

Course: SURFACES OF GENERAL TYPE: GEOGRAPHICAL AND BOTANICAL ASPECTS.

This series of 4 lectures will focus on some aspects of the fine classification of complex surfaces of general type.

Two important numerical invariants of minimal surfaces of general type are the self-intersection of the canonical divisor, K^2 , and the holomorphic characteristic, χ . Both K^2 and χ are strictly positive integers that satisfy the "geographical" restriction

$$2\chi - 6 \leq K^2 \leq 9\chi$$

and minimal surfaces in special regions of the plane (K^2, χ) have different features - for instance minimal surfaces satisfying $K^2 < 2\chi$ are always regular and minimal surfaces satisfying $K^2 < 3\chi - 10$ have always canonical map generically finite of degree 2.

The two first lectures will be devoted to:

- the behaviour of surfaces in some special regions of the plane;
- discussion of several standard techniques used on the study of surfaces
- construction and analysis of some examples of surfaces.

The two last lectures will be focused in the following topics:

- geography of irregular surfaces;
- algebraic fundamental groups of surfaces with low K^2 .

1. PREREQUISITES

The basic language of algebraic geometry will be assumed to be known and a general knowledge of the rough classification of complex surfaces will be assumed to be known.

2. MAIN REFERENCES

1. A. Beauville *L'application canonique pour les surfaces de type général*. Inv. Math. **55** (1979), 121–140.
2. A. Beauville *L'inégalité $p_g \geq 2q - 4$ pour les surfaces de type général*, Appendix to Bull. Soc. Math. de France, vol. **110** (1982), 343–346.
3. C. Ciliberto, M. Mendes Lopes, R. Pardini *Surfaces with $K^2 < 3\chi$ and finite fundamental group*, Math. Res. Lett. **14** (2007), 1081–1098.
4. O. Debarre *Inégalités numériques pour les surfaces de type général*, Bull. Soc. Math. France **110** 3 (1982), 319–346.
5. M. Mendes Lopes, R. Pardini *On the algebraic fundamental group of surfaces with $K^2 \leq 3\chi$* , Journal of Differential Geometry, **77**, no. 2, (2007), 189–199.

6. M. Mendes Lopes, R. Pardini *The order of finite algebraic fundamental groups of surfaces with $K^2 \leq 3\chi - 2$* , in “Algebraic geometry and Topology” Suurikaiseki kenkyusho Koukyuuroku, No. 1490 (2006), 69–75, math.AG/0605733
7. M. Mendes Lopes, R. Pardini *On surfaces with $p_g = 2q - 3$* , to appear in Advances in Geometry, arXiv:0811.0390
8. M. Mendes Lopes, R. Pardini *Severi type inequalities for irregular surfaces with ample canonical class*, to appear in Comment. Math. Helvetici, arXiv:0904.1004
9. M. Mendes Lopes, R. Pardini *The geography of irregular surfaces*, arXiv:0909.5195.

3. BOOKS ON ALGEBRAIC SURFACES

1. W. Barth, C. Peters, A. Van de Ven, *Complex algebraic surfaces*, Ergeb. Math. Grenzgeb. (3) Band 4, Springer, Berlin (2004).
2. A. Beauville, *Complex algebraic surfaces*, LMS, Student Texts, Vol. 34 (1966), C.U.P.
3. M. Reid *Chapters on algebraic surfaces*, Complex Algebraic Geometry Park City, UT, 1993, IAS/Park City Math. Ser. vol. 3.