# The Sum Formula, the Restricted Sum Formula, Their Generalizations and Applications 

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#### Abstract

The sum formula $$
\sum_{\substack{|\boldsymbol{\alpha}|=m+r+1 \\ \alpha_{j} \geq 1}} \zeta\left(\alpha_{0}, \alpha_{1}, \ldots, \alpha_{r}+1\right)=\zeta(m+r+2)
$$


asserted that the sum of all multiple zeta values of the same depth $r+1$ and weight $m+r+2$ is equal to a single zeta values $\zeta(m+r+2)$. In particular, when $m=0$, it implies that

$$
\zeta\left(\{1\}^{r}, 2\right)=\zeta(r+2)
$$

which is a special case of duality. It was originally conjectured by C.Moen and M.Schmidt independently around 1994. It was proved for the case of depth $2(r=1)$ by Euler long time ago and for the case of depth $3(r=2)$ by Hoffman and Moen in 1996. A.Granville proved the general cases in 1996 and he mentioned that it was also proved independently by D.Zagier in one of his unpublished papers. Zagier had also made a remark: Although this proof is not very long, it seems too complicated compared with the elegance of the statement.

