Finite Quantum Corrections to the Neutrino Mixing Matrix

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<u>PLB699, arXiv: 1012.2970 [hep-ph]</u>
<u>JHEP1109, arXiv: 1108.3175 [hep-ph]</u>

Outline

- Introduction
  - neutrino oscillation experiments
  - theoretical studies
- Motivation
- Finite Quantum Corrections
  - framework & Numerical analysis
  - simple realization
- Summary

Introduction - experiment -

Neutrinos do oscillate, have masses, and mix.

$$V_{PMNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_\nu$$

[T.Schwets, et al, *arXiv:1108.1376*]

<u>Two large angles</u>

$$\theta_{23} = 45.6^\circ \ {}^{+3.4^\circ}_{-3.5^\circ}$$

$$\theta_{12} = 34.0^\circ \pm 1.0^\circ$$

<u>One small angle</u>

1 σ

$$\theta_{13} = 5.7^\circ \ {}^{+2.2^\circ}_{-2.1^\circ}$$

$$\theta_{13}^{\mathrm{T,M,D}} = 8.3^\circ \ (8.6^\circ)$$

[Minakata, et al, *arXiv:1111.3330*]

We don't know even the ordering of neutrino masses

$m_3 \gg m_2 > m_1$ [Normal] **or** $m_2 > m_1 \gg m_3$ [Inverted] **or** $m_1 \simeq m_2 \simeq m_3$ [Degenerate]

and what is even worse, no info. about CP violations.

Introduction - theory -

Constant-number-parametrizations:

$$V_{DC} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{6} & 1/\sqrt{6} & 2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \Rightarrow \begin{array}{l} \theta_{23} \simeq 54.7^\circ \\ \theta_{12} = 45^\circ \\ \theta_{13} = 0^\circ \end{array}$$

[H.Fritzsch and Z.Z.Xing, PLB372(1996)]

$$V_{BM} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \Rightarrow \begin{array}{l} \theta_{23} = 45^\circ \\ \theta_{12} = 45^\circ \\ \theta_{13} = 0^\circ \end{array}$$

[F.Vissani, *hep-ph*/9708483]

$$V_{TBM} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \Rightarrow \boxed{\begin{array}{l} \theta_{23} = 45^\circ \\ \theta_{12} \simeq 35.3^\circ \\ \theta_{13} = 0^\circ \end{array}}$$

[P.F.Harrison, et.al, *PLB*530(2002)]

Not perfect...

These special mixing matrices can be derived from non-Abelian discrete flavor symmetres: e.g., S3, S4, A4(T') ...

Broken symmetries

[Y.Koide, PRD71(2005)]

We define the diagonalizations of mass matrices as

$$U_\nu^\dagger M_\nu U_\nu^* = D_\nu = \mathrm{Diag}(m_1, m_2, m_3)$$

$$U_e^\dagger M_e M_e^\dagger U_e = D_e^2 \quad \Rightarrow \quad V_{PMNS} = U_e^\dagger U_\nu$$

Let us consider the following (flavor) symmetry:

$$G_L^\dagger M_e M_e^\dagger G_L = M_e M_e^\dagger, \quad G_L^\dagger M_\nu G_L^* = M_\nu \quad \underline{(G_L \equiv G_{eL} = G_{\nu L})}$$

Then, we obtain two conditions for G_L.

$$[U_e^\dagger G_L U_e]^\dagger D_e^2 [U_e^\dagger G_L U_e] = D_e^2$$

$$\Rightarrow \quad P_e = \mathrm{Diag}(e^{i\phi_1^e}, e^{i\phi_2^e}, e^{i\phi_3^e})$$

$$[U_\nu^\dagger G_L U_\nu]^\dagger D_\nu [U_\nu^\dagger G_L U_\nu]^* = D_\nu$$

$$\Rightarrow \quad P_\nu = \mathrm{Diag}(e^{i\phi_1^\nu}, e^{i\phi_2^\nu}, e^{i\phi_3^\nu}) \qquad (\phi_i = 0,\ \pi)$$

Broken symmetries

[Y.Koide, PRD71(2005)]

From the two conditions

$$G_L = U_e P_e U_e^\dagger = U_\nu P_\nu U_\nu^\dagger$$

$(V_{PMNS} = U_e^\dagger U_\nu)$ $\Rightarrow$ $P_e V_{PMNS} = V_{PMNS} P_\nu$

$\Rightarrow$ $\underline{(e^{i\phi_i^e} - e^{i\phi_j^\nu})(V_{PMNS})_{ij} = 0}$

Only $P_e = P_\nu = e^{i\phi}$ $\Rightarrow$ $G_L = 1$ is possible,

otherwise some elements of V_{PMNS} must be vanishing.

The conclusion is based on the assumption: $\underline{G_{eL} = G_{\nu L}}$.

$G_{eL} \neq G_{\nu L}$

G_{eL} $(G_{\nu L})$ is only valid in the charged lepton (neutrino) sector, and it is **broken** in the neutrino (charged lepton) sector.

Motivation

- Constant-number mixings may approximately be reasonable, but they are not completely consistent with experiments.

Especially, $\theta_{13}^{theory} = 0^\circ$ vs $\theta_{13}^{exp} \simeq 10^\circ$.

- These mixings can be derived from family symmetries, but they should be **broken symmetries**.

Motivated by these facts, here we consider

$$M_\nu = M_\nu^{TB} + \delta M_\nu \ (\ V_{MNS} = V^{TB} + \delta V \ )$$

TB is exact at leading order
e.g., discrete symmetries

small corrections
(breaking term)

Motivation

Question: What is the correction term?

We know nothing about the correction term.

How many parameters?

How small?

$$M_\nu = M_\nu^0 + \,?$$

Exception: RGE

V^0 vs? V^{exp}, V_{DC} vs? V_{BM} vs? V_{TBM}, No prediction.

It is very important to study not only M^0 but also δM.

We would like to focus on the origin of δM and propose a new mechanism.

Finite Quantum Corrections

- **Tree Level**

Suppose, at tree-level (and arbitrary energy scale), the TBM mixing is exact and ensured by a family symmetry,

$$M_\nu^0 = \frac{m_1^0 e^{i\rho}}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2^0 e^{i\sigma}}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3^0}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

in the basis of $D_e = \mathrm{Diag}(m_e, m_\mu, m_\tau)$.

Spontaneously broken S_4 [C.S.Lam, $PLB656(2007)$]

$$\boxed{S_4} \to Z_2 : G_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad \text{for } \mathcal{L}_\nu$$

$$\boxed{S_4} \to Z_3 : G_\ell = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 1 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad \omega = e^{i\frac{2\pi}{3}} \quad \text{for } \mathcal{L}_\ell$$

Finite Quantum Corrections

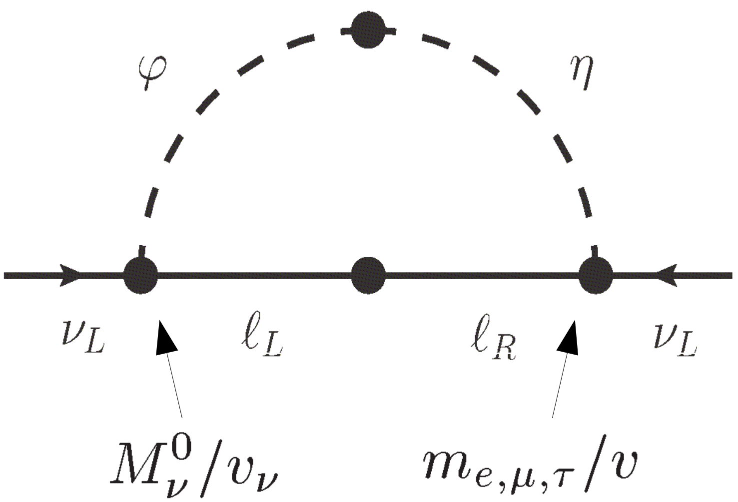
- **1 loop level**

There may exist 1-loop corrections

$$\delta M_\nu = \frac{M_\nu^0 D_e^2 + D_e^2 M_\nu^0}{v^2} \times I^{\text{loop}}$$

I^{loop} : Dim-less para. from loop $\int$

v : electroweak scale



No new Yukawa coupling, no new energy scale.

- **Tree + 1 loop**

Hence, the full mixing matrix departs from the tree-level mixing after including the finite quantum effects.

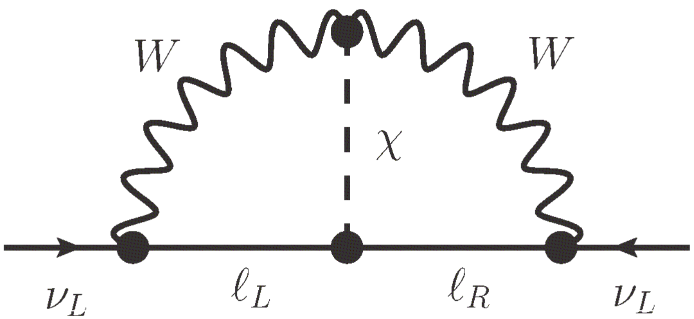
Finite Quantum Corrections

- A specific correction term is automatically obtained once a specific mixing is given at tree level.
- No new energy scale; the deviation from the tree-level mixing would naturally be small.
- This is a *finite* quantum correction, not RGEs, and is defined at an arbitrary energy scale. (No running.)
- Only an overall factor (I^{loop}) depends on the model details.
- The Dirac CP phase is radiatively induced from two Majorana phases: ρ and σ.

Finite Quantum Corrections

More higher order loop corrections are possible.

$$\delta M_\nu = \frac{D_e M_\nu^0 D_e}{v^2} \times I^{\mathrm{loop}}$$

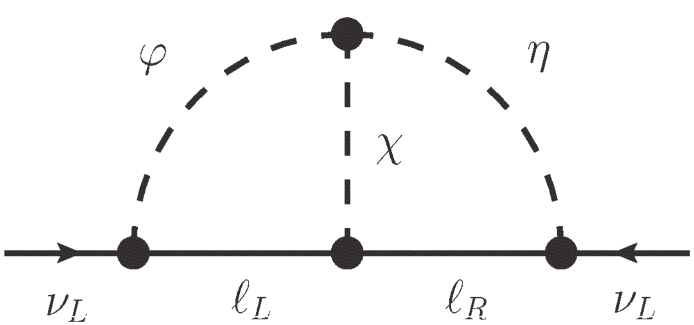


$$\delta M_\nu = \frac{\tilde{M}_\nu^0 D_e (M_\nu^0)^* D_e \tilde{M}_\nu^0}{v^2} \times I^{\mathrm{loop}}$$

$\tilde{M}_\nu^0 = M_\nu^0/(1 \text{ eV})$

I^{loop} : Dim-less para. from loop $\int$

v : electroweak scale



Setup of numerical analysis

Setup: $M_\nu = M_\nu^0 + \delta M_\nu$

$$M_\nu^0 = (V^0)^T D_\nu V^0, \quad \delta M_\nu = \frac{M_\nu^0 D_e^2 + D_e^2 M_\nu^0}{v^2} \times I^{\text{loop}}$$

For V_{DC}^0, V_{BM}^0 and V_{TBM}^0.

Input parameters:

$$\Delta m_{31}^2 = \begin{cases} +\left(2.45^{+0.28}_{-0.27}\right) \times 10^{-3} \ \ \text{eV}^2 & \text{for NO} \\ -\left(2.34^{+0.30}_{-0.26}\right) \times 10^{-3} \ \ \text{eV}^2 & \text{for IO} \end{cases}$$

$$\Delta m_{21}^2 = \left(7.59^{+0.60}_{-0.50}\right) \times 10^{-5} \ \ \text{eV}^2 \qquad \theta_{12} = \left(34.0^{+2.9}_{-2.7}\right)^\circ$$

$$m_e = 0.486 \text{ MeV}, \quad m_\mu = 102.718 \text{ MeV}, \quad m_\tau = 1746.24 \text{ MeV}$$

$$\underline{m_\nu^{\text{heaviest}} = 0.2 \text{ eV}} \qquad \rho,\ \sigma = 0 \sim 2\pi \qquad I^{loop} = -125 \sim 125$$

Outputs: θ_{13}, θ_{23} and J_{CP}.

$\delta M_\nu / M_\nu^0 \simeq 0.004$

Perturbative Expansion

$$M_\nu = M_\nu^0 + \delta M_\nu :$$

$$M_\nu^0 = (V^0)^T D_\nu V^0, \quad \delta M_\nu = \frac{M_\nu^0 D_e^2 + D_e^2 M_\nu^0}{v^2} \times I^{loop}$$

By regarding δM_ν as small perturtabations

$$\sin\theta_{13}^{TBM} \simeq \frac{m_\tau^2}{3\sqrt{2}v^2}\left|\frac{m_3^0 + m_1^0 e^{i\rho}}{m_3^0 - m_1^0 e^{i\rho}} - \frac{m_3^0 + m_2^0 e^{i\sigma}}{m_3^0 - m_2^0 e^{i\sigma}}\right| I^{loop}$$

$$\tan\theta_{23}^{TBM} \simeq \left|1 - \frac{m_\tau^2}{3v^2}\left[\frac{m_3^0 + m_1^0 e^{i\rho}}{m_3^0 - m_1^0 e^{i\rho}} + 2\frac{m_3^0 + m_2^0 e^{i\sigma}}{m_3^0 - m_2^0 e^{i\sigma}}\right] I^{loop}\right|$$

$$\tan\theta_{12}^{TBM} \simeq \frac{1}{\sqrt{2}}\left|1 - \frac{m_\tau^2}{2v^2}\frac{m_2^0 e^{i\sigma} + m_1^0 e^{i\rho}}{m_2^0 e^{i\sigma} - m_1^0 e^{i\rho}} I^{loop}\right|$$

Most sensitive

The corrections can be enhanced due to $m_1 \simeq m_2 \simeq m_3$.

Perturbative Expansion

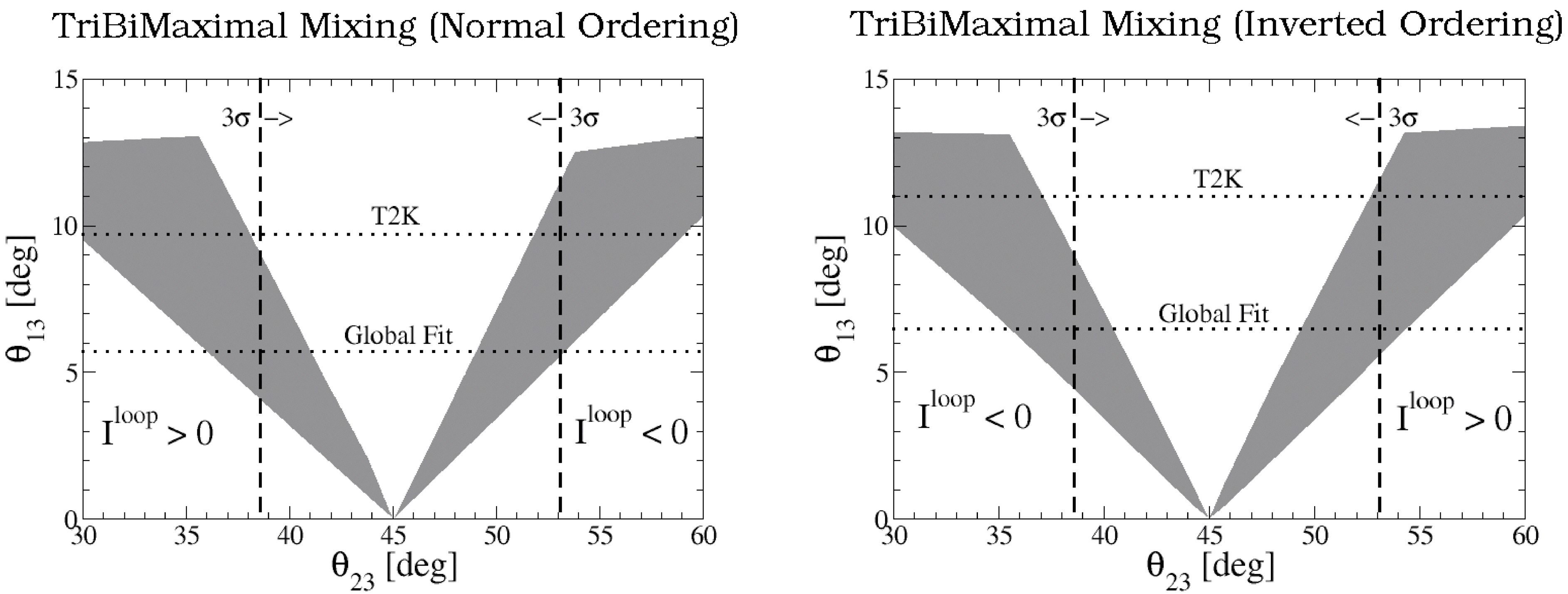
$$\sin\theta_{13}^{TBM} \simeq \frac{m_\tau^2}{3\sqrt{2}v^2}\left|\frac{m_3^0+m_1^0e^{i\rho}}{m_3^0-m_1^0e^{i\rho}}-\frac{m_3^0+m_2^0e^{i\sigma}}{m_3^0-m_2^0e^{i\sigma}}\right| I^{loop}$$

$$\tan\theta_{23}^{TBM} \simeq \left|1-\frac{m_\tau^2}{3v^2}\left[\frac{m_3^0+m_1^0e^{i\rho}}{m_3^0-m_1^0e^{i\rho}}+2\frac{m_3^0+m_2^0e^{i\sigma}}{m_3^0-m_2^0e^{i\sigma}}\right] I^{loop}\right|$$

$$\tan\theta_{12}^{TBM} \simeq \frac{1}{\sqrt{2}}\left|1-\frac{m_\tau^2}{2v^2}\frac{m_2^0e^{i\sigma}+m_1^0e^{i\rho}}{m_2^0e^{i\sigma}-m_1^0e^{i\rho}}I^{loop}\right|$$

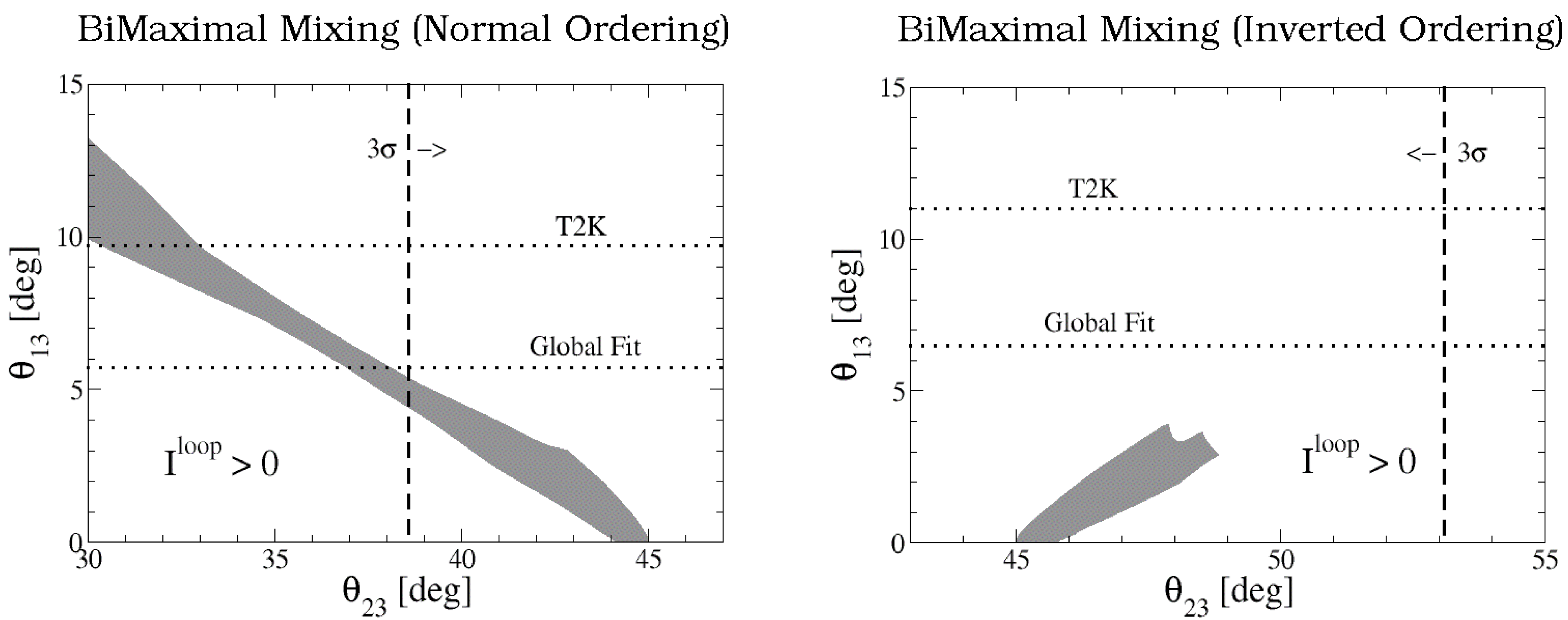
- $I^{loop}>0 \rightarrow \theta_{12}<\theta_{12}^0, \quad I^{loop}<0 \rightarrow \theta_{12}>\theta_{12}^0.$
- NO with $I^{loop}>0$ and IO with $I^{loop}<0 \rightarrow \theta_{23}<\theta_{23}^0.$
- NO with $I^{loop}<0$ and IO with $I^{loop}>0 \rightarrow \theta_{23}>\theta_{23}^0.$

Tri-bimaximal mixing



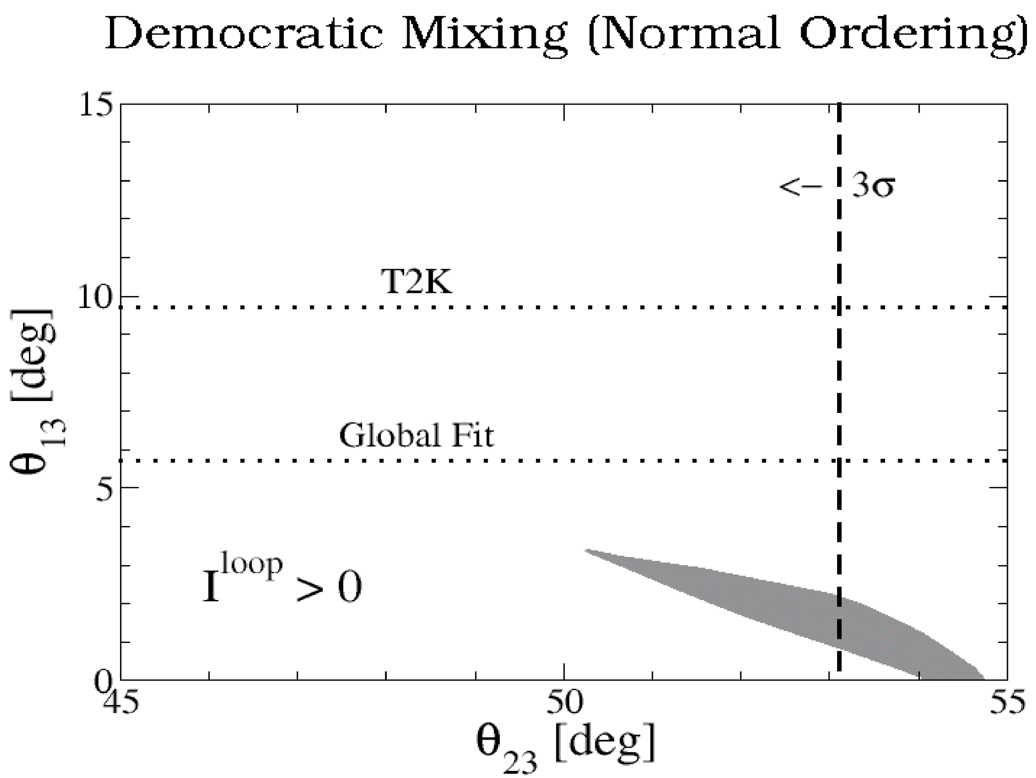
- θ_{13} can largely depart from 0, but it leads to $|\theta_{23} - 45^\circ| \gg 1$.
- $I^{loop} > 0 \;\rightarrow\; \theta_{23} < 45^\circ$ and $I^{loop} < 0 \;\rightarrow\; \theta_{23} > 45^\circ$ for NO.
- $I^{loop} < 0 \;\rightarrow\; \theta_{23} < 45^\circ$ and $I^{loop} > 0 \;\rightarrow\; \theta_{23} > 45^\circ$ for IO.

Bimaximal mixing



- θ_{13} can largely depart from 0, but it leads to $|\theta_{23} - 45| \gg 1$.
- Because of $\theta_{12}^{BM} = 45^\circ$, the allowed regions are limited.
- $I^{loop} < 0$ results in $\theta_{12} > 45^\circ$.

Democratic mixing



- Because of $\theta_{12}^{DC} = 45^\circ$, the allowed regions are limited.
- $I^{loop} < 0$ results in $\theta_{12} > 45^\circ$.
- The IO with $I^{loop} > 0$ results in $\theta_{23} > 54.7^\circ$.

New mixing

In summary

- It is possible to account for $\theta_{13}^{T2K} \simeq 10^\circ$, but it is accompanied with a large deviation of θ_{23} from θ_{23}^0.

$$\Delta\theta_{23} \simeq \Delta\theta_{13} \simeq 10^\circ$$

- θ_{12} cannot drastically deviate from θ_{12}^0.

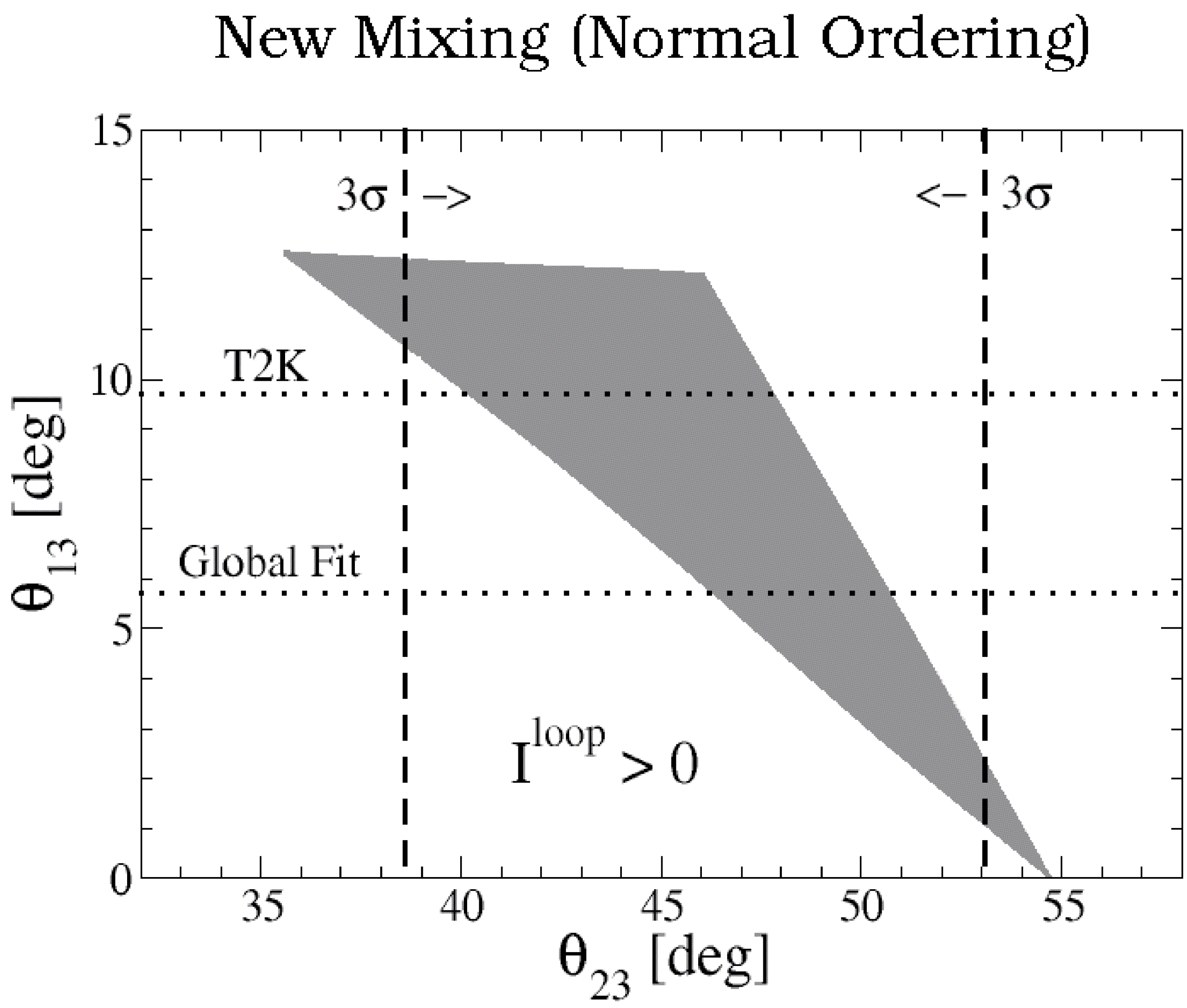
$$\theta_{12}^0 \simeq \theta_{12}^{exp} \simeq 34^\circ$$

We invent a new mixing

$$V_{new}^0 = \frac{1}{3}\begin{pmatrix} \sqrt{6} & \sqrt{3} & 0 \\ -1 & \sqrt{2} & -\sqrt{6} \\ -\sqrt{2} & 2 & \sqrt{3} \end{pmatrix} P_\nu$$

$$\theta_{23}^0 = 54.7^\circ$$
$$\theta_{12}^0 \simeq 35.3^\circ$$
$$\theta_{13}^0 = 0^\circ$$

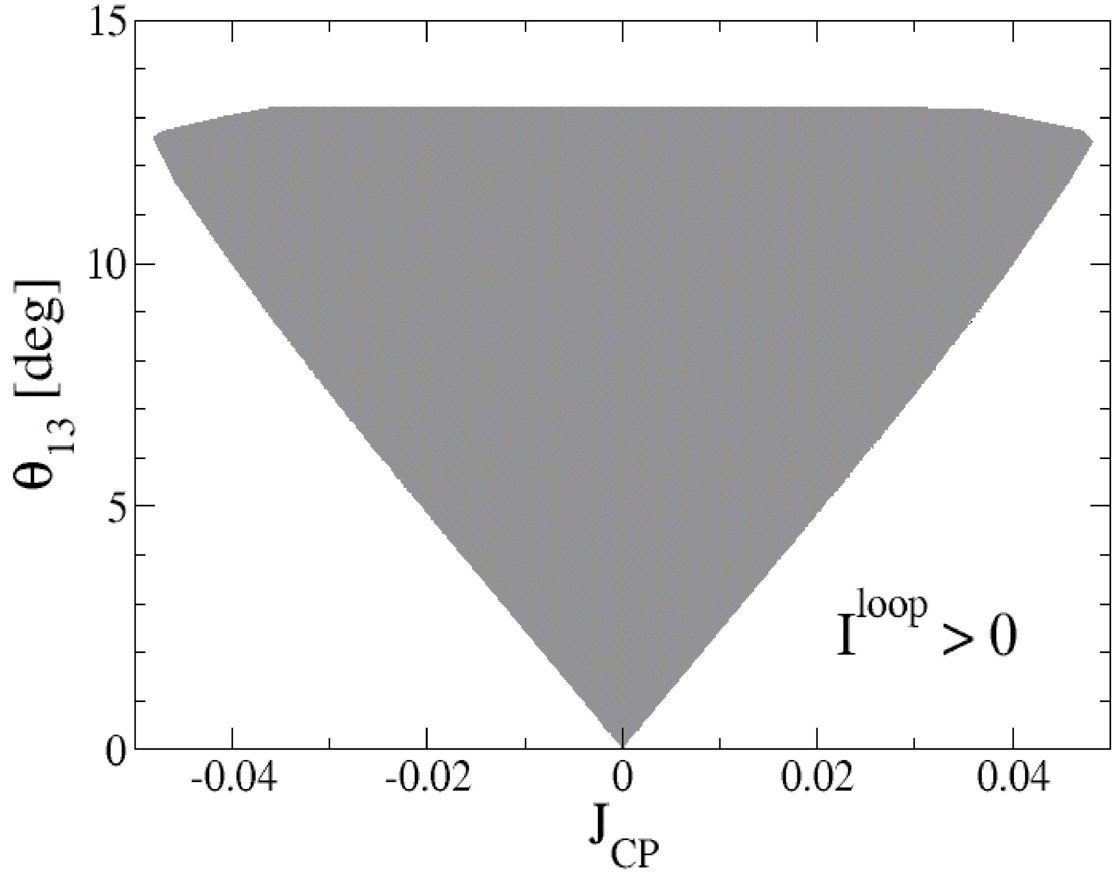
New mixing


New Mixing (Normal Ordering)
3σ ->
<- 3σ
T2K
Global Fit
$I^{loop} > 0$
15
10
5
0
35
40
45
50
55
θ_{13} [deg]
θ_{23} [deg]

New mixing

$$J_{CP} \simeq \frac{m_\tau^2}{9v^2}\left[\frac{m_2 m_3 \sin\sigma}{m_2^2+m_3^2-2m_2m_3\cos\sigma} - \frac{m_1 m_3 \sin\rho}{m_1^2+m_3^2-2m_1m_3\cos\rho}\right] I^{\rm loop}$$

New Mixing (Normal Ordering)



Possible Model

	Q_L	d_R	u_R	L_L	ℓ_R	H_u	H_d	Δ
$SU(2)_L$	2	1	1	2	1	2	2	3
$U(1)_Y$	1/3	$-2/3$	4/3	-1	-2	1	1	2
Z_4	0	1	1	0	1	1	3	0

Consider a SM extension by 2HDs + 1HT with Z_4 .

$$\mathcal{L}_y = Y_d\ \overline{Q}_L H_d d_R + Y_u\ \overline{Q}_L (i\sigma_2 H_u^*) u_R$$

Same as MSSM
No FCNCs

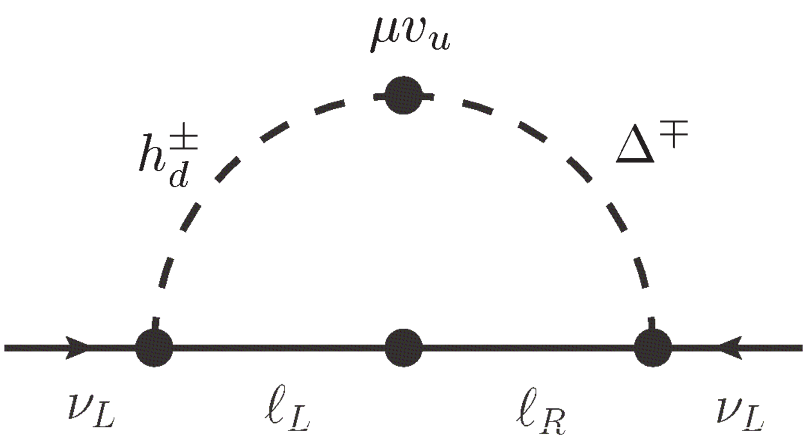
$$+Y_\ell\ \overline{L}_L H_d \ell_R + Y_\Delta\ L_L^T C (i\sigma_2 \Delta) L_L + h.c.$$

$$V = n_u^2 H_u^\dagger H_u + n_d^2 H_d^\dagger H_d + n_\Delta^2 \mathrm{Tr}[\Delta\Delta^\dagger]$$

$$+\underline{\mu \left[H_u^T (i\sigma_2 \Delta^\dagger) H_d + h.c.\right]} + \lambda_3 \left[(H_u^\dagger H_d)^2 + h.c.\right]$$

$$+\cdots$$

Possible Model



- A tree-level mass term is induced from a VEV of the triplet(type-II):

$$M_\nu^0 = Y_\Delta v_\Delta = Y_\Delta \frac{\mu v_u v_d}{2 n_\Delta^2}$$

$$v_\Delta = \langle \Delta^0 \rangle$$

$$\tan\beta = v_u / v_d$$

$$v^2 = v_u^2 + v_d^2 = (174\ \mathrm{GeV})^2$$

- Moreover, from a 1-loop diagram

$$\delta M_\nu \simeq \frac{M_\nu^0 D_e^2 + D_e^2 M_\nu^0}{v^2} \times \underbrace{\frac{\tan^2\beta}{16\pi^2} \frac{1}{1 - M_{h_d^\pm}^2 / M_{\Delta^\pm}^2} \ln \frac{M_{\Delta^\pm}^2}{M_{h_d^\pm}^2}}_{I^{loop}}$$

$$M_{h_d^\pm} = 10^2\ \mathrm{GeV},\ M_{\Delta^\pm} = 10^5\ \mathrm{GeV}\ \text{and}\ \tan\beta = 38 \to I^{loop} = 125.$$

Summary

- We have proposed a new mechanism which naturally generates a small correction term based on *finite* quantum effects.
- Once a tree-level mixing is given, a specific correction term is automatically obtained (up to an overall factor).
- Unfortunately, the mechanism does not work well with the TBM, BM and DC mixings.
- Instead, we have proposed a new mixing pattern.
- We have shown a simple realization of the finite quantum corrections.

Comments

A conclusion and results depend a model.

$$V^0_{TBM},\ V^0_{BM},\ V^0_{DC},\ \cdots$$

$$\delta M_i,\ \delta M_j, \delta M_k,\ \cdots$$

Many combinations

- Once a tree-level mixing is given, we obtain a specific correction term (upto I^{loop}). Very predictive!!
- Such (finite quantum) correction terms indeed appear in some simple new physics models.

Perturbative Expansion

$$V^0 = \begin{pmatrix} c_{12}^0 & s_{12}^0 & 0 \\ -c_{23}^0 s_{12}^0 & c_{23}^0 c_{12}^0 & -s_{23}^0 \\ -s_{23}^0 s_{12}^0 & s_{23}^0 c_{12}^0 & c_{23}^0 \end{pmatrix}$$

$$\sin\theta_{13} \simeq \left| 2 s_{23}^0 c_{23}^0 s_{12}^0 c_{12}^0 \frac{m_\tau^2}{v^2} \left\{ \frac{\lambda_3^2[\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2\cos(\rho-\sigma)]}{[\lambda_1^2 + \lambda_3^2 - 2\lambda_1\lambda_3\cos\rho][\lambda_2^2 + \lambda_3^2 - 2\lambda_2\lambda_3\cos\sigma]} \right\}^{\frac{1}{2}} I^{\rm loop} \right|$$

$$\tan\theta_{12} \simeq t_{12}^0 \left[1 + (s_{23}^0)^2 \frac{m_\tau^2}{v^2} \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2\cos(\rho-\sigma)} I^{\rm loop} \right]$$

$$\tan\theta_{23} \simeq t_{23}^0 \left\{ 1 + \frac{m_\tau^2}{v^2} \left[\frac{(s_{12}^0)^2(\lambda_1^2 - \lambda_3^2)}{\lambda_1^2 + \lambda_3^2 - 2\lambda_1\lambda_3\cos\rho} + \frac{(c_{12}^0)^2(\lambda_2^2 - \lambda_3^2)}{\lambda_2^2 + \lambda_3^2 - 2\lambda_2\lambda_3\cos\sigma} \right] I^{\rm loop} \right\}$$

$$J_{\rm CP} \simeq 2(s_{23}^0 c_{23}^0 s_{12}^0 c_{12}^0)^2 \frac{m_\tau^2}{v^2} \left[\frac{\lambda_2\lambda_3\sin\sigma}{\lambda_2^2 + \lambda_3^2 - 2\lambda_2\lambda_3\cos\sigma} - \frac{\lambda_1\lambda_3\sin\rho}{\lambda_1^2 + \lambda_3^2 - 2\lambda_1\lambda_3\cos\rho} \right] I^{\rm loop}$$

Small I^{loop} case

For $|I^{loop}| < 0.5$, only $|\tan^2\theta_{12} - 0.5|$ is appreciable.

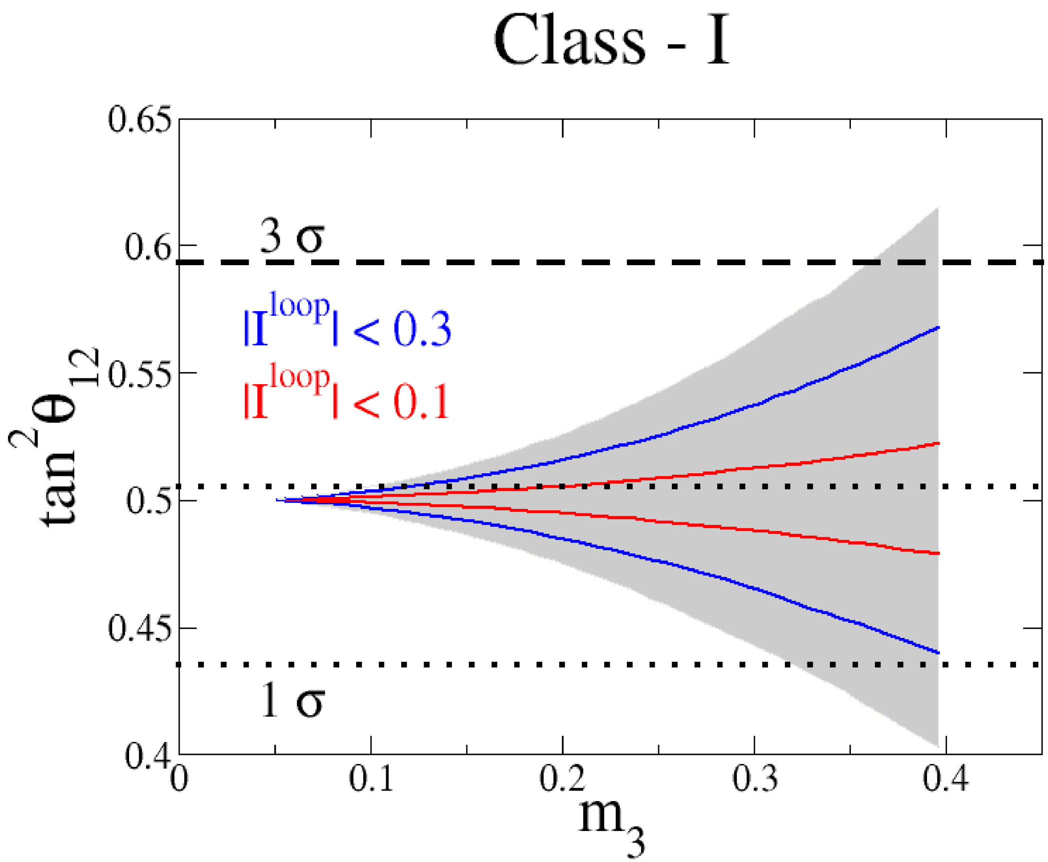
Conditions

$$m_e = 0.486 \text{ MeV}$$
$$m_\mu = 102.718 \text{ MeV}$$
$$m_\tau = 1746.24 \text{ MeV}$$

$$m_1 + m_2 + m_3 < 1.19 \text{ eV}$$

$$\Delta m^2_{31} = (2.46 \pm 0.12)10^{-3} \text{ eV}^2$$

$$\Delta m^2_{21} = (7.59 \pm 0.20)10^{-5} \text{ eV}^2$$



Deviations of the other mixing angles are very small:

$$|\tan^2\theta_{23} - 1| < \mathcal{O}(10^{-3}) \quad \text{and} \quad \sin^2\theta_{13} < \mathcal{O}(10^{-6}) \ .$$