Holographic superconductors

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Prelude – big picture motivation

- The AdS/CFT correspondence has been quantitatively useful in understanding the strongly coupled quark gluon plasma.
- This approach works despite using $\mathcal{N} = 4$ Super Yang Mills theory, with many colors, as an uncontrolled approximation to QCD.
- In condensed matter setups, many strongly coupled systems of interest. There is a wealth of experimental data.
- These systems can exhibit quantum criticality (especially in 2+1 dimensions). Description has an underlying conformal invariance
  $\Rightarrow$ Ideal for AdS/CFT!

Structure of this talk

Part I: Review of superconductivity

1. Conventional superconductors (BCS theory, Landau-Ginzburg)
2. Unconventional superconductors (High T_c, heavy fermions)

Part II: An AdS/CFT superconductor

1. A black hole instability
2. Computation of the conductivity
3. Magnetically induced currents: London equation

Part III: p wave superconductivity

1. A Yang-Mills black hole instability
2. Pseudogap and breaking time reversal invariance

Part I: Review of superconductivity

1. Basic experimental facts
2. BCS theory
3. Landau-Ginzburg theory
4. Unconventional superconductors: High T_c.
5. Unconventional superconductors: Heavy fermions.

Basic experimental facts

- Superconductivity occurs in certain materials when cooled down below a critical temperature: $T < T_c$.
- The DC ($\omega = 0$) conductivity is infinite.
- There is an energy gap E_g in the system. Processes at energies less than E_g are dissipationless.
- Magnetic fields are expelled from superconductors (Meissner effect)

$$\nabla^2 H = \frac{1}{\lambda^2} H \,.$$

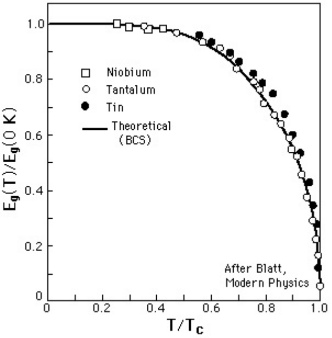
- This relation can be obtained from the London equation

$$j = -\frac{1}{\lambda^2} A \,.$$

Reveals the key physics: diamagnetic currents.

BCS theory (1957)

- Superconductors are charged superfluids, condensation of Cooper pairs of electrons: $\mathcal{O} \sim \psi^\dagger \psi^\dagger$.
- Repulsive Coulomb force between electrons is screened. Attractive force due to the lattice phonons. Instability of Fermi surface.
- Many predictions, for instance $E_g(0) \approx 3.5 T_c$.
- Data:



Landau-Ginzburg theory (1950)

- Phenomenological effective field theory for superconductors close to $T = T_c$.
- Field theory for $\varphi = \langle \mathcal{O} \rangle$ coupled to an electromagnetic field.

$$\Delta f_{\text{L-G}} = \frac{1}{2m^*}|(\nabla + iqA)\varphi|^2 + a|\varphi|^2 + \frac{b}{2}|\varphi|^4 \,.$$

- Experimental quantities: e.g. superconducting coherence length and magnetic penetration depth

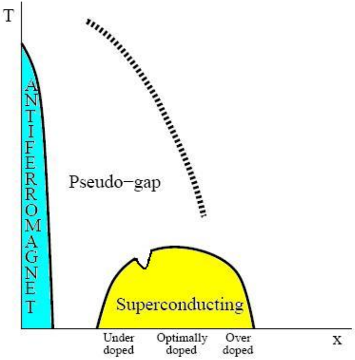
$$\xi \sim \frac{1}{(am^*)^{1/2}} \,, \qquad \lambda \sim \frac{(m^*)^{(1/2)}}{q^2 \varphi_0} \,.$$

- Difference between type I and type II superconductors depends on λ/ξ.

The High T_c cuprate superconductors (1986)

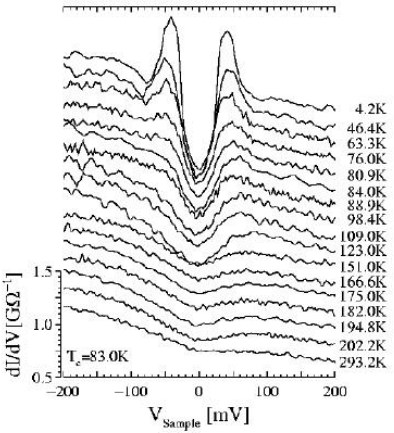
- Copper oxide compound La_2CuO_4: (antiferromagnetic) insulator.

  Hole doping: substitute some of the *La* with *Sr*, to obtain $La_{2-x}Sr_xCuO_4$, removing available conduction electrons.
- Phase diagram:



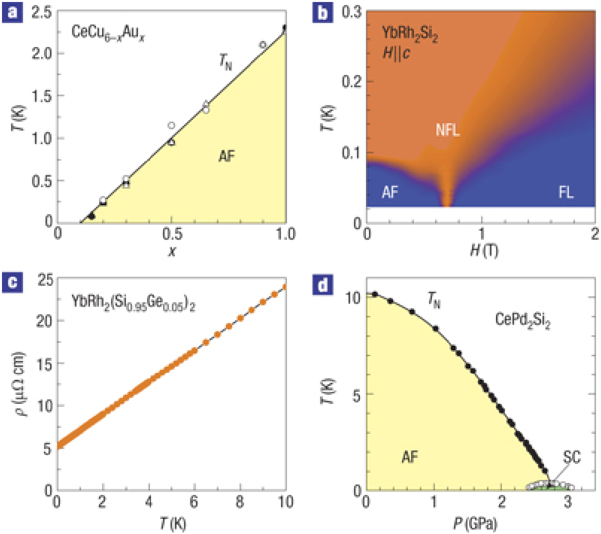
The pseudogap in $Bi_2Sr_2CaCu_2O_{8+\delta}$

- Depletion of states at Fermi energy continuous across $T_c \approx 83K$.



[Renner, Revaz, Genoud, Kadowaki, Fischer '98]

Heavy fermion compounds



Part II: An AdS/CFT superconductor

1. Ingredients for a holographic superconductor
2. A black hole instability
3. Hairy black holes
4. Computation of the conductivity
5. Magnetically induced currents: London equation

Ingredients for a holographic superconductor

- What are the minimal ingredients to capture superconducting physics in the bulk?
  - Continuum theory $\Rightarrow$ have $T^{\mu\nu}$ $\Rightarrow$ need bulk g_{ab}.
  - Conserved charge $\Rightarrow$ have J^{μ} $\Rightarrow$ need bulk A_a.
  - 'Cooper pair' operator $\Rightarrow$ have $\mathcal{O}$ $\Rightarrow$ need bulk ψ.

- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2 \,.$$

- To get a critical temperature, need a scale. Will work at constant charge density ρ. By dimensional analysis $T_c \propto \sqrt{\rho}$.

A black hole instability

[Gubser '08]

- Finite temperature and charge density ⇒ AdS Reissner-Nordstrom black hole

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)\,,$$

where

$$g(r) = r^2 - \frac{1}{r}\left(r_+^3 + \frac{\rho^2}{4r_+}\right) + \frac{\rho^2}{4r^2}\,,$$

- Scalar potential

$$A_0 = \rho\left(\frac{1}{r_+} - \frac{1}{r}\right)\,,$$

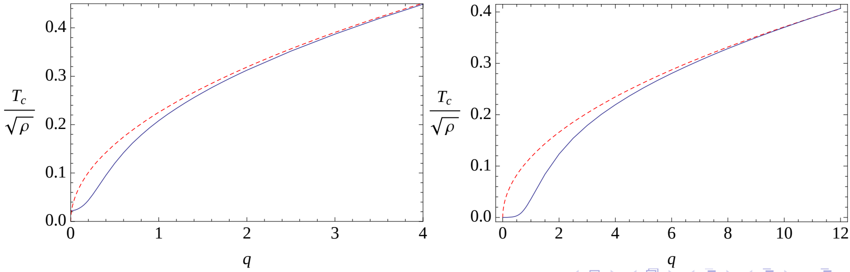
- Hawking temperature

$$T = \frac{12r_+^4 - \rho^2}{16\pi r_+^3}\,.$$

A black hole instability

- For simplicity: $V(\psi) = -2\psi^2$. The dual operator $\mathcal{O}$ can have conformal dimension $\Delta = 1$ or $\Delta = 2$.
- Reissner-Nordstrom-AdS will describe the 'normal phase' of the theory. However it becomes unstable against fluctuations in ψ for $T < T_c \sim \sqrt{\rho}$. Intuitively

$$m^2_{\text{eff.}} \sim m^2 - q^2 A_0^2 \,.$$



Hairy black holes

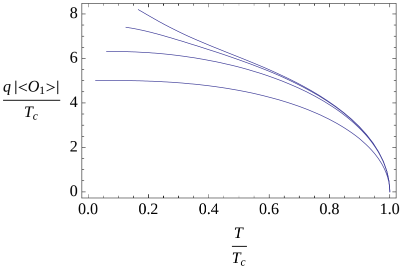
[Hartnoll, Herzog, Horowitz '08 + in progress]

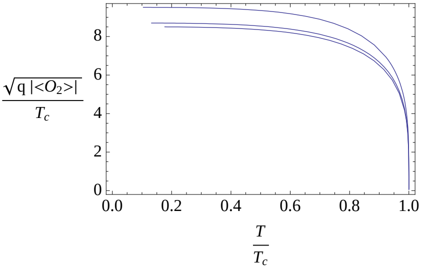
- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right) ,$$

$$A = \phi(r)dt , \qquad \psi = \psi(r) .$$

- Solve numerically. Can obtain $\langle \mathcal{O} \rangle$:





- Compare 8 to ~ 3.5 for BCS and ~ 7 for High-T_C.

Electrical conductivity - some experimental expectations

- BCS theory prediction and some data

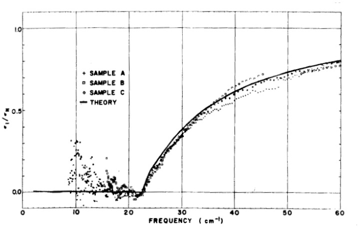
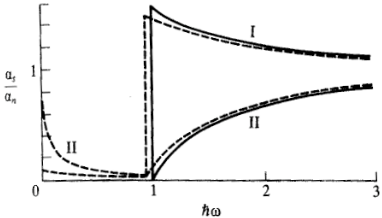


Fig. 2. Measured and calculated values of the conductivity ratio σ_1/σ_n of three superconducting lead films as a function of the photon frequency. [After Palmer (39).]

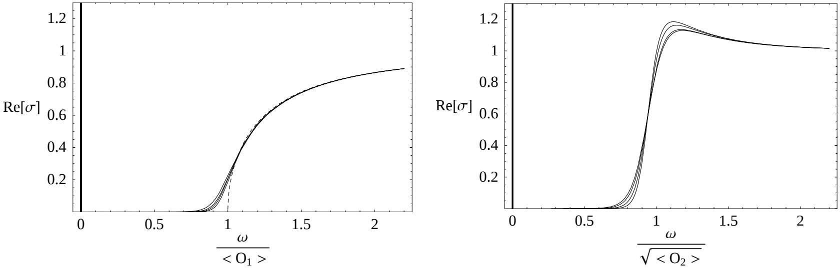
- A figure from a texbook (Tinkham)



Electrical conductivity - AdS/CFT

[Hartnoll, Herzog, Horowitz '08 + in progress]

- Let's focus on the probe limit ($q \to \infty$) for simplicity.
- We computed the conductivity (2 cases). At $T \sim 0$:



- If the gap is 2Δ then we found that

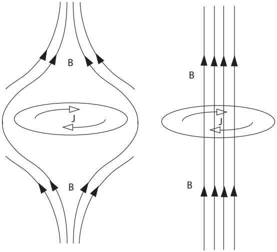
$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\Delta/T}.$$

- Strongly suggests a 'pairing mechanism'. What is it?

Magnetically induced currents

[Hartnoll, Herzog, Horowitz – in progress]

- First note that for a thin film, a magnetic field always penetrates



- It is hard to solve the full system with both a condensate and an external magnetic field. Even numerically, it requires PDEs.
- Simplifies in two limits: small magnetic field or small condensate.
- Small magnetic fields: Take the $B = 0$ solution and perturb by A_i.

Magnetically induced currents

[Hartnoll, Herzog, Horowitz – in progress]

- In the bulk at large radius

$$A_i = A_i^{(0)} + \frac{1}{r} A_i^{(1)} + \cdots .$$

- In AdS/CFT: $A_i^{(0)}$ will give the boundary background gauge potential (hence magnetic field) and $A_i^{(1)}$ gives the current $\langle J^i \rangle$.
- At low temperature we were able to show that

$$\langle J_i \rangle = -q \langle \mathcal{O}_1 \rangle A_i .$$

  This is the London equation and determines the magnetic penetration depth when the theory is coupled to an external photon.
- [some related work appeared last week: Maeda and Okamura]

Part III: p wave superconductivity

1. A Yang-Mills black hole instability
2. Pseudogap and breaking time reversal invariance

Yang-Mills black hole instability

[Gubser '08]

- The Reissner-Nordstrom-AdS black hole is also a solution of $SU(2)$ Yang-Mills theory in AdS, by taking a charge in $U(1) \subset SU(2)$

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} \,.$$

- The W bosons are charged fields. Similar to the scalars before, they have an instability and condense at low temperatures.
- The W bosons are not scalars! The system will be dual to a theory where the 'Cooper pairs' are not spin singlets, in particular, to 'p wave' superconductors [cf. Helium 3 and Strontium Ruthenate].
- For simplicity, work in probe limit $g \to \infty$.

Black holes with Yang-Mills hair

[Gubser and Pufu '08; Hartnoll and Roberts '08]

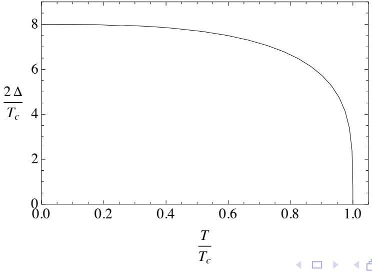
- The ground state of this system breaks rotational invariance. Consider here a phase in which rotational invariance is preserved.

$$A = \phi(r)\, dt\, \tau^3 + w(r)\, (dz\tau^- + d\bar{z}\tau^+)\,.$$

Invariant under combined spatial and gauge rotation

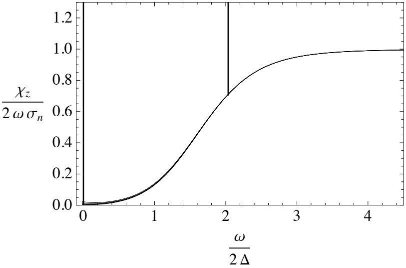
$$z \to e^{i\theta} z\,, \qquad \tau^{\pm} \to e^{\pm i\theta}\tau^{\pm}\,.$$

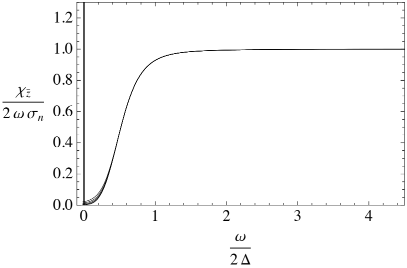
- Solution gives order parameter as a function of T:



Pseudogap and breaking of time reversal invariance

- To look for a gap, compute the spectral function of the electric current at $T \sim 0$:





- This Yang-Mills condensate spontaneously breaks time reversal invariance. A consequence is a nonzero Hall conductivity:

$$H = \lim_{\omega \to 0} \mathrm{Re}\, \sigma_{xy} \approx 0.36\, \sigma_n \,.$$

Summary

1. Conventional superconductivity is well described by BCS theory. Many materials of theoretical and technological interest are not conventional.
2. It is possible to construct a strongly coupled theory that has a low T superconducting phase using AdS/CFT.
3. We computed the gap as function of T and the frequency dependent conductivity.
4. Diamagnetic currents are induced by an external magnetic field.
5. It is possible to construct 'p wave' superconductors in AdS/CFT.