Title:

Surfaces of general type: some construction technique and investigation of moduli spaces

Abstract:

The aim of these lectures will be to outline general methods for the determination of moduli spaces of surfaces, especially of their connected components. It will be shown how one can do this in several concrete cases where one has explicitly constructed some algebraic surface.

The first Lecture should be devoted to the construction of surfaces through bidouble covers , or through more general abelian covers. And to the calculation of the invariants of the surfaces thus constructed, also in the singular case.

The second lecture shall rehearse the basic tools from deformation theory of smooth and singular surfaces, with special attention to the problem of smoothing. It will then be shown how these methods can be used to show that a given family yields an open set of the moduli space.

The third lecture should cover some topics from the theory of fibred surfaces, especially with applications to the theme of surfaces with large fundamental groups. Some examples, like Keum-Naie, primary Burniat surfaces, or some Inoue surface will be discussed.

Theme of the fourth lecture shall be the closure of an open set in the moduli space through the method of degenerating 1-parameter families. In particular, the moduli space of secondary Burniat surfaces should be discussed here as a concrete example where the deformation and degeneration method both apply quite successfully.

Some of the topics mentioned here, or neighbouring ones, should be also covered in the lectures by Ingrid Bauer and Roberto Pignatelli.

Some references:

I. Bauer, F. Catanese, A volume maximing canonical surface in 3-space, Comment. Math. Helv. 83 (2008), no. 2, 387-406.

I. Bauer, F. Catanese, The moduli space of Keum-Naie surfaces, 20 pages, arXiv:0909.1733, to appear in Groups, Geometry and dynamics.

I. Bauer, F. Catanese, Burniat surfaces I: fundamental groups and moduli of primary

Burniat surfaces, 30 pages, arXiv:0909.3699.

I. Bauer, F. Catanese, Burniat surfaces II: secondary Burniat surfaces form three connected components of the moduli space, arXiv:0911.1466.

Burniat, Pol Sur les surfaces de genre $P\not{W}sb{12}>1$. Ann. Mat. Pura Appl. (4) 71 1966 1-24.

Catanese, F. On the moduli spaces of surfaces of general type. J. Differential Geom. 19 (1984), no. 2, 483-515.

Catanese, F. Moduli of algebraic surfaces. Theory of moduli (Montecatini Terme, 1985), 1-83, Lecture Notes in Math., 1337, Springer, Berlin, 1988.

Catanese, Fabrizio, Singular bidouble covers and the construction of interesting algebraic surfaces. Algebraic geometry: Hirzebruch 70 (Warsaw, 1998), 97-120, Contemp. Math., 241, Amer. Math. Soc., Providence, RI, 1999.

Catanese, Fabrizio Differentiable and deformation type of algebraic surfaces, real and symplectic structures. Symplectic 4-manifolds and algebraic surfaces, 55-167, Lecture Notes in Math., 1938, Springer, Berlin, 2008.

Catanese, Fabrizio; LeBrun, Claude On the scalar curvature of Einstein manifolds. Math. Res. Lett. 4 (1997), no. 6, 843-854.

Catanese, Fabrizio; Pignatelli, Roberto Fibrations of low genus. I. Ann. Sci. Ecole Norm. Sup. (4) 39 (2006), no. 6, 1011-1049.

Inoue, Masahisa Some new surfaces of general type. Tokyo J. Math. 17 (1994), no. 2, 295-319.

Kollár, J.; Shepherd-Barron, N. I. Threefolds and deformations of surface singularities. Invent. Math. 91 (1988), no. 2, 299-338.

Lee, Yongnam; Park, Jongil A simply connected surface of general type with $p_g=0$ and $K^2=2$. Invent. Math. 170 (2007), no. 3, 483--505.

Manetti, Marco Smoothings of singularities and deformation types of surfaces. Symplectic 4-manifolds and algebraic surfaces, 169-230, Lecture Notes in Math., 1938, Springer, Berlin, 2008.

Pardini, Rita Abelian covers of algebraic varieties. J. Reine Angew. Math. 417 (1991), 191--213.