Titles and Abstracts

Exploring self-affine tilings with and without coincidences

Shigeki Akiyama

With J.-Y. Lee, we gave a simple algorithm to check pure discreteness of self-affine tiling dynamical systems. Our program simply requires IFS data of self-affine tilings, and we are enjoying example computations. In this talk, we briefly explain the idea of our algorithm and discuss several examples of self-affine tilings having pure discrete spectrum, together with non pure cases. We are expecting your critical discussion to formulate higher dimensional Pisot conjecture, or to find an ideal counter example.

Rauzy fractals associated with a sequence of substitutions

Pierre Arnoux
Collaboration: Masahiro Mizutani and Tarek Sellami

In several contexts, it would be interesting to describe a Rauzy fractal associated, not with a substitution, but with an infinite sequence of substitutions. We explain how this can be realized in a specific case, when the substitutions have the same matrix of the irreducible Pisot type. The generalization of this contruction poses difficult questions.

Close-packed dimers on the line – diffraction versus dynamical spectrum

Michael Baake

TBA

Wannier transform for aperiodic solids: preliminary results

Jean Bellissard
Collaboration: Giuseppe De Nittis, Vida Milani

The talk will present some preliminary results on a work in progress concerning the possible extension of the Bloch theory to aperiodic Schrödinger operators on $L^2(\mathbb{R}^d)$. The operator giving the potential energy in this specific case will be seen as a sum of short range atomic potentials with atoms located on a Delone set $Ll \subset \mathbb{R}^d$, which is aperiodic, repetitive with finite local complexity. The main properties of such sets will be summarized, examples will be given, and their associated tilings will be described. The Penrose, the octagonal tilings and the icosahedral phase of several quasicrystals are important examples. The Anderson-Putnam complex and the Lagarias group will be defined. This will lead to a definition of the Wannier transform that is shown to be a unitary map onto a Hilbert space analog to the space of Bloch functions used in Solid State Physics. The Schrödinger operator will be shown to have a Bloch like representation and its domain will be characterized. Comments will be made about the potential applications of such method of analysis depending upon how far this work has progressed.

Aperiodic structures and notions of order and disorder

Shelomo I. Ben-Abraham
Collaboration: Alexander Quandt

Artificial aperiodic structures have recently been the subject of extensive and intensive research, resulting in layered quasiregular heterostructures, as well as photonic and phononic metamaterials with possible applications such as optical and acoustic bandpassfilters or photonic waveguides. Our main interests are centered on rather fundamental questions concerning determinism, or the old problem of order vs. disorder. We seek to establish possible quantifications of such concepts, to develop basic notions of complexity and entropy, and beyond. Our specific objects of study are multidimensional generalizations of the standard substitution sequences. Here we present and discuss some two-dimensional instances of the Prouhet-Thue-Morse and paperfolding systems. We compute their rectangle complexities and find them to be at most polynomial, which means that the corresponding entropy basically vanishes. We also point out that the perfectly deterministic Champernowne sequence has entropy $\log 2$. Thus the notion of entropy cannot serve as an unqualified measure of disorder, and there still remain many unanswered questions.

Toward S-adic Rauzy fractals

Valerie Berthe

Rauzy fractals are compact sets that are associated with substitutive dynamical systems with Pisot type in order to produce a geometric and spectral representation of the underlying symbolic dynamical system. In particular, discrete spectrum property can be stated in terms of tiling properties for the Rauzy fractals. The aim of this lecture is to discuss a possible generalization of this situation to more general symbolic dynamical systems, namely symbolic systems generated by iterating a finite number of substitutions according to a given sequence. Such systems (called S-adic) arise for instance when applying a multidimensional continued fraction algorithm.

Aperiodic order and quantum mechanics

David Damanik

We review recent developments in the spectral theory of Schrödinger operators with potentials exhibiting aperiodic order. We will discuss in detail the shape of the spectrum, especially the presence of gaps and their structure.

Fusion tilings

Natalie Priebe Frank

We introduce a class of tilings of Euclidean space that generalizes self-similar tilings. Larger and larger patches of tiles are constructed according to fusion rules that play a similar role to the substitution rules of symbolic dynamics. The construction is extremely general, but conditions can be chosen that allow us to investigate the measure theory and topology. We will discuss which standard results for self-similar tilings hold and if time permits, sketch the proofs.

Topological invariants of aperiodic tilings

Franz Gahler

An overview is given on different ways to construct a space of aperiodic tilings as an inverse limit of a sequence of simpler spaces, and how this can be used to compute topological invariants like Cech cohomology. For two-dimensional canonical projection tilings it is then analysed to what extent the different MLD classes of such tiling spaces can be distinguished by their cohomology.

Some comments on a hexagonal monotile

Uwe Grimm

TBA

Stochastic processes with pure point diffraction

Daniel Lenz

We study how stochastic processes with pure point diffraction are determined by their moments. We give a sufficient condition for such processes to be determined by finitely many moments and show how they can be classified via a certain group. (This is based on joint work with Robert V. Moody.)

Quasicrystals - Some of nature's most intriguing forms of matter (KIAS Physics School colloquium talk)

Ron Lifshitz

The discovery of quasicrystals in 1982 signaled the beginning of a remarkable scientific revolution, in which some of the most basic notions of condensed matter physics and material science have undergone a thorough reexamination. Almost three decades later, the field continues to intrigue us with scientific puzzles, surprising discoveries, and new possibilities for applications. I will focus on some current issues from my own research - including soft matter quasicrystals and photonic applications based on metamaterials - but only after giving a concise overview for nonspecialists of what quasicrystals are, and why their discovery was so important.

On Fibonacci electrons and the stability of soft quasicrystals

Ron Lifshitz

I would like to take advantage of the informal nature of this workshop to describe three (not necessarily related) results of ongoing research on quasicrystals in my group, which may be of interest to mathematicians. The first is the observation of log-periodic oscillations in quantities characterizing the quantum dynamics of Fibonacci electrons, such as the probability of an electron to remain in its initial position as a function of time. The second is a conjecture suggesting that the addition of weak disorder, to an otherwise perfect quasiperiodic potential, may provide the necessary conditions for the existence of a generalized Bloch theorem for quasicrystals. The third result is our successful explanation of the thermodynamic stability of quasicrystals, composed of soft isotropic particles. We have demonstrated that, with some help from entropy, very simple (even purely-repulsive) isotropic pair-potentials can lead to the formation of stable dodecagonal quasicrystals.

Words, decomposition rules and invariants

Jun Morita

For bi-infinite words, we introduce some algebraic structure using Kellendonk product, which allows us to discuss their standard modules and decomposition rules. We also obtain several deformations of the dimension map. Using this, we define certain formal power series as word invariants.

Complexity of entropy zero topological systems

Kyewon Koh Park

The notion of topological entropy dimension is introduced to measure the complexity of entropy zero systems. It measures the superpolynomial, but subexponential growth rate of orbits. We introduce the dimension sets to investigate the complexity of their factors. We discuss the disjointness among entropy zero systems.

Rank and directional entropy

E Arthur Robinson

The rank of an ergodic measure preserving transformation is an invariant that measures how well it is approximated by periodic transformations. Implicit in the notion of rank is the shape of a Folner sequence. Even in the $\mathbb{Z}$ case, this leads to the notion of funny rank 1. When the concept of rank is generalized to the $\mathbb{Z}^n$ or $\mathbb{R}^n$ case, the role of the Folner sequence becomes more prominent. Of particular interest is the effect the Folner sequence has on directional entropy. In this talk we discuss the notion of rank in the case of $\mathbb{Z}^n$ and $\mathbb{R}^n$ actions, and mention the ranks of some well known examples of aperiodic order. We also discuss the relation between rank and directional entropy.

Quotient cohomology for tiling spaces (Cancelled talk)

Lorenzo Sadun

The Cech cohomology of a tiling space has long been studied as an invariant of that space, and much work has been done to associate that cohomology with properties of tilings in the space. However, little has been done with cohomology to study factor maps between spaces, the way that relative (co)homology is used to study inclusions of one topological space in another. In this work, joint with Marcy Barge, I present a relative version of tiling cohomology, called "quotient cohomology", and use it to relate the chair tiling, a variant of the chair tiling, and the 2-dimensional dyadic solenoid.

Discrete planes, two dimensional placement rules and connectivity of Rauzy fractals

Anne Siegel

Rauzy fractals - or central tiles - can be considered as geometrical representations of self-similar processes. Their topological properties are related to several questions in number theory, ergodic theory or quasi-crystals. Most of methods allowing studying the topological properties of fractals are connected to an ad-hoc description of the boundary of the tiles. Unfortunately, this description of the graph boundary is not uniform and prevents one to study the properties of families of fractals. In this talk, we investigate the connections between Rauzy fractals and discrete geometry, especially polygonal tilings induced by discrete plane. We will explain how these polygonal tilings can be generated thanks to placement rules inspired by a duality process. As an example of application, we will prove Rauzy fractal connectivity for the full family of Arnoux-Rauzy substitutions. This is a common work with Valerie Berthe and Timo Jollivet.

Tilings associated with shift radix systems

Wolfgang Steiner

Shift radix systems form a collection of dynamical systems depending on a parameter which varies in the d-dimensional real vector space. They generalize well-known numeration systems such as beta-expansions, expansions with respect to rational bases, and canonical number systems. In this talk, we discuss collections of fractal tiles defined by shift radix systems. For certain classes of parameters, they provide natural families of tiles for beta-expansions with (non-unit) Pisot numbers as well as for canonical number systems with (non-monic) expanding polynomials. In the unit and monic case respectively, they coincide with the well-known Rauzy fractals or fundamental domains of canonical number systems. The case of non-monic expanding polynomials is intimately related to the self-affine tiles with p-adic factors discussed in Jörg Thuswaldner's talk.

Under some algebraic conditions on the parameter, the tiles provide multiple (weak) tilings and in many cases, e.g. for the dense family of all rational parameters, even (weak) tilings of the d-dimensional real vector space. Here, we mean by weak tiling that the tiles are compact, cover the space, and the interiors of distinct tiles do not intersect. The obtained weak tilings can have a complicated structure: They may consist of tiles having infinitely many different shapes, the tiles need not be self-affine (or graph directed self-affine), and we do not know whether the boundary of the tiles has zero measure.

Number systems and tilings with p-adic factors

Joerg Thuswaldner

Many results are known about tilings induced by “integral self-similar” tiles T. These tiles are defined as fixed point of the set equation $AT = T + D$ where A is a $d \times d$ integer matrix and $D \subset \mathbb{Z}^d$. We consider a more general setting. Indeed, we allow A to be a matrix with rational entries. It turns out that in this case the proper space for the tiles is a d-dimensional space times certain P-adic factors. The prime ideals P determining these factors are related to the root β of the minimal polynomial of A. Our main result is a tiling theorem which generalizes a result of Lagarias and Wang for the integer case to rational matrices. This result has consequences for tilings related to so-called shift radix systems. These objects are discussed in Wolfgang Steiner's talk.

Arithmetics of self-similarities of Finitely Generated Uniformly Discrete Sets

Jean-Louis Verger-Gaugry

Canonical cut-and-project schemes associated with uniformly discrete sets $\Lambda \subset \mathbb{R}^n$, $n \geq 1$, such that $m := rk\mathbb{Z}[\Lambda - \Lambda]$, $m \geq 1$, is finite are studied. The self-similarities λ of Λ are algebraic integers of degree d with d dividing m, and each of them, associated with a "center", provides a Jordan decomposition of the ambiant space. We discuss the various types of collections of centers which can be encountered. Given such a c-a-p scheme, the internal space is the product of two spaces, the first one (shadow space) deduced from the real and complex embeddings of the number field $\mathbb{K} = \mathbb{Q}(\lambda)$ and the $\mathbb{R}$-linearization $\mathbb{R}[\Lambda]$ of Λ, and the second one deduced from the space of relations between the generators of the basis in the Jordan decomposition. The orthogonal projection of the lattice of the c-a-p scheme contains $\mathbb{Z}[\Lambda - \Lambda]$. The c-a-p scheme is endowed with an Euclidean structure given by a real trace-like symmetric bilinear form (Ideal Lattices - E. Bayer) for which the two spaces (factors) of the internal space are orthogonal. Links between sphere packings, cluster geometries in tilings and collections of "centers" are evoked in this context.

J-L Verger-Gaugry, On self-similar finitely generated uniformly discrete (SFU-) sets and sphere packings, in Physics and Number Theory, IRMA Lectures inMathematics and Theoretical Physics Vol. 10, Ed. L. Nyssen, European Mathematical Society (2006), 39.78. 1

Coincidences of lattices and beyond

Peter Zeiner

Coincidence site lattices, i.e. lattices $\Gamma \cap R\Gamma$, where R is a linear isometry, are a well known tool in the analysis of grain boundaries. Several generalizations have been investigated as well, like coincidences of modules and colour coincidences. In this talk, we want to concentrate on some other generalizations. On the one hand, we generalize the linear isometry R to affine isometries. On the other hand, we generalize Γ by considering shifted lattices and modules, i.e. sets $t + \Gamma$, where $t \notin \Gamma$, and more generally multilattices and "multimodules", i.e. finite unions of translates of a fixed lattice or module, respectively. We point out the connection with colourings and discuss applications to the problem of solving the coincidence problem of sublattices of lattices, whose coincidence problem has already been solved.