Seoul-Tokyo Conference on
Elliptic and Parabolic PDEs and Related Topics

Date : Nov 30-Dec 1, 2012

Place : 5F Seminar room / KIAS, Seoul, Korea

Time Table:

Time	30-Nov	01-Dec
9:30-10:10	Norbert Pozar	H. Matano
10:20-11:00	K. Lee	Jinmyoung Seok
11:20-12:00	Yutaka Terasawa	M. Yamamoto
12:00-14:00	Lunch	Lunch
14:00-14:40	Jihoon Lee	Post session
14:50-15:30	Ken Abe	Nao Hamamuki
15:50-16:30	Soojung Kim	Minha Yoo

Chairman:

Time	30-Nov	01-Dec
Morning	D. Chae	K. Lee
Afternoon	Norbert Pozar	Giga

Title & Abstract

Ken Abe (Univ. of Tokyo)

Title: Stokes resolvent estimates in spaces of bounded functions

Abstract:
We consider a direct resolvent approach for the analyticity of the Stokes semigroup in spaces of bounded functions. This was recently proved by an indirect argument for admissible domains including bounded domains of class C^3. Our approach is based on an adjustment of a standard resolvent estimate method for general elliptic operators introduced by K. Masuda (1972) and H. B. Stewart (1974). The resolvent approach in particular clarifies the sectorial region, Re $z > 0$ for $z \in \mathbf{C}$ for which the Stokes semigroup has an analytic continuation in spaces of bounded functions. This talk is based on a joint work with Professor Yoshikazu Giga and Professor Matthias Hieber.

Nao Hamamuki (Univ. of Tokyo)

Title: Asymptotically self-similar solutions to curvature flow equations with prescribed contact angle

Abstract:
We study the asymptotic behavior of solutions to fully nonlinear second order parabolic equations including a generalized curvature flow equation which was introduced by Mullins in 1957 as a model of evaporation-condensation.
We prove that, in the multi-dimensional half space, solutions of the Problem with prescribed contact angle asymptotically converge to a self-similar solution of the associated problem. We also study the depth of the groove, which is represented by the value of the self-similar solution at the boundary.
It turns out that, as the contact angle tends to zero, the depth of the Groove is well approximated by the linearized problem.

Soojung Kim (Seoul National Univ.)

Title: Asymptotic Behavior in Parabolic Fully Nonlinear equations and its application to Elliptic Eigenvalue Problems

Abstract : We study the fully nonlinear parabolic equation $$F(D^2 u^m)-u_t=0\quad\mbox{in}\,\,\,\Omega\times(0,+\infty),\quad m\geq1,$$ with the Dirichlet boundary condition and positive initial data in a smooth bounded domain $\Omega\subset\mathbb{R}^n,$ provided that the operator F is uniformly elliptic and positively homogeneous of order one. We prove that the renormalized limit of parabolic flow $u(x,t)$ as $t\rightarrow +\infty$ is the corresponding positive eigenfunction which satisfies
$$F(D^2 \varphi) +\mu\varphi^{p}=0\quad\mbox{in}\,\,\,\Omega,$$
where $0<p=\frac{1}{m}\leq 1$ and $\mu>0$ is the corresponding eigenvalue.
We also show that some geometric property of the positive initial data is preserved by the parabolic flow, under the additional assumptions that Ω is convex and F is concave. As a consequence, the eigenfunction has such geometric property, that is,
$ \log (\varphi)$ is concave in the case $p=1$ and $\varphi^{\frac{1-p}{2}}$ is concave for $0<p<1.$
This is a joint work with Ki-ahm Lee.

Jihoon Lee (SungKyunKwan Univ.)

Title: Global existence of Navier-Stokes-Vlaosv Fokker-Planck equations

Abstract: In this talk, we consider two and three dimensional Navier-Stokes-Vlaosv-Fokker Planck equations and some related equations.
We consider both incompressible and compressible Navier-Stokes-Vlaosv-Fokker-Planck equations. We also show global existence of solution near equilibrium and 2 dimensional case.
This is the joint work with Myeongju Chae and Kyungkeun Kang.

Ki-Ahm Lee (Seoul National Univ.)

Title: Regularity theory on nonlinear integro-differential operators

In this talk , we are going to introduce *integro-differential operators* with possibly nonsymmetric kernel:

$$\mathcal{L}u(x) = \text{p.v.} \int_{\mathbb{R}^n} \mu(u, x, y) K(y)\, dy \tag{1}$$

where $\mu(u,x,y) = u(x+y) - u(x) - (\nabla u(x) \cdot y)\chi_{B_1}(y)$ in symmetric case and $\mu(u,x,y) = u(x+y) - u(x) - (\nabla u(x) \cdot y)\chi_{B_1}(y)$ in general , which describes the infinitesimal generator of given purely jump processes. We also discuss nonlinear integro-differential operators come from the stochastic control theory related with

$$\mathcal{I}u(x) = \sup_{\alpha} \mathcal{L}_\alpha u(x),$$

or game theory associated with

$$\mathcal{I}u(x) = \inf_{\beta} \sup_{\alpha} \mathcal{L}_{\alpha\beta} u(x), \tag{2}$$

when the stochastic process is of Lèvy type allowing jumps. We want to introduce recent development on the regularity theory on these nonlinear nonlocal equations and our results

Hiroshi Matano (Univ. of Tokyo)

Title: Mathematical analysis of a 3D model of cellular electrophysiology

Abstract: In this talk I will consider an evolution equation that describes the electrical activity of cells with 3D structure and discuss the existence and properties of time-global solutions. The model is written as a pseudodifferential equation on a closed surface, which represents the cell membrane.
Previously the existence of a global classical solution was not known, due mainly to the lack of a uniform L^\infty bound.
The difficulty lies in the fact that the principal part of the pseudo differential operator does not satisfy the maximum principle unless the shape of the cell is close to a perfect sphere.
In our earlier work, we considered the case where the cell membrane is a compact surface and, using what we call the ``quasipositivity principle'', we were able to prove the uniform bound (hence the global existence) of solutions for both FitzHugh-Nagumo and the Hodgkin-Huxley kinetics, as well as for the simpler Allen-Chan equation.
In this talk I will first review our earlier results on the compact surface case and then discuss the case where the cell membrane is an infinite cylinder. This is joint work with Yoichiro Mori

Norbert Pozar (Univ. of Tokyo)

Title: Homogenization of a Hele-Shaw-type problem in periodic timedependent media

Abstract: In this talk we discuss a generalization of the Hele-Shaw problem, a popular model of the flow in a porous medium, and study the behavior of its solutions as the size of inhomogeneity of the medium approaches zero. The main new feature of this nonlocal free boundary problem is the periodic dependence of the free boundary velocity on time as well as on position. Unfortunately, this dependence breaks the obstacle problem structure, which is usually used to define weak solutions and facilitates homogenization, and therefore a notion of viscosity solutions is necessary. We extend the comparison principle to allow for time-dependence and even discontinuity of the free boundary velocity, and obtain well-posedness of viscosity solutions. This provides a reasonable setting for homogenization. Since the standard technique of correctors is not available, we improve the geometric method of I. Kim (2007), based on the pioneering work of Caffarelli, Souganidis and Wang (2005), featuring an auxiliary obstacle problem.
In contrast to the previous works, we identify the homogenized free boundary velocity via a ``mesoscopic flatness'' of the free boundary.
We develop new tools to handle this quantity, in particular a boundary cone flatness estimate. Finally, we show that, in the homogenization limit, the solutions converge to the solution of a homogenized Hele-Shaw-type problem.

Jinmyoung Seok(Seoul National Univ.)

Title: Existence results about the nonlinear schrodinger-poisson equations

Abstract:

The system of the Nonlinear Schrödinger-Poisson equations consists of a Schrödinger equation coupled with a Poisson term as the following:

$$\begin{cases} -\Delta u + \omega u + \lambda \phi u = f(u) & \text{in } \mathbb{R}^3 \\ -\Delta \phi = u^2, \quad \lim_{|x|\to\infty} \phi(x) = 0 & \text{in } \mathbb{R}^3 \end{cases}$$

It describes standing waves of a system of identical quantum particles which have electric charges, for examples, a system of electrons.

In this talk, I will present several recent results obtained so far by the variational methods. Our main concern is the existence with respect to the parameter $\lambda \in \mathbb{R}$ representing the strength of the coupling effect.

Yutaka Terasawa (Univ. of Tokyo)

Title: On a diffuse interface model for non-Newtonian fluids with matched densities

Abstract: We consider a diffuse interface model for power-law fluids with matched densities. We construct weak solutions of the system for a certain range of the powers associated with power-law fluids. For the construction of the solutions, we consider an approximate system and pass those solutions to the limit, using an adaptation of the Lipschitz truncation method, which was used for the construction of weak solutions of the power-law fluid equations with low powers in Diening-Ruzicka-Wolf ('10). This talk is based on a joint work with Helmut Abels and Lars Diening.

Masahiro Yamamoto (Univ. of Tokyo)

Title: Mathematical analysis for several inverse problems for fractional diffusion equations

Abstract: We consider fractional diffusion equations where the time derivative is replaced by the Caputo fractional time derivative, which is a model equation for anomalous diffusion in heterogeneous media.
We briefly review properties of solutions to the forward problems and discuss several types of inverse problems:
backward problem, determination of coefficients, the determination of boundary value, etc. and prove uniqueness and some stability results.

Minha Yoo (Seoul National Univ.)

Title: Highly oscillating thin obstacles

ABSTRACT. The focus of this talk is on a thin obstacle problem where the obstacle is defined on the intersection between a hyperplane Γ in $\mathbb{R}^n$ and a periodic perforation $\mathcal{T}_\epsilon$ of $\mathbb{R}^n$, depending on a small parameter $\epsilon > 0$. As $\epsilon \to 0$, it is crucial to estimate the frequency of intersections and to determine this number locally. This is done using strong tools from uniform distribution. By employing classical estimates for the discrepancy of sequences of type $\{k\alpha\}_{k=1}^{\infty}$, $\alpha \in \mathbb{R}$, we are able to extract rather precise information about the set $\Gamma \cap \mathcal{T}_\epsilon$. As $\epsilon \to 0$, we determine the limit u of the solution u_ϵ to the obstacle problem in the perforated domain, in terms of a limit equation it solves. We obtain the typical "strange term" behaviour for the limit problem, but with a different constant taking into account the contribution of all different intersections, that we call the averaged capacity. Our result depends on the normal direction of the plane, but holds for a.e. normal on the unit sphere in $\mathbb{R}^n$.