Holographic Monopole Catalysis of Baryon Decay

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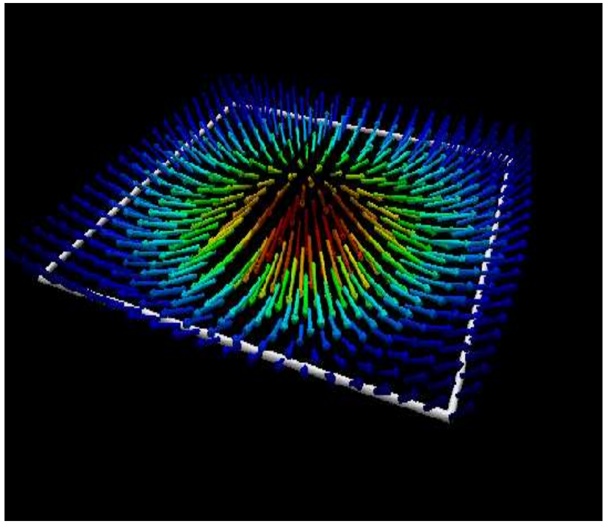


Based on arXiv:0804.1326, done with K. Lee, C. Park, and H.U. Yee

I. Introduction

- Gauge/String duality has opened up a new paradigm to understand both gauge theory and string theory. (Maldacena, Witten, Gubser-Klebanov-Polyakov, ...)
- Using the gauge/string duality, there have been attempts to understand QCD. (Witten, Sakai-Sugimoto, EKSS, DaRold-Pomarol, HIY, ...)
- They are called holographic QCD, which is a 5D gauge theory with warped geometry of hadrons like baryons, mesons, and glueballs.
- QCD is a gauge theory that describes strong interactions with quarks and gluons as fundamental degrees of freedom:

$$\mathcal{L} = -\frac{1}{4} {F^a_{\mu\nu}}^2 + \bar{q}_i \left(i \not{D} - M_q \right) q_i + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \, . \tag{1}$$

- Because of asymptotic freedom, hQCD is therefore more proper in describing strong interactions at low energy.

- The asymptotic freedom, predicted by QCD, is well tested and QCD was thus accepted as the theory of strong interactions (2004):

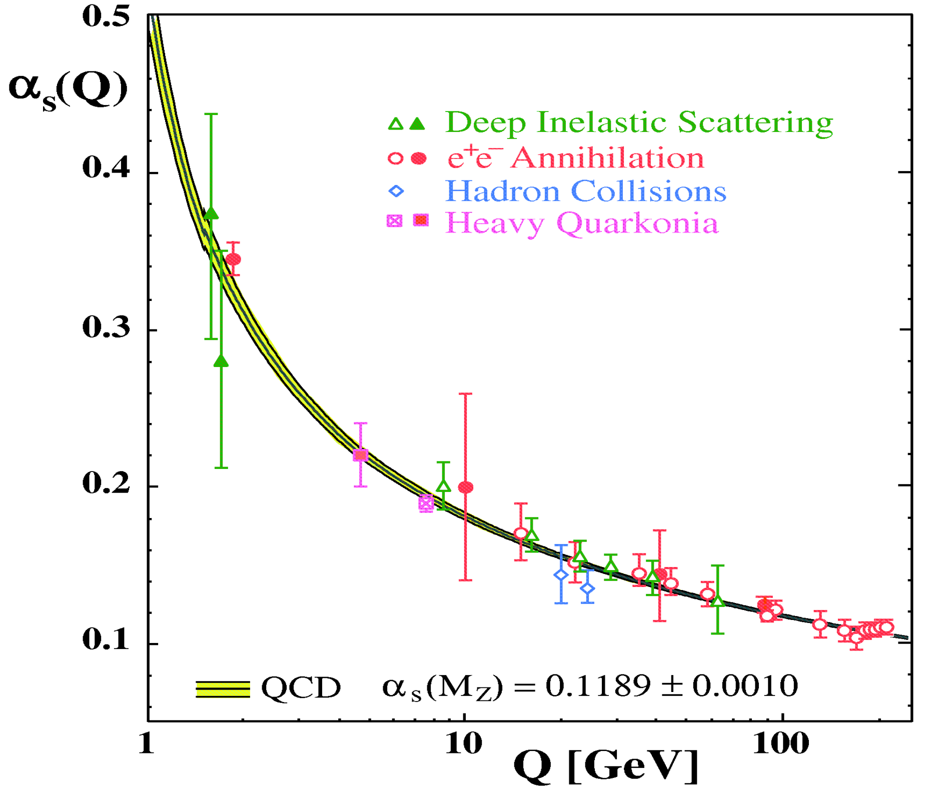
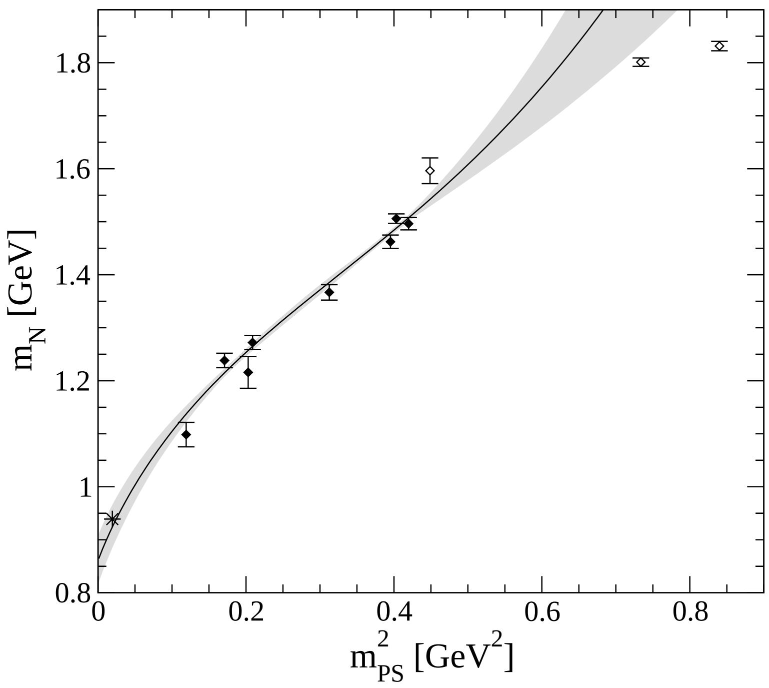


Figure 1: Running coupling (Bethke et al, 2006)

- Solving QCD is hard, since it's strongly coupled and has no expansion parameter.
- Lattice Calculation of Nucleon Mass (QCDSF 2008)



- We need a theory whose degrees freedoms are mesons and baryons. Holographic QCD is such a theory, based on gauge/string duality.

Key Features of holographic QCD

- (New) Vector Meson Dominance and Instantonic nature of Baryons.
- If we expand the nonnormalizable photon field (or EW fields for EW form factors) in terms of the normalizable vector meson ψ_{2k+1} of mass m_{2k+1} as

$$A(q,w) = \sum_k \frac{g_{v^{(k)}}\psi_{(2k+1)}(w)}{Q^2 + m_{2k+1}^2}\,, \qquad (2)$$

the EM form factors of Nucleon become

$$F_1(Q^2) \;=\; \sum_{k=1}^{\infty}\left(g_{V,min}^{(k)} Q_{\rm em} + \frac{5}{3} g_{V,mag}^{(k)}\,\tau^3\right)\frac{\zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2}\,, \qquad (3)$$

$$F_2(Q^2) \;=\; \sum_{k=1}^{\infty}\frac{g_2^{(k)}\zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2}\,. \qquad (4)$$

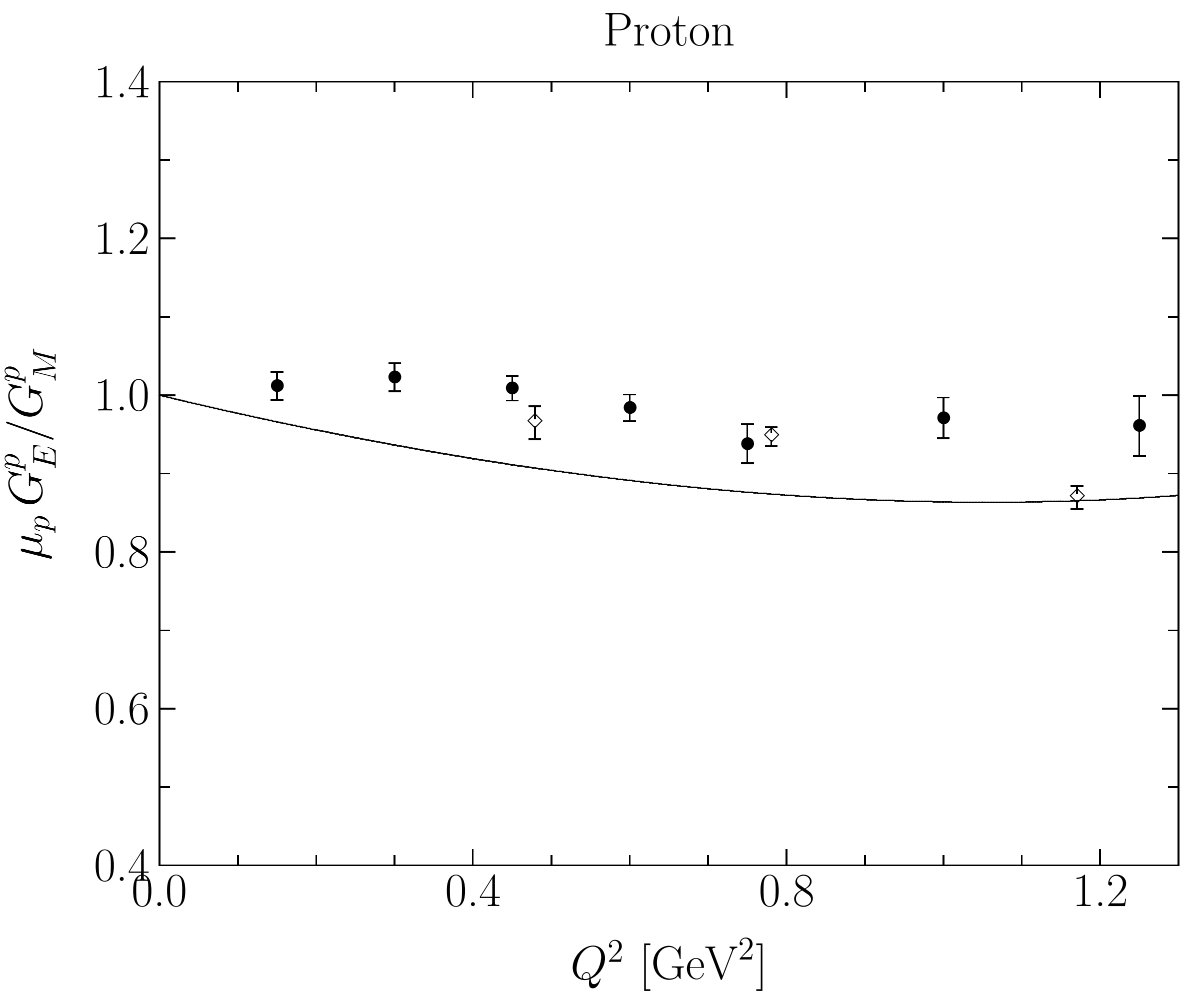


Figure 2: Open circles are JLab data (1999) and closed circles are from Walker (1993). The solid curve are the HRYY calculations in SS model

- We plot the ratio with the dipole form factors, $G_D = 1/(1+Q^2/0.71)^2$.

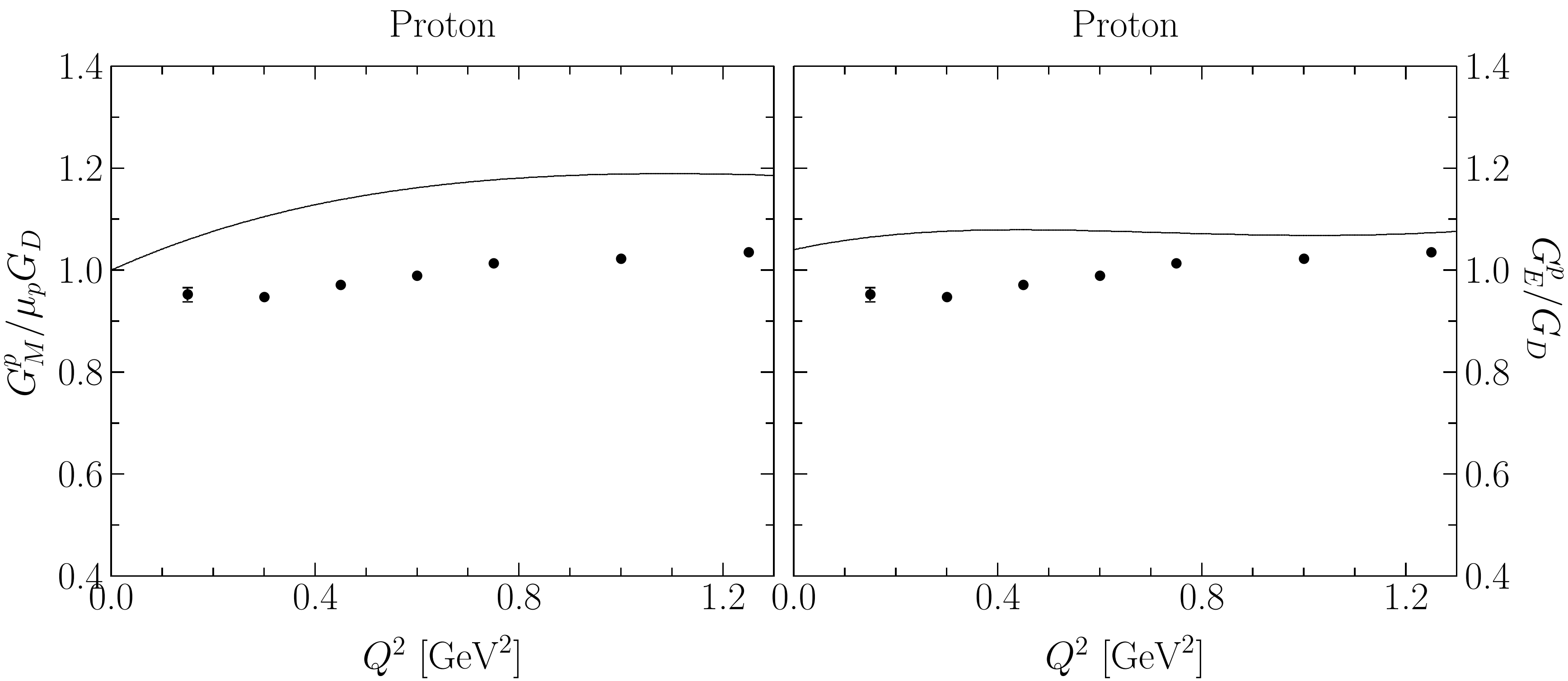


Figure 3: The closed circles are data from R. C. Walker *et al.* (1994) and Jones et al (2000).

- New sum rules due to instantonic nature of baryons (HRYY 2007):

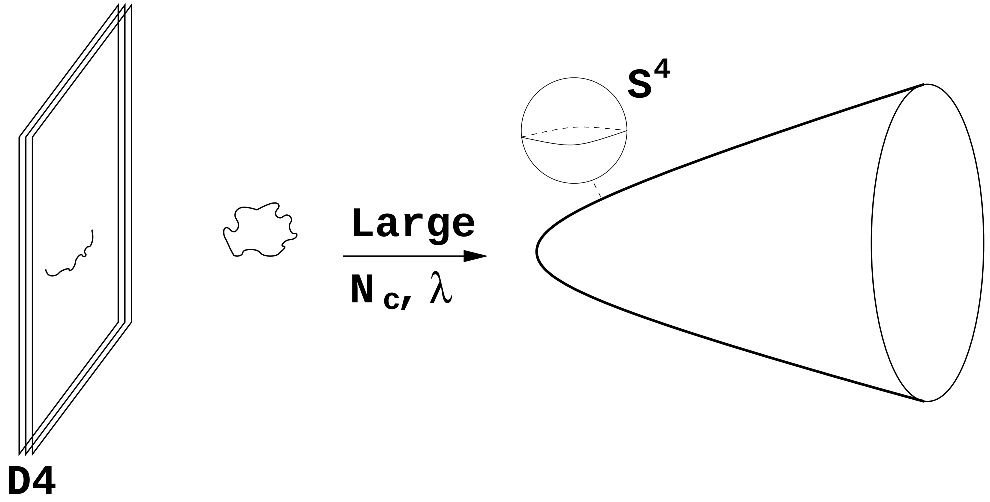
$$\mu^p_{\rm an} + \mu^n_{\rm an} = 0 \;\; (1.79 - 1.91 = 0.12\mu_N, \;\text{experiments}),$$
$$d_p + d_n = 0 \;\; (0.26 - 0.21 = 0.05\,\bar{\theta}\,e\cdot\text{fm}, \;\text{Shintani et al 07})\,.$$

- New relations among low energy parameters (some of them, known phenomenologically).

- Holographic QCD gives a new insight on QCD process or EW process of hadrons. For instance the baryon number violation under magnetic monopoles or by sphalerons is unified as a single process in hQCD, leading to a new mechanism for the baryon number violation, as we will see in the next slides.

II. . Baryons as 5D Instanton Solitons in hQCD

- N_c stack of $D4$ brane over $R^3 \times S^1$ describes pure $SU(N_c)$ YM. (Witten '98)

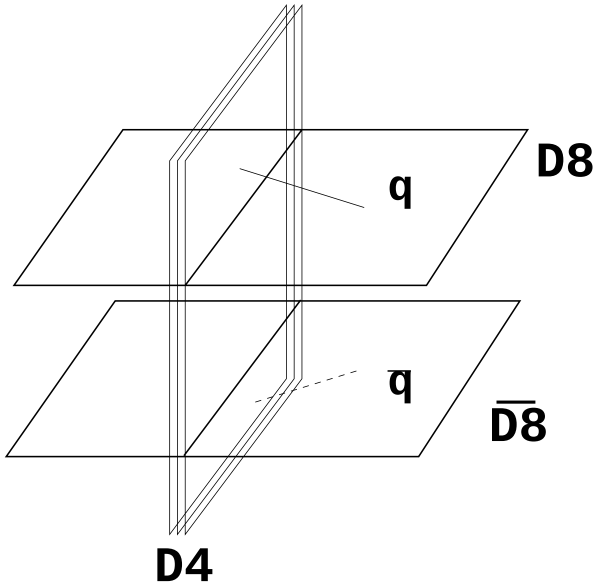


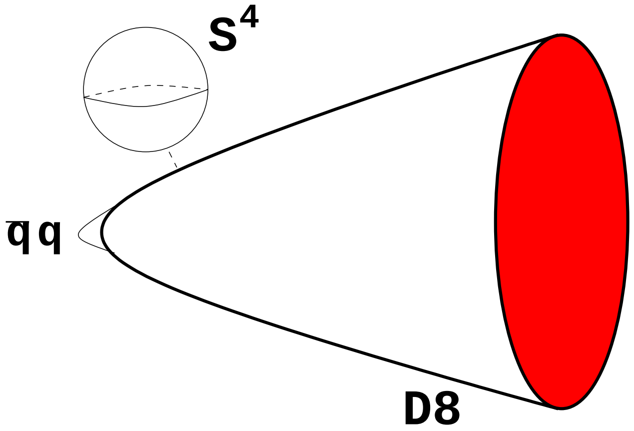
$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

with $e^\phi = g_s(\frac{U}{R})^{3/4}$, $\mathrm{d}C_3 = \frac{2\pi N_C}{V_4}\epsilon_4$, and $f(U) = 1 - \frac{U_{KK}^3}{U^3}$

- The ratios of glueball mass agree with the lattice data rather well. (Csaki et al ’99; Brower et al ’99)

- Adding flavors was done by Sakai-Sugimoto (2004) (Cf. Karch and Katz in $D3$-$D7$, probe approximation,’02).





- Spontaneous chiral symmetry breaking is geometrically realized:

$$SU(N_F)_L \times SU(N_F)_R \mapsto SU(N_F)_V \, . \tag{5}$$

- Effective action on D8 is a $U(N_F)$ gauge theory,

$$
\begin{aligned}
S_{D8} &= -\mu_8 \int \mathrm{d}^9 x \, e^{-\phi} \sqrt{-\mathbf{det} \left(g_{MN} + 2\pi\alpha' F_{MN} \right)} \\
&\quad + \mu_8 \int \sum C_{p+1} \wedge \mathrm{Tr} \, e^{2\pi\alpha' F} \, ,
\end{aligned}
$$

- The gauge fields contain pions and whole tower of vector mesons:

$$
A_\mu(x,z) = \alpha_\mu(x)\psi_0(z) + \beta_\mu(x) + \sum_{n\geq 1} B_\mu^{(n)} \psi_n(z) \, , \qquad (6)
$$

where with $\xi = \exp(i\pi(x)/f_\pi)$

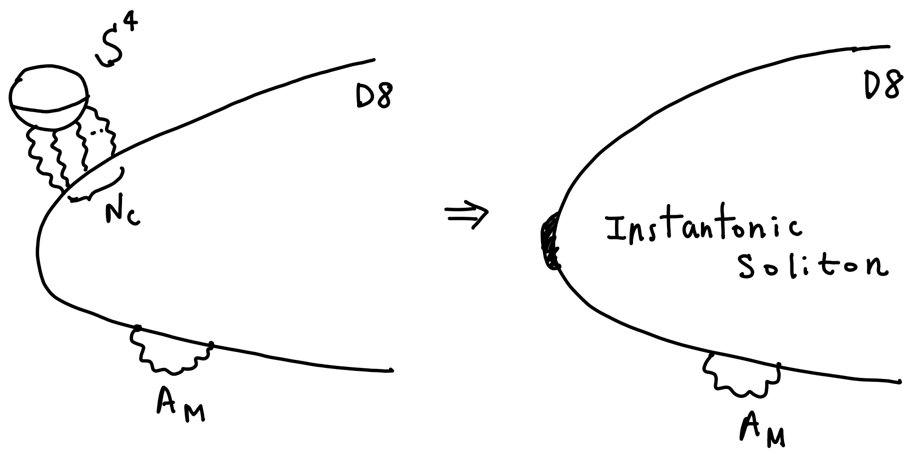
$$
\alpha_\mu = \left\{ \xi^\dagger, \partial_\mu \xi \right\} , \quad \beta_\mu = \left[\xi^\dagger, \partial_\mu \xi \right] \, . \qquad (7)
$$

- The effective action successfully describes all the interactions of pions and vector mesons, including the Skyrme term.

- What are baryons in hQCD? It must be solitons:

$$m_{\text{baryon}} \sim N_c \,. \tag{8}$$

- In SS model, D4 brane wrapping S^4 is the baryon vertex (Witten).
- $D4$ brane becomes instanton in $D8$ (Douglas '95).



Baryons are instanton solitons in hQCD

- In 5D YM there is a topologically conserved current, $d\,^*J = 0 = DF$,

$$J^M = \frac{1}{24\pi^2}\epsilon^{MNLPQ}\,\mathrm{tr}F_{NL}F_{PQ}\,. \tag{9}$$

- One can define a 3D current

$$B^\mu = \frac{1}{8\pi^2}\int \mathrm{d}z\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\nu\rho}F_{\sigma z}\,. \tag{10}$$

- The current conservation is guaranteed when the sphaleron vanishes

$$\partial_\mu B^\mu = \int_{-\infty}^{\infty}\mathrm{d}z\partial_\mu J^\mu = -\int_{-\infty}^{\infty}\mathrm{d}z\partial_5 J^5 = \frac{1}{24\pi^2}\mathrm{Tr}F\tilde{F}|_{\infty}^{-\infty}\,, \tag{11}$$

where $J^5 = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(F_{\mu\nu}F_{\alpha\beta}\right)$.

- In the gauge $A_z = 0$ one may write $U = \exp(2i\pi/f_\pi)$

$$A_\mu(x,z) = U^{-1}\partial_\mu U \psi_0(z) + \sum_{n\geq 1} B_\mu^{(n)}\psi_n(z)\,. \tag{12}$$

  Then the current becomes the Skyrme current

$$B^\mu = \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}U^{-1}\partial_\nu U U^{-1}\partial_\rho U U^{-1}\partial_\sigma U \tag{13}$$

- Unlike Skyrme model we know that it must be the baryon current.

- We first note that the $D4$ brane, wrapping S^4, couples to the U(1) quark-number potential with N_C unit via the Chern-Simons term,

$$S_{CS}^{D4} = \int_{R\times S^4} C \wedge e^{F/2\pi} \sim N_c \int_R A\,. \tag{14}$$

The D4 brane, wrapping S^4, must carry one unit of baryon number.

- From $D8$ point of view the charge density, carried by the D4 brane, is nothing but the instanton density

$$S_{CS}^{D8} = \frac{N_c}{24\pi^2}\int_{M^4\times R}\omega_5(A)\,, \quad \rho(x) = \frac{\delta S^{D8}}{\delta A_0(x)} = \frac{N_c}{24\pi^2}\int \mathrm{d}z F\tilde{F}\,.$$

- Baryons are therefore realized as instantons, which is a generic feature of hQCD.

III. Holographic Monopole Catalysis

- Fermions numbers are often not conserved under the external fields due to level crossing. Well known examples are violation of axial fermion number due to instantons and the B+L non-conservation due to sphalerons. ('t Hooft 1976)

$$\Delta N_F = q \tag{15}$$

- Baryon number is not conserved under magnetic monopole. (Rubakov '81; Callan '82)
- In the Skyrme model the baryon number is topologically conserved. How can it be broken?
- When skyrmion passes through the magnetic monopole, it unwinds the topological number due to the angular momentum barrier at the core. (Callan-Witten 1983)

- In hQCD monople catalysis is easily understood, since the instanton number is not conserved in the presence of monopole:

$$DF \neq 0 \longrightarrow d^*J \neq 0 \, . \tag{16}$$

- In SS model the magnetic monopole background gives a BC for the gauge fields, for monopoles $A^{EM} = -\frac{i}{2}\,(1 - \cos\theta)\, d\varphi$,

$$A(+\infty) = A(-\infty) = QA^{EM} , \quad Q = \mathrm{diag}\,(2/3, -1/3) \; . \tag{17}$$

- In a general background A_L and A_R with $\xi_\pm^{-1} = P\mathrm{exp}(-\int_0^{\pm\infty} A_z)$ and $\xi_+^{-1}\xi_- = U$ we write

$$A_\mu(x,z) = A_{L\mu}^{\xi_+}(x)\psi_+(z) + A_{R\mu}^{\xi_-}(x)\psi_-(z) + (\text{excited modes}),$$

where $A_{L\mu}^{\xi_+} = \xi_+ A_{L\mu}\xi_+^{-1} + \xi_+\partial_\mu\xi_+^{-1}$, $A_{R\mu}^{\xi_-} = \xi_- A_{R\mu}\xi_-^{-1} + \xi_-\partial_\mu\xi_-^{-1}$.

- Then the baryon current becomes

$$B^\mu = \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\,\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right)$$
$$-\frac{1}{8\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\,\partial_\nu\,\left(U^{-1}A_{L\alpha}\partial_\beta U + A_{R\alpha}U^{-1}\partial_\beta U - U^{-1}A_{L\alpha}U A_{R\beta}\right)$$
$$-\frac{1}{8\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\left(\partial_\nu A_{L\alpha}\,A_{L\beta} + \frac{2}{3}A_{L\nu}A_{L\alpha}A_{L\beta} - (L\leftrightarrow R)\right)\,.$$

- The background gauge field that satisfies BC becomes with normalizable mode $\tilde{A}$

$$A = QA^{EM} + \tilde{A} \tag{18}$$

- The 5D current is no longer conserved, since

$$d\mathrm{Tr}(F\wedge F) = 2\mathrm{Tr}(DF\wedge F) = -4\pi i\mathrm{Tr}(QF_{tz})\wedge\delta^3(\vec{x})dx^1\wedge dx^2\wedge dx^3$$

- Then the 4D baryon currents is not conserved under external fields

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left(\mathrm{Tr} F_L \tilde{F}_L - \mathrm{Tr} F_R \tilde{F}_R\right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi}\int_{-\infty}^{+\infty} dz\, \mathrm{Tr}\left(Q F_{tz}\right).$$

The first term giving the sphaleron contribution and the second term is the effect of magnetic monopoles.

- Upon the integration the magnetic monopole effects on the baryon number violation becomes

$$\begin{aligned}\partial_\mu B^\mu &= -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{Tr}\left(Q A_t\right)\Big|_{-\infty}^{+\infty} \\ &= -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\left[\mathrm{Tr}\left(Q U^{-1}\partial_t U\right) + \mathrm{Tr}\left(Q U^{-1} A_{Lt} U\right) - \mathrm{Tr}\left(Q A_{Rt}\right)\right] \quad .\end{aligned}$$

- The first term is the usual monopole catalysis but the second and third term are new mechanism of baryon number violation due to the (chiral) chemical potential in the presence of magnetic monopole.

- For the monopole catalysis of instanton-baryon decay, $U = \exp(2i\pi/f_\pi)$ we have from the first term

$$\partial_\mu B^\mu = -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{Tr}\left(Q\sigma^3\right)\frac{2i(\partial_t\pi^0)}{F_\pi} = \frac{(\partial_t\pi^0)}{\pi F_\pi}\delta^{(3)}(\vec{x}) \quad , \qquad (19)$$

- Upon integration, when the baryons passes the core of the monopole, we get

$$\frac{dB}{dt} = \frac{1}{\pi f_\pi}(\partial_t\pi^0)\,, \qquad (20)$$

which reproduces the Callan-Witten effect.

IV. Conclusion and Outlook

- Baryons are realized as 4D instanton solitons in holographic QCD, which are made of pions and whole towers of vector mesons.
- The effective chiral Lagrangian for baryons is uniquely determined up to the Pauli term.
- New VMD is a key feature of holgraphic QCD: Form factors, $\cdots$.
- Since it is based on principle, it gives relations to low energy parameters of hadrons. Low energy parameters of hadrons are unified into a few parameters in 5D:
  1. Magnetic moments of baryons and g_A: $g_A \sim \mu_{\rm an}$
  2. Various couplings with vector mesons: $g_{\omega NN} \approx N_c\, g_{\rho NN}, \cdots$.
- Furthermore, it has model-independent predictions, insensitive to $1/N_c$ corr: New sum rules due to the instanton nature of baryons:

$$\mu_{\rm an}^p + \mu_{\rm an}^n = 0, \quad d_n + d_p = 0. \tag{21}$$

- The instanton soliton has the 3D baryon current, given as

$$B^\mu = \frac{1}{8\pi^2}\int \mathrm{d}z \epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\nu\rho}F_{\sigma z}\,. \tag{22}$$

- In hQCD the baryon number is conserved ($\partial_\mu B^\mu = 0$) due to the Bianchi idensity, which holds if monopoles are absent:

$$\partial_M \epsilon^{MNOPQ}\mathrm{Tr}F_{NO}F_{PQ} = \epsilon^{MNOPQ}\mathrm{Tr}D_M F_{NO}F_{PQ} = 0\,. \tag{23}$$

- In hQCD we find a unified and new formula for baryon number violation in the presence of magnetic monopole or sphaleron:

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left[\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(F_{\nu\nu}F_{\alpha\beta}\right)\Big|_R - \left(F_{\nu\nu}F_{\alpha\beta}\right)\Big|_L\right] - \frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{Tr}\left[QU^{-1}\partial_t U + QU^{-1}A_{Lt}U - QA_{Rt}\right]\,.$$

- The new mechanism of baryon number violation might be important in early universe.