Sasaki-Einstein Manifolds and Chern-Simons Quivers

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based on
0808.0904 [hep-th], 0808.0912 [hep-th]
with James Sparks

Recent Developments in String/M Theory
KIAS, Seoul, September, 2008

Outline

1. Motivations
2. Infinite families of AdS_4 backgrounds
3. Chern-Simons quivers and their moduli spaces
4. Examples of AdS_4/CFT_3 duals with $\mathcal{N} = 2$
5. Conclusions

Motivations

- Understand M2-branes and M-theory
- CFT_3 description of AdS_4 vacua in the "landscape"
- Extension of nice $\mathrm{AdS}_5/\mathrm{CFT}_4$ results to $\mathrm{AdS}_4/\mathrm{CFT}_3$
- Learn something about strongly coupled $\mathcal{N} = 2$ CFTs
- Possible condensed matter applications

From BLG to ABJM

BL/G model

- New 3D Lagrangian with $\mathcal{N} = \mathbf{8}$ supersymmetry
- Proposed to describe M2-branes in flat $\mathbb{R}^{\mathbf{8}}$ space
- Slightly mysterious... e.g. $\mathbf{N = 2}$ M2-branes
- 3-algebras, Lorentzian signature, extensions to $\mathcal{N} = \mathbf{6, 5, 4}$

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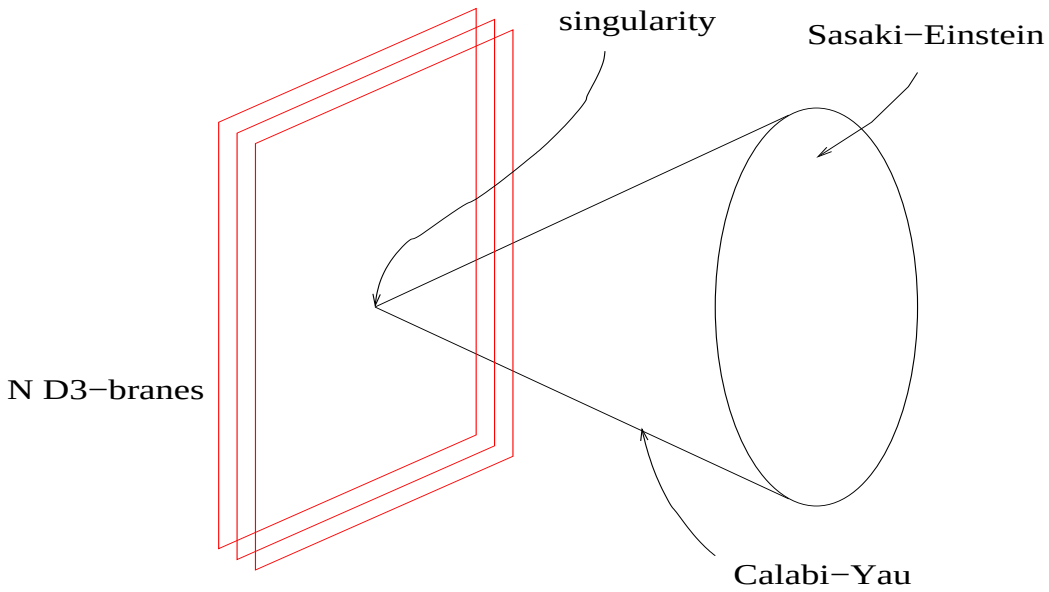
ABJM model

- New 3D Lagrangian with manifest $\mathcal{N} = 6$ supersymmetry
- $U(N)_k \times U(N)_{-k}$ Chern-Simons quiver gauge theory
- $k = 1, 2$ susy enhancement to $\mathcal{N} = 8$; $N = 2$ recover BLG
- Gravity duals: $AdS_4 \times S^7/\mathbb{Z}_k$ M-theory backgrounds
- First check: VMS of ABJM model is $\mathbb{C}^4/\mathbb{Z}_k \cong C(S^7/\mathbb{Z}_k)$

D3-branes at Calabi-Yau cone singularities $\mathbf{C(Y_5)}$



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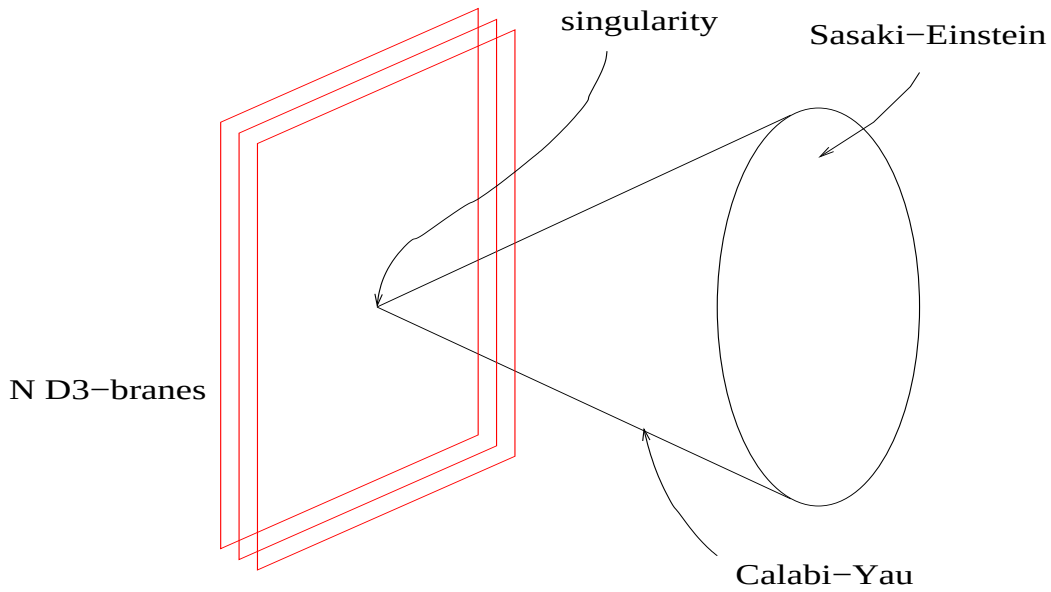


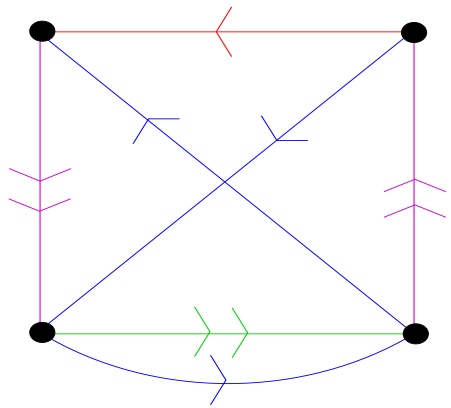
Gravity solutions

- Near-horizon solution: $\mathrm{AdS}_5 \times \mathbf{Y_5}$, $\mathbf{Y_5}$ Sasaki-Einstein 5-manifolds
- Preserve $\mathcal{N} = 1$ supersymmetry (8 supercharges)

Infinite family of explicit Sasaki-Einstein 5-manifolds:
$\mathbf{Y^{p,q}}$ [Gauntlett,Martelli,Sparks,Waldram] (later generalized to $\mathbf{L^{a,b,c}}$)

Gauge theories: quivers

- Constructed from microscopic open string d.o.f. on D3-branes
- Best understood examples: $\mathcal{N} = 1$ "toric" quiver gauge theories
- $\mathbf{G = \prod_{i}^{n} U(N_i)}$ with bi-fundamental chiral fields and superpotential



node $= \mathbf{U(N_i)}$ arrow $= \mathbf{(\bar{N}_i, N_j)}$ chiral $\mathbf{X_{ij}}$ **W**= polynomial in $\mathbf{X_{ij}}$

Checks of AdS_5/CFT_4

Classical moduli spaces of susy vacua (VMS)

- F-terms: $\mathcal{Z} = \{\partial W/\partial X_{ij} = 0\}$
- D-terms/mod gauge symmetries: $\mathcal{M}_{4d} = \mathcal{Z}//U(1)^{n-1} \cong C(Y_5)$
- VMS of point-like D3 transverse to $C(Y_5)$ is $C(Y_5)$

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- Similarly: field theory on M2-branes probing an 8-fold conical singularity $C(Y_7)$, must have $C(Y_7)$ as (part of) its VMS
- I will come back to this in the second part

Checks of AdS_5/CFT_4

R and central charges at the IR fixed point

- These theories flow to strongly-coupled IR fixed points (SCFT)
- Exact NSVZ beta-functions vanish $\longrightarrow$ constraints on R-charges
- Residual ambiguities fixed by a-maximization [Intrilligator,Wecht]
- Determines exact R-charges of bi-fundamentals and central charge **a**
- On the gravity side these may be computed from volume of $\mathbf{Y_5}$ and supersymmetric sub-manifolds
- Explicit metric not necessary [Martelli,Sparks,Yau]

$AdS_4 \times Y_7$ M-theory solutions

- 11-dimensional version of Freund-Rubin solutions
- Y_7 Einstein 7-manifolds of various types

$\mathcal{N}$	Y_7	Cone(Y_7)
1	weak G_2	Spin(7)
2	Sasaki-Einstein	Calabi-Yau
3	tri-Sasakian	hyper-Kähler
> 3	S^7/Γ	$\mathbb{C}^4/\Gamma$

- $\mathcal{N} = 1$: squashed $\tilde{S}^7$ is an example. Dual CS theory proposed by [Ooguri-C.S.Park]. More examples, e.g. Aloff-Wallach cosets $N^{k,l} = SU(3)/U(1)_{k,l}$. I will not discuss $\mathcal{N} = 1$
- $\mathcal{N} = 3$: tri-Sasakian metrics easy to derive. Examples of CS quivers duals proposed by [Jafferis-Tomasiello]. $\mathcal{N} = 3$ CS theories are rigid $\longrightarrow$ possible to show they are SCFT

$\mathcal{N} = 2$ families of $AdS_4 \times Y_7$ M-theory solutions

- Known examples of Sasaki-Einstein in **7D** until 2004:
  $M^{3,2}$ (a.k.a. $M^{1,1,1}$), $Q^{1,1,1}$ (close relatives of $T^{1,1}$ in **5D**), $V_{5,2}$

- Proposals for gauge theory duals (end '90s) $\longrightarrow$ problematic;
  but not Chern-Simons gauge theories

- [Gauntlett,Martelli,Sparks,Waldram] (2004) constructed explicit
  Sasaki-Einstein manifolds $Y^{p,k}(B_{2n})$ in any $D = 2n + 3$ dimension

- In $D = 7$: families $Y^{p,k}(B_4)$ – generalisations of $Y^{p,q}(\mathbb{C}P^1)$
  $B_4 = \mathbb{C}P^2, \mathbb{C}P^1 \times \mathbb{C}P^1, dP_m$ with $3 \leq m \leq 8$

- "Irregular" type: volumes are not rational multiples of volume of S^7.
  Not coset spaces

Toric Sasaki-Einstein manifolds $\mathbf{Y^{p,k}}$

- When $\mathbf{B_4 = \mathbb{C}P^2}$ and $\mathbf{B_4 = \mathbb{C}P^1 \times \mathbb{C}P^1}$ we have two families of toric SE manifolds, labeled by two integers $\mathbf{p, k}$
- Toric [isometry group $\supset \mathbf{U(1)^4}$] $\longrightarrow$ use methods of toric geometry
- Old examples are special cases:

$$\mathbf{Y^{2,3}(\mathbb{C}P^2) = M^{3,2}} \qquad \mathbf{Y^{1,1}(\mathbb{C}P^1 \times \mathbb{C}P^1) = Q^{1,1,1}}$$

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Strategy: analyze associated Calabi-Yau cones $\longrightarrow$ find CS theories with such geometries as VMS

Calabi-Yau cones from $Y^{p,k}$ 7-manifolds

- [MS-0808.0904] studied geometry of two families

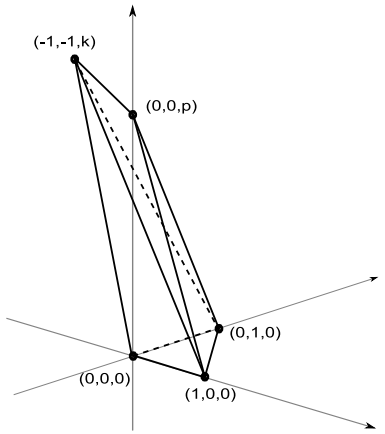
$$Y^{p,k}(\mathbb{C}P^2) \qquad Y^{p,k}(\mathbb{C}P^1 \times \mathbb{C}P^1)$$

Summary of the analysis

- Explicit volumes and sub-volumes: roots of cubic polynomials in p, k
- Homology: $b_2(Y^{p,k}) = 1, 2$ respectively
- GLSM description of toric Calabi-Yau cones $C(Y^{p,k})$
- Reduction to $AdS_4 \times M_6$ $\mathcal{N} = 2$ solutions of type IIA

Toric diagrams and GLSM charges: $Y^{p,k}(\mathbb{C}P^2)$

- Calabi-Yau cones $C(Y^{p,k})$ are toric. Geometry encoded in toric diagrams: set of d vertices $v_a \in \mathbb{Z}^4$ of a polytope



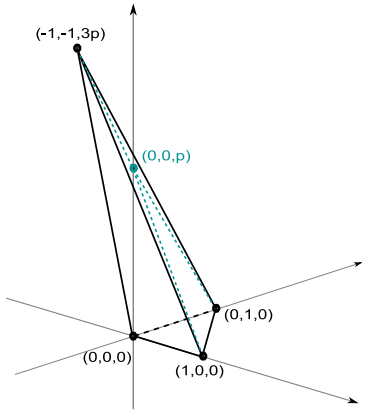
- Kernel of the map $\mathbb{R}^5 \ni E_a \longrightarrow v_a \in \mathbb{R}^4$ gives charges of GLSM quotient (Kähler quotient):

$$C(Y^{p,k}(\mathbb{C}P^2)) = \mathbb{C}^5 // U(1)_Q \qquad Q = (-3p+k, -k, p, p, p)$$

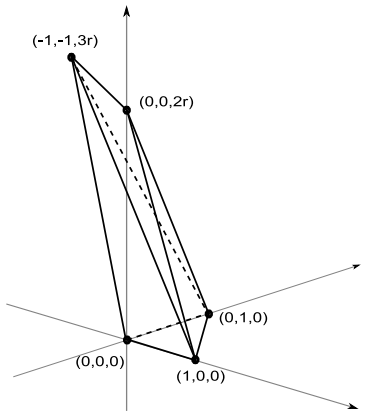
$$\mathrm{vol}\left(Y_7^{1,2}(\mathbb{C}P^2)\right) = \frac{3\pi^4}{64}\left[\frac{107}{27} + \left(\frac{521}{54} - \sqrt{78}\right)\left(53+6\sqrt{78}\right)^{1/3} + \left(\frac{2341}{54} - \frac{44}{9}\sqrt{78}\right)\left(53+6\sqrt{78}\right)^{2/3}\right]$$

Special cases

Toric diagram for the orbifold $\mathbf{Y^{p,3p}(\mathbb{C}P^2) = S^7/\mathbb{Z}_{3p}}$



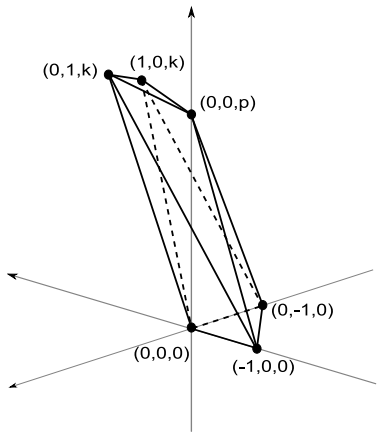
Toric diagram for $\mathbf{Y^{2r,3r}(\mathbb{C}P^2) = M^{3,2}/\mathbb{Z}_r}$



Toric diagrams and GLSM charges: $\mathbf{Y^{p,k}(\mathbb{C}P^1 \times \mathbb{C}P^1)}$

- Kernel of the map $\mathbb{R}^{\mathbf{6}} \ni \mathbf{E_a} \longrightarrow \mathbf{v_a} \in \mathbb{R}^{\mathbf{4}}$ gives GLSM charges:

$$\mathbf{C(Y^{p,k}(\mathbb{C}P^1 \times \mathbb{C}P^1)) = \mathbb{C}^6//U(1)_Q} \quad \mathbf{Q} = \begin{pmatrix} -2p+k & -k & p & p & 0 & 0 \\ -2p+k & -k & 0 & 0 & p & p \end{pmatrix}$$



- $\mathbf{p = k = 1} \longrightarrow$ old $\mathbf{Q^{1,1,1}}$ with $\mathbf{SU(2)^3 \times U(1)}$ isometry

$AdS_4 \times M_6$ type IIA backgrounds

- Can be reduced to type IIA along a circle with non-constant (but non-vanishing) norm

$$ds^2_{st}(IIA) = \sqrt{w(\rho)}\,\ell R^3 \left(\frac{1}{4}ds^2(AdS_4) + ds^2(M_6)\right)$$

$$F_4 = \frac{3}{8}R^3\,\mathrm{vol}(AdS_4)$$

- Radius of curvature $R^6 = \dfrac{(2\pi l_p)^6 N}{6\,\mathrm{vol}[Y_7](p,k)}$. $\qquad \ell = \ell(p,k)$

- $M_6(B_4)$ are bundles $S^2 \longrightarrow B_4$. Not Einstein manifolds

- Cf. with ABJM: $Y_7 = S^7$ and $M_6 = \mathbb{C}P^3$ with k units of RR F_2

$AdS_4 \times M_6$ type IIA backgrounds

- We have non-constant dilaton and F_2 on various two-cycles in M_6

$$e^{2\Phi} = \ell^3 R^3 (w(\rho))^{3/2} \qquad \text{bounded}$$

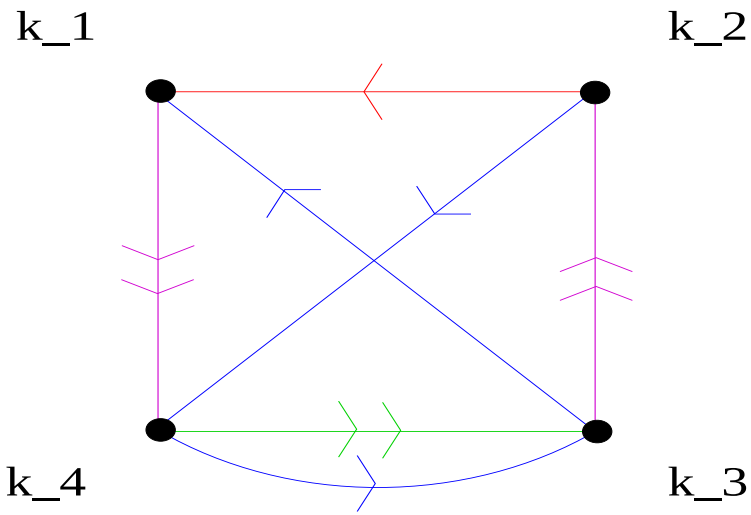
$$\int_{\Sigma} \frac{F_2}{2\pi} = p \qquad \int_{\sigma^N \Sigma_i} \frac{F_2}{2\pi} = k$$

$$R_{11} = \ell R \sim \frac{\ell N^{1/6}}{\text{vol}(Y_7)^{1/6}}$$

- M-theory description valid when $N >> p^5$. Type IIA description valid when $p^5 >> N >> p$

$\mathcal{N} = 2$ Chern-Simons quivers

- [MS-0808.0912] studied the VMS of general $\mathcal{N} = 2$ Chern-Simons quiver gauge theories
- Gauge group $\mathbf{G} = \prod_{\mathbf{i}}^{\mathbf{n}} \mathbf{U(N_i)_{k_i}}$ (will take $\mathbf{N_i = N}$ here)
- **node** = CS term at level $\mathbf{k_i}$ arrow = $\mathbf{(\bar{N}_i, N_j)}$ "chiral" field



$\mathcal{N} = 2$ Chern-Simons Lagrangian

- General $\mathcal{N} = 2$ Lagrangian

$$S = S_{CS} + S_{matter} + S_{potential}$$

$$S_{CS} = \sum_{i=1}^{n} \frac{k_i}{4\pi} \int \mathrm{Tr} \left(A_i \wedge dA_i + \frac{2}{3} A_i \wedge A_i \wedge A_i - \bar{\chi}_i \chi_i + 2 D_i \sigma_i \right)$$

$$S_{matter} = \int d^3x d^4\theta \sum_a \mathrm{Tr}\, \bar{\Phi}_a e^V \Phi_a$$

$$S_{potential} = \int d^3x d^2\theta\, W(\Phi) + c.c.$$

- $\mathcal{N} = 3$ requires special (quartic) W. We keep it general
- Same Lagrangian for $\mathcal{N} = 1$ quivers in 4D, with $S_{YM} \longrightarrow S_{CS}$

Moduli spaces

- Consider Abelian theories for simplicity: $\mathbf{G = U(1)^n}$
- After integrating out the auxiliary fields $\mathbf{D_i}$, the total (bosonic) potential is $\mathcal{V} = \mathcal{V}_D + \mathcal{V}_F$

$$\mathcal{V}_F = \sum_{a \in \mathcal{A}} \left| \frac{\partial W}{\partial \phi_a} \right|^2$$

$$\mathcal{V}_D = \sum_{a \in \mathcal{A}} |\phi_a|^2 (\sigma_{h(a)} - \sigma_{t(a)})^2$$

- In the process we get effective D-terms:

$$- \sum_{a|h(a)=i} |\phi_a|^2 + \sum_{a|t(a)=i} |\phi_a|^2 = \frac{k_i \sigma_i}{2\pi} \quad \forall \quad i$$

- The usual 4d D-terms are LHS $= 0$

Supersymmetric vacua

- $\mathcal{V}_D, \mathcal{V}_F$ must vanish separately
  - F-terms: $\dfrac{\partial W}{\partial \phi_a} = 0 \quad \rightarrow \quad \mathcal{Z} = \{dW = 0\} \subset \mathbb{C}^{|\mathcal{A}|}$
  - $\sigma_1 = \sigma_2 = \cdots = \sigma_n \equiv s$
- D-terms:

$$\mathcal{D}_i = -\sum_{a|h(a)=i} |\phi_a|^2 + \sum_{a|t(a)=i} |\phi_a|^2 = \frac{s k_i}{2\pi} \quad \forall \quad i$$

Necessary condition

$$\sum_{i=1}^{n} k_i = 0$$

Gauge symmetries

- We should mod by gauge transformations. Naively mod by: $\mathbf{U(1)^{n-1} \cong U(1)^n/U(1)}$. Problematic...
- A particular $\mathbf{U(1)}$ gauge symmetry is broken to a discrete subgroup $\mathbb{Z}_\mathbf{h}$ by background monopoles. Defining

$$\mathbf{a} = \sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{A_i} \qquad \mathbf{b} = \frac{\mathbf{1}}{\mathbf{h}} \sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{k_i A_i} \qquad \mathbf{h} = \mathbf{hcf(k_i)}$$

the (Abelian) CS action is

$$\mathbf{S_{CS}(A_i)} = \frac{\mathbf{h}}{\mathbf{2\pi n}} \int \mathbf{b} \wedge \mathbf{f} + \mathbf{S'}$$

where under $\mathbf{A_i} \longrightarrow \mathbf{A_i} + \lambda$, $\delta \mathbf{S'} = \mathbf{0}$. Then we can dualize $\mathbf{b}$ to a periodic scalar τ integrating out $\mathbf{f}$

$$\mathbf{b} = \frac{\mathbf{n}}{\mathbf{h}} \mathrm{d}\tau \qquad \tau \in [0, \frac{2\pi}{\mathbf{n}}] \quad (*)$$

- Gauge transformations:

$$A_i \to A_i + d\theta_i \ , \qquad \sum_{i=1}^{n} k_i \theta_i = 0 \ ; \qquad \tau \to \tau + \frac{h}{n}\theta$$

Thus $\theta = \frac{2\pi l}{h}$ $l = 1, \ldots, h$ is a residual gauge symmetry. Kernel of the character

$$\chi_k : U(1)^n \to U(1)$$

$$\left(e^{i\theta_1}, \ldots, e^{i\theta_n}\right) \mapsto \exp\left(i \sum_{i=1}^{n} k_i \theta_i\right)$$

- The gauge symmetries are then the group

$$H_k = \ker \chi_k / U(1) \cong U(1)^{n-2} \times \mathbb{Z}_h$$

- The $U(1)^{n-2}$ part is the same group for which we imposed D-terms

$$\sum_{i=1}^{n} v_i \mathcal{D}_i = 0, \qquad v \in \ker(k)$$

Periodicity of τ

- Remains to justify $\tau \in [0, \frac{2\pi}{n}]$

$$\mathbf{b} = \frac{1}{h}\sum_{i=1}^{n} k_i \mathbf{A}_i \qquad S_{CS}(\mathbf{A}_i) = \frac{h}{2\pi n}\int \mathbf{b}\wedge\mathbf{f} + S'$$

- $\mathbf{b}$ is dualized adding S_τ

$$S_\tau = -\frac{1}{2\pi}\int d\tau \wedge \mathbf{f} \quad \Rightarrow \quad \mathbf{b} = \frac{n}{h} d\tau$$

- Periodicity fixed by $\int_{\sigma_2} \mathbf{f} \neq 0$.
  - $\int_{\sigma_2} \mathbf{F}_i \in 2\pi\mathbb{Z}$
  - $\sum_{a\in\mathcal{A}} |\phi_a|^2 (\mathbf{A}_{h(a)} - \mathbf{A}_{t(a)})^2 = 0 \quad \Rightarrow \quad \mathbf{F}_1 = \cdots = \mathbf{F}_n$

$$\int_{\sigma_2} \mathbf{f} \in 2\pi n\,\mathbb{Z}$$

Moduli space: summary of main results

- The VMS contains a $\mathbf{4}_{\mathbb{C}}$-dimensional branch

$$\mathscr{M}_{\mathbf{3d}}(\mathbf{k}) = \mathscr{Z}//\mathbf{H_k}$$

- A further quotient by the broken $\mathbf{U(1)}$ gives a $\mathbf{3}_{\mathbb{C}}$-dimensional space

$$\mathscr{M}_{\mathbf{4d}} = \mathscr{M}_{\mathbf{3d}}(\mathbf{k})//\mathbf{U(1)}$$

- $\mathscr{Z}$ is the baryonic VMS of the parent 4d $\mathcal{N} = \mathbf{1}$ quiver theory

- $\mathscr{M}_{\mathbf{4d}}$ Calabi-Yau 3-fold $\Rightarrow$ $\mathscr{M}_{\mathbf{3d}}(\mathbf{k})$ Calabi-Yau 4-fold!

- For $\mathbf{N} > \mathbf{1}$: $\mathscr{M}_{\mathbf{3d,N}}(\mathbf{k}) = \mathbf{Sym}^{\mathbf{N}}\, \mathscr{M}_{\mathbf{3d,1}}(\mathbf{k})$

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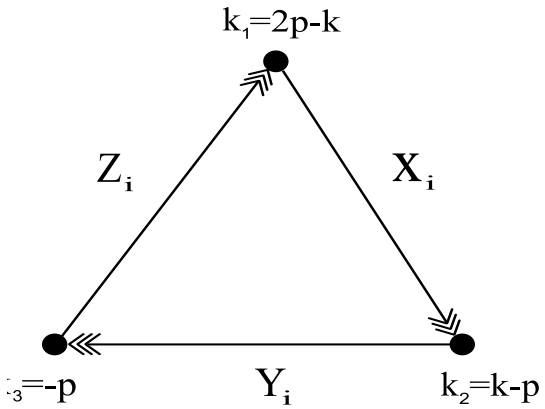
- For $\mathbf{N} > \mathbf{1}$: $\mathscr{M}_{\mathbf{3d,N}}(\mathbf{k}) = \mathbf{Sym}^{\mathbf{N}}\, \mathscr{M}_{\mathbf{3d,1}}(\mathbf{k})$

- Let's look at the ABJM model: $\mathbf{G} = \mathbf{U(1)_k} \times \mathbf{U(1)_{-k}}$

  $\mathscr{Z} = \mathbb{C}^4$, $\mathbf{H_K} = \mathbb{Z}_\mathbf{k}$, $\mathscr{M}_{\mathbf{3d}}(\mathbf{k}) = \mathbb{C}^4/\mathbb{Z}_\mathbf{k}$, $\mathscr{M}_{\mathbf{4d}}(\mathbf{k})$ =conifold

CS quivers for $Y^{p,k}(\mathbb{C}P^2)$ Sasaki-Einstein metrics

- Consider the following quiver gauge theory: 3 nodes, 9 fields X_i, Y_i, Z_i $(i = 1, 2, 3)$ and cubic superpotential $W = \epsilon_{ijk} \mathrm{Tr}\,(X_i Y_j Z_k)$



- In 4d language this is dual to a simple orbifold model $\mathbb{C}^3/\mathbb{Z}_3$

- The space of F-term solutions is

$$\mathcal{Z} = \mathbb{C}^6 // \mathrm{U}(1)_Q \qquad Q = (1, 1, 1, -1, -1, -1)$$

- Group of residual gauge symmetries is (take hcf($\mathbf{k_i}$)=1 to begin with)

$$\mathbf{H_k = U(1)_{Q'}} \qquad \mathbf{Q' = (p, -2p + k, p - k, 0, 0, 0)}$$

$$\mathscr{M}_{\mathbf{3d}}\mathbf{(k)} = \mathbb{C}^{\mathbf{6}}//\mathbf{U(1)^2} \cong \mathbf{C(Y^{p,k}(\mathbb{C}P^2))} \qquad \mathbf{p \leq k \leq 2p}$$

- Can assign geometric R-charges using the formula

$$\mathbf{R_a} = \frac{\pi\,\mathbf{vol}[\Sigma_\mathbf{a}]}{\mathbf{6\,vol(Y_7)}} \qquad \Sigma_\mathbf{a} \text{ supersymmetric submanifolds}$$

$$\mathbf{R[X_i] = R_1 = R_2 = R_3} \quad \mathbf{R[Y_i] = R_1 + R_4} \quad \mathbf{R[Z_i] = R_1 + R_5}$$

- These satisfy automatically $\mathbf{R[X_i] + R[Y_j] + R[Z_k] = 2}$
- At present it is not known how to compute these R-charges in the field theory :-(

Summary

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- Analyzed geometry and topology of concrete examples $\longrightarrow$ found they correspond to M2-branes at toric Calabi-Yau singularities
- Studied the VMS of general $\mathcal{N} = 2$ Chern-Simons quivers. For large classes there is a branch which is a (toric) Calabi-Yau four-fold
- Discussed examples of families of **3D** $\mathcal{N} = 2$ CS quivers, whose VMS are Calabi-Yau cone metrics $\longrightarrow$ explicit AdS_4 gravity duals (TBC)

Directions

- Useful to construct more examples of CS quivers with CY_4 moduli spaces [Hanany-Vegh-Zaffaroni]

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- Useful to construct more examples of CS quivers with CY_4 moduli spaces [Hanany-Vegh-Zaffaroni]
- Improve algorithms for constructing the VMS of CS quivers, and vice-versa. Tilings [Ueda-Yamazaki,Imamura-Kimura] and crystals [Sa. Lee,S. Kim-Sa. Lee-Su. Lee-J. Park,Sa. Lee-Su. Lee-J. Park]

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- Improve algorithms for constructing the VMS of CS quivers, and vice-versa. Tilings [Ueda-Yamazaki,Imamura-Kimura] and crystals [Sa. Lee,S. Kim-Sa. Lee-Su. Lee-J. Park,Sa. Lee-Su. Lee-J. Park]

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- Different reductions to type IIA $\longrightarrow$ different $\mathbf{M_6}$'s with same M-theory geometry (should be related to previous point)

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- Field theory duals of other classes of AdS_4 geometries. For example: weak $\mathbf{G_2}$; warped AdS_4; massive type IIA