Symmetry behind Flavor Physics: the Structure of Mixing Matrix

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INTRODUCTION

- Flavor Issues in Standard Model:
  1. Mass hierarchy of quarks and leptons
  2. Origin of petterns of mixing matrices
     (CKM and PMNS matrix)
  3. Origin of CP violation in weak interaction

They come from Yukawa couplings,

free parameters of SM

Representations under gauge group are flavor universal

- Model of particle physics beyond the SM should visit such flavor issues.

- How? :

  Extension of symmetry including group(s) with flavor dependent representations

NONABEIAN DISCRETE SYMMETRY

Explains the pattern of mixing matrices

(Review: H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1)

Hints from PMNS matrix:

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.58	0.306	0.021	0.42	2.35
		(0.312)	(0.025)		
1σ range	7.32 – 7.80	0.291 – 0.324	0.013 – 0.028	0.39 – 0.50	2.26 – 2.47
		(0.296 – 0.329)	(0.018 – 0.032)		
2σ range	7.16 – 7.99	0.275 – 0.342	0.008 – 0.036	0.36 – 0.60	2.17 – 2.57
		(0.280 – 0.347)	(0.012 – 0.041)		
3σ range	6.99 – 8.18	0.259 – 0.359	0.001 – 0.044	0.34 – 0.64	2.06 – 2.67
		(0.265 – 0.364)	(0.005 – 0.050)		

(G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A. M. Rotunno, arXiv:1106.6028)

This comes from PMNS matrix parametrization

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note the maximal mixing between 2nd and 3rd generations

Recent T2K report (T2K collaboration, Phys. Rev. Lett. **107** (2011) 041801)

This result converted into a confidence interval yields $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ at 90% C.L. for $\sin^2 2\theta_{23} = 1.0$, $|\Delta m^2_{23}| = 2.4 \times 10^{-3}\ \mathrm{eV}^2$, $\delta_{CP} = 0$ and for normal (inverted) neutrino mass hierarchy. Under the same assumptions, the best fit points are 0.11(0.14), respectively.

Maximal mixing is easily obtained in nonAbelian discrete symmetry model. It comes from the Yukasa coupling Y or $YY^\dagger$ of the form

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

For example,

1. Permutation group : It has the structure of $1 \to 2, 2 \to 1$ for indices i, j of Y_{ij}
2. In Dihedral group, U(1) subgroup property allows this pattern.

In Doublet representation, Dihedral group D(2N) action is generated by

$$a = \begin{pmatrix} e^{2\pi ik/2N} & 0 \\ 0 & e^{-2\pi ik/2N} \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Then, tensor product is given by

For $k + k' \neq N$ and $k - k' = 0$,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}(\mathbf{2}_k)\times\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(\mathbf{2}_{k'})=(x_1y_2+x_2y_1)(\mathbf{1}_{++})+(x_1y_2-x_2y_1)(\mathbf{1}_{--})+\begin{pmatrix} x_1y_1 \\ x_2y_2 \end{pmatrix}(\mathbf{2}_{k+k'}).$$

Suppose x and y be quarks or leptons. Then, the existence of flavons in 1(++) and 2(k+k') with two components of 2(k+k') has the same vacuum expectation value (VEV) give the structure.

In many discrete symmetry model building, θ_{13} is zero , but as T2K reports the possibility of large θ_{13} , many efforts putting this correction has been made. (we do not treat this in detail here)

The solar mixing angle, θ_{12} can be determined from physics model builder favors.

a) fitting the measurement + permutation structure (S4, S3, A4...)
: Tri-bi maximal mixing

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

(For comparison of various models with measurement, see
C. H. Albright, A. Dueck, W. Rodejohann, Eur. Phys. J. C. **70** (2010), 1099)

b) Some special numerical relations

e.g. Quark-Lepton complementarity

$$\theta_{\rm sol} + \theta_C \simeq 45^{\rm o}.$$

To use this relation, we have to think of representation of quark sector, as well as lepton sector under nonAbelian discrete group.

Taking $\theta_C = \frac{\pi}{12}$, $\theta_{\rm sol} = \frac{\pi}{6}$

: It is possible to construct the model with Dihedral group, D(12) (J. E. Kim and M. S. Seo, JHEP 02(2011) 0907)

It is not very close to measurement, but it may leave the space for expansion with respect to θ_{13} to fit theory and experiment.
(J. E. Kim and M. S. Seo, arXiv:1106.6117)

$$\begin{pmatrix}
\frac{1}{2}(\sqrt{3}-\beta-\frac{\sqrt{3}}{2}(1+B^2)\beta^2), & \frac{1}{2}(1+\sqrt{3}\beta-\frac{1}{2}(1+B^2)\beta^2), & B\beta, \\
\begin{matrix} -\frac{1}{2\sqrt{2}}(1+\sqrt{3}(1+Be^{-i\delta})\beta \\ -(A+\frac{1}{2}+Be^{-i\delta})\beta^2) \end{matrix}, & \begin{matrix} \frac{1}{2\sqrt{2}}\Big(\sqrt{3}-(1+Be^{-i\delta})\beta \\ -\sqrt{3}(A+\frac{1}{2}+Be^{-i\delta})\beta^2\Big) \end{matrix}, & \frac{1}{\sqrt{2}}(1+(A-\frac{B^2}{2})\beta^2)e^{-i\delta} \\
\begin{matrix} \frac{1}{2\sqrt{2}}\Big(e^{i\delta}+\sqrt{3}(e^{i\delta}-B)\beta \\ +([A-\frac{1}{2}]e^{i\delta}+B)\beta^2\Big) \end{matrix}, & \begin{matrix} -\frac{1}{2\sqrt{2}}\Big(\sqrt{3}e^{i\delta}-[e^{i\delta}-B]\beta \\ +\sqrt{3}([A-\frac{1}{2}]e^{i\delta}+B)\beta^2\Big) \end{matrix}, & \frac{1}{\sqrt{2}}\Big(1-(A+\frac{B^2}{2})\beta^2\Big)
\end{pmatrix}$$

NonAbelian discrete symmetry provides the pattern of PMNS matrix (and possibly CKM matrix) well, but does not tell us much about mass hierarchy.

One Effort: Extremely heavy third generation comes from "democratic" Yukawa matrix of the form

$$\frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

It is, in fact, not so democratic because it is equivalent to Diag. (0,0,1) via unitary matrix

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} \qquad \text{where } \omega = \exp(i2\pi/3).$$

When it is combined with maximal mixing, tri-bimaximal mixing is naturally obtained.

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

In fact, the unitary matrix $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}$ generally diagonalizes the Hermitian matrix ($YY^\dagger$) of the form

$$\begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix}$$

Is it possible to put the correction to democratic mass matrix to show the mass hierarchy as $\mathrm{Diag.}(\epsilon^3, \epsilon, 1)$ without changing unitary matrix too much i.e.

keeping the form or

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}$$

additional matrix controlled by ϵ as

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} + \epsilon \mathrm{V}$$

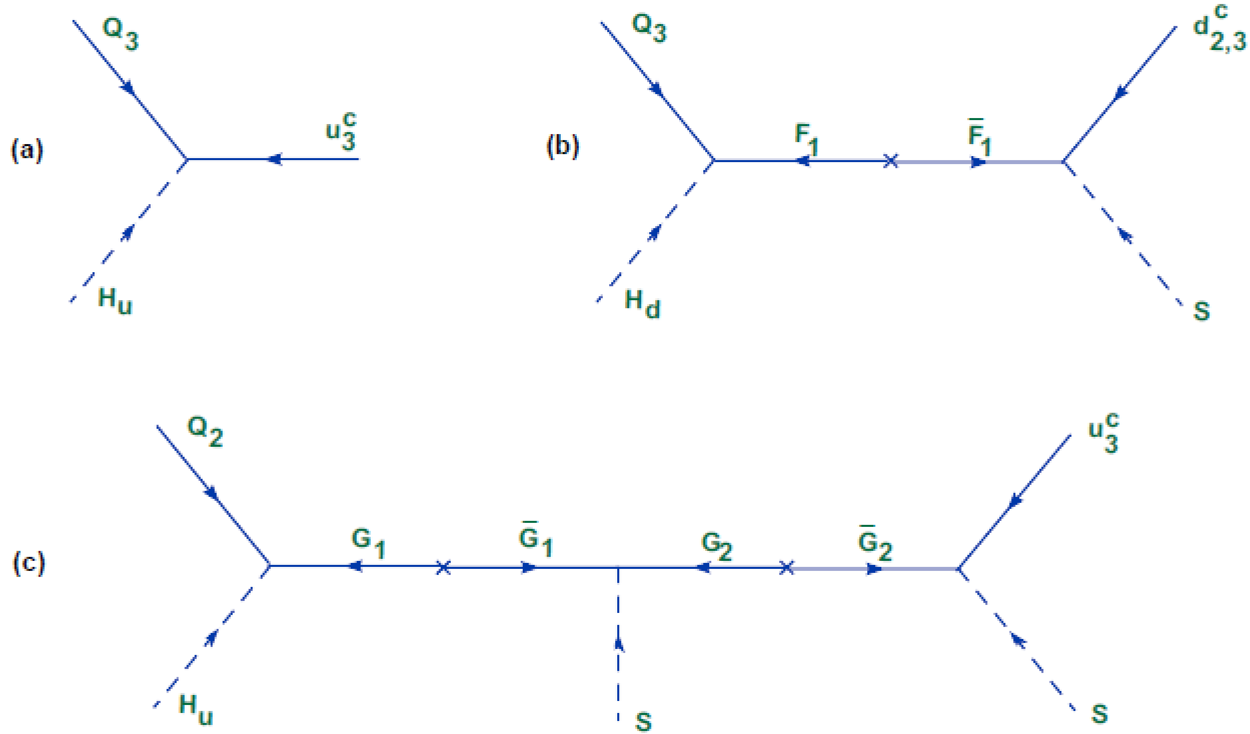
: We hope so.
But we have to be more systematic by specifying remaining symmetry after introducing ϵ explicitly.

MASS HIERARCHY AND CP VIOLATION

Another Plausible approach to mass hierarcy and mixing angle is Froggatt and Nielsen mechanism
(C.D. Frogatt, H. B. Nielsen, Nucl. Phys. B**147** (1979) 277)

$$\begin{aligned}\mathcal{L}_{FN}^{\text{eff}} &= \left[Q_3u_3^cH_u + Q_2u_3^cH_uS^2 + Q_3u_2^cH_uS^2 + Q_2u_2^cH_uS^4\right] \\ &+ \left[Q_3d_3^cH_dS + Q_3d_2^cH_dS + Q_2d_2^cH_dS^3 + Q_2d_3^cH_dS^3\right] + h.c.\end{aligned}$$

where S = ϕ/M_{pl}



(Adopted from K. S. Babu's TASI lecture [arXiv:0910.2948])

Such power of S requires the introducing new symmetry which is flavor non-universal.

Since we do not know this symmetry, we have to deduce it from the structure of mixing matrices and mass hierarchy.

But they are not enough to fix the form of Yukawa matrices, because

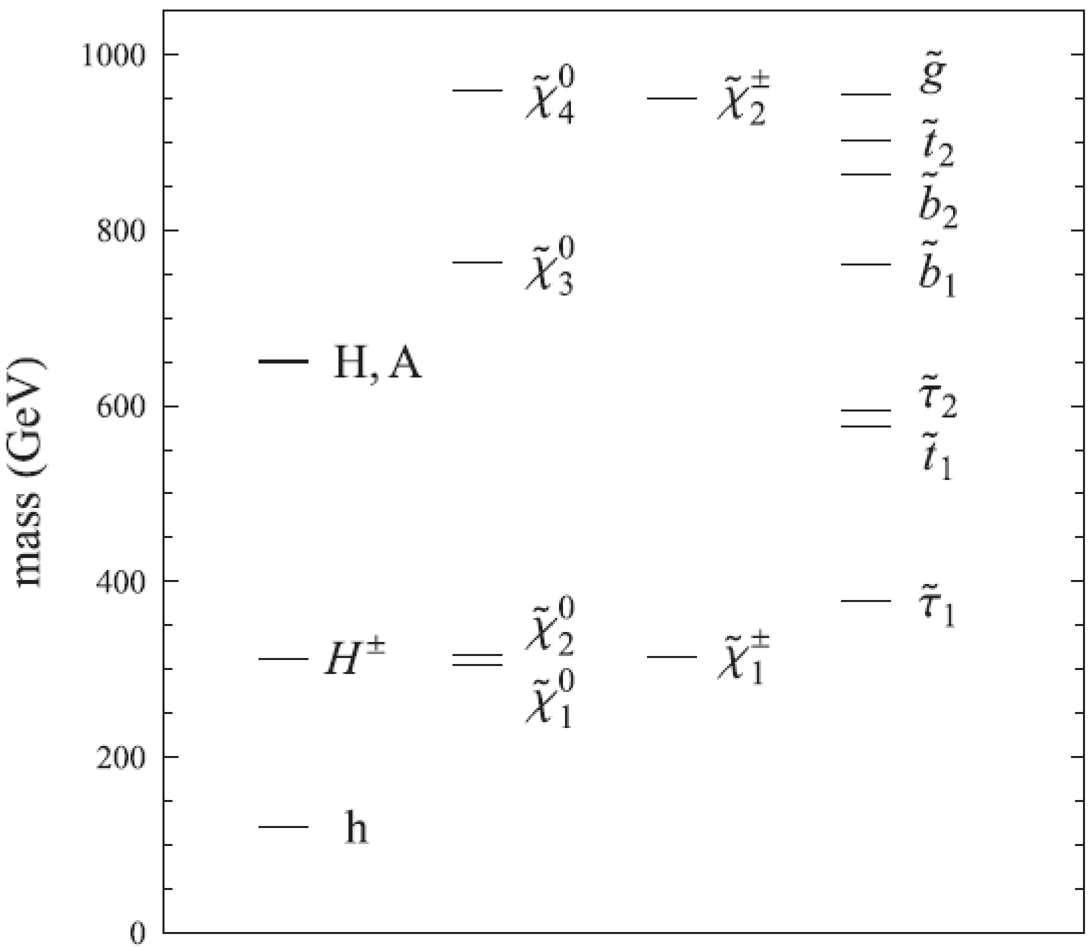
1) We do not know how to divide $V_{\mathrm{CKM}} = U_u^\dagger U_d$ or $V_{\mathrm{PMNS}} = U_l^\dagger U_\nu$ into unitary matrices for u- d-, l-, v sectors

2) Since each unitary matrix rotates left-handed fermions, we do not know how to rotate right-handed fermions.

If flavor dependent symmetry is found, we can have more information about the flavor structure.

For example, low energy supersymmetry, one of representative solution to gauge hierarchy problem has not been found yet.
: What if only the third generation squark (stop) which is essential for stable Higgs mass correction, or possibly third generation sparticles, remain around the experimental search bound, but others are very heavy?

<u>Effective Supersymmetry</u> (A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys . Lett. B**388** (1996) 588)



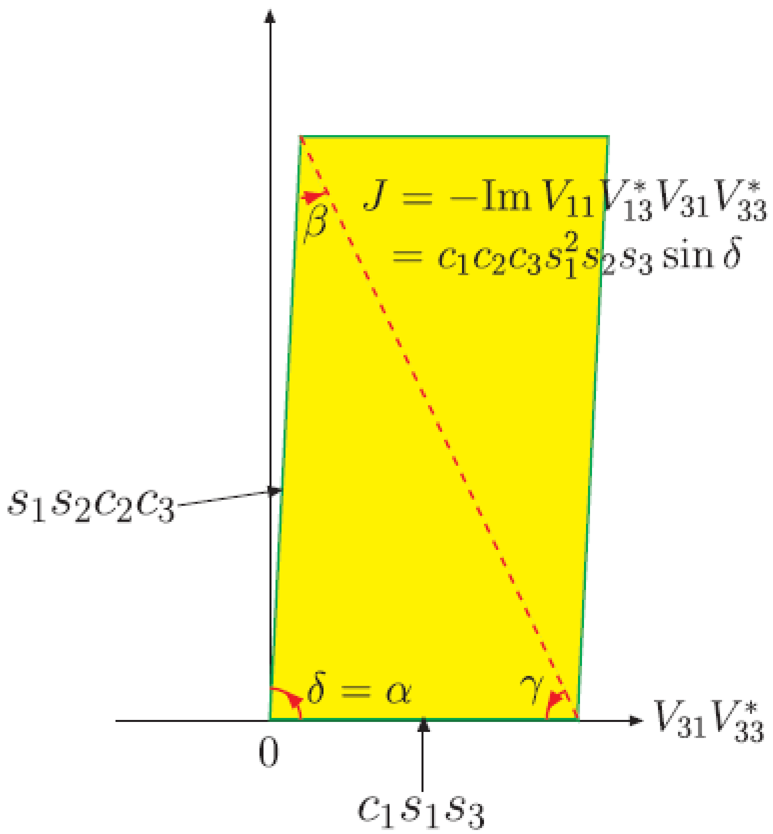
(K. S. Jeong, J. E. Kim and M. S. Seo, Phys . Rev. D**84** (2011) 075008)

Since we do not know them, let us probe the structure of mixing matrices more detail.

1. If only three generations exist, it is unitary.

From the unitarity condition, we can deduce the unique imaginary part property(=CP violation property) of the mixing matrix:

Jarlskon determinant (C. Jarlskog, Phys. Rev. Lett. **55** (1985) 1039)



(J. E. Kim and M. S. Seo, Phys . Rev. D**84** (2011) 037303)

2. What if determinant of mixing matrix is not unity but has a complex phase?

: In fact, the determinant of CKM matrix in the original paper (M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**(1973), 652) has a phase.

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}.$$

$$\mathrm{Det}V = e^{i\delta}$$

However, phase of determinant of CKM matrix is related to Arg. Det. M, M is quark mass matrix, and it can be absorbed into QCD θ term ,

$$\frac{\theta_{\rm QCD}}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}.$$

by chiral transformation, to make

$$\theta_{\rm QCD} + {\rm Arg.Det.}V_{\rm CKM}.$$

The invariance of physics is guaranteed by Peccei-Quinn symmetry.

The parametrization of CKM matrix in the original paper can be written in the real determinant form

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix},$$

To make the determinant of complex(one phase in CKM matrix is unremovable) matrix real, cancellation of imaginary number between “components of determinant” occurs, and this complex number is just the Jarlskog determinant

$$
\begin{aligned}
V_{11}V_{22}V_{33} &= c_1^2c_2^2c_3^2 + c_1^2s_2^2s_3^2 + 2c_1c_2c_3s_2s_3\cos\delta \\
&\quad - c_1c_2c_3s_1^2s_2s_3e^{i\delta}, \\
-V_{11}V_{23}V_{32} &= c_1^2c_2^2s_3^2 + c_1^2s_2^2c_3^2 - 2c_1c_2c_3s_2s_3\cos\delta \\
&\quad + c_1c_2c_3s_1^2s_2s_3e^{i\delta}, \\
V_{12}V_{23}V_{31} &= s_1^2s_2^2c_3^2 - c_1c_2c_3s_1^2s_2s_3e^{i\delta}, \\
-V_{12}V_{21}V_{33} &= s_1^2c_2^2c_3^2 + c_1c_2c_3s_1^2s_2s_3e^{i\delta}, \\
V_{13}V_{21}V_{32} &= s_1^2c_2^2s_3^2 - c_1c_2c_3s_1^2s_2s_3e^{i\delta}, \\
-V_{13}V_{22}V_{31} &= s_1^2s_2^2s_3^2 + c_1c_2c_3s_1^2s_2s_3e^{i\delta}.
\end{aligned}
$$

So, if you want to know the Jalskog determinant, read off the imaginary number of $V_{13}V_{22}V_{31}$!

Of course, in lepton sector, we cannot use the strong CP violation argument, and in general, determinant is complex due to Majorana phase. But such a parametrization may be useful since Jarlskog determinant is observable from neutrino mixing.

$$
\begin{aligned}
A_{CP}^{\alpha'\alpha} &\equiv P(\nu_\alpha \to \nu_{\alpha'}) - P(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) \\
&= 4 \sum_{j>k} \mathrm{Im}(V_{\alpha' j} V_{\alpha j}^* V_{\alpha k} V_{\alpha' k}^*) \sin \frac{\Delta m_{jk}^2}{2p} L
\end{aligned}
$$

Especially,

$$
\begin{aligned}
A_{CP}^{(\mu e)} &= -A_{CP}^{(\tau e)} = A_{CP}^{(\tau \mu)} \\
&= 4J(\sin \frac{\Delta m_{32}^2}{2p} L + \sin \frac{\Delta m_{21}^2}{2p} L + \sin \frac{\Delta m_{13}^2}{2p} L)
\end{aligned}
$$

3. Wolfenstein parametrization

In CKM matrix, $\lambda \simeq \sin\theta_C$ is a good expansion parameter

$$V_{\text{Wolf}} = \begin{pmatrix} 1-\lambda^2/2, & \lambda, & A\lambda^3(\rho - i\eta) \\ -\lambda, & 1-\lambda^2/2, & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta), & -A\lambda^2, & 1 \end{pmatrix}$$

In a view of Froggatt-Nielesn, S= ϕ/M_{pl} would have VEV λ

But how many Ss do we need?

If left- and right- handed fermions are rotated by the same unitary matrix, Yukawa coupling would be Hermitian, and we may guess

For $R = L$,

$$\tilde{M}^{(u)} = \begin{pmatrix} (c+\kappa_t^2\lambda)\lambda^6, & -(c+\kappa_t^2)\lambda^5, & \kappa_t\lambda^3\left(1+\frac{1}{3}\lambda^2\right) \\ -(c+\kappa_t^2)\lambda^5, & c\lambda^4\left(1-\frac{1}{3}\lambda^2\right), & -\kappa_t\lambda^2+\frac{\kappa_t}{6}\lambda^4+O(\lambda^6) \\ \kappa_t\lambda^3\left(1+\frac{1}{3}\lambda^2\right), & -\kappa_t\lambda^2+\frac{\kappa_t}{6}\lambda^4+O(\lambda^6), & 1-\kappa_t^2\frac{\lambda^4}{2}-\kappa_t^2\frac{\lambda^6}{3} \end{pmatrix}$$

$$\tilde{M}^{(d)} = \begin{pmatrix} d\lambda^4\left(1+\frac{2}{3}\lambda^2\right), & 0, & 0 \\ 0, & s\lambda^2+\left(\kappa_b+\frac{s}{3}\right)\lambda^4+\left(\frac{8}{45}s+\frac{2\kappa_b^2}{3}\right)\lambda^6, & \kappa_b e^{i\delta}\left(-\lambda^2+\left(s-\frac{1}{3}\right)\lambda^4\right)+O(\lambda^6) \\ 0, & \kappa_b e^{-i\delta}\left(-\lambda^2+\left[s-\frac{1}{3}\right]\lambda^4\right)+O(\lambda^6), & 1-\kappa_b^2\lambda^4+\kappa_b^2\left(s-\frac{2}{3}\right)\lambda^6 \end{pmatrix}.$$

In this case, we need at least three Froggatt-Niesen fields :
Their VEVs have phases 0, δ., and - δ.

In CKM matrix, δ. is almost 90 degree
(maximal CP violation)

$$\delta = 89.0° \pm 4.4°.$$

Question:
a) Is it possible to explain maximal CP phase δ. naturally?
(J. E. Kim, Phys. Lett. B**704** (2011) 360) :
Abelian discrete symmetry Z(12), Z(4) and Z(3) are used to explain leading order structure of Hermitian Yukawa matrix and maximal CP violation

b) Is it possible to relate such Froggatt-Nielsen parametrization of CKM matrix with that of lepton sector?

(e.g. Quark-lepton complementarity)

(N. Qin and B. Q. Ma, Phys. Rev. D**83** (2011) 033006
J. E. Kim and M. S. Seo, working in progress):

One interesting feature is the possibility of observation of CP phase in PMNS matrix from leptogenesis because CP violation in heavy right handed neutrino decay plays an essential role in leptogenesis.

If we can explain the structure of PMNS matrix and mass hierarchy of lepton with the same Froggatt-Nielsen fields in quark sector, it has a predictivity of weak CP violation in lepton sector related to maximal CP violation in quark sector

FLAVOR PROBLEM BEYOND SM

- Lepton Flavor Violation (LFV) is the one more physics related to the flavor structure in lepton sector
- It is strongly suppressed in SM.

-For example $B(\mu \to e\gamma)$ is less then 10^{-46}

-It is the effect of the suppression of flavor changing neutral current (FCNC) in SM. (unitarity of mixing matrix cancel FCNC in the degenerate neutrino mass limit)

$$B(l_j \to l_i\gamma) \sim \frac{3\alpha}{32\pi} |\sum_i V^*_{j\beta} V_{i\alpha} \frac{m^2_{\nu_\alpha}}{M^2_W}|^2$$

Current status of LFV:

- $l_j^- \to l_i^- \gamma$

  $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$

  $B(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$

  $B(\tau \to e\gamma) < 3.3 \times 10^{-8}$

- $l_j^- \to l_i^- l_i^- l_i^+$

  $B(\mu \to e^- e^- e^+) < 1.0 \times 10^{-12}$

- $\mu - e$ conversion in nuclei

  $\frac{\Gamma(\mu^- \mathrm{Ti} \to e^- \mathrm{Ti})}{\Gamma(\mu^- \mathrm{Ti} \to \mathrm{capture})} < 4.3 \times 10^{-12}$

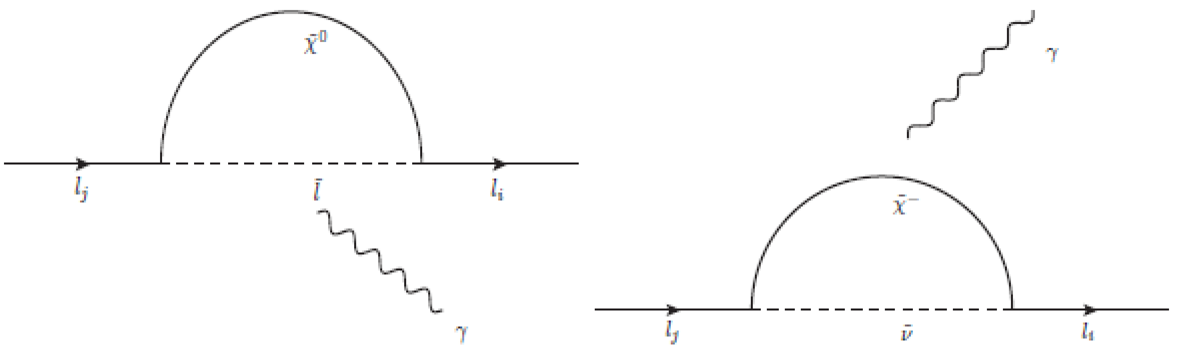
  $\frac{\Gamma(\mu^- \mathrm{Au} \to e^- \mathrm{Au})}{\Gamma(\mu^- \mathrm{Au} \to \mathrm{capture})} < 7 \times 10^{-13}$

(particle data group)

Recent MEG experiment reports more stringent bound on $B(\mu \to e\gamma) < 2.4 \times 10^{-12}$ (90% C.L.)

(MEG collaboration, [arXiv:1107.5547])

If sizable LFV is observed, it is the signal of new physics.
 : For example, in supersymmetric extension of SM, sparticle mass differnences is not the same as those in SM, hence LFV enhanced.



In gauge mediated supersymmetry(SUSY) breaking mechanism, supersymmetry is broken in the hidden sector, and it is transfered through gauge interaction, which is flavor universal :LFV not enhanced

Then, can we control the effects of additional SUSY breaking mechanism in LFV in a small parameter?

One trial: Suppose that SUSY breaking is mediated also through Yukawa coupling in seesaw mechanism.

(G. F. Giudice, R. Rattazzi, A. Strumia, Phys. Lett. B**694** (2010) 26)

Question:

Effects on LFV?

Is it possible to relate it to large (13) element of PMNS matrix?

(H. D. Kim, D. Y. Mo, and M. S. Seo, working in progress)

SUMMARY

- Flavor physics requires new symmetry structure to explain mass hierarchy, pattern of mixing matrix, and CP violation in weak interaction
- Recent experiments are expected to reveal many mysterious aspects in flavor physics

After all, our purpose in theoretical physics is not just describe the world as we find it, but to explain – in terms of a few funcdamental principles – why the world is the way it is.

- S. Weinberg