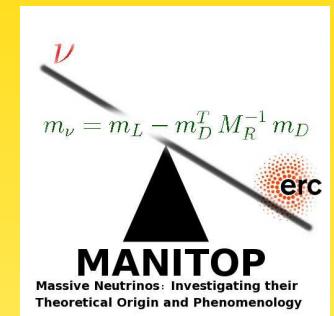


# Gauged $L_\mu - L_\tau$ Symmetry



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(MPIK, HEIDELBERG)  
KIAS, 15/11/11



based on

Julian Heeck, W.R., JPG38 (2011) 085005; PRD84 (2011) 075007

## Outline

- $Z-Z'$  mixing
- Gauged  $L_\alpha - L_\beta$
- first case:  $L_\mu - L_\tau$  and low  $Z'$  mass; application to MINOS anomaly
- second case:  $L_\mu - L_\tau$  and large  $Z'$  mass

## Z-Z' mixing

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{Z'} + \mathcal{L}_{\text{mix}}$$

with

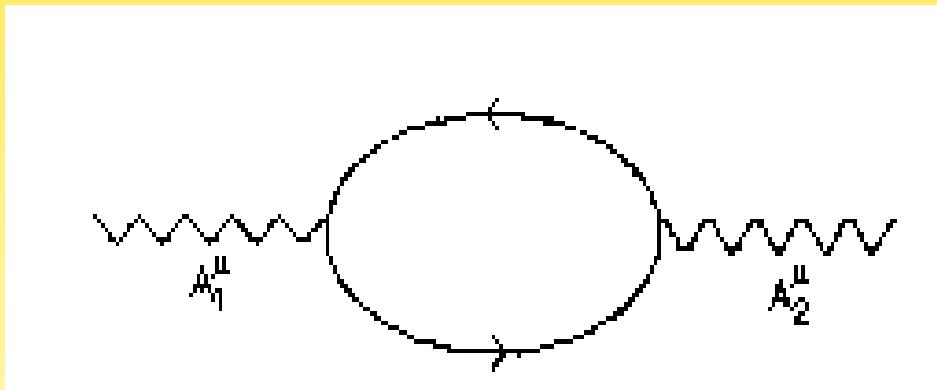
$$\begin{aligned}
 \mathcal{L}_{\text{SM}} &= -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}_{\mu\nu}^a\hat{W}^{a\mu\nu} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu \\
 &\quad - \hat{e} \sum_i \bar{\psi}_i \gamma^\mu \left( \frac{1}{\hat{c}_W} (Y_L^i P_L + Y_R^i P_R) \hat{B}_\mu + \frac{1}{\hat{s}_W} P_L T^a \cdot \hat{W}_\mu^a \right) \psi_i \\
 \mathcal{L}_{Z'} &= -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}_{Z'}^2\hat{Z}'_\mu\hat{Z}'^\mu - \frac{\hat{g}'}{2} \sum_i \bar{\psi}_i \gamma^\mu (f_V^i - f_A^i \gamma^5) \psi_i \hat{Z}'_\mu \\
 \mathcal{L}_{\text{mix}} &= -\frac{\sin \chi}{2}\hat{Z}'_{\mu\nu}\hat{B}^{\mu\nu} + \delta\hat{M}^2 \hat{Z}'_\mu\hat{Z}^\mu
 \end{aligned}$$

Babu, Kolda, March-Russell, PRD**57**

## Z-Z' mixing

$$\mathcal{L}_{\text{mix}} = - \frac{\sin \chi}{2} \hat{Z}'_{\mu\nu} \hat{B}^{\mu\nu} + \delta \hat{M}^2 \hat{Z}'_\mu \hat{Z}^\mu$$

- $\delta \hat{M}^2$  when one  $Z'$  is charged under both groups (couple to same scalar);
- $\sin \chi$  arises directly or radiatively ([Holdom](#), [PLB166](#))



- same (with  $\chi = 0$ ) for non-Abelian symmetries

## Z-Z' mixing

- diagonalize field strength

$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} 1 & \sin \chi \\ 0 & \cos \chi \end{pmatrix} \begin{pmatrix} \hat{B}_\mu \\ \hat{Z}'_\mu \end{pmatrix}$$

- orthogonal  $O(3)$  rotation to diagonalize mass terms

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \hat{c}_W \sin \chi \\ 0 & \cos \xi & -\hat{s}_W \cos \xi \sin \chi + \sin \xi \cos \chi \\ 0 & -\sin \xi & \cos \xi \cos \chi + \hat{s}_W \sin \xi \sin \chi \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{Z}' \end{pmatrix}$$

physical fields  $A, Z_1, Z_2$

## Z-Z' mixing

$$M_{1,2}^2 = \frac{a+c}{2} \pm \sqrt{b^2 + \left(\frac{a-c}{2}\right)^2} \quad \text{and} \quad \tan 2\xi = \frac{2b}{a-c}$$

with

$$\begin{aligned} a &= \hat{M}_Z^2, & b &= \hat{s}_W \tan \chi \hat{M}_Z^2 + \frac{\delta \hat{M}^2}{\cos \chi} \\ c &= \frac{1}{\cos^2 \chi} \left( \hat{M}_Z^2 \hat{s}_W^2 \sin^2 \chi + 2 \hat{s}_W \sin \chi \delta \hat{M}^2 + \hat{M}_{Z'}^2 \right) \end{aligned}$$

## Z-Z' mixing

The gauge boson couplings to fermions are hence changed to

$$ej_{\text{EM}}\hat{A} + \frac{e}{2s_W c_W} j_{\text{NC}} \hat{Z} + g' j' \hat{Z}' \rightarrow \\ \left( ej_{\text{EM}}, \frac{e}{2\hat{s}_W \hat{c}_W} j_{\text{NC}}, g' j' \right)^T \begin{pmatrix} 1 & -\hat{c}_W \sin \xi \tan \chi & -\hat{c}_W \cos \xi \tan \chi \\ 0 & \cos \xi + \hat{s}_W \sin \xi \tan \chi & \hat{s}_W \cos \xi \tan \chi - \sin \xi \\ 0 & \frac{\sin \xi}{\cos \chi} & \frac{\cos \xi}{\cos \chi} \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix}$$

with  $j_{\text{EM}} \equiv j_W^3 + \frac{1}{2} j_Y$  and  $j_{\text{NC}} \equiv 2j_W^3 - 2\hat{s}_W^2 j_{\text{EM}}$

From Lagrangian:

$$\alpha_{\text{EM}} S \simeq 4\xi c_W^2 s_W \tan \chi$$

$$\alpha_{\text{EM}} T \simeq \xi^2 \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} - 1 \right) + 2\xi s_W \tan \chi$$

## Z-Z'-Z'' mixing

for 3  $U(1)$  symmetries

$$\begin{aligned}\mathcal{L}_{\text{mix}} = & -\frac{\sin \alpha}{2} \hat{B}_{\mu\nu} \hat{X}_1^{\mu\nu} - \frac{\sin \beta}{2} \hat{B}_{\mu\nu} \hat{X}_2^{\mu\nu} - \frac{\sin \gamma}{2} \hat{X}_1{}_{\mu\nu} \hat{X}_2^{\mu\nu} \\ & + m_1^2 \hat{Z}_\mu \hat{X}_1^\mu + m_2^2 \hat{Z}_\mu \hat{X}_2^\mu + m_3^2 \hat{X}_1{}_\mu \hat{X}_2^\mu\end{aligned}$$

Heeck, W.R., PLB**705**

canonical kinetic terms by

$$\begin{pmatrix} 1 & -t_\alpha & (t_\alpha s_\gamma - s_\beta/c_\alpha)/D \\ 0 & 1/c_\alpha & (t_\alpha s_\beta - s_\gamma/c_\alpha)/D \\ 0 & 0 & c_\alpha/D \end{pmatrix}^{-1} \begin{pmatrix} \hat{B} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \begin{pmatrix} B \\ X_1 \\ X_2 \end{pmatrix}$$

where  $D \equiv \sqrt{1 - s_\alpha^2 - s_\beta^2 - s_\gamma^2 + 2s_\alpha s_\beta s_\gamma}$

## The mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} \hat{M}_Z^2 & m_1^2/c_\alpha + \hat{M}_Z^2 \hat{s}_W t_\alpha & M_{13}^2 \\ \cdot & \hat{M}_{X_1}^2/c_\alpha^2 + \hat{s}_W t_\alpha (2m_1^2 + \hat{M}_Z^2 \hat{s}_W s_\alpha)/c_\alpha & M_{23}^2 \\ \cdot & \cdot & M_{33}^2 \end{pmatrix}$$

$$M_{13}^2 \cdot c_\alpha D \equiv (\hat{M}_Z^2 \hat{s}_W (s_\beta - s_\alpha s_\gamma) + m_1^2 (s_\alpha s_\beta - s_\gamma) + m_2^2 c_\alpha^2)$$

$$\begin{aligned} M_{23}^2 \cdot c_\alpha^2 D \equiv & \hat{M}_{X_1}^2 (s_\alpha s_\beta - s_\gamma) + \hat{M}_Z^2 \hat{s}_W^2 s_\alpha (s_\beta - s_\alpha s_\gamma) + m_1^2 \hat{s}_W (s_\beta - 2s_\alpha s_\gamma + s_\beta s_\alpha^2) \\ & + m_2^2 \hat{s}_W s_\alpha c_\alpha^2 + m_3^2 c_\alpha^2 \end{aligned}$$

$$\begin{aligned} M_{33}^2 \cdot c_\alpha^2 D^2 \equiv & \hat{M}_{X_2}^2 c_\alpha^4 + \hat{M}_{X_1}^2 (s_\gamma - s_\alpha s_\beta)^2 + \hat{M}_Z^2 \hat{s}_W^2 (s_\beta - s_\alpha s_\gamma)^2 \\ & - 2m_1^2 \hat{s}_W (s_\alpha s_\beta - s_\gamma) (s_\alpha s_\gamma - s_\beta) + 2m_2^2 c_\alpha^2 \hat{s}_W (s_\beta - s_\alpha s_\gamma) \\ & + 2m_3^2 c_\alpha^2 (s_\alpha s_\beta - s_\gamma) \end{aligned}$$

is diagonalized by  $O$

the 4 physical neutral bosons are

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & O^T & \\ 0 & & & \end{pmatrix} \begin{pmatrix} 1 & 0 & \hat{c}_W s_\alpha & \hat{c}_W s_\beta \\ 0 & 1 & -\hat{s}_W s_\alpha & -\hat{s}_W s_\beta \\ 0 & 0 & c_\alpha & (s_\gamma - s_\alpha s_\beta)/c_\alpha \\ 0 & 0 & 0 & D/c_\alpha \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix}$$

with oblique parameters

$$\alpha_{\text{EM}} T \simeq \frac{s_W^2 \alpha^2 - m_1^4/M_1^4}{1 - M_2^2/M_1^2} + \frac{s_W^2 \beta^2 - m_2^4/M_1^4}{1 - M_3^2/M_1^2}$$

$$\alpha_{\text{EM}} S \simeq 4s_W c_W^2 \alpha \frac{s_W \alpha + m_1^2/M_1^2}{1 - M_2^2/M_1^2} + 4s_W c_W^2 \beta \frac{s_W \beta + m_2^2/M_1^2}{1 - M_3^2/M_1^2}$$

## Generic comments

- break the symmetry  $\leftrightarrow$  scalars  $\leftrightarrow$  massive  $Z'$

**Mass scale?**

- avoid anomalies
- gauge what?

## Gauged $L_\alpha - L_\beta$

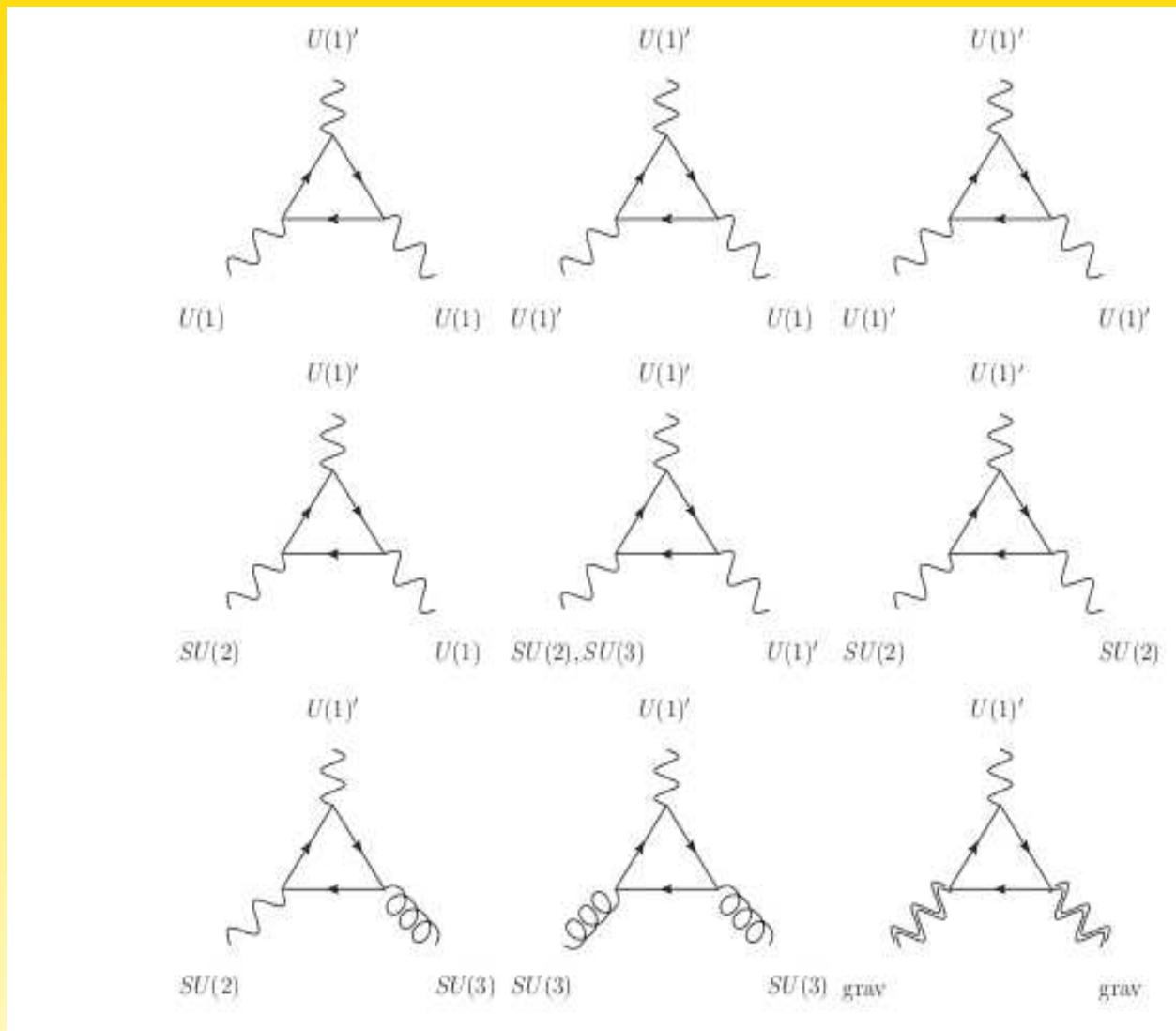
$L_e - L_\mu$  or  $L_e - L_\tau$  or  $L_\mu - L_\tau$  can be gauged **without anomaly** in the Standard Model (Foot, 1991)

Leptons get new charge, e.g. for  $L_\mu - L_\tau$

$$L_e \sim (1, 2, -1)(0) \text{ and } \overline{e_R} \sim (1, 1, 2)(0)$$

$$L_\mu \sim (1, 2, -1)(+1) \text{ and } \overline{\mu_R} \sim (1, 1, 2)(-1)$$

$$L_\tau \sim (1, 2, -1)(-1) \text{ and } \overline{\tau_R} \sim (1, 1, 2)(+1)$$



## Gauged $L_\alpha - L_\beta$

for instance  $U(1)$ - $U(1)$ - $U(1)'$  diagram

$$\begin{aligned}\text{Tr}(Y'Y^2) &= \sum_\ell Y'(\ell)Y^2(\ell) \\ &= [Y'(\mu_L)Y^2(\mu_L) + Y'(\nu_\mu)Y^2(\nu_\mu) + Y'(\mu_R)Y^2(\mu_R)] \\ &\quad + [Y'(\tau_L)Y^2(\tau_L) + Y'(\nu_\tau)Y^2(\nu_\tau) + Y'(\tau_R)Y^2(\tau_R)] \\ &= [(+1) \cdot 2 \cdot (-1)^2 + (-1) \cdot (+2)^2] + [(-1) \cdot 2 \cdot (-1)^2 + (+1) \cdot (+2)^2] \\ &= 0\end{aligned}$$

or gravity-gravity- $U(1)'$  diagram

$$\text{Tr}(Y') = [(+1) \cdot 2 + (-1)] + [(-1) \cdot 2 + (+1)] = 0$$

note: anomalies cancel among different (lepton) generations

## Why $L_\mu - L_\tau$ ?

If symmetry exact: neutrino mass matrix looks like

$$m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$$

and has masses  $a, \pm b$  with mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

close-to-degeneracy and  $\mu-\tau$  symmetry !

gauge  $\leftrightarrow$  flavor

(Choubey, W.R., EPJC40)

## The neutrino mass matrix

Assume  $\theta_{23} = \pi/4$  and  $\theta_{13} = |U_{e3}| = 0$ :

$$U = U(\theta_{23} = \pi/4, \theta_{13} = 0) = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}}{\sqrt{2}} & -\frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\text{and } m_\nu = U m_\nu^{\text{diag}} U^T = \begin{pmatrix} A & B & B \\ . & \frac{1}{2}(D+E) & \frac{1}{2}(D-E) \\ . & . & \frac{1}{2}(D+E) \end{pmatrix} \text{ with}$$

$$A = m_1 \cos^2 \theta_{12} + e^{2i\alpha} m_2 \sin^2 \theta_{12}$$

$$B = \frac{\sin \theta_{12} \cos \theta_{12}}{\sqrt{2}} (e^{2i\alpha} m_2 - m_1)$$

$$D = (m_1 \sin^2 \theta_{12} + e^{2i\alpha} m_2 \cos^2 \theta_{12})$$

$$E = e^{2i\beta} m_3$$

## The neutrino mass matrix

$\mu-\tau$  symmetric mass matrix simplifies further for certain mass hierarchies

- NH:  $m_3^2 \simeq \Delta m_A^2 \gg m_2^2 \gg m_1^2$ :

$$\frac{m_\nu}{\frac{1}{2}\sqrt{\Delta m_A^2}} \simeq \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix} \text{ conserves } L_e$$

- IH:  $m_2 \simeq m_1 \simeq \sqrt{\Delta m_A^2}$  and  $m_3 \simeq 0$ :

$$\frac{m_\nu}{\frac{1}{2}\sqrt{\Delta m_A^2}} \simeq \begin{pmatrix} 1 + e^{2i\alpha} & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) \\ \cdot & e^{i\alpha} \cos \alpha & e^{i\alpha} \cos \alpha \\ \cdot & \cdot & e^{i\alpha} \cos \alpha \end{pmatrix} \xrightarrow{\alpha=\pi/2} \begin{pmatrix} 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$$

conserves  $L_e - L_\mu - L_\tau$

## The neutrino mass matrix

QD:  $m_3 \simeq m_2 \simeq m_1 \equiv m_0$ :

$$m_\nu \simeq \frac{m_0}{2} \begin{pmatrix} 1 + e^{2i\alpha} & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) & \sqrt{\frac{1}{2}}(e^{2i\alpha} - 1) \\ \cdot & \frac{1}{2}(1 + e^{2i\alpha} + 2e^{2i\beta}) & \frac{1}{2}(1 + e^{2i\alpha} - 2e^{2i\beta}) \\ \cdot & \cdot & \frac{1}{2}(1 + e^{2i\alpha} + 2e^{2i\beta}) \end{pmatrix}$$

$$\xrightarrow{\alpha=\beta=0} m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 1 & 0 \\ \cdot & \cdot & 1 \end{pmatrix} \text{ unit matrix}$$

$$\xrightarrow{\alpha=0, \beta=\pi/2} m_0 \begin{pmatrix} 1 & 0 & 0 \\ \cdot & 0 & 1 \\ \cdot & \cdot & 0 \end{pmatrix} \text{ conserves } L_\mu - L_\tau$$

## The case of ultra-light $Z'$

considered in the past:  $L_e - L_\mu$

$$\begin{aligned} & g' \overline{\nu}_e Z'_\rho \gamma^\rho (1 - \gamma_5) \nu_e - g' \overline{\nu}_\mu Z'_\rho \gamma^\rho (1 - \gamma_5) \nu_\mu \\ & + g' \overline{e} Z'_\rho \gamma^\rho (1 - \gamma_5) e - g' \overline{\mu} Z'_\rho \gamma^\rho (1 - \gamma_5) \mu \end{aligned}$$

if  $m'_Z \leq 1/\text{A.U.} \simeq 10^{-18}$  eV: all electrons in the Sun generate potential for terrestrial  $e, \nu_e, \mu, \nu_\mu$

$$V = \frac{g'^2}{4\pi} \frac{N_e}{R} \equiv \alpha \frac{N_e}{R}$$

(**Joshipura, Mohanty, PLB584**; **Bandyopadhyay, Dighe, Joshipura, PRD75**)

$V$  must be added to Hamiltonian:

$$\Rightarrow \mathcal{H}_{e\mu} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix}$$

- $\leftrightarrow$  looks like NSI, but **does not depend on matter density!**
- $\leftrightarrow$  also works for vacuum oscillations!
- $V$  changes sign for anti-neutrinos!

$$\Rightarrow P(\nu_\alpha \rightarrow \nu_\alpha) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \text{ without CPT violation}$$

$$V = \alpha \frac{N_e}{R_{\text{A.U.}}} = \alpha \frac{6 \times 10^{56}}{8 \times 10^{17} \text{ eV}^{-1}} \simeq 8 \times 10^{38} \alpha \text{ eV}$$

- atmospheric neutrinos

$$\frac{\Delta m_A^2}{4E} \simeq 6 \times 10^{-13} \left( \frac{\text{GeV}}{E} \right) \text{ eV}$$

Limits  $\alpha_{e\mu} \leq 5.5 \times 10^{-52}$  and  $\alpha_{e\tau} \leq 6.4 \times 10^{-52}$

(**Joshipura, Mohanty, PLB584**)

- solar neutrinos

$$\frac{\Delta m_\odot^2}{4E} \simeq 2 \times 10^{-11} \left( \frac{\text{MeV}}{E} \right) \text{ eV}$$

Limits ( $\theta_{13} = 0$ )  $\alpha_{e\mu} \leq 3.4 \times 10^{-53}$  and  $\alpha_{e\tau} \leq 2.5 \times 10^{-53}$

(**Bandyopadhyay, Dighe, Joshipura, PRD75**)

**stronger than limits from equivalence principle!**

Obvious problem

$$m_\nu = \begin{pmatrix} 0 & a & 0 \\ \cdot & 0 & 0 \\ \cdot & \cdot & b \end{pmatrix} \quad \text{or} \quad m_\nu = \begin{pmatrix} 0 & 0 & a \\ \cdot & b & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$$

$\Rightarrow$  use  $L_\mu - L_\tau$

but: no reasonable amount of muons or tauons

## Solution

$Z-Z'$  mixing!

$$\mathcal{L} = -\frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{1}{2} M_Z'^2 Z'_\mu Z'^\mu - g' j'^\mu Z'_\mu - \frac{\sin \chi}{2} Z'^{\mu\nu} B_{\mu\nu} + \delta M^2 Z'_\mu Z^\mu$$

with new current

$$j'^\mu = \bar{\mu} \gamma^\mu \mu + \bar{\nu}_\mu \gamma^\mu P_L \nu_\mu - \bar{\tau} \gamma^\mu \tau - \bar{\nu}_\tau \gamma^\mu P_L \nu_\tau$$

Diagonalizing kinetic and mass terms gives

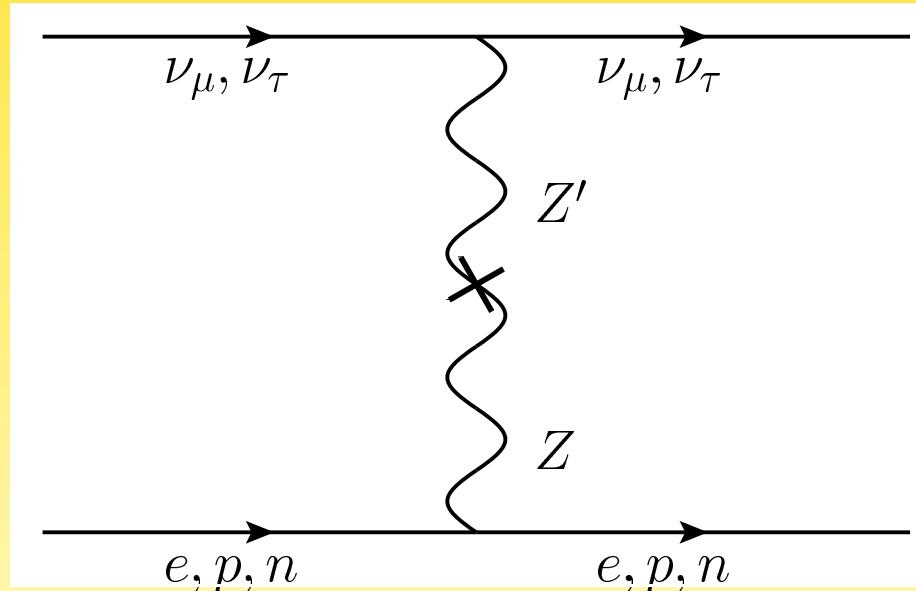
$$\mathcal{L}_A = -e (j_{\text{EM}})_\mu A^\mu$$

$$\mathcal{L}_{Z_1} = - \left( \frac{e}{s_W c_W} ((j_3)_\mu - s_W^2 (j_{\text{EM}})_\mu) + g' \xi (j')_\mu \right) Z_1^\mu$$

$$\mathcal{L}_{Z_2} = - \left( g' (j')_\mu - \frac{e}{s_W c_W} (\xi - s_W \chi) ((j_3)_\mu - s_W^2 (j_{\text{EM}})_\mu) - e c_W \chi (j_{\text{EM}})_\mu \right) Z_2^\mu$$

Potential through  $Z$ - $Z'$  mixing:

$$V = g' (\xi - s_W \chi) \frac{e}{4 s_W c_W} \frac{N_n}{4\pi R_{A.U.}} \equiv \alpha \frac{e}{4 s_W c_W} \frac{N_n}{4\pi R_{A.U.}}$$
$$= 3.60 \times 10^{-14} \text{ eV} \left( \frac{\alpha}{10^{-50}} \right)$$



(Heeck, W.R., JPG38)

## Simple Oscillation Phenomenology

$$\Rightarrow \mathcal{H}_{\mu\tau} = \frac{\Delta m_A^2}{4E} \begin{pmatrix} -\cos 2\theta_{23} & \sin 2\theta_{23} \\ \sin 2\theta_{23} & \cos 2\theta_{23} \end{pmatrix} + \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix}$$

With  $\eta = 2E V / \Delta m_A^2 \simeq 0.025 \left( \frac{\alpha}{10^{-50}} \right) \left( \frac{E}{\text{GeV}} \right)$ :

$$\sin^2 2\theta_V = \frac{\sin^2 2\theta_{23}}{1 - 4\eta \cos 2\theta_{23} + 4\eta^2}$$

$$\Delta m_V^2 = \Delta m_A^2 \sqrt{1 - 4\eta \cos 2\theta_{23} + 4\eta^2} = \Delta m_A^2 \sqrt{\frac{\sin^2 2\theta_{23}}{\sin^2 2\theta_V}}$$

Recall:  $V$  changes sign for anti-neutrinos!!

## Simple Oscillation Phenomenology

Recall:  $V$  changes sign for anti-neutrinos!!

$$\begin{aligned}\Delta m_V^2 - \overline{\Delta m_V^2} &= \Delta m_A^2 \sqrt{1 - 4\eta \cos 2\theta_{23} + 4\eta^2} - \Delta m_A^2 \sqrt{1 + 4\eta \cos 2\theta_{23} + 4\eta^2} \\ &\simeq -4\eta \Delta m_A^2 \cos 2\theta_{23} \\ &\neq 0 \text{ for } \theta_{23} \neq \pi/4\end{aligned}$$

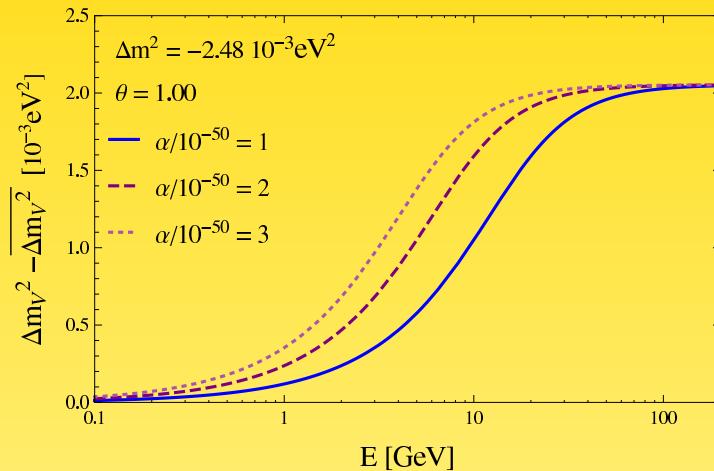
same for mixing

$$\frac{\sin^2 2\theta_V - \sin^2 2\bar{\theta}_V}{\sin^2 2\theta_{23}} \simeq 8\eta \cos 2\theta_{23}$$

probabilities

$$P \equiv P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_V \sin^2 \frac{\Delta m_V^2}{4E} L$$

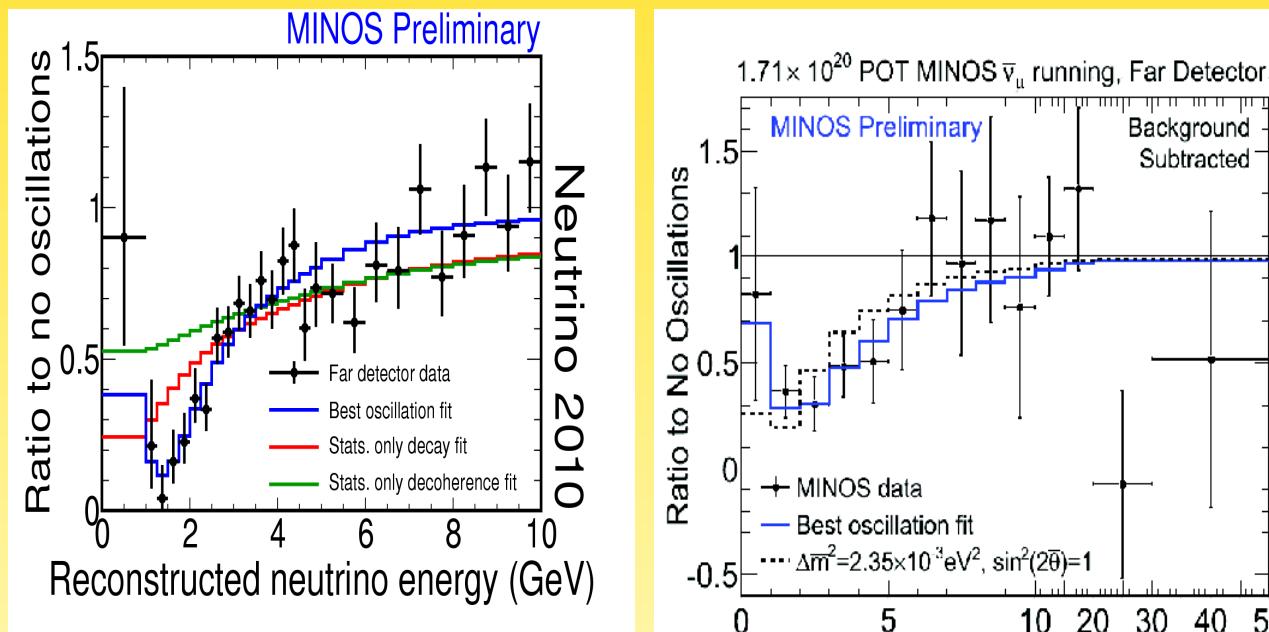
## Simple Oscillation Phenomenology



$$\begin{aligned}
 \Delta m_V^2 - \overline{\Delta m_V^2} &= \Delta m_A^2 \sqrt{1 - 4 \eta \cos 2\theta_{23} + 4 \eta^2} - \Delta m_A^2 \sqrt{1 + 4 \eta \cos 2\theta_{23} + 4 \eta^2} \\
 &\simeq -4 \eta \Delta m_A^2 \cos 2\theta_{23} \\
 &\neq 0 \text{ for } \theta_{23} \neq \pi/4
 \end{aligned}$$

## MINOS data

- $7.24 \times 10^{20}$  POT for  $\nu_\mu$ ,  $1.71 \times 10^{20}$  POT for  $\bar{\nu}_\mu$
- $\nu_\mu$ : 2451 if no oscillations, 1986 observed
- $\bar{\nu}_\mu$ : 155 if no oscillations, 97 observed



$$\Delta m^2 = (2.35_{-0.08}^{+0.11}) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta > 0.91$$

$$\overline{\Delta m^2} = (3.36_{-0.40}^{+0.45}) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\bar{\theta} = 0.86 \pm 0.11$$

## Is this... .

- ...Non-Standard Interaction ([Mann, Cherdack, Musial, Kafka, 1006.5720; Kopp, Machado, Parke, 1009.0014](#))?
- ...sterile neutrino (plus gauged  $Z'$  from  $U(1)$  according to  $B - L$ ) ([Engelhardt, Nelson, Walsh, 1002.4452](#))?
- ...gauged ultra-light  $Z'$  from  $U(1)$  according to  $L_\mu - L_\tau$  ([Heeck, W.R., 1007.2655](#))?
- ...CPT violation? ([Barenboim, Lykken, 0908.2993; Choudhury, Datta, Kundu, 1007.2923](#))?
- ...nothing and will go away ([common sense](#))?

## Gauged $L_\mu - L_\tau$ and MINOS

First, fit MINOS data with  $\alpha = 0$ :

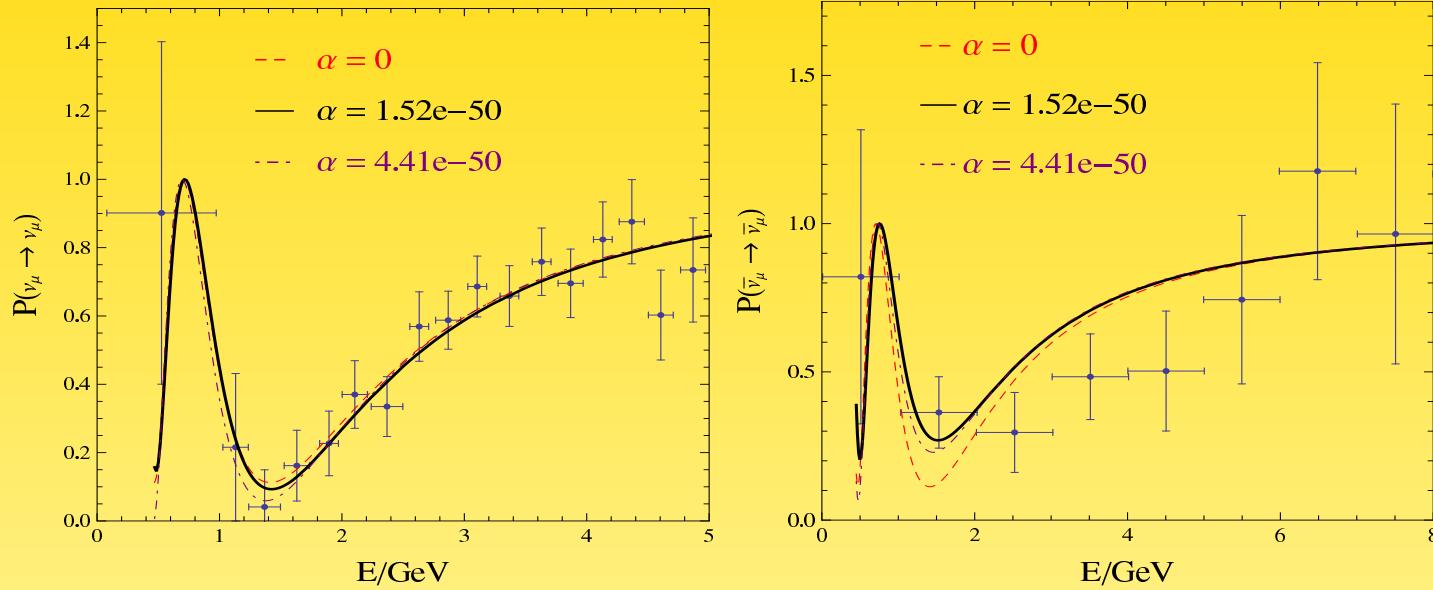
$$\Delta m^2 = 2.28 \times 10^{-3} \text{ eV}^2, \sin^2 2\theta = 0.94 \text{ for neutrinos}$$

$$\overline{\Delta m^2} = 3.38 \times 10^{-3} \text{ eV}^2, \sin^2 2\bar{\theta} = 0.81 \text{ for anti-neutrinos}$$

to be compared with official MINOS result:

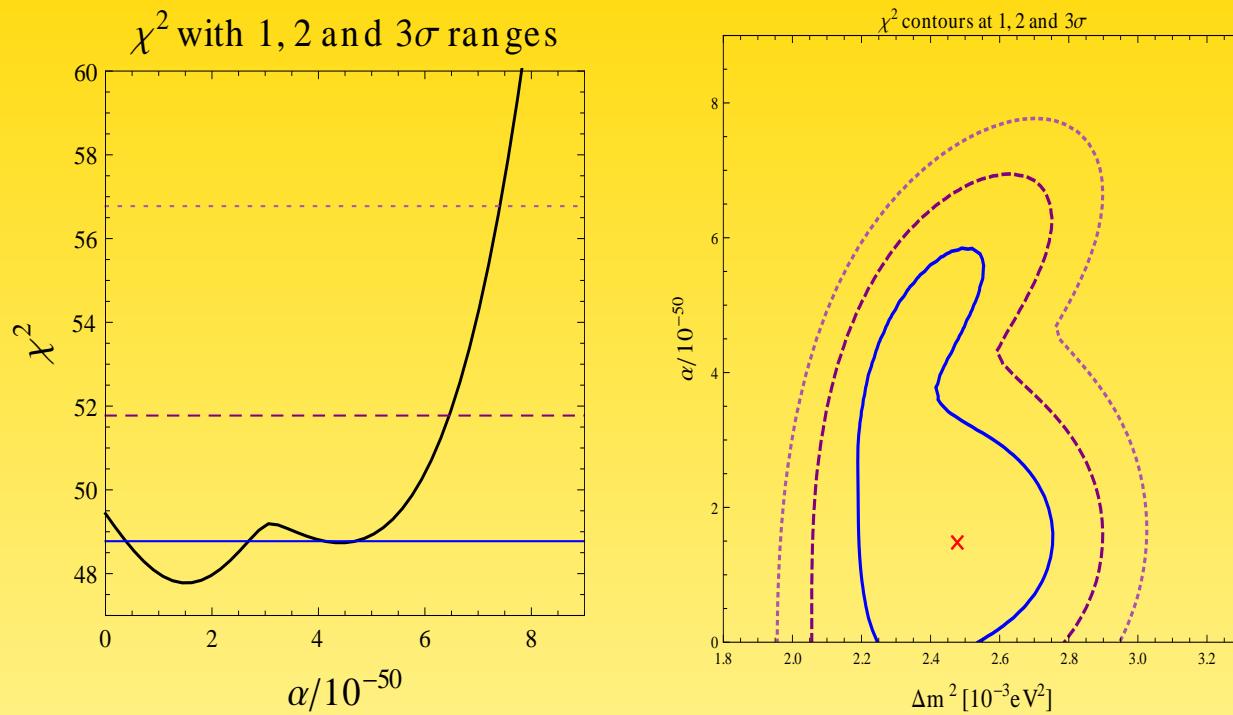
$$\Delta m^2 = 2.35 \times 10^{-3} \text{ eV}^2, \sin^2 2\theta > 0.91 \text{ for neutrinos}$$

$$\overline{\Delta m^2} = 3.36 \times 10^{-3} \text{ eV}^2, \sin^2 2\bar{\theta} = 0.86 \text{ for anti-neutrinos}$$

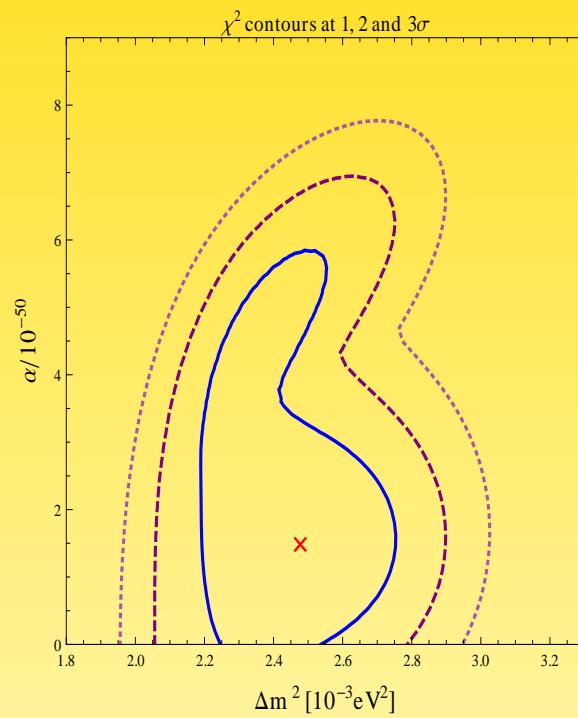
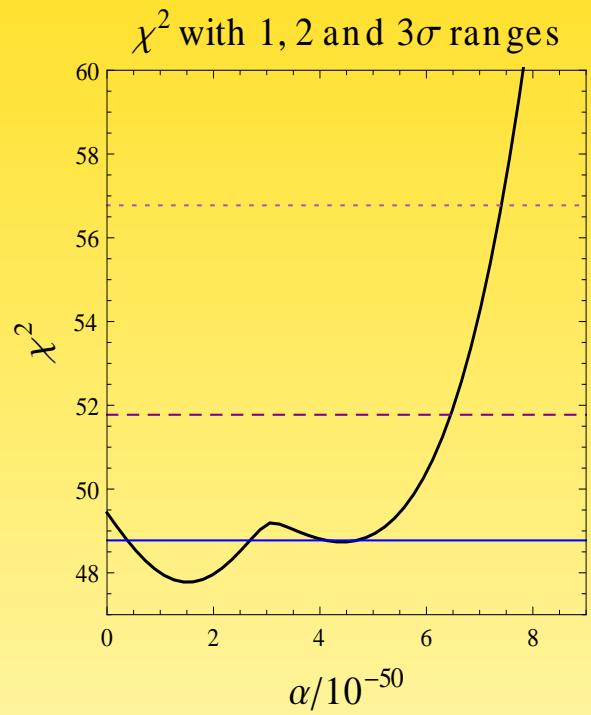


$$\sin^2 2\theta = 0.83 \pm 0.08, \quad \Delta m^2 = (-2.48 \pm 0.19) \times 10^{-3} \text{ eV}^2, \quad \alpha = (1.52^{+1.17}_{-1.14}) \times 10^{-50}$$

with  $\chi^2_{\min}/N_{\text{dof}} = 47.77/50$ , (without  $\alpha$ :  $\chi^2_{\min}/N_{\text{dof}} = 49.43/51$ )



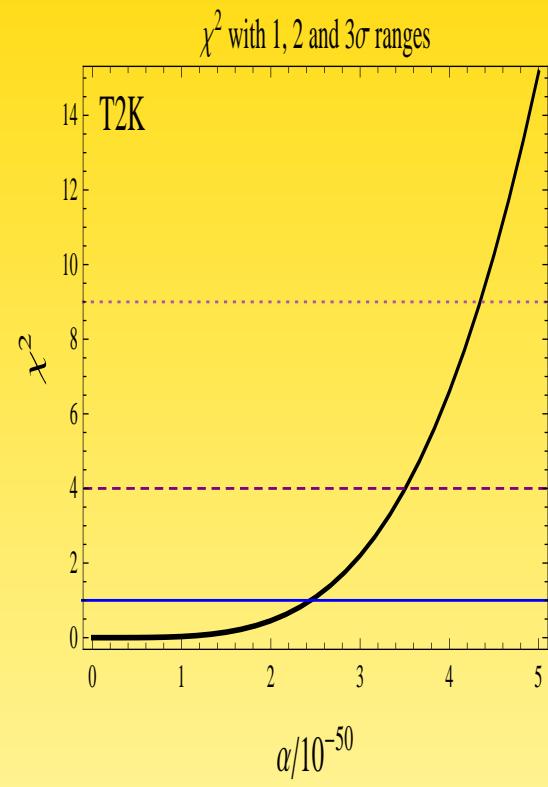
$$P(\theta, \Delta m^2, \alpha) = P(\theta, -\Delta m^2, -\alpha) = P(\theta + \pi/2, \Delta m^2, -\alpha) = P(\theta + \pi/2, -\Delta m^2, \alpha)$$



## GLoBES:

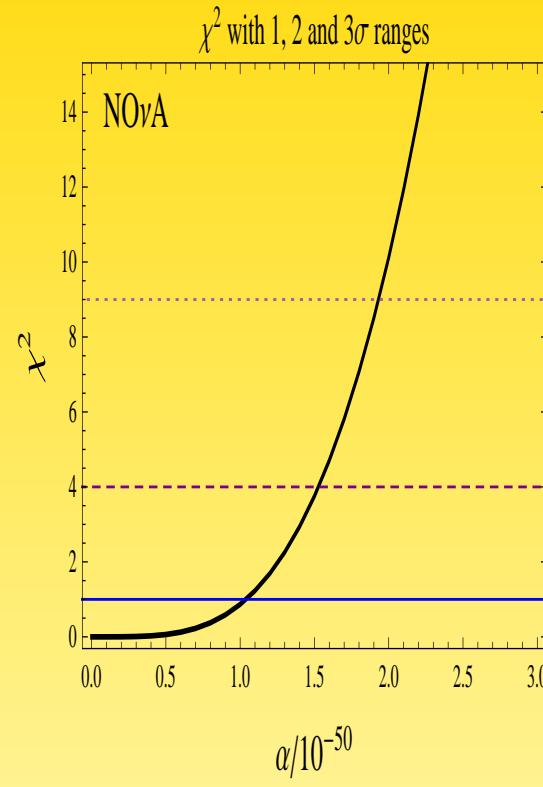
Experiment	Baseline	Running-time [years]	Beam-energy [GeV]	mass
T2K	295 km	$5 \nu + 5 \bar{\nu}$	$0.2 - 2$	22.5 kt
T2HK	295 km	$4 \nu + 4 \bar{\nu}$	$0.4 - 1.2$	500 kt
SPL	130 km	$2 \nu + 8 \bar{\nu}$	$0.01 - 1.01$	500 kt
NO $\nu$ A	812 km	$3 \nu + 3 \bar{\nu}$	$0.5 - 3.5$	15 kt
Nufact	3000 km	$4 \nu + 4 \bar{\nu}$	$4 - 50$	50 kt

$\theta_{12}$	$\arcsin \sqrt{0.318} \pm 0.02$ (3%)
$\theta_{13}$	$0 \pm 0.2$
$\theta_{23}$	$\arcsin \sqrt{0.500} \pm 0.07$ (9%)
$\delta_{\text{CP}}$	$\in [0, 2\pi]$
$\Delta m_{21}^2$ [10 $^{-5}$ eV $^2$ ]	$7.59 \pm 0.23$ (3%)
$\Delta m_{31}^2$ [10 $^{-3}$ eV $^2$ ]	$2.40 \pm 0.12$ (5%)



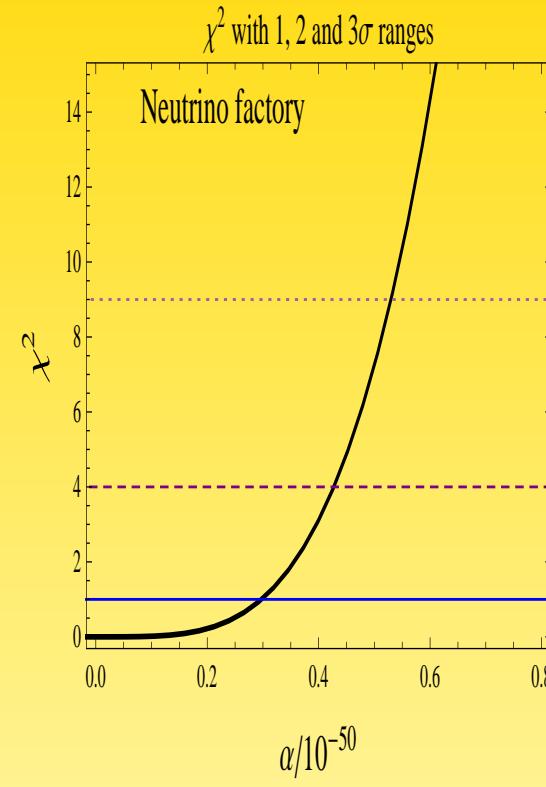
$$\alpha \leq 4.30$$

$$(\alpha = 1.52 \pm 0.46)$$



$$\alpha \leq 1.93$$

$$(\alpha = 1.52 \pm 0.27)$$



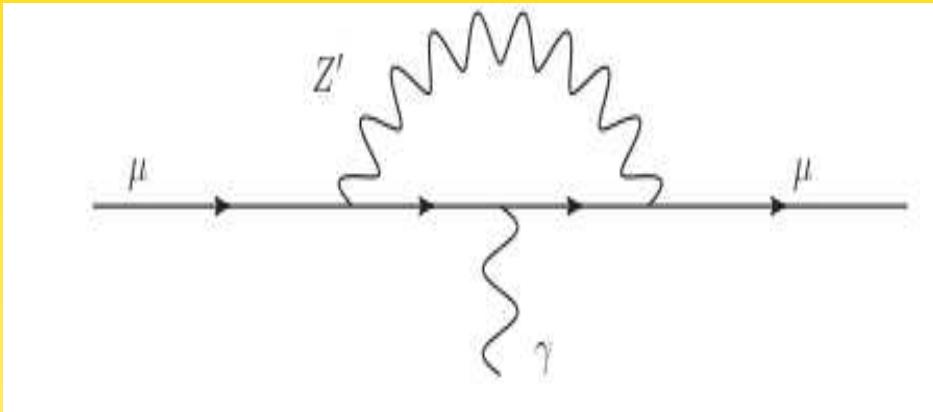
$$\alpha \leq 0.53$$

$$(\alpha = 1.52^{+0.11}_{-0.21})$$

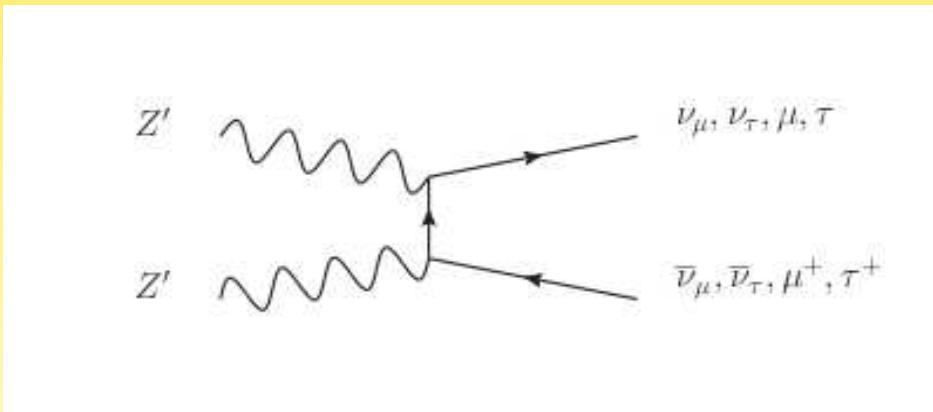
Experiment	Sensitivity to $\alpha/10^{-50}$ at 99.73% CL
T2K ( $\nu$ -run)	11.8
T2K	4.3
T2HK	1.7
SPL	7.5
NO $\nu$ A	1.9
Combined Superbeams	1.4
Nufact	0.53

## Other aspects/limits of $L_\mu - L_\tau$

- $\Delta a_\mu = g'^2 / (8\pi^2)$



- BBN:  $\Gamma(Z' Z' \rightarrow \nu_{\mu,\tau} \bar{\nu}_{\mu,\tau}) \propto g'^2 T \Rightarrow g' \lesssim 10^{-5}$



- other EW precision: there are only  $\sim 10^8 Z$  ...

## Other aspects/limits of $L_\mu - L_\tau$

- coupling of  $Z'$  with electromagnetic current gives modified charge

$$\frac{Q(\mu^+)}{Q(e^+)} \simeq 1 + \frac{g'}{e} \left( (\xi - s_W \chi) \left( \frac{1}{4} - s_W^2 \right) / (s_W c_W) + c_W \chi \right)$$

measured to be  $1 \pm 10^{-9}$

Equivalence principle is violated:

$$V(r) = \frac{e(\xi - s_W \chi)}{4 s_W c_W} N_n \frac{e^{-rM_2}}{4\pi r}$$

gravitational potential between 2 bodies with neutron content  $N_{n_1}$  and  $N_{n_2}$ :

$$V_{\text{grav}}(r) = -G_N \frac{m_1 m_2}{r} \left( 1 - \left( \frac{e(\xi - s_W \chi)}{4 s_W c_W} \right)^2 \frac{N_{n_1}}{m_1} \frac{N_{n_2}}{m_2} \frac{1}{4\pi G_N} e^{-rM_2} \right)$$

Use the limits from Adelberger *et al.*, PPNP **62**, 102 (2009) who analyze

$$V_{\text{grav}}(r) = -G_N \frac{m_1 m_2}{r} \left( 1 + \tilde{\alpha} \frac{N_{n_1}}{\mu_1} \frac{N_{n_1}}{\mu_2} e^{-r/\lambda} \right)$$

this gives limits depending on range:

$$\alpha/g' \leq 5 \times 10^{-24} \quad \text{Sun-Earth}$$

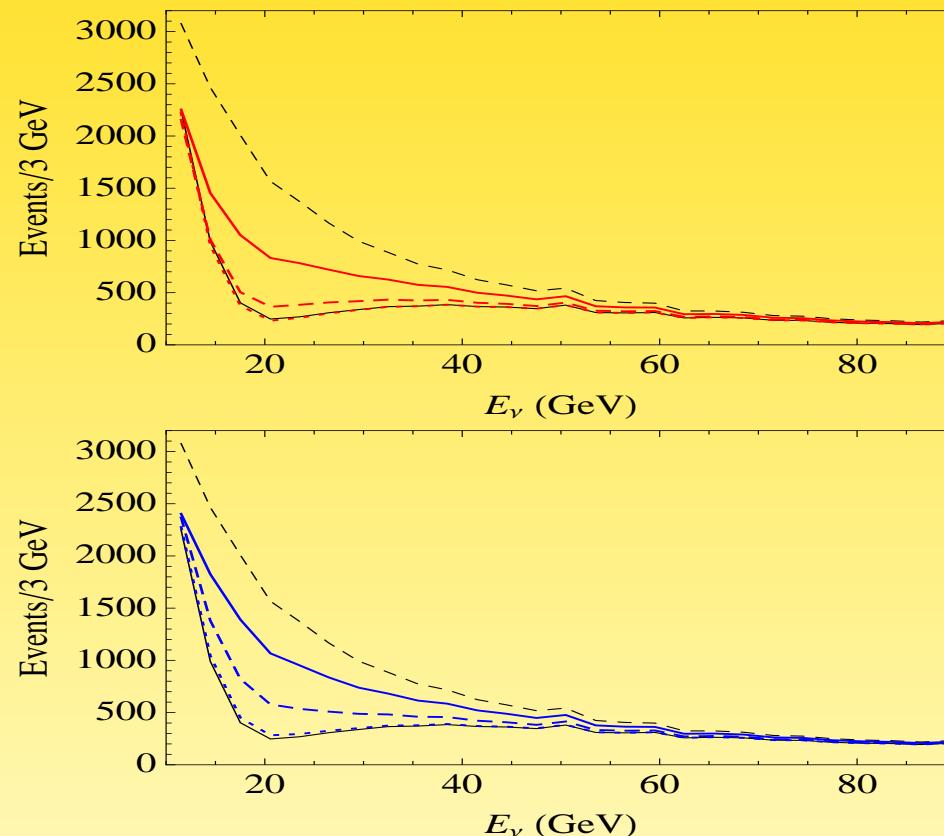
$$\alpha/g' \leq 1 \times 10^{-22} \quad \text{Earth}$$

$\Rightarrow$  neutrinos give best limits on leptonic fifth forces :-)

## More Oscillation Phenomenology

Davoudiasl, Lee, Marciano, PRD84

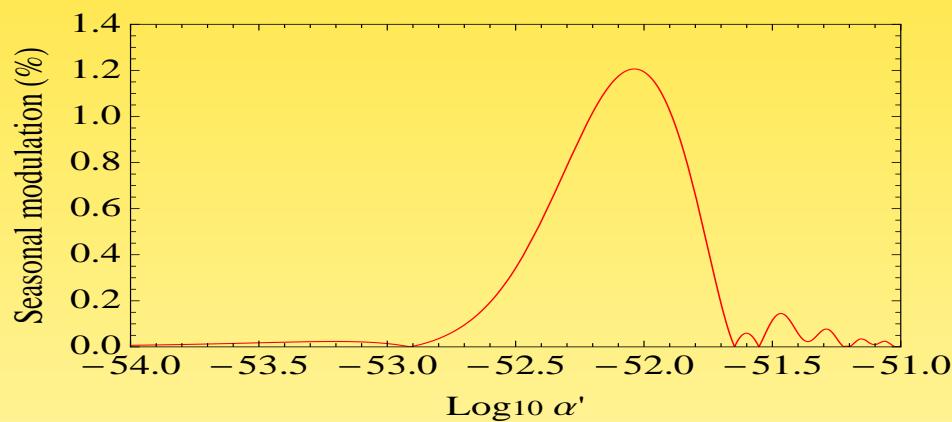
used  $B - L + (L_\mu - L_\tau) = B - L_e - 2L_\tau$



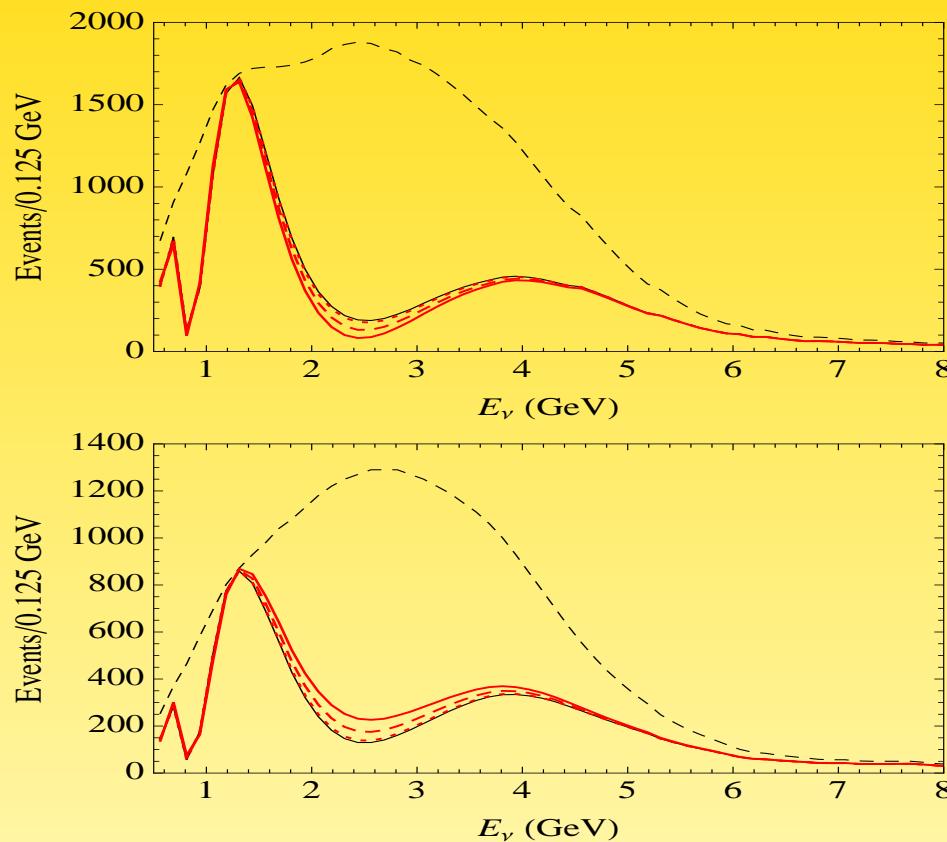
Deep-core atmospheric neutrinos

## More Oscillation Phenomenology

seasonal modulation



## More Oscillation Phenomenology



LBNE

## Problems

- Neutrino masses tend to be quasi-degenerate

$$m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$$

breaking generates both  $m_{Z'}$  and non-zero entries in  $m_\nu$

$m_{Z'} \sim g' \langle \Phi' \rangle$  and e.g.,  $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle^2 / \Lambda$  or  $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle \langle \Phi \rangle / \Lambda$  or  $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle$

- neutral scalar  $\chi$  is present,  $m_\chi \simeq \lambda \langle \Phi' \rangle$ , with dangerous  $Z \rightarrow Z' \chi$

## Comparison of MINOS result

$$\begin{aligned}\mathcal{H}_{e\mu} &= \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V_{e\mu} & 0 \\ 0 & -V_{e\mu} \end{pmatrix} \\ &= \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & V_{\mu\tau} \end{pmatrix}\end{aligned}$$

and

$$\begin{aligned}\mathcal{H}_{\mu\tau} &= \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -V_{e\tau} \end{pmatrix} \\ &= \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V_{\mu\tau} & 0 \\ 0 & -V_{\mu\tau} \end{pmatrix}\end{aligned}$$

okay with atm. limits, in conflict with solar ( $\alpha \leq 4 \times 10^{-51}$ )

- looks in  $\mathcal{H}$  like NSI, hence apply NSI limits

$$\alpha = 10^{-50} \Rightarrow |\epsilon_{\mu\mu}^\oplus| \simeq 0.25$$

current limit

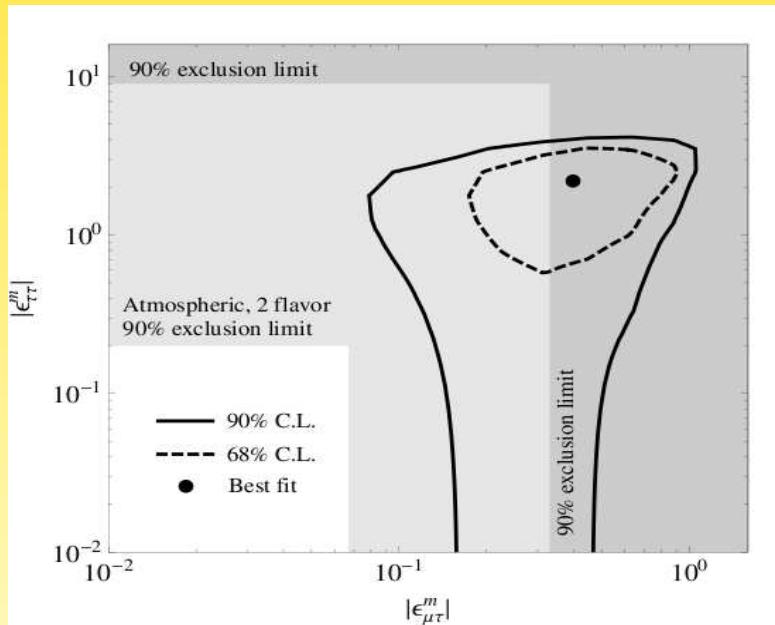
$$|\epsilon_{\mu\mu}^\oplus| \lesssim 0.068 \Rightarrow \alpha \simeq 10^{-51}$$

## Non-Standard Interactions

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \epsilon_{\alpha\beta}^f (\bar{\nu}_\alpha \gamma_\mu \nu_\beta) (\bar{f} \gamma^\mu f)$$

and  $\epsilon_{\alpha\beta} \rightarrow \epsilon_{\alpha\beta}^*$  for anti-neutrinos

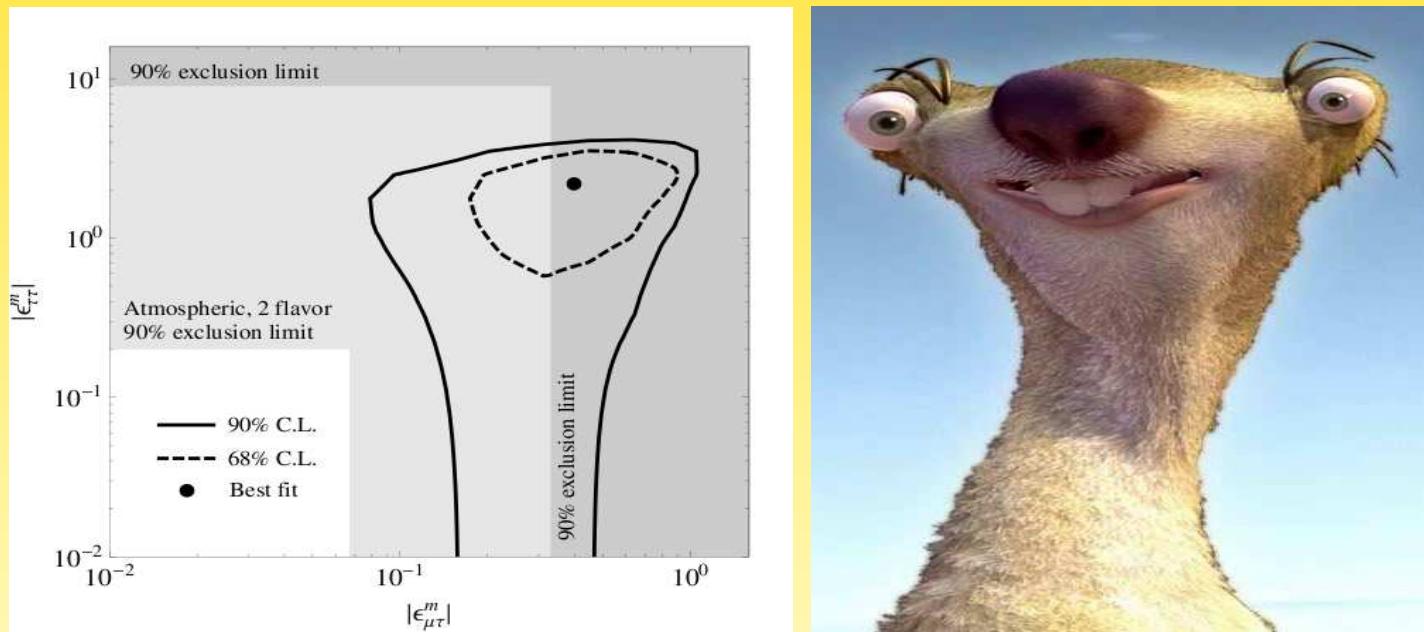
$$\mathcal{H} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & 0 \\ 0 & \Delta m_{32}^2 \end{pmatrix} U^\dagger + A \begin{pmatrix} \epsilon_{\mu\mu}^\oplus & \epsilon_{\mu\tau}^\oplus \\ \epsilon_{\mu\tau}^{\oplus*} & \epsilon_{\tau\tau}^\oplus \end{pmatrix} \right]$$



Kopp, Machado, Parke, 1009.0014 (only  $\epsilon_{\mu\tau}^\oplus$ : Mann *et al.*, 1006.5720)

## NSIs

$$\mathcal{H} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & 0 \\ 0 & \Delta m_{32}^2 \end{pmatrix} U^\dagger + A \begin{pmatrix} \epsilon_{\mu\mu}^\oplus & \epsilon_{\mu\tau}^\oplus \\ \epsilon_{\mu\tau}^{\oplus*} & \epsilon_{\tau\tau}^\oplus \end{pmatrix} \right]$$

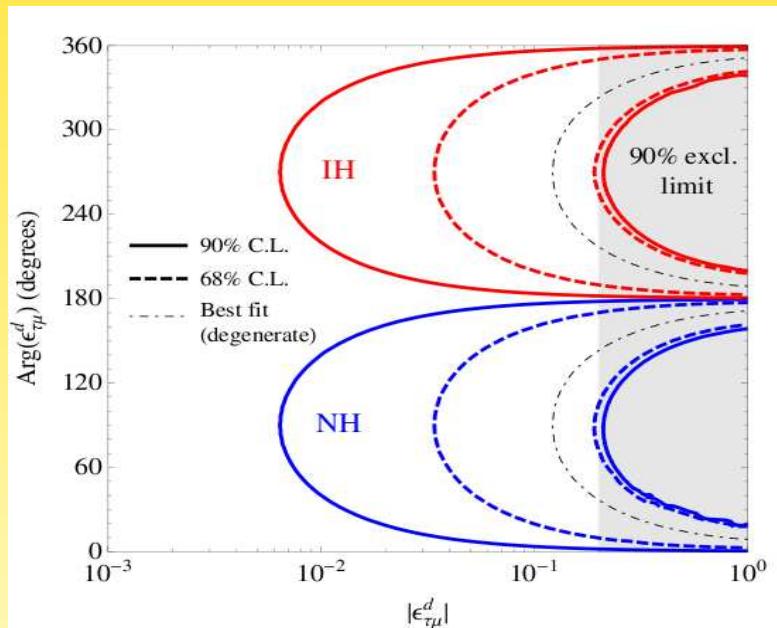


## Charged Current NSIs

$$\mathcal{L}_{\text{NSI}} \supset -2\sqrt{2} G_F \epsilon_{\tau\mu}^d V_{ud} [\bar{u}\gamma^\mu d] [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

leads to interference of

$$\nu_\mu \rightsquigarrow \nu_\tau + N \rightarrow X + \mu \text{ and } \nu_\mu + N \rightarrow X + \mu$$



Kopp, Machado, Parke, 1009.0014

## Gauge Invariance strikes back!

$$\mathcal{L}_{\text{NSI}} \supset -2\sqrt{2} G_F \epsilon_{\tau\mu}^d V_{ud} [\bar{u}\gamma^\mu d] [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

gives 1-loop diagram for  $\tau \rightarrow \mu \pi^0$ :  $|\epsilon_{\tau\mu}^d| \leq 0.2$

Kopp, Machado, Parke, 1009.0014

BUT: gauge invariant term

$$\mathcal{L}_{\text{NSI}} \supset -2\sqrt{2} G_F \epsilon_{\tau\mu}^d V_{ud} [\bar{Q}\gamma^\mu \tau Q] [\bar{L}_\mu \gamma_\mu \tau L_\tau]$$

gives tree-level diagram for  $\tau \rightarrow \mu \pi^0$ :  $|\epsilon_{\tau\mu}^d| \leq 10^{-4}$

Gavela, Talk@NOW2010

$\Leftrightarrow$  this argument does not apply to gauged  $U(1)$ !

## Sterile Neutrino (plus $Z'$ )

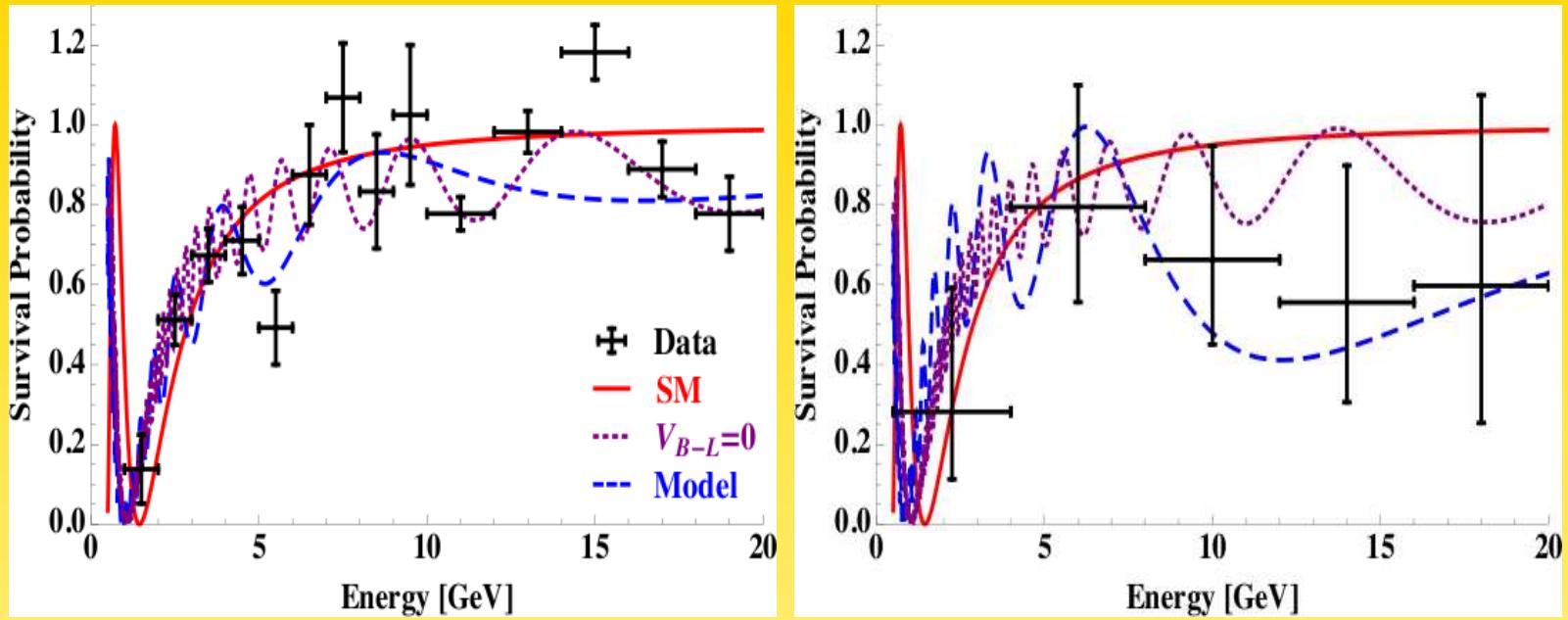
Engelhardt, Nelson, Walsh, PRD**81**

$$V = \begin{pmatrix} V_{CC} - V_{NC} & 0 & 0 & 0 \\ \cdot & -V_{NC} & 0 & 0 \\ \cdot & \cdot & -V_{NC} & 0 \\ \cdot & \cdot & \cdot & 0 \end{pmatrix}$$

if you add a massive  $Z'$  due to  $B - L$  ( $V_{B-L} = N_n g'^2/M_{Z'}^2$ )

$$V = \begin{pmatrix} V_{CC} - V_{NC} - V_{B-L} & 0 & 0 & 0 \\ \cdot & -V_{NC} - V_{B-L} & 0 & 0 \\ \cdot & \cdot & -V_{NC} - V_{B-L} & 0 \\ \cdot & \cdot & \cdot & V_{B-L} \end{pmatrix}$$

$V_x \rightarrow -V_x$  for antineutrinos



$$m = 0.0394 \text{ eV}, \quad M = 0.157 \text{ eV}, \quad V_{B-L} = 2.01 \times 10^{-4} \text{ neV}$$

(or  $g'/M_Z \lesssim 6 \times 10^{-12} \text{ eV}^{-1}$ ) with  $\chi^2 = 24.8 = 1.24/\text{dof}$

Fit without  $V_{B-L}$ :

$$m = 0.0420 \text{ eV}, \quad M = 0.309 \text{ eV}$$

with  $\chi^2 = 28.1 = 1.34/\text{dof}$

## Summary so far



It's very hard to explain MINOS data

The more natural case: Heavy  $Z'$  and gauged  $L_\mu - L_\tau$

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}_{\mu\nu}^a\hat{W}^{a\mu\nu} + \frac{1}{2}\hat{M}_Z^2\hat{Z}_\mu\hat{Z}^\mu - \frac{\hat{e}}{\hat{c}_W}j_Y^\mu\hat{B}_\mu - \frac{\hat{e}}{\hat{s}_W}j_W^{a\mu}\hat{W}_\mu^a$$

$$\mathcal{L}_{Z'} = -\frac{1}{4}\hat{Z}'_{\mu\nu}\hat{Z}'^{\mu\nu} + \frac{1}{2}\hat{M}'_Z^2\hat{Z}'_\mu\hat{Z}'^\mu - \hat{g}'j'^\mu Z'_\mu$$

$$\mathcal{L}_{\text{mix}} = -\frac{\sin\chi}{2}\hat{Z}'^{\mu\nu}\hat{B}_{\mu\nu} + \delta\hat{M}^2\hat{Z}'_\mu\hat{Z}^\mu$$

with currents

$$j_Y^\mu = -\sum_\ell [\bar{L}_\ell\gamma^\mu L_\ell + 2\bar{\ell}_R\gamma^\mu\ell_R] + \frac{1}{3}\sum_q [\bar{Q}_L\gamma^\mu Q_L + 4\bar{u}_R\gamma^\mu u_R - 2\bar{d}_R\gamma^\mu d_R]$$

$$j_W^{a\mu} = \sum_\ell \bar{L}_\ell\gamma^\mu\frac{\sigma^a}{2}L_\ell + \sum_{\text{quarks}} \bar{Q}_L\gamma^\mu\frac{\sigma^a}{2}Q_L$$

$$j'^\mu = \bar{\mu}\gamma^\mu\mu + \bar{\nu}_\mu\gamma^\mu P_L\nu_\mu - \bar{\tau}\gamma^\mu\tau - \bar{\nu}_\tau\gamma^\mu P_L\nu_\tau$$

The more natural case: Heavy  $Z'$  and gauged  $L_\mu - L_\tau$

mass eigenstates  $A, Z_1, Z_2$

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \hat{c}_W \sin \chi \\ 0 & \cos \xi & -\hat{s}_W \cos \xi \sin \chi + \sin \xi \cos \chi \\ 0 & -\sin \xi & \cos \xi \cos \chi + \hat{s}_W \sin \xi \sin \chi \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{Z}' \end{pmatrix}$$

couple to currents

$$ej_{\text{EM}}\hat{A} + \frac{e}{2s_W c_W} j_{\text{NC}}\hat{Z} + g' j' \hat{Z}' \rightarrow$$

$$\left( ej_{\text{EM}}, \frac{e}{2\hat{s}_W \hat{c}_W} j_{\text{NC}}, g' j' \right)^T \begin{pmatrix} 1 & -\hat{c}_W \sin \xi \tan \chi & -\hat{c}_W \cos \xi \tan \chi \\ 0 & \cos \xi + \hat{s}_W \sin \xi \tan \chi & \hat{s}_W \cos \xi \tan \chi - \sin \xi \\ 0 & \frac{\sin \xi}{\cos \chi} & \frac{\cos \xi}{\cos \chi} \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix}$$

The more natural case: Heavy  $Z'$  and gauged  $L_\mu - L_\tau$

$$\begin{aligned}
 ej_{\text{EM}}\hat{A} + \frac{e}{2s_W c_W} j_{\text{NC}}\hat{Z} + g' j' \hat{Z}' &\rightarrow \\
 \left( ej_{\text{EM}}, \frac{e}{2\hat{s}_W \hat{c}_W} j_{\text{NC}}, g' j' \right)^T &\begin{pmatrix} 1 & -\hat{c}_W \sin \xi \tan \chi & -\hat{c}_W \cos \xi \tan \chi \\ 0 & \cos \xi + \hat{s}_W \sin \xi \tan \chi & \hat{s}_W \cos \xi \tan \chi - \sin \xi \\ 0 & \frac{\sin \xi}{\cos \chi} & \frac{\cos \xi}{\cos \chi} \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} \\
 &\simeq \left( ej_{\text{EM}}, \frac{e}{2\hat{s}_W \hat{c}_W} j_{\text{NC}}, g' j' \right)^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\xi \\ 0 & \xi & 1 \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix}
 \end{aligned}$$

The more natural case: Heavy  $Z'$  and gauged  $L_\mu - L_\tau$

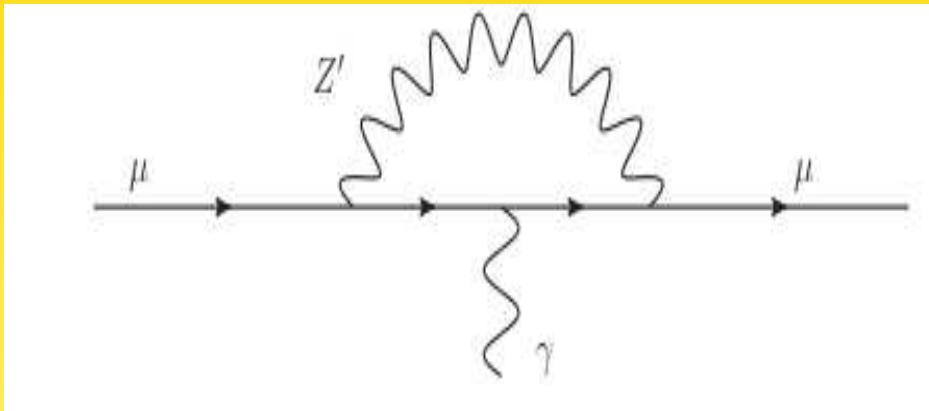
$$\simeq \left( ej_{\text{EM}}, \frac{e}{2\hat{s}_W \hat{c}_W} j_{\text{NC}}, g' j' \right)^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\xi \\ 0 & \xi & 1 \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix}$$

note:  $Z_1$  couples with strength  $g'\xi$  to  $j' = \bar{\mu}\gamma_\mu\mu + \bar{\tau}\gamma_\tau\tau$

modifies asymmetry parameter  $A^\ell = 2g_V^\ell g_A^\ell / ((g_V^\ell)^2 + (g_A^\ell)^2)$

$$A^\mu \rightarrow A^\mu \left( 1 - g'\xi \frac{4s_W c_W / e}{1 - 4s_W^2} \right) , \quad A^\tau \rightarrow A^\tau \left( 1 + g'\xi \frac{4s_W c_W / e}{1 - 4s_W^2} \right)$$

The more natural case: Heavy  $Z'$  and gauged  $L_\mu - L_\tau$



$$\begin{aligned}\Delta a_\mu &= \frac{g'^2}{4\pi} \frac{1}{2\pi} \int_0^1 dx \frac{2m_\mu^2 x^2(1-x)}{m_\mu^2 x^2 + M_{Z'}^2(1-x)} \\ &\simeq \frac{g'^2}{4\pi} \frac{1}{2\pi} \begin{cases} 1 & \text{for } M_{Z'} \ll m_\mu \\ 2m_\mu^2/3M_{Z'}^2 & \text{for } M_{Z'} \gg m_\mu \end{cases}\end{aligned}$$

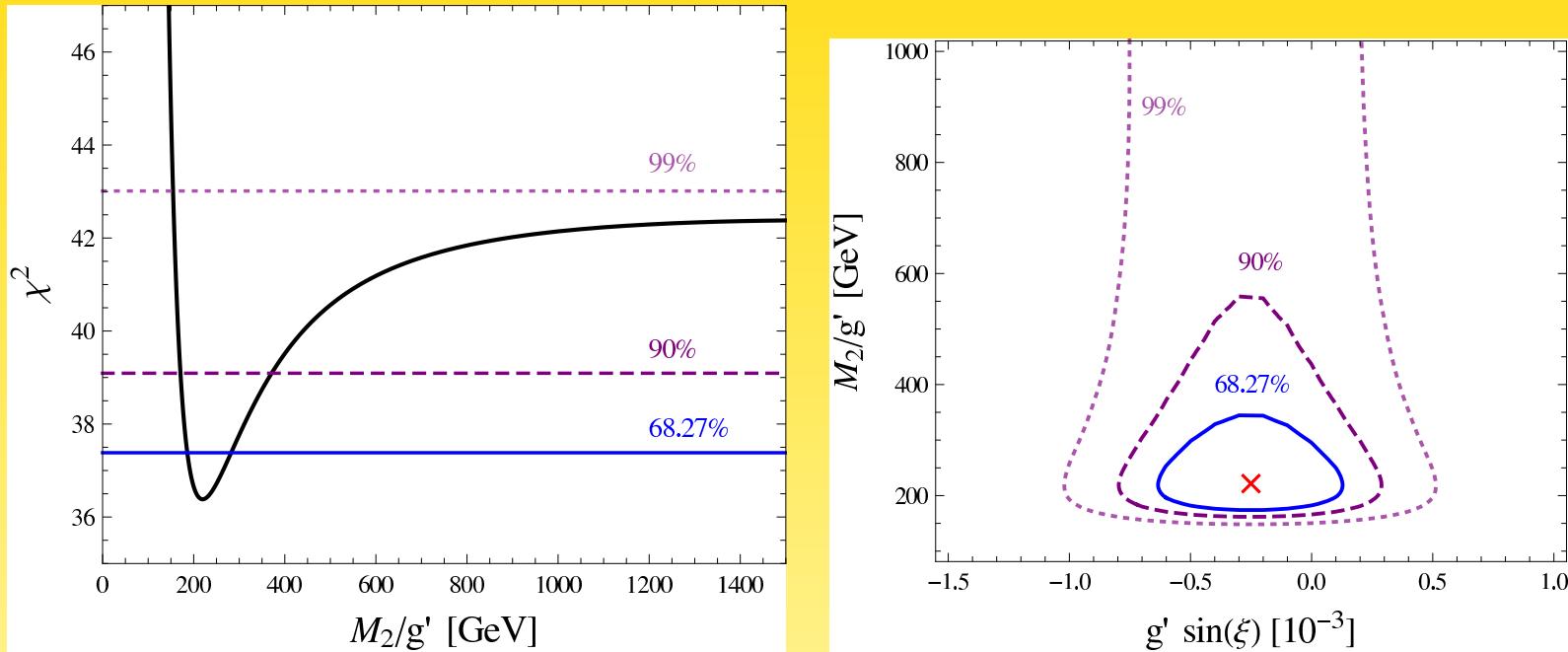
$$\text{thus: } \Delta a_\mu \simeq 236 \times 10^{-11} \left( \frac{200 \text{ GeV}}{M_{Z'}/g'} \right)^2$$

## Fit to EWPD

modify GAPP software ([Erler](#))

	SM	SM+ $Z'$
$M_1$ [GeV]	91.1877	91.1877
$m_t$ [GeV]	164.0	164.0
$m_b$ [GeV]	4.199	4.200
$m_c$ [GeV]	1.270	1.278
$\alpha_s$	0.1183	0.1185
$\Delta\alpha_{\text{had}}^{(3)}(1.8 \text{ GeV})$	$5.75 \times 10^{-3}$	$5.72 \times 10^{-3}$
$m_H$ [GeV]	114.4	114.4
$M_2/g'$ [GeV]	—	219.6
$g' \sin \xi$	—	$-2.5 \times 10^{-4}$
$\chi^2_{\text{min}}/N_{\text{d.o.f.}}$	43.8/44	36.4/42

## Fit to EWPD



$$160 \text{ GeV} \leq M_2/g' \leq 560 \text{ GeV}$$

$$-0.0008 < g'/\sin \xi < +0.0003$$

at 90% C.L.

Integrating out  $Z_2$ :

$$\begin{aligned}\mathcal{L}_{Z_2}^{\text{eff}} = & \frac{-1}{2M_2^2} \left( g' \frac{\cos \xi}{\cos \chi} j' - e \hat{c}_W \cos \xi \tan \chi j_{\text{EM}} \right. \\ & \left. + \frac{e}{2\hat{s}_W \hat{c}_W} (\hat{s}_W \cos \xi \tan \chi - \sin \xi) j_{\text{NC}} \right)^2\end{aligned}$$

compare with

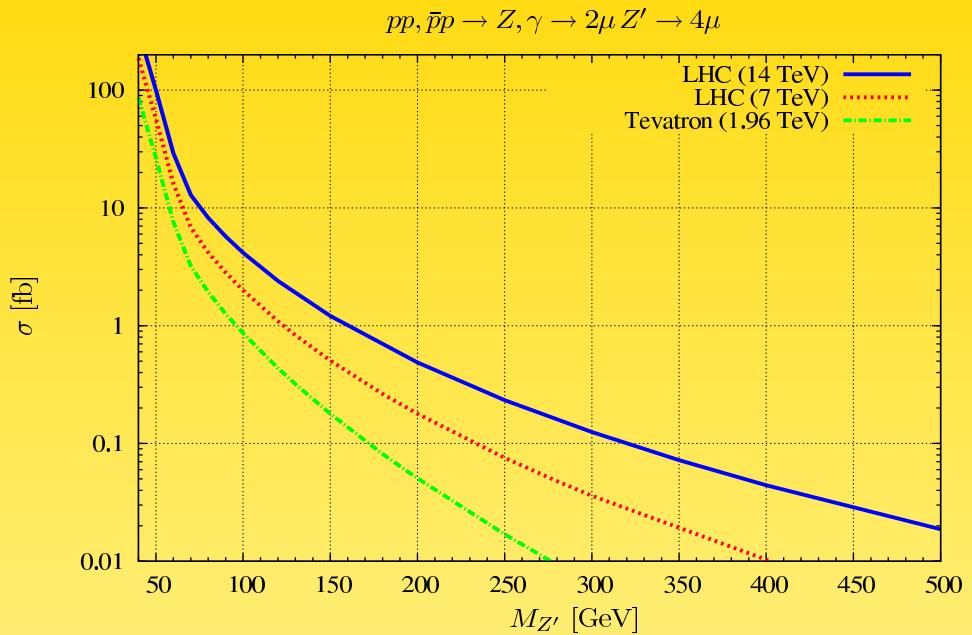
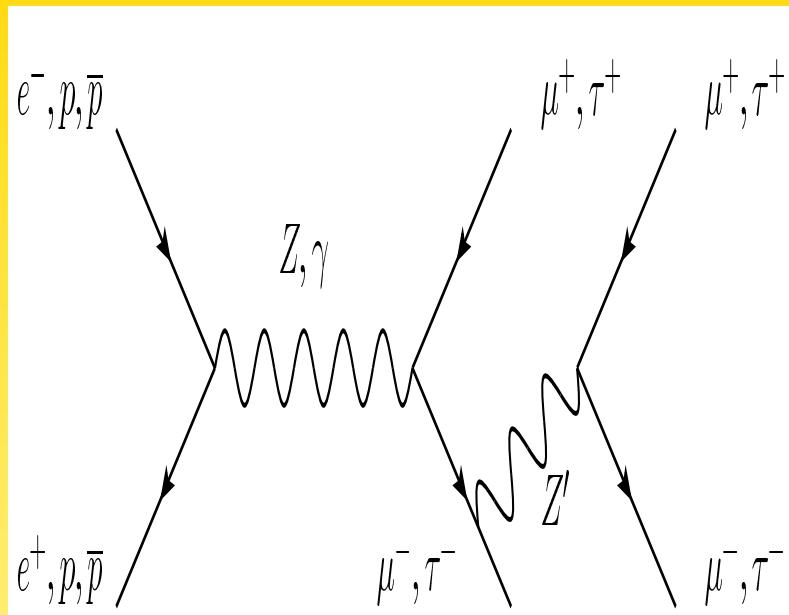
$$\mathcal{L}_{\text{NSI}}^{\text{eff}} = -2\sqrt{2}G_F \epsilon_{\alpha\beta}^{fP} [\bar{f} \gamma^\mu f] [\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta]$$

gives for  $\epsilon_{\alpha\beta}^{\oplus} = \epsilon_{\alpha\beta}^{eV} + 3\epsilon_{\alpha\beta}^{uV} + 3\epsilon_{\alpha\beta}^{dV}$ :

$$\begin{aligned}\epsilon_{\mu\mu}^{\oplus} = & \frac{-g'}{4\sqrt{2}G_F \cos \chi} \frac{e}{\hat{s}_W \hat{c}_W} \left[ \cos \xi \sin \xi \left( \frac{1}{M_1^2} - \frac{1}{M_2^2} \right) + \hat{s}_W \tan \chi \left( \frac{\sin^2 \xi}{M_1^2} + \frac{\cos^2 \xi}{M_2^2} \right) \right] \\ & \simeq 10^{-2} - 10^{-3}\end{aligned}$$

for  $\chi = 0$ ,  $g' \simeq 1/4$ ,  $M_2 \simeq 50 \text{ GeV}$  and  $\xi \simeq 4 \times 10^{-3}$

## Production at colliders



LHC 2012 (7 TeV, 5  $\text{fb}^{-1}$ ): 100 GeV

LHC full (14 TeV, 100  $\text{fb}^{-1}$ ): 350 GeV

linear 0.5 TeV: 300 GeV

linear 1 TeV: 500 GeV

## Model building

add right-handed neutrinos

$$N_1 \sim (\mathbf{1}, \mathbf{1}, 0)(0), \quad N_2 \sim (\mathbf{1}, \mathbf{1}, 0)(+1), \quad N_3 \sim (\mathbf{1}, \mathbf{1}, 0)(-1)$$

gives seesaw

$$\mathcal{M}_R = \begin{pmatrix} X & 0 & 0 \\ 0 & 0 & Y \\ 0 & Y & 0 \end{pmatrix} \quad m_D = \begin{pmatrix} m_{\nu_e} & 0 & 0 \\ 0 & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau} \end{pmatrix}$$

and effective mass matrix

$$\mathcal{M}_\nu \simeq -m_D \mathcal{M}_R^{-1} m_D^T = - \begin{pmatrix} \frac{m_{\nu_e}^2}{X} & 0 & 0 \\ \cdot & 0 & \frac{m_{\nu_\mu} m_{\nu_\tau}}{Y} \\ \cdot & \cdot & 0 \end{pmatrix}$$

and diagonal charged leptons

## Model building

Breaking of  $U(1) \leftrightarrow$  filling of zeros  $\leftrightarrow$  massive  $Z'$

economical and phenomenologically interesting choice:

$$\phi = (\phi^+, \phi^0)^T \sim (\mathbf{1}, \mathbf{2}, +1)(-1) \text{ and } S \sim (\mathbf{1}, \mathbf{1}, 0)(-1)$$

giving  $(f, d \ll m_{\nu_\alpha} \text{ and } s, t \ll X, Y)$

$$m_D = \begin{pmatrix} m_{\nu_e} & 0 & d \\ f & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau} \end{pmatrix} \quad \mathcal{M}_R = \begin{pmatrix} X & s & t \\ s & 0 & Y \\ t & Y & 0 \end{pmatrix}$$

and effective mass matrix

$$\mathcal{M}_\nu \simeq \begin{pmatrix} -\frac{m_{\nu_e}^2}{X} & -\frac{m_{\nu_e}f}{X} - \frac{m_{\nu_\mu}d}{Y} + \frac{m_{\nu_e}m_{\nu_\mu}t}{XY} & \frac{m_{\nu_e}m_{\nu_\tau}s}{XY} \\ . & 0 & -\frac{m_{\nu_\mu}m_{\nu_\tau}}{Y} \\ . & . & 0 \end{pmatrix}$$

## Model building

$$\mathcal{M}_\nu \simeq \begin{pmatrix} -\frac{m_{\nu_e}^2}{X} & -\frac{m_{\nu_e} f}{X} - \frac{m_{\nu_\mu} d}{Y} + \frac{m_{\nu_e} m_{\nu_\mu} t}{XY} & \frac{m_{\nu_e} m_{\nu_\tau} s}{XY} \\ . & 0 & -\frac{m_{\nu_\mu} m_{\nu_\tau}}{Y} \\ . & . & 0 \end{pmatrix}$$

we know that  $s, t \simeq \langle S \rangle \simeq M'_Z/g' \simeq 200 \text{ GeV}$

Thus:  $X, Y \simeq \text{TeV}$

$\Rightarrow \text{TeV seesaw}$

can be written as

$$\mathcal{M}_\nu / \left( \frac{m_{\nu_e}^2}{Y} \right) = \mathcal{M}_\nu^0 + \Delta \mathcal{M} = - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & ab \\ 0 & ab & 0 \end{pmatrix} + \Delta \mathcal{M}$$

with perturbation

$$\Delta \mathcal{M} = \begin{pmatrix} \delta \epsilon (2 - \delta \epsilon) & b \alpha (1 + \delta \epsilon) + \gamma (-1 + \delta \epsilon) + b \delta (k - 1) & a \epsilon (1 - \delta \epsilon) \\ \cdot & -(\gamma - b \alpha)^2 & a (b \alpha \epsilon + \epsilon \gamma + b k) \\ \cdot & \cdot & -a^2 \epsilon^2 \end{pmatrix}$$

resulting in (including small charged lepton corrections)

$$\sin \theta_{13} = \mathcal{O}(\epsilon) \quad \text{and} \quad \sin \theta_{23} = 1/\sqrt{2} + \mathcal{O}(\epsilon^2)$$

## Scalar Sector

$$\begin{aligned}
V = & -\mu_1|H|^2 + \lambda_1|H|^4 - \mu_2|\phi|^2 + \lambda_2|\phi|^4 - \mu_3|S|^2 + \lambda_3|S|^4 \\
& + \delta_1|H|^2|\phi|^2 + \delta_2|H^\dagger\phi|^2 + \delta_3|H|^2|S|^2 + \delta_4|\phi|^2|S|^2 \\
& - \left( \sqrt{2}|\mu|e^{i\kappa}H^\dagger\phi\bar{S} \right)
\end{aligned}$$

$$\text{Re } S \rightarrow \langle S \rangle \equiv v_S/\sqrt{2}, \quad \text{Re } \phi^0 \rightarrow \langle \phi^0 \rangle \equiv v_\phi/\sqrt{2}, \quad \text{Re } h^0 \rightarrow v/\sqrt{2}$$

in total: 6 physical fields,  $\sigma$ ,  $\sigma^\pm$ ,  $\text{Re } S$ ,  $\text{Re } \Phi^0$ ,  $\text{Re } h^0$ , for instance

$$m_\sigma^2 = \frac{|\mu|v v_S}{v_\phi} + \frac{|\mu|v_\phi v_S}{v} + \frac{|\mu|v v_\phi}{v_S}$$

$$m_{\sigma^\pm}^2 = \frac{|\mu|v v_S}{v_\phi} + \frac{|\mu|v_\phi v_S}{v} - \frac{1}{2}\delta_2(v_\phi^2 + v^2)$$

safe and phenomenologically relevant with  $v_\Phi \sim \text{GeV} \ll v_S \sim v$

### example mixing terms

$$\begin{pmatrix} \text{Im } S \\ \text{Im } \phi^0 \\ \text{Im } h^0 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\cos \beta \sin \theta & \cos \beta \cos \theta & \sin \beta \\ \sin \beta \sin \theta & -\sin \beta \cos \theta & \cos \beta \end{pmatrix}^T \begin{pmatrix} 0 \\ 0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \sin \beta \sin \theta \sigma \\ -\sin \beta \cos \theta \sigma \\ \cos \beta \sigma \end{pmatrix}$$

with angles

$$\tan \theta \equiv \frac{v_\phi}{v_S}, \quad \tan \beta \equiv \frac{v}{v_S v_\phi} \sqrt{v_S^2 + v_\phi^2} = \frac{v}{v_S \sin \theta}$$

and with  $v_\phi \ll v_S \sim v$  this implies  $\sin \theta, \cos \beta \ll 1$

$$\begin{pmatrix} \phi^- \\ h^- \end{pmatrix} = \begin{pmatrix} -\sin \beta^- \sigma^- \\ \cos \beta^- \sigma^- \end{pmatrix}$$

with  $\tan \beta^- \equiv v/v_\phi = \cos \theta \tan \beta \simeq \tan \beta \gg 1$

## Yukawa interactions

$$\begin{aligned} -\mathcal{L} \supset & \sum_i \frac{m_i^d}{v} \bar{d}_L^i d_R^i (h + i c_\beta \sigma) + \sum_i \frac{m_i^u}{v} \bar{u}_L^i u_R^i (h - i c_\beta \sigma) \\ & + c_\beta^- \frac{\sqrt{2}}{v} \sum_{i,j} m_j^d V_{ij} \bar{u}_L^i d_R^j \sigma^+ - c_\beta^- \frac{\sqrt{2}}{v} \sum_{i,j} m_j^u V_{ji}^* \bar{d}_L^i u_R^j \sigma^- \end{aligned}$$

FCNC suppressed by  $m_q/v$  and  $\cos \beta^- \simeq v_\Phi/v$

similar to 2HDM type I:  $m_{\sigma^\pm} \gtrsim 80$  GeV

## Kinetic Terms

$$\begin{aligned}
\mathcal{L} \supset & (D_\mu H)^\dagger (D^\mu H) + (D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu S)^\dagger (D^\mu S) \\
= & |\partial_\mu h^0 - i \frac{e}{2s_W c_W} Z_\mu h^0 - i \frac{e}{\sqrt{2}s_W} W_\mu^- h^+|^2 \\
& + |\partial_\mu h^+ - i \frac{e}{s_W c_W} (c_W^2 - s_W^2) Z_\mu h^+ - ie A_\mu h^+ - i \frac{e}{\sqrt{2}s_W} W_\mu^+ h^0|^2 \\
& + |\partial_\mu \phi^0 + ig' Z'_\mu \phi^0 - i \frac{e}{2s_W c_W} Z_\mu \phi^0 - i \frac{e}{\sqrt{2}s_W} W_\mu^- \phi^+|^2 \\
& + |\partial_\mu \phi^+ + ig' Z'_\mu \phi^+ - i \frac{e}{s_W c_W} (c_W^2 - s_W^2) Z_\mu \phi^+ - ie A_\mu \phi^+ - i \frac{e}{\sqrt{2}s_W} W_\mu^+ \phi^0|^2 \\
& + |\partial_\mu S + ig' Z'_\mu S|^2
\end{aligned}$$

with boson masses

$$\begin{array}{ll}
M_W^2 = \frac{e^2}{4s_W^2} (v^2 + v_\phi^2) & M_Z^2 = \frac{e^2}{4s_W^2 c_W^2} (v^2 + v_\phi^2) \\
M_{Z'}^2 = g'^2 (v_\phi^2 + v_S^2) & \delta \hat{M}^2 = -\frac{e}{2s_W c_W} g' v_\phi^2
\end{array}$$

$h^3$	$h^4$	$S^3$	$S^4$	$\phi^3$	$\phi^4$	$\sigma^4$	
$\sigma^2 h$	$h\phi S$	$\sigma^2 \phi$	$\sigma^2 S$	$\phi^2 S^2$	$h^2 S^2$	$h^2 \phi^2$	
$\sigma^2 \phi^2$	$\sigma^2 h^2$	$h^2 S$	$h^2 \phi$	$\phi^2 h$	$\sigma^2 S^2$	$\phi^2 S$	
$S^2 \phi$	$h S^2$	$\sigma^+ \sigma^- \sigma^+ \sigma^-$	$h^2 \sigma^+ \sigma^-$	$\phi^2 \sigma^+ \sigma^-$	$\sigma^2 \sigma^+ \sigma^-$	$h \sigma^+ \sigma^-$	
$\phi \sigma^+ \sigma^-$	$S \sigma^+ \sigma^-$	$h \phi \sigma^+ \sigma^-$	$S^2 \sigma^+ \sigma^-$				
$Z^2 h$	$W^+ W^- \phi$	$Z^2 \sigma^+ \sigma^-$	$Z A \sigma^+ \sigma^-$	$Z W^- \sigma^+ \phi$	$A^2 \sigma^+ \sigma^-$	$Z^2 h^2$	
$W^- A \sigma^+ \phi$	$W^+ W^- \sigma^2$	$W^+ W^- \phi^2$	$Z W^- \sigma^+ \sigma$	$W^+ A \sigma^- \sigma$	$W^+ W^- \sigma^+ \sigma^-$	$Z W^- \sigma^+ h$	
$W^- A \sigma^+ h$	$W^+ W^- h$	$W^+ W^- h^2$	$Z^2 \sigma^2$	$Z^2 \phi$	$Z^2 \phi^2$		
$Z'^2 \phi^2$	$Z'^2 S^2$	$W^- Z' \sigma^+ \phi$	$ZZ' \sigma^+ \sigma^-$	$A Z' \sigma^+ \sigma^-$	$W^+ Z' \sigma^- \sigma$	$Z'^2 \sigma^+ \sigma^-$	
$W^+ Z' \sigma^-$	$Z'^2 \phi$	$Z'^2 S$	$Z'^2 \sigma^2$	$ZZ' \sigma^2$	$ZZ' \phi$	$ZZ' \phi^2$	
$A \sigma^+ \sigma^-$	$Z \sigma^+ \sigma^-$	$Z \sigma h$	$Z \sigma \phi$	$W^- \sigma^+ \sigma$	$W^- \sigma^+ h$	$W^- \sigma^+ \phi$	
$Z' \phi \sigma$	$Z' \sigma S$	$Z' \sigma^+ \sigma^-$					

## Lepton Flavor Violation

$$\mathcal{M}_{\text{leptons}} = \frac{1}{\sqrt{2}} \begin{pmatrix} \lambda_H^{ee} v & \lambda_\phi^{e\mu} v_\phi & 0 \\ 0 & \lambda_H^{\mu\mu} v & 0 \\ \lambda_\phi^{\tau e} v_\phi & 0 & \lambda_H^{\tau\tau} v \end{pmatrix}$$

diagonalization with  $U_L$  and  $U_R$  modifies  $Z'$  coupling:

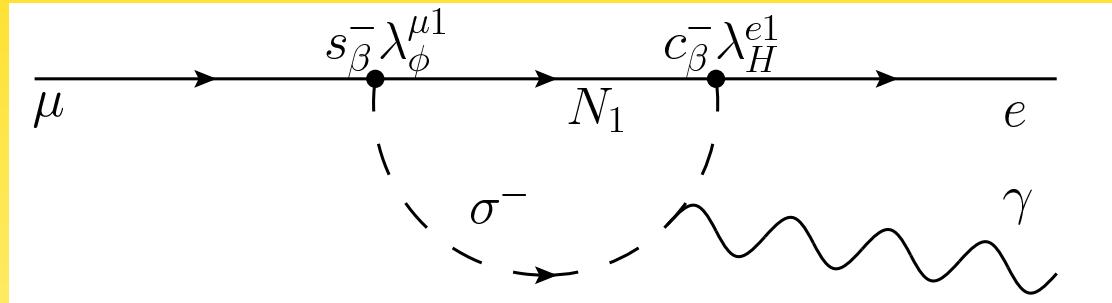
$$j'_\mu Z'^\mu = \sum_{i=L,R} \bar{\ell}_i \begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \end{pmatrix} \gamma_\mu \ell_i Z'^\mu \rightarrow \sum_{i=L,R} \bar{\ell}_i U_i^\dagger \begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \end{pmatrix} U_i \gamma_\mu \ell_i Z'^\mu$$

introduces new decays

$$\tau \rightarrow e \bar{\mu} \mu, \quad \tau \rightarrow e \bar{\nu}_{\mu,\tau} \nu_{\mu,\tau}, \quad \mu \rightarrow e \bar{\nu}_{\mu,\tau} \nu_{\mu,\tau}$$

and constrains  $\lambda_\phi^{\tau e} < 10^{-3}\text{--}10^{-4}$

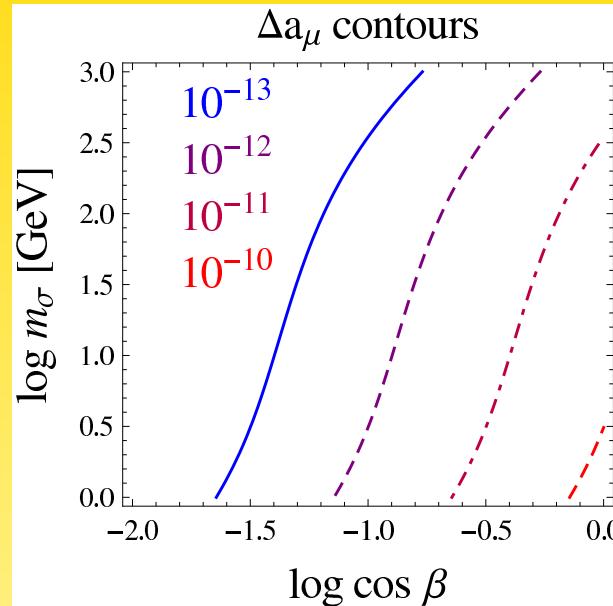
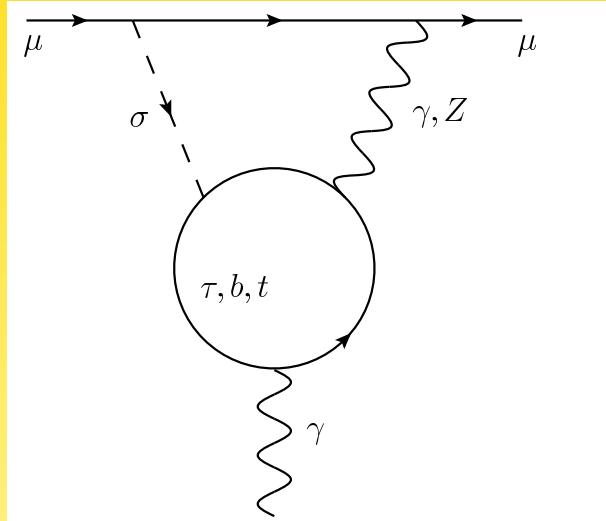
## Lepton Flavor Violation



$$\text{BR}(\mu \rightarrow e\gamma) \simeq \frac{1}{192\pi^2} \left( \frac{s_\beta^- \lambda_\phi^{\mu 1} c_\beta^- \lambda_H^{e 1}}{2G_F m_N^2} \right)^2 \simeq \frac{\gamma^2}{96\pi^2} \left( \frac{m_{\nu_e}^2/Y}{Y} \right)^2 \simeq 10^{-29} \gamma^2$$

where  $\gamma = f/m_{\nu_e}$  in effective mass matrix

## Magnetic Moment



$$\Delta a_\mu^{\text{2-loop}} = \frac{\alpha}{8\pi^3} \frac{m_\mu^2}{v^2} c_\beta^2 \sum_{f=t,b,\tau} N_{\text{color}}^f Q_f^2 \frac{m_f^2}{m_\sigma^2} \int_0^1 dx \frac{\ln \left( \frac{m_f^2/m_\sigma^2}{x(1-x)} \right)}{m_f^2/m_\sigma^2 - x(1-x)}$$

## Extension to $SU(2)'$ : “Leptospin”

include electron in the game

$$\mathbf{L} \equiv (L_\mu, L_e, L_\tau) \sim (\mathbf{1}, \mathbf{2}, -1)(\mathbf{3}), \quad \ell_R \equiv (\mu_R, e_R, \tau_R) \sim (\mathbf{1}, \mathbf{1}, -2)(\mathbf{3})$$

becomes nasty..., e.g. charged lepton masses

$$\mathcal{L} \supset Y_H \overline{\mathbf{L}} H \ell_R$$

are degenerate, but if  $SU(2)'$  is broken

$$H \sim (\mathbf{1}, \mathbf{2}, +1)(\mathbf{1}), \quad \Delta \sim (\mathbf{1}, \mathbf{2}, +1)(\mathbf{3}), \quad \Sigma \sim (\mathbf{1}, \mathbf{2}, +1)(\mathbf{5})$$

with

$$\Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta^0 & \Delta^+ & 0 \\ \Delta^- & 0 & \Delta^+ \\ 0 & \Delta^- & -\Delta^0 \end{pmatrix}, \quad \Sigma = \frac{1}{\sqrt{6}} \begin{pmatrix} \Sigma^0 & \sqrt{3}\Sigma^+ & \sqrt{6}\Sigma^{++} \\ \sqrt{3}\Sigma^- & -2\Sigma^0 & -\sqrt{3}\Sigma^+ \\ \sqrt{6}\Sigma^{--} & -\sqrt{3}\Sigma^- & \Sigma^0 \end{pmatrix}$$

## Extension to $SU(2)'$ : “Leptospin”

then this is allowed:

$$\mathcal{L} \supset \overline{\boldsymbol{L}} (Y_H H + Y_\Delta \Delta + Y_\Sigma \Sigma) \boldsymbol{\ell}_R$$

and after symmetry breaking

$$m_\mu = Y_H \langle H \rangle + Y_\Delta \langle \Delta^0 \rangle / \sqrt{2} + Y_\Sigma \langle \Sigma^0 \rangle / \sqrt{6}$$

$$m_e = Y_H \langle H \rangle - 2Y_\Sigma \langle \Sigma^0 \rangle / \sqrt{6}$$

$$m_\tau = Y_H \langle H \rangle - Y_\Delta \langle \Delta^0 \rangle / \sqrt{2} + Y_\Sigma \langle \Sigma^0 \rangle / \sqrt{6}$$

## Extension to $SU(2)'$ : “Leptospin”

right-handed neutrinos  $\mathbf{N} \equiv (N_\mu, N_e, N_\tau) \sim (\mathbf{1}, \mathbf{1}, 0)(\mathbf{3})$ , or

$$N = \begin{pmatrix} N_e/\sqrt{2} & N_\mu \\ N_\tau & -N_e/\sqrt{2} \end{pmatrix}$$

mass degeneracy broken by  $\Omega \sim (\mathbf{1}, \mathbf{1}, 0)(\mathbf{5})$ , allows Majorana mass term

$$\frac{Y_\Omega}{2\sqrt{6}} \begin{pmatrix} -\bar{N}_\mu^c & \bar{N}_e^c & \bar{N}_\tau^c \end{pmatrix} \begin{pmatrix} \sqrt{6}\Omega^{--} & -\sqrt{3}\Omega^- & \Omega^0 \\ -\sqrt{3}\Omega^- & 2\Omega^0 & \sqrt{3}\Omega^+ \\ \Omega^0 & \sqrt{3}\Omega^+ & \sqrt{6}\Omega^{++} \end{pmatrix} \begin{pmatrix} -N_\mu \\ N_e \\ N_\tau \end{pmatrix}$$

and vev  $\langle \Omega^0 \rangle$  breaks it to  $L_\mu - L_\tau$  invariant mass matrix

## $L_\mu - L_\tau$ and Dark Matter

Baek, Ko, JCAP **0910**, 011 (2009)

add Dirac fermion  $\psi$  charged under  $L_\mu - L_\tau$

relic density from  $\psi\bar{\psi} \rightarrow Z' \rightarrow (\mu^+\mu^-, \tau^+\tau^-, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$

annihilation as well

annihilation is automatically leptophilic  $\leftrightarrow$  PAMELA frenzy

## Summary

- $L_\mu - L_\tau$  can be gauged in the SM without anomaly
- two main cases:
  - ultra-light  $Z'$ : long-range force
  - electroweak scale  $Z'$ : lots of physics
- gauge vs. flavor

## Derivation of Potential

Consider the time-like components, note that  $j_{\text{EM}}^0 = 0$  and use

$$j_3^0 = -\frac{1}{2} \bar{e}_L \gamma^0 e_L + \frac{1}{2} \bar{p}_L \gamma^0 p_L - \frac{1}{2} \bar{n}_L \gamma^0 n_L = -\frac{1}{4} (n_e - n_p + n_n) = -\frac{n_n}{4}$$

since the axial-part will result in a spin-operator in the non-relativistic limit and we assume the Sun is not polarized. The equation of motion for  $Z_2^0$ , following from the Euler-Lagrange equation

$$\partial_\nu \frac{\delta}{\delta(\partial_\nu Z_{2\mu})} \left( -\frac{1}{4} Z_{2\alpha\beta} Z_2^{\alpha\beta} \right) - \frac{\delta}{\delta Z_{2\mu}} \left( \frac{1}{2} M_2^2 Z_{2\alpha} Z_2^\alpha + \mathcal{L}_{Z_2} \right) = 0$$

is therefore

$$(\partial^2 + M_2^2) Z_2^0 = (\xi - s_W \chi) \frac{e}{s_W c_W} \frac{n_n}{4}$$

In the static case outside of the Sun this is ( $n_n(\mathbf{x}) = N_n \delta^{(3)}(\mathbf{x})$ ):

$$(\Delta - M_2^2) Z_2^0 = -(\xi - s_W \chi) \frac{e}{s_W c_W} \frac{1}{4} N_n \delta^{(3)}(\mathbf{x})$$

with the well-known solution

$$V(r) = Z_2^0 = (\xi - s_W \chi) \frac{e}{s_W c_W} \frac{1}{4} N_n \times \frac{e^{-rM_2}}{4\pi r}$$

In the limit  $M_2 \rightarrow 0$  the potential, for  $\nu_\mu$  and  $\nu_\tau$  respectively, on Earth is:

$$V_{\mu,\tau} = \pm g' (\xi - s_W \chi) \frac{e}{4 s_W c_W} \frac{N_n}{4\pi R_{\text{A.U.}}}$$