for the case of depth 2 ($r=1$) by Euler long time ago and for the case of depth 3 ($r=2$) by Hoffman and Moen in 1996. A.Granville proved the general cases in 1996 and he mentioned that it was also proved independently by D.Zagier in one of his unpublished papers. Zagier had also made a remark: Although this proof is not very long, it seems too complicated compared with the elegance of the statement.

JACOBI MODULAR FORMS AND BORCHERDS AUTOMORPHIC PRODUCTS (60 MINUTES $\times$ 3)

VALERY GRITSENKO

1. INTRODUCTION: The orthogonal group O(2,n) and moduli spaces.

A) Modular varieties of orthogonal type and moduli spaces of elliptic curves and polarized K3 surfaces.
B) Rational quadratic divisors and ramification divisor of modular varieties.
C) Pluri-canonical differential forms and modular forms on orthogonal group.
D) Projective and affine definition of modular forms of orthogonal type.
E) Fourier-Jacobi expansion.

2. JACOBI LIFTING.

A) Modular forms on a parabolic subgroup, Heisenberg group and its modular character.
B) Weil representation, vector valued modular forms and Jacobi forms.
C) Singular weight and classical Jacobi theta-series.
D) Jacobi lifting and the simplest modular forms for orthogonal groups.

3. BORCHERDS AUTOMORPHIC PRODUCTS.

A) The first automorphic correction of Jacobi forms.
B) Borcherds products in terms of Jacobi forms.
C) Examples and applications.

MODULI SPACES OF ALGEBRAIC CURVES AND AUTOMORPHIC FORMS (60 MUNITES $\times$ 3)

TAKASHI ICHIKAWA

In this series of three lectures, we will review some results on arithmetic geometry of algebraic curves and their moduli space. In particular, we will explain how Schottky uniformization of Riemann surfaces is extended in arithmetic geometry, and is applied to studying Teichmüller modular forms which are defined as automorphic forms on the moduli space of curves.

In the first lecture, we consider the arithmetic Schottky uniformization theory which constructs generalized Tate curves, and show that their multiplicative periods, called universal periods, are computable integral power series. In the second lecture, using the evaluation theory on the generalized Tate curves we study arithmetic properties of Teichmüller modular forms, and apply our result to the geometry of the moduli space of curves via Mumford's isomorphism and Klein's amazing formula. In the third lecture, by Teichmüller modular forms and nonarchimedean theta functions, we consider the Schottky problem characterizing the Jacobian locus and Jacobian varieties, and give algebraic and rigid analytic versions of results of Shiota and Krichever.

APPLICATION OF TWO GEOMETRIC THETA LIFTS TO TRACES OF SINGULAR MODULI (60 MINUTES)

CHANG HEON KIM

In this talk I will investigate Borcherds lifts of vector valued modular forms of weight 1/2 and Kudla Milson theta lifts of weakly holomorphic modular forms of weight 0. As their application I will show the modularity of traces of singular moduli.

THETA LIFTING ON WEAK MAASS FORMS (60 MINUTES)

SUBONG LIM

We study a theta lifting from weak Maass forms of half integral weight to those of even integral weight. This is an extension of Shimura lifting for a holomorphic modular forms.

TWO LECTURES BY RICCARDO SALVATI MANNI

Modular forms, hyperelliptic loci and applications (80 minutes)

We will explain a ring homomorphism from the ring of modular forms to the ring of binary invariant. We will report about classical results due to Thomae, Igusa, Mumford and some recent consequences that lead to some interesting applications concerning the slope of divisors and covering of the moduli spaces of curves in low genera.

Siegel modular varieties with a Calabi-Yau model (80 minutes)

We will describe some Siegel modular varieties which admit a weak Calabi–Yau model. Not all of them admit a projective model. We will exhibit criterions for the projectivity. We will discuss a significative example which cannot admit a projective model. We shall give a method for computing (in the projective case) Hodge number.

SOLITON EQUATIONS AND THE RIEMANN-SCHOTTKY PROBLEM (80 MINUTES $\times$ 2)

TAKAHIRO SHIOTA

A one-to-one correspondence between commutative subring of certain non-commutative ring of operators, and geometric data consisting of an algebraic curve, one or more points on it and a torsion-free sheaf over it, was discovered and applied to a construction of explicit solutions of certain nonlinear equations by Krichever in 1970s. The correspondence also leads very naturally to a solution of the Riemann-Schottky problem, i.e., a characterization of Jacobians among all ppav's, using the same class of equations: an indecomposable ppav is a Jacobian of a curve iff its Riemann theta function solves those equations. One can reduce the number of equations and characterize the Jacobians using just one PDE called the KP equation, or as proved by Krichever recently, its difference analogue called Fay's trisecant identity. I would also like to point out that since those equations involve derivatives or differences of the theta function, we have not obtained (vector-valued) modular forms that characterize the Jacobian locus.