Minimal 4 zero Seesaw Mass Matrices and Large θ_{13}

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Based on Work in Preparation
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TINP Meetings, November 2011

Plan of the talk

- Introduction and motivation for 4 zero Yukawa textures.
- Phenomenology of these textures with 4 zeros.
- Symmetry realization of these textures.
- Connecting low energy CP violation and leptogenesis.
- Summary.

Motivation for 4 zero textures

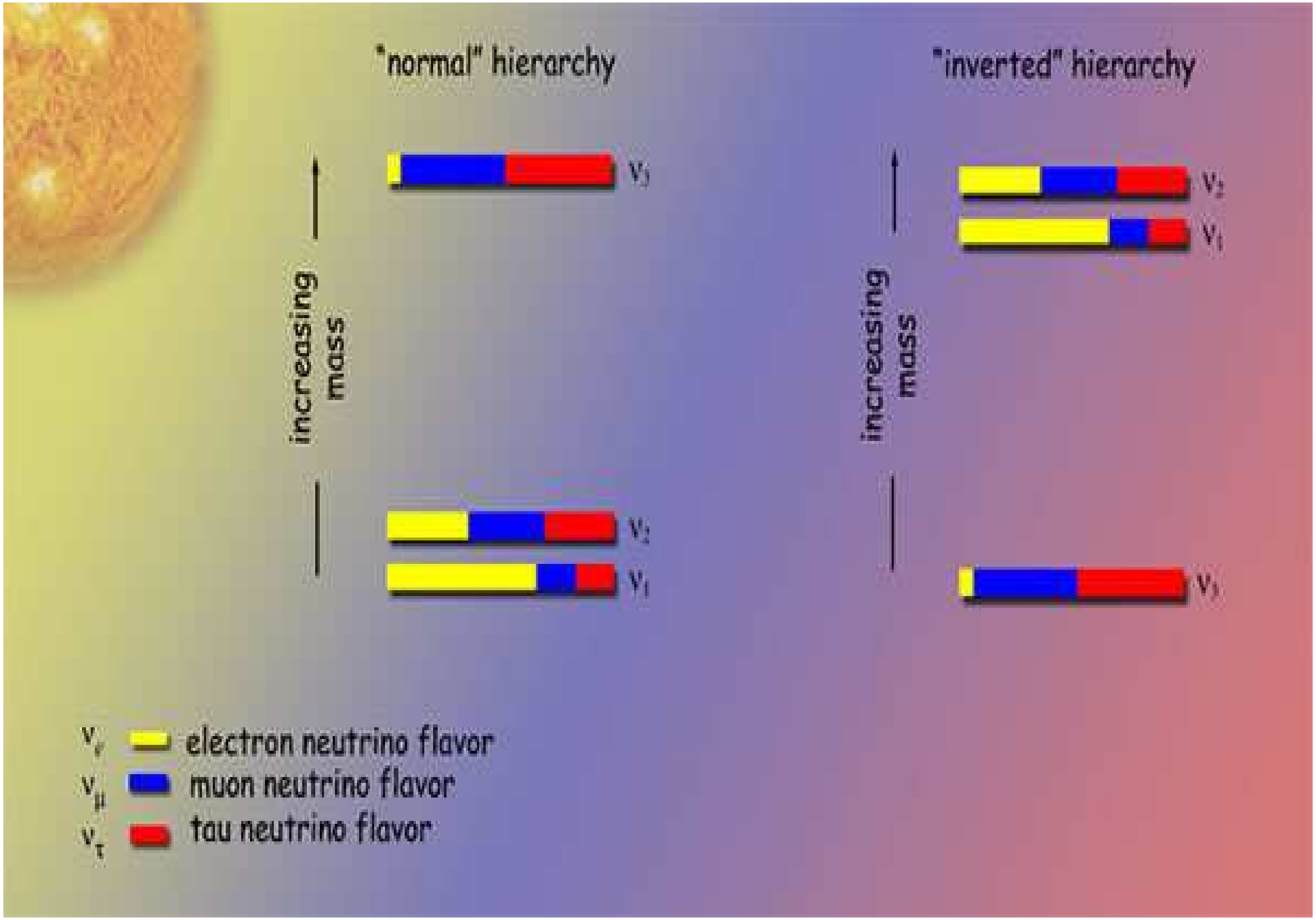
- In the Standard Model neutrinos are massless .
- But the neutrino oscillation experiments showed that neutrinos are massive.
- Data from neutrino oscillations experiments and cosmology
  - Recent data on neutrino oscillation data at 3σ level gives us (arXiv:1106.6028 by Fogli et al)

$$\begin{gathered}\triangle m^2_{\odot}(10^{-5}eV^2) = (6.99 - 8.18)\\ |\triangle m^2_A(10^{-3}eV^2)| = (2.06 - 2.67)\\ \sin^2\theta_{12} = (0.259 - 0.359)\\ \sin^2\theta_{23} = (0.34 - 0.64)\\ \sin^2\theta_{13} = (0.001 - 0.004)\end{gathered}$$

  - The Double Chooz data :
    $\sin^2 2\theta_{13} = 0.085 \pm 0.029(stat) \pm 0.042(syst)$ at 90% CL
  - The bounds on the absolute neutrino masses are

$$\begin{gathered}m_{\nu_e} = (\textstyle\sum_i m_i^2|U_{ei}|^2)^{1/2} < 2.3eV \text{ (Tritium } \beta \text{ decay)}\\ m_{ee} = |\textstyle\sum_i m_i U_{ei}^2| < 0.3 - 1.0eV \text{ (}0\nu\beta\beta\text{ decay)}\\ m_{cosmo} = \textstyle\sum_i |m_i| < 0.61eV \text{ (cosmological bounds)}\end{gathered}$$

- But still we do not know the absolute neutrino mass and the exact hierarchy.


"normal" hierarchy
"inverted" hierarchy
increasing mass
increasing mass
ν_3
ν_2
ν_1
ν_2
ν_1
ν_3
electron neutrino flavor
ν_μ muon neutrino flavor
ν_τ tau neutrino flavor

All these data leads to three possible patterns of neutrino mass ordering:

- Normal hierarchy $m_1 < m_2 < m_3$.

$$\text{with } m_2 = \sqrt{m_1^2 + \Delta m_\odot^2}$$
$$m_3 = \sqrt{m_1^2 + \Delta m_\mathrm{A}^2} \ ,$$

- Inverted hierarchy $m_1 \geq m_2 > m_3$.

$$\text{with } m_2 = \sqrt{m_3^2 + \Delta m_\odot^2 + \Delta m_\mathrm{A}^2} \ ;$$
$$m_1 = \sqrt{m_3^2 + \Delta m_\mathrm{A}^2} \ .$$

- Degenerate model $m_1 \simeq m_2 \simeq m_3$.

Type-I Seesaw

- The simplest way to write mass terms for the neutrino is just by adding a heavy SU(2) singlet right handed neutrino N_R.
- In the *Type I Seesaw Formula* addition of one singlet heavy right handed neutrino per generation gives masses to the neutrinos.

$$\mathcal{L}_y = -Y^l_{ij}\overline{L_i}\phi l_{Rj} - Y^\nu_{ij}\overline{L_i}\tilde{\phi}N_{Rj} + \frac{1}{2}\overline{N_R{}^c}M_N N_R - \ +\text{h.c.} \ \ .$$

where $\tilde{\phi} = i\sigma_2\phi$, $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

- The mass matrix for the neutral fermions can be written as

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

- The light neutrino mass matrix can be written as
  $m_\nu = -m_D M_R^{-1} m_D^T$
  where
  - m_D is the Dirac neutrino mass matrix.
  - M_R is the right handed Majorana mass matrix.
  - m_ν is the light neutrino mass matrix.

The Mixing Matrix

- The low energy neutrino mass matrix can in general be diagonalized as $V_\nu^{\mathrm{T}}\, m_\nu\, V_\nu = D_\nu$ where $D_\nu = \mathrm{diag}(m_1, m_2, m_3)$.
- The leptonic mixing matrix can be obtained from the charged current for the leptons

$$J^{\mu +} = \bar{l}\gamma_\mu\,(1-\gamma_5)\,\nu = \bar{l}'\gamma_\mu\,(1-\gamma_5)\,V_l^\dagger V_\nu$$

$$\nu = V_\nu \nu' \quad l = V_l l'$$

- $U_{PMNS} = V_l^\dagger V_\nu$. If charged lepton mass matrix is real and diagonal then $V_l = I$, $\rightarrow U_{PMNS} = V_\nu$
- The matrix V_ν is unitary and includes 3 angles and 6 phases in general.

$$U_{PMNS} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -c_{23}\, s_{12} - s_{23}\, s_{13}\, c_{12}\, e^{i\delta} & c_{23}\, c_{12} - s_{23}\, s_{13}\, s_{12}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{23}\, s_{12} - c_{23}\, s_{13}\, c_{12}\, e^{i\delta} & -s_{23}\, c_{12} - c_{23}\, s_{13}\, s_{12}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix} P\,.$$

where $P = \mathrm{diag}(e^{i\alpha}, e^{i\beta}, 1)$ is the diagonal matrix containing the Majorana phases ,$s_{ij} = sin\theta_{ij}$,$c_{ij} = cos\theta_{ij}$ and δ is the Dirac CP-phase.

Motivation of the present work

- In the seesaw $m_\nu = -m_D M_R^{-1} m_D^T$ there are (18-3)+(12-3)=24 parameters.
- The low scale parameters are 9. So there is a mismatch in the number of parameters which prohibits definite predictions for the low energy neutrino parameters.
- One of the possible solution is imposing “texture zeros"
  i.e, some entries in m_ν much smaller than the other entries.
- Texture zeros in the low energy Majorana mass matrix (P. H. Frampton, S. L. Glashow and D. Marfatia, Phys. Lett. B **536**, 79 (2002) ,A. Merle and W. Rodejohann, Phys. Rev. D **73**, 073012 (2006) ,S. Dev, S. Kumar, S. Verma and S. Gupta, Phys. Rev. D **76**, 013002 (2007).)
- Texture zeros in both the charged lepton and neutrino mass matrices (Z. Z. Xing and H. Zhang, Phys. Lett. B **569**, 30 (2003), Z. Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004), S. Zhou and Z. Z. Xing, Eur. Phys. J. C **38**, 495 (2005)...)

Our Work

- It is more natural to consider the zeros appearing in the fundamental mass matrix in the lagrangian i.e, m_D amd M_R.
- So to reconstruct the seesaw we need to take some assumptions about m_D and M_R.
- 4 zero Yukawa textures have been investigated in detail for diagonal M_R and all the light neutrinos massive.
  (G. C. Branco, D. Emmanuel-Costa, M. N. Rebelo and P. Roy,Phys. Rev. D **77**, 053011 (2008))
- We consider 4 zero textures in m_D and non diagonal form of M_R with the Type I seesaw mechanism and *s*till having all 3 light neutrinos massive
- We enumerate all such possible matrices and obtain the ones that are consistent with low energy phenomenology and study the neutrino parameter predictions.
- We construct a possible model based on U(1) symmetry which can lead to the textures under study.
- We try to connect the low energy CP violation and leptogenesis in the allowed textures.

Texture Analysis

- The most general form for the Dirac matrix m_D is

$$m_D = \begin{pmatrix} a_1\, e^{i\alpha_1} & a_2\, e^{i\alpha_2} & a_3\, e^{i\alpha_3} \\ b_1\, e^{i\beta_1} & b_2\, e^{i\beta_2} & b_3\, e^{i\beta_3} \\ c_1\, e^{i\gamma_1} & c_2\, e^{i\gamma_2} & c_3\, e^{i\gamma_3} \end{pmatrix}.$$

- The number of parameters in m_D is (18-3)=15. There are ${}^9C_4$=126 possibilities of putting 4 zeros in m_d.
- Heavy neutrino mass matrices in non-diagonal form contain four independent zeros.

$$M_R = \begin{pmatrix} p & 0 & 0 \\ 0 & 0 & u \\ 0 & u & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & t \\ 0 & q & 0 \\ t & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & r \end{pmatrix}$$

  correspond to flavor symmetries $L_\mu - L_\tau$, $L_e - L_\tau$ and $L_e - L_\mu$.
- In total there are 378 forms of m_ν.

Allowed textures

- In terms of the allowed forms of the left handed Majorana mass matrix m_ν. We consider the following forms of m_ν to be allowed
  - m_ν with one zero entry i.e, one vanishing entry.
    (A. Merle and W. Rodejohann; Phys. Rev. D 73, 073012 (2006))
  - m_ν with less than or equal to two zeros.
    (P.H. Frampton et. al. Phys. Lett. B 536, 79 (2002))
  - m_ν obeying scaling property.
    (R. N. Mohapatra and W. Rodejohann; Phys. Lett. B 644, 59 (2007))

  Applying the above criterion,the allowed textures can be classified into two categories
  - m_ν of rank 3 with all lght neutrinos massive.
  - m_ν of rank 2 with one light neutrino massless.
- We find that there are total 18 allowed textures which give rise to m_ν with one eigenvalue =0 i.e m_ν with rank 2(We have not analysed them here).
- There are total 62 allowed textures with rank 3 for each form of M_R which we have analysed.

Allowed textures of rank 3 with 2 zeros

- Textures with two zeros:
  1. 6 cases with $m_{ee} = 0$ and $m_{e\mu} = 0$
     (Identical to Case A1 of Frampton et al)
  2. 6 cases with $m_{ee} = 0$ and $m_{e\tau} = 0$
     (Identical to Case A2 of Frampton et al)
  3. 6 cases with $m_{\mu\mu} = 0$ and $m_{e\mu} = 0$
     (Identical to Case B3 of Frampton et al)
  4. 6 cases with $m_{\tau\tau} = 0$ and $m_{e\tau} = 0$
     (Identical to Case B4 of Frampton et al)
  5. 2 cases with $m_{\mu\mu} = 0$ and $m_{\tau\tau}$ =0
     (Identical to Case C of Frampton et al)
- 26 total allowed cases. Out of these 13 cases can be obtained from the other 13 by 2-3 exchange of column in m_D and they have similar predictions.

Allowed textures of rank 3 with 1 zeros

- 6 cases with $(m_\nu)_{\alpha\alpha}$=0 for each α where $\alpha = e, \mu, \tau$. In total 18 cases for each M_R.
  1. Under 2$\leftrightarrow$ 3 column exchange of m_D these reduces to 9.
  2. Of these there are 3 cases with vanishing minor and 1 texture zero.
     (S. Dev, S. Verma, S. Gupta, R. R. Gautam, Phys. Rev. **D81**, 053010 (2010).)
  3. 6 cases have a vanishing minor like condition

$$(m_\nu)_{\alpha\beta}(m_\nu)_{\beta\gamma} - (1/2)(m_\nu)_{\beta\beta}(m_\nu)_{\alpha\gamma} = 0.$$

- 12 cases for each M_R have $(m_{\nu\,\alpha\beta}) = 0$ and one zero texture
  1. 4 cases with $m_{e\mu} = 0$ and vanishing 1-3 minor.
  2. 4 cases with $m_{e\tau} = 0$ and vanishing 1-2 minor.
  3. 4 cases with $m_{\mu\tau} = 0$ and vanishing 1-2 minor.

  These are identical to cases analysed by S Dev et al.

Allowed textures of rank 3 with no zeros

- There are 6 such cases in total:
  1. 2 case with two vanishing minors where the minors corresponding to 33 and 22 vanish.
  2. 2 case with two vanishing minors where the minors corresponding to 33 and 11 vanish.
  3. 2 case with two vanishing minors where the minors corresponding to 22 and 11 vanish.
- Cases 1, 2 and 3 are identical to the cases D, F_1 and F_2 analysed in E. I. Lashin, N. Chamoun, Phys. Rev. **D78**, 073002 (2008).

Total new scenarios allowed

- Two solutions with $(m_\nu)_{ee} = 0$ and a vanishing minor like condition are allowed for the normal hierarchy.
- One solution with $(m_\nu)_{\mu\mu} = 0$ and a vanishing minor like condition is allowed for inverted hierarchy.
- One with $(m_\nu)_{\tau\tau} = 0$ and vanishing minor like condition is allowed for inverted hierarchy

Solution Type I

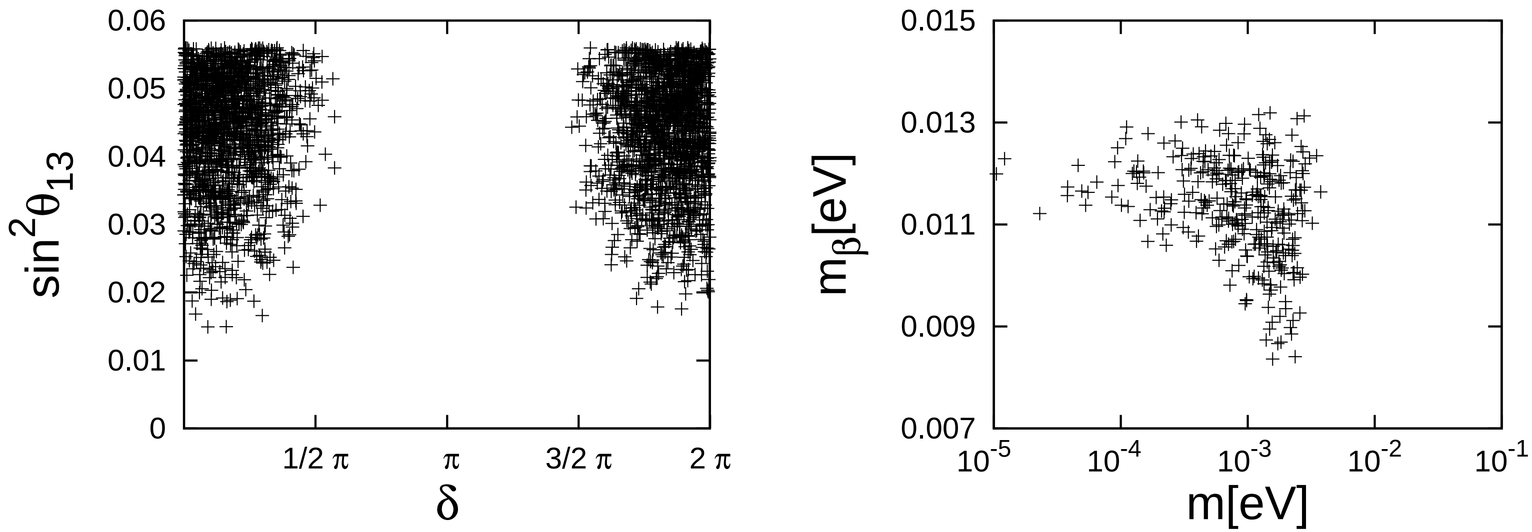
- $$m_\nu = \begin{pmatrix} 0 & x & y \\ \cdot & 2xz & zy \\ \cdot & \cdot & w \end{pmatrix},$$

  with complex x, y, z, w.

- The matrix satisfies the following two conditions:

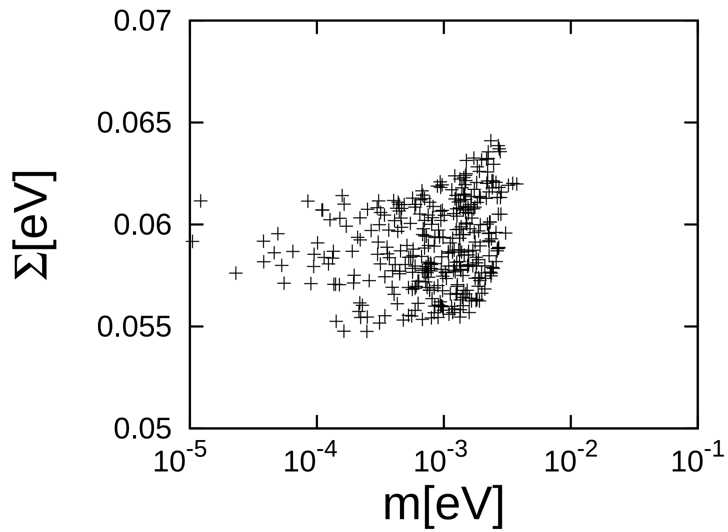
$$\begin{aligned} m_{ee} &= 0, \\ m_{e\mu} m_{\mu\tau} - \frac{1}{2} m_{\mu\mu} m_{e\tau} &= 0. \end{aligned}$$

Analysis of Solution Type I



- As ee element of m_ν being zero so the IH not allowed.
- Left panel shows $\sin^2\theta_{13} \geq 0.014$ for lightest mass (0- 0.1)eV.
- Right panel shows that m_β is in the range (0.008 - 0.013) eV within the KATRIN sensitivity.

Analysis of Solution Type I



- A plot of the smallest mass m_1 vs. the sum of the neutrino masses Σ.
- Σ is always bounded from below by 0.054 eV and above by 0.064 eV.
- Much smaller then the current bounds of $\Sigma <$0.28 eV(95)C.L

Approximate analytic estimates- Type I

$$m_{ee} = 0,$$
$$m_{e\mu} m_{\mu\tau} - \frac{1}{2} m_{\mu\mu} m_{e\tau} = 0.$$

$$X m_1 + Y m_2 + Z m_3 = 0$$
$$a m_2^2 + b m_2^2 + c m_3^2 + d m_1 m_2 + e m_1 m_3 + f m_2 m_3 = 0$$

$$\left(\frac{m_1}{m_2}\right) = \frac{-A_{12} + \sqrt{A_{12}^2 - 4A_{11}A_{22}}}{2A_{11}}$$
$$\left(\frac{m_1}{m_3}\right) = \frac{(-B_{13} + \sqrt{B_{13}^2 - 4B_{11}B_{33}})}{2B_{11}}$$

Approximate analytic estimates- Type I

$$aZ^2 + cX^2 - eZX = A_{11}$$
$$2XYc + dZ^2 - eYZ - fXZ = A_{12}$$
$$bZ^2 + CY^2 - fYZ = A_{22}$$

$$aY^2 + bX^2 - dXY = B_{11}$$
$$2XZb + eY^2 - dYZ - fXY = B_{13}$$
$$bZ^2 + CY^2 - fYZ = B_{33}$$

$\theta_{23} = \frac{\pi}{4}$ and $\theta_{12} = \sin^{-1}(\frac{1}{\sqrt{3}})$, and $\sin^2\theta_{13} = 0.04$, δ=0.5, α=2.848 and $\beta = 4.284$ from the allowed values of parameters, one finds for the neutrino mass ratios:

$$R \equiv \frac{\Delta m^2_{\odot}}{\Delta m^2_{\mathrm{A}}} == 2.3 \times 10^{-2}$$

Solution Type II

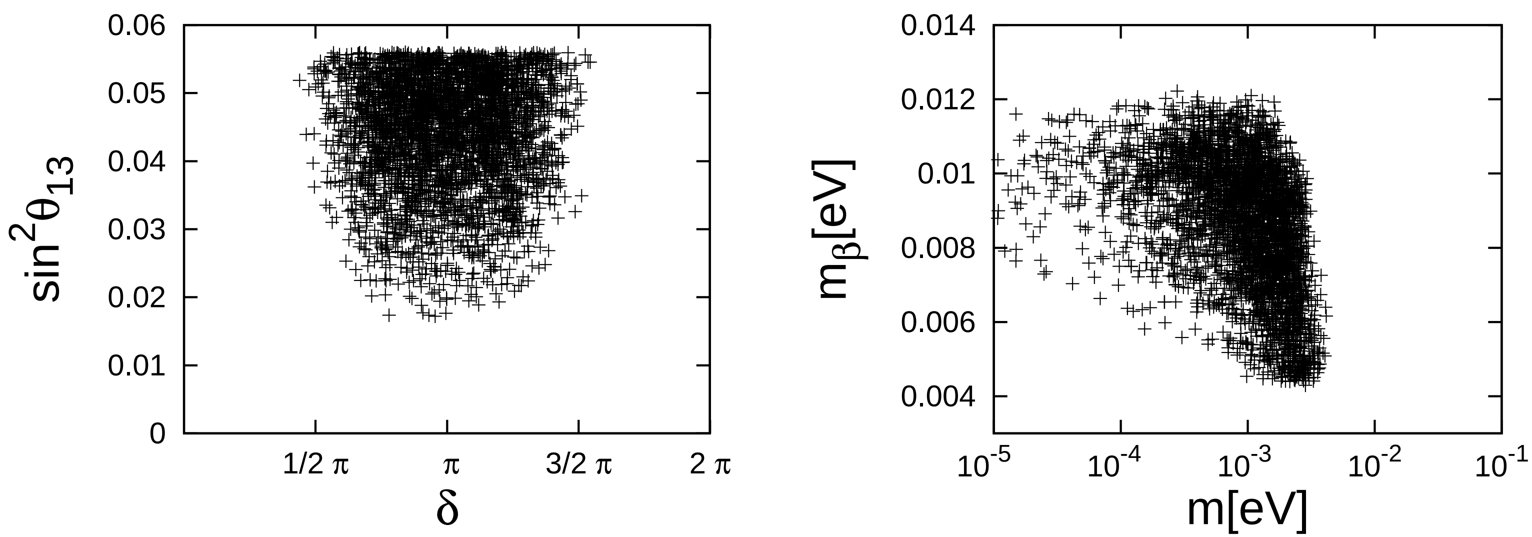
- $$m_\nu = \begin{pmatrix} 0 & y & x \\ \cdot & w & zy \\ \cdot & \cdot & 2zx \end{pmatrix}$$

- The matrix satisfies the following two conditions:

$$(m_\nu)_{ee} = 0, \tag{1}$$

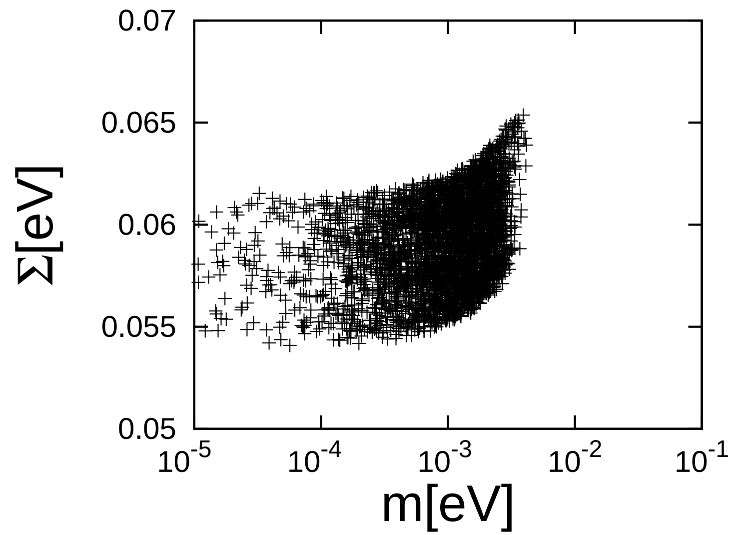
$$(m_\nu)_{e\tau}(m_\nu)_{\mu\tau} - (1/2)(m_\nu)_{\tau\tau}(m_\nu)_{e\tau} = 0. \tag{2}$$

Analysis of Solution Type II



- ee element of m_ν being zero so the IH not allowed.
- The lower bound on $sin^2\theta_{13}$ is 0.017.
- δ lies within ($\frac{\pi}{2} \leq \delta_{CP} \leq \frac{3\pi}{2}$).
- Right panel shows that m_β is in the range (0.004 - 0.012) eV within the KATRIN sensitivity.

Analysis of Solution Type II



- A plot of the smallest mass m_1 vs. the sum of the neutrino masses Σ.
- Σ is always bounded from below by 0.056 eV and above by 0.065 eV.
- Analytic estimate of the mass squared difference ratio R following the preceeding procedure gives

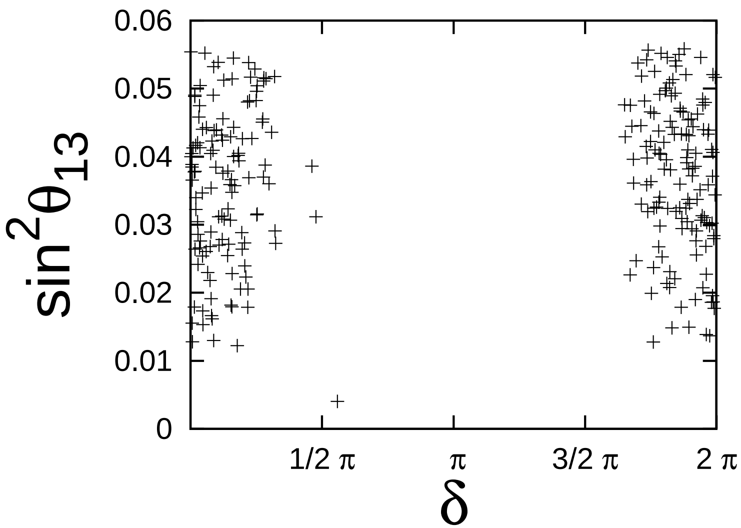
$$R \equiv \frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm A}} = 1.7 \times 10^{-2}$$

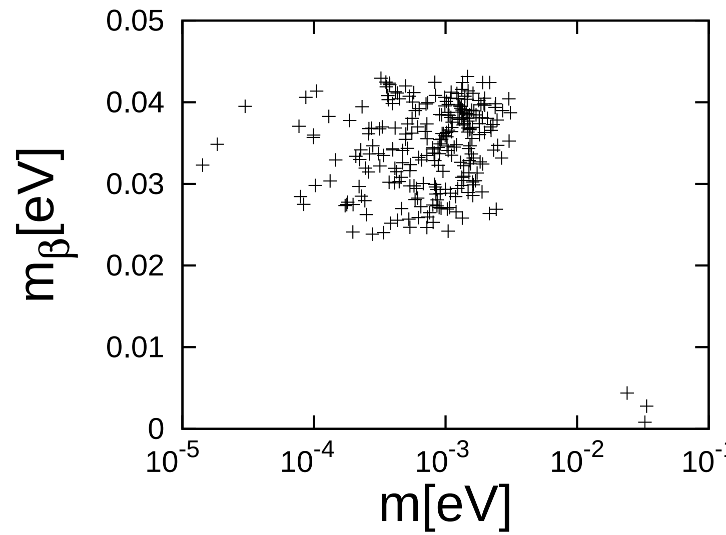
Solution Type III

- $$m_\nu = \begin{pmatrix} w & z & x \\ \cdot & 0 & yz \\ \cdot & \cdot & 2xy \end{pmatrix}$$
- The matrix satisfies the following two conditions:

$$\begin{aligned} (m_\nu)_{\mu\mu} &= 0, \\ (m_\nu)_{e\tau}(m_\nu)_{\tau\mu} - (1/2)(m_\nu)_{\tau\tau}(m_\nu)_{e\tau} &= 0. \end{aligned}$$

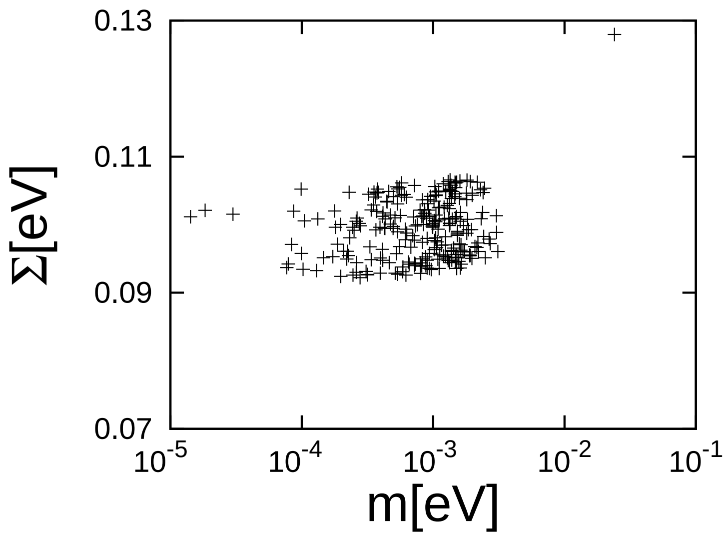
Analysis of Solution Type III

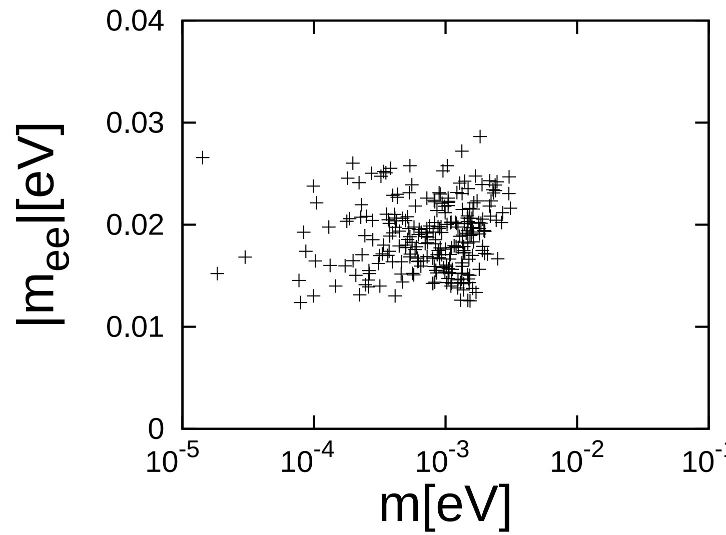




- This is of inverted hierarchical ordering.
- The lower bound on $sin^2\theta_{13}$ is 0.012.
- δ within limits of ($0 \leq \delta_{CP} \leq \frac{\pi}{2}$) and ($\frac{3\pi}{4} \leq \delta_{CP} \leq 2\pi$).
- Right panel shows that m_β is in the range (0.024 - 0.034) eV.

Analysis of Solution Type III





- Left panel shows Σ is always bounded from below by 0.090 eV and above by 0.107 eV.
- Effective mass m_{ee} probed in $0\nu 2\beta$ decay against the smallest neutrino mass m_3 lies within 0.012 eV $< m_{ee} < 0.028$ eV.
- $$R \equiv \frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm A}} = 1.7 \times 10^{-2}$$

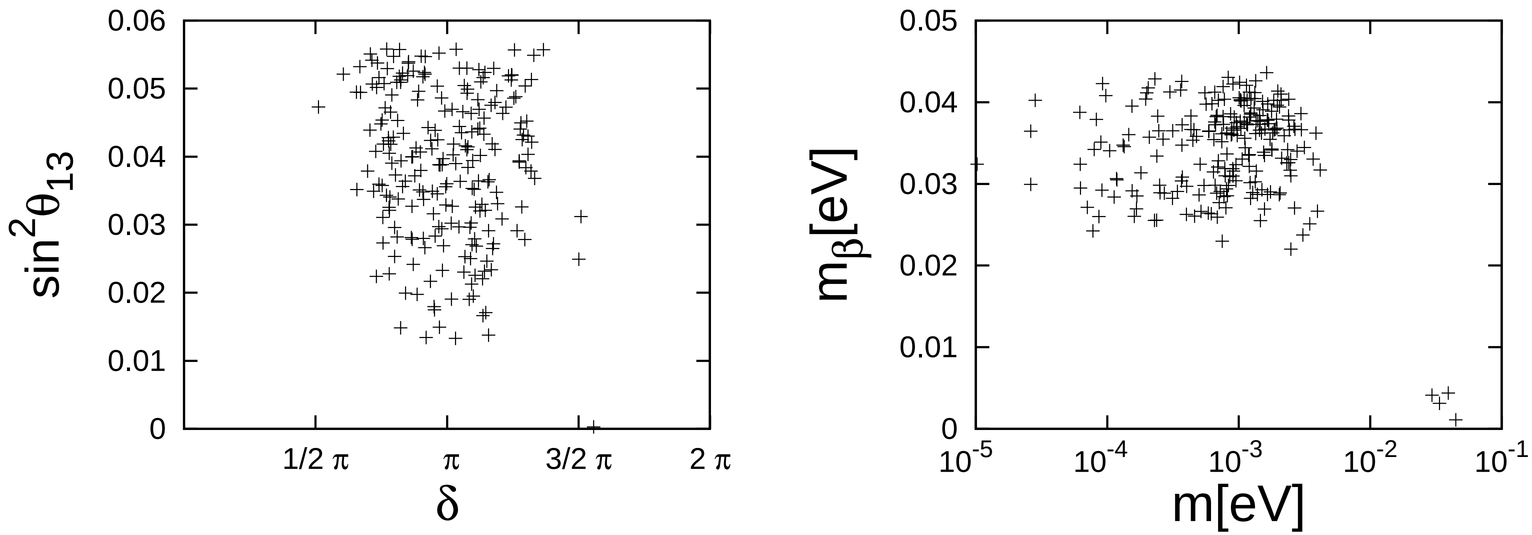
Solution Type IV

- $$m_\nu = \begin{pmatrix} w & x & z \\ \cdot & 2xy & yz \\ \cdot & \cdot & 0 \end{pmatrix}$$

- The matrix satisfies the following two conditions:

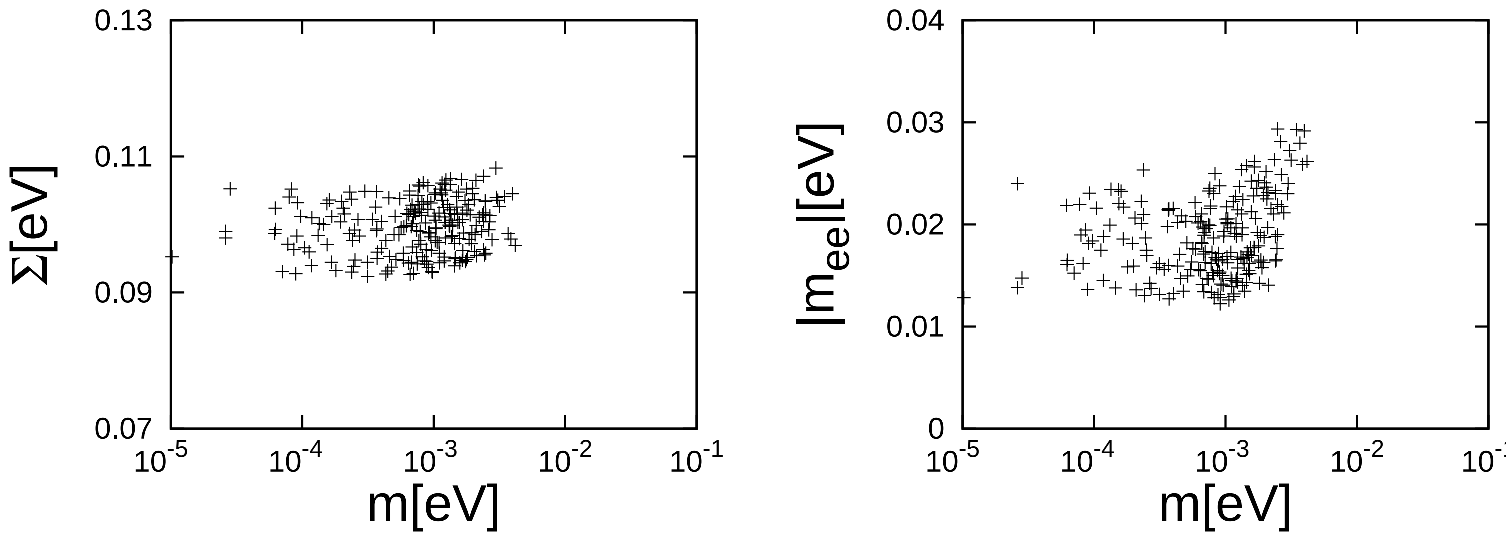
$$\begin{aligned} (m_\nu)_{\tau\tau} &= 0, \\ (m_\nu)_{e\tau}(m_\nu)_{\mu\tau} - (1/2)(m_\nu)_{\mu\mu}(m_\nu)_{e\tau} &= 0. \end{aligned}$$

Analysis of Solution Type IV



- This is of inverted hierarchical ordering.
- The lower bound on $sin^2\theta_{13}$ is 0.013.
- δ within limits of ($\frac{\pi}{2} \leq \delta_{CP} \leq \frac{3\pi}{4}$)
- Right panel shows that m_β is in the range (0.022 - 0.043) eV.

Analysis of Solution Type IV



- Left panel shows Σ is always bounded from below by 0.091 eV and above by 0.108 eV.
- Effective mass m_{ee} lies within 0.012 eV $< m_{ee} < 0.029$ eV.
- $$R \equiv \frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm A}} = 3.8 \times 10^{-2}$$

Symmetry Realization of the Textures

- Texture zeros are a basis-dependent concept can be justified by an underlying model.
- Apply $U(1)$ symmetry - illustrate Solution Type I.
  Add right-handed singlets N_{Ri} and new scalar weak doublets of the form
  χ_{ij}= $\begin{pmatrix} \chi_{ij}^{+} \\ \chi_{ij}^{0} \end{pmatrix}$, ij indicates the position of non zero entry in m_D.

$$\begin{aligned} L &\to e^{i\gamma n_L} L, \\ l_R &\to e^{i\gamma n_R} l_R, \\ N_R &\to e^{i\gamma n_\nu} N_R \\ \chi_{ij} &\to e^{i\gamma Q_{ij}} \chi_{ij} \end{aligned}$$

$$\mathcal{L}_{\mathcal{Y}} = -Y_{ij} \bar{L}_i \phi l_{Rj} - Y_{ij}^{\nu} \bar{L}_i \tilde{\chi_{ij}} N_{Rj} - Y_{ij}^{\nu'} \bar{L}_i \tilde{\phi} N_{Rj} + \frac{1}{2} \bar{N^c}_{Ri} M_{Rij} N_{Rj} + h.c.$$

  where $\tilde{\chi} = i\sigma_2 \chi^*$ and the Higgs doublet $\tilde{\phi} = i\sigma_2 \phi^*$
- Charged lepton mass matrix is diagonal and all SM fermions get their masses through the Yukawa coupling with the SM Higgs ϕ but are not allowed to couple with χ_{ij}.

Symmetry Realization of the Textures

- $$m_D = \begin{pmatrix} 0 & a_2 & 0 \\ 0 & b_2 & b_3 \\ c_1 & 0 & c_3 e^{i\gamma} \end{pmatrix},$$
- For every non-zero entry in m_D we need a new scalar doublet except for the (22) entry which can be generated from the SM Higgs $\tilde{\phi}$.
- There are five non zero entries in m_D we require four new scalar doublets.

Fermions	U(1) charge
$\bar{L}_1$	-1
$\bar{L}_2$	+2
$\bar{L}_3$	-4
N_{R1}	0
N_{R2}	-2
N_{R3}	2

Scalar Particle	U(1) charge
$\tilde{\chi_{13}}$	-1
$\tilde{\phi}$	0
$\tilde{\chi_{23}}$	-4
$\tilde{\chi_{31}}$	4
$\tilde{\chi_{32}}$	6

CP violation and Leptogenesis

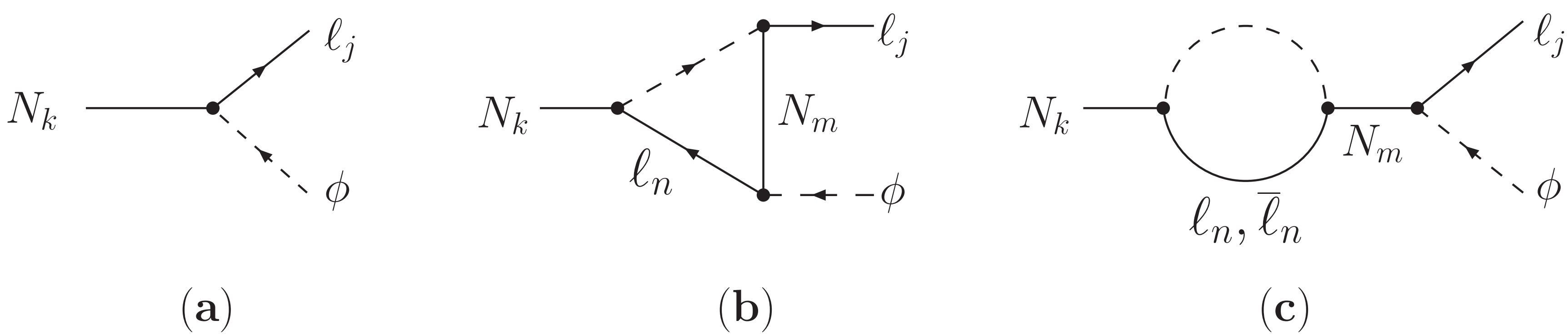
- Since θ_{13} is large and non-zero it should be possible to relate low energy CP violation with the phases of the Yukawa matrices.
- The neutrino oscillation can measure the Dirac CP phase and hence can be relegated to the Jarkslog invariant J_{CP}

$$J_{CP} = Im\{U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}\} = \frac{Im\{h_{12}h_{23}h_{13}\}}{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}$$

  where $h = m_\nu m_\nu^\dagger$
- A non-zero J_{CP} which indicate a possible low energy CP violation.

CP violation and Leptogenesis

- CP violation also plays a very crucial role in the leptogenesis.
- The RH neutrino can decay into a lepton and a Higgs or an antilepton and Higgs.
- The difference in the decay rates of the lepton and anti lepton generates a CP asymmetry through the interference of the the tree level and loop correction diagrams



CP violation and Leptogenesis

- In the daiagonal basis of M_R ,
  $m_D \to \tilde{m_D} = m_D U_R$
- The CP asymmetry is in general given by

$$\epsilon_i^\alpha \equiv \frac{\Gamma(N_i \to \phi\, \bar{l}_\alpha) - \Gamma(N_i \to \phi^\dagger\, l_\alpha)}{\sum_\beta \Big[\Gamma(N_i \to \phi\, \bar{l}_\beta) + \Gamma(N_i \to \phi^\dagger\, l_\beta)\Big]}$$

$$= \frac{1}{8\pi\, v_u^2}\, \frac{1}{(\tilde{m}_D^\dagger\, \tilde{m}_D)_{ii}} \sum_{j\neq i} \left(\mathcal{I}_{ij}^\alpha\, f(M_j^2/M_i^2) + \mathcal{J}_{ij}^\alpha\, \frac{1}{1 - M_j^2/M_i^2} \right),$$

where

$$\mathcal{I}_{ij}^\alpha = Im\Big[\big(\tilde{m}_D^\dagger\big)_{i\alpha}\, \big(\tilde{m}_D\big)_{\alpha j}\, \big(\tilde{m}_D^\dagger \tilde{m}_D\big)_{ij}\Big]\,, \quad \mathcal{J}_{ij}^\alpha = \mathrm{Im}\Big[\big(\tilde{m}_D^\dagger\big)_{i\alpha}\, \big(\tilde{m}_D\big)_{\alpha j}\, \big(\tilde{m}_D^\dagger \tilde{m}_D\big)_{ji}\Big]\,.$$

α= e, μ, τ and i,j=1,2,3 and $f(x) = \sqrt{x}\left[\frac{2}{1-x} - \ln\left(\frac{1+x}{x}\right)\right]$.

- The second term vanishes when summed over all the flavours and the only relevent contribution comes from the first term .

CP violation and Leptogenesis

- $$\epsilon_i = \frac{1}{8\pi\, v_u^2} \frac{1}{(\tilde{m}_D^\dagger\, \tilde{m}_D)_{ii}} \mathcal{I}_{ij}\ , \text{ where } \mathcal{I}_{ij} = \sum_\alpha \mathcal{I}_{ij}^\alpha\ f(\frac{M_j^2}{M_i^2})\ .$$

- $$\tilde{m}_1^\alpha = \frac{(\tilde{m}_D^\dagger)_{1\alpha}\, (\tilde{m}_D)_{\alpha 1}}{M_1}\ .$$
$$\tilde{m} = \sum_\alpha \tilde{m}_1^\alpha$$

Solution-Type-I

	m_D	M_R	J_{CP}	$\mathcal{I}^{\alpha}_{ij}$ and $\mathcal{J}^{\alpha}_{ij}$
Ia	$\begin{pmatrix} 0 & a_2 & 0 \\ 0 & b_2 & b_3 \\ c_1 & 0 & c_3 e^{i\gamma} \end{pmatrix}$	$\begin{pmatrix} p & 0 & 0 \\ 0 & 0 & u \\ 0 & u & 0 \end{pmatrix}$	$\neq 0$	$\mathcal{I}^{\tau}_{12} = -c_1^2 c_3^2 cos(\gamma) sin(\gamma)$ $\mathcal{I}^{\tau}_{13} = c_1^2 c_3^2 cos(\gamma) sin(\gamma)$
Ib	$\begin{pmatrix} 0 & a_2 & 0 \\ b_1 e^{i\gamma} & b_2 & 0 \\ c_1 & 0 & c_3 \end{pmatrix}$	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & r \end{pmatrix}$	$\neq 0$	$\mathcal{I}^{e}_{12} = \mathcal{J}^{e}_{12} = \frac{1}{2} a_2^2 b_1 b_2 sin(\gamma)$ $\mathcal{I}^{\mu}_{12} = \frac{1}{2} b_1 b_2 (a_2^2 - 2b_1^2 + 2b_2^2 - c_1^2) sin(\gamma)$ $\mathcal{J}^{\mu}_{12} = \frac{1}{2} b_1 b_2 (a_2^2 - c_1^2) sin(\gamma)$ $\mathcal{I}^{\tau}_{12} = \mathcal{J}^{\tau}_{12} = -\frac{1}{2} b_1 b_2 c_1^2 sin(\gamma)$
Ic	$\begin{pmatrix} 0 & 0 & a_3 \\ b_1 & 0 & b_3 \\ c_1 e^{i\gamma} & c_2 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & t \\ 0 & q & 0 \\ t & 0 & 0 \end{pmatrix}$	$\neq 0$	$\mathcal{I}^{\tau}_{12} = -c_1^2 c_2^2 cos(\gamma) sin(\gamma)$ $\mathcal{I}^{\tau}_{13} = c_1^2 c_2^2 cos(\gamma) sin(\gamma)$

Solution-Type-II

	m_D	M_R	J_{CP}	$\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$
IIa	$\begin{pmatrix} 0 & a_2 & 0 \\ b_1 & 0 & b_3 \\ 0 & c_2 & c_3 e^{i\gamma} \end{pmatrix}$	$\begin{pmatrix} p & 0 & 0 \\ 0 & 0 & u \\ 0 & u & 0 \end{pmatrix}$	$\neq$0	0
IIb	$\begin{pmatrix} 0 & a_2 & 0 \\ b_1 & 0 & b_3 \\ c_1 e^{i\gamma} & c_2 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & r \end{pmatrix}$	$\neq$0	$\mathcal{I}_{12}^{e}$= $\mathcal{J}_{12}^{e}$=$\frac{1}{2}a_2^2 c_1 c_2 sin(\gamma)$ $\mathcal{I}_{12}^{\mu}$= $\mathcal{J}_{12}^{\mu}$=-$\frac{1}{2}b_1^2 c_1 c_2 sin(\gamma)$ $\mathcal{I}_{12}^{\tau}$=-$\frac{1}{2}c_1 c_2 (a_2^2 - b_1^2 - 2c_1^2 + 2c_2^2) sin(\gamma)$ = $\mathcal{J}_{12}^{\tau}$=-$\frac{1}{2}c_1 c_2 (a_2^2 - b_1^2) sin(\gamma)$
IIc	$\begin{pmatrix} 0 & 0 & a_3 \\ b_1 & b_2 & 0 \\ c_1 e^{i\gamma} & 0 & c \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & t \\ 0 & q & 0 \\ t & 0 & 0 \end{pmatrix}$	$\neq$0	0

Solution-Type-III

	m_D	M_R	J_{CP}	$\mathcal{I}^{\alpha}_{ij}$ and $\mathcal{J}^{\alpha}_{ij}$
IIIa	$\begin{pmatrix} a_1 & 0 & a_3 \\ 0 & b_2 & 0 \\ 0 & c_2 & c_3 e^{i\gamma} \end{pmatrix}$	$\begin{pmatrix} p & 0 & 0 \\ 0 & 0 & u \\ 0 & u & 0 \end{pmatrix}$	$\neq 0$	0
IIIb	$\begin{pmatrix} a_1 & 0 & a_3 \\ 0 & b_2 & 0 \\ c_1 e^{i\gamma} & c_2 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & r \end{pmatrix}$	$\neq 0$	$\mathcal{I}^{e}_{12}=\mathcal{J}^{e}_{12}=-\frac{1}{2}a_1^2 c_1 c_2 sin(\gamma)$ $\mathcal{I}^{\mu}_{12} = \mathcal{J}^{\mu}_{12} = \frac{1}{2} b_2^2 c_1 c_2 sin(\gamma)$ $\mathcal{I}^{\tau}_{12}=\frac{1}{2}c_1 c_2(-a_1^2 + b_2^2 - 2c_1^2 + 2c_2^2) sin(\gamma)$ $\mathcal{J}^{\tau}_{12}=\frac{1}{2}c_1 c_2(a_1^2 - b_2^2) sin(\gamma)$
IIIc	$\begin{pmatrix} a_1 & a_2 & 0 \\ 0 & 0 & b_3 \\ c_1 e^{i\gamma} & 0 & c_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & t \\ 0 & q & 0 \\ t & 0 & 0 \end{pmatrix}$	$\neq 0$	0

Solution-Type-IV

	m_D	M_R	J_{CP}	$\mathcal{I}_{ij}^{\alpha}$ and $\mathcal{J}_{ij}^{\alpha}$
IVa	$\begin{pmatrix} a_1 & 0 & a_3 e^{i\gamma} \\ 0 & b_2 & 0 \\ 0 & c_2 & c_3 \end{pmatrix}$	$\begin{pmatrix} p & 0 & 0 \\ 0 & 0 & u \\ 0 & u & 0 \end{pmatrix}$	$\neq 0$	$\mathcal{I}_{12}^{e} = -a_1^2 a_3^2 cos(\gamma) sin(\gamma)$ $\mathcal{I}_{13}^{e} = a_1^2 a_3^2 cos(\gamma) sin(\gamma)$
IVb	$\begin{pmatrix} a_1 & 0 & a_3 e^{i\gamma} \\ b_1 & b_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & s & 0 \\ s & 0 & 0 \\ 0 & 0 & r \end{pmatrix}$	$\neq 0$	$\mathcal{I}_{12}^{e} = \mathcal{J}_{12}^{e} = -\frac{1}{2} a_1^2 b_1 b_2 sin(\gamma)$ $\mathcal{I}_{12}^{\mu} = -\frac{1}{2} b_1 b_2 (-a_1^2 - 2b_1^2 + 2b_2^2 + c_2^2) sin(\gamma)$ $\mathcal{J}_{12}^{\mu} = \frac{1}{2} b_1 b_2 (a_1^2 - c_2^2)$ $\mathcal{I}_{12}^{\tau} = \mathcal{J}_{12}^{\tau} = \frac{1}{2} b_1 b_2 c_2^2 sin(\gamma)$
IVc	$\begin{pmatrix} a_1 e^{i\gamma} & a_2 & 0 \\ b_1 & 0 & b_3 \\ 0 & 0 & c_3 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & t \\ 0 & q & 0 \\ t & 0 & 0 \end{pmatrix}$	$\neq 0$	$\mathcal{I}_{12}^{e} = -a_1^2 a_2^2 cos(\gamma) sin(\gamma)$ $\mathcal{I}_{13}^{e} = a_1^2 a_2^2 cos(\gamma) sin(\gamma)$

Conclusion and Summary

- When we consider the m_ν with all the light neutrinos massive we get 62 possibilities according to our required classification.
- We find six new cases of m_ν which have not been analyzed in the literature earlier and out of these only four are allowed by low energy phenomenology.
- Type I and Type II with $m_{ee} = 0$ are of normal hierarchical ordering.
- There is a constrain on the Dirac CP phase for Type I and Type II but they are completely non-overlapping.
- Type III with $m_{\mu\mu} = 0$ and Type IV with $m_{\tau\tau} = 0$ are of inverted hierarchy.
- In Type III and IV also we find some constrain in the Dirac CP phase.
- The models constrain $\sin^2\theta_{13}$ in the range 0.02-0.05 in all the four cases which is consistent with the recent observations in T2K and Double Chooz experiments.
- These models follow a U(1) symmetry.
- There are only cases with M_R corresponding to $L_e - L_\tau$ where we can draw a correlation between the high and the low energy CP violation.

Thank You