The D3-D7 Model in Electric and Magnetic Fields

formerly known as

"Two Uses of Kalb-Ramond Fields in AdS/CFT with Flavour"

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September 24, 2008

JHEP 0712:091,2007 (with J. Erdmenger & J. P. Shock)
JHEP 0807:068,2008 (with M. Ammon, JE, S. Höhne & D. Lüst)

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Main Message: JHEP 0712:091,2007

- The effect of a pure gauge Kalb-Ramond Field

$$B_{mag} = B\mathrm{d}y \wedge \mathrm{d}z\,, \quad B_{el} = B\mathrm{d}t \wedge \mathrm{d}x$$

on one probe D7 brane in $AdS_5 \times S^5$:

1. Magnetic Field: Induced SCSB and Prevention of Meson Melting
2. Electric Field: Dissociation of Mesons

Outline

Outline

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The AdS/CFT Correspondence

The Original Correspondence (weakest form)

IIB Supergravity on $AdS_5 \times S^5$ with $(R/l_s)^4 = 4\pi\lambda \gg 1$

$$\mathrm{d}s^2_{AdS_5\times S^5} = R^2\left(\frac{\mathrm{d}x^{\mu 2} + \mathrm{d}u^2}{u^2} + \mathrm{d}\Omega_5^2\right)$$

$$\Leftrightarrow$$

large N_c limit of $\mathcal{N} = 4$ $SU(N_c)$ Super Yang-Mills with $\lambda = g^2_{YM}N_c$

1. Strong-Weak Coupling Duality
2. Operator-Field Dictionary:

$$\phi_{m^2=\Delta(\Delta-4)} \simeq u^{4-\Delta}J_{\mathcal{O}} + u^{\Delta}\langle\mathcal{O}\rangle$$

3. Symmetries are important: $SO(4,2)\times SU(4)$

Introducing Flavour

Extended Correspondence: N_f D7 Branes in $AdS_5 \times S^5$

4d $\mathcal{N} = 4$ SU(N) Super Yang-Mills theory
coupled to
4d N_f $\mathcal{N} = 2$ hypermultiplets

$\leftrightarrow$

type IIB SUGRA on $AdS_5 \times S^5$
+
Dirac-Born-Infeld theory on D7

The Right Embedding

$$\mathrm{d}s^2_{AdS_5 \times S^5} = \frac{\rho^2 + L^2}{R^2} \mathrm{d}x^{\mu 2} + \frac{R^2}{\rho^2 + L^2} \left(\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega_3^2 + \mathrm{d}L^2 + L^2 \mathrm{d}\Phi^2 \right)$$

Embedding: $\xi^\alpha = (x^\mu, \rho, S^3)\,, \quad L = L(\rho)$

$$\mathcal{L}_{DBI} \propto \rho^3 \sqrt{1 + L'^2} \Rightarrow L = 2\pi\alpha' m_q = const.$$

Introducing Flavour: Field Theory

$\mathcal{N}=4$ Glue & N_f $\mathcal{N}=2$ Quarks

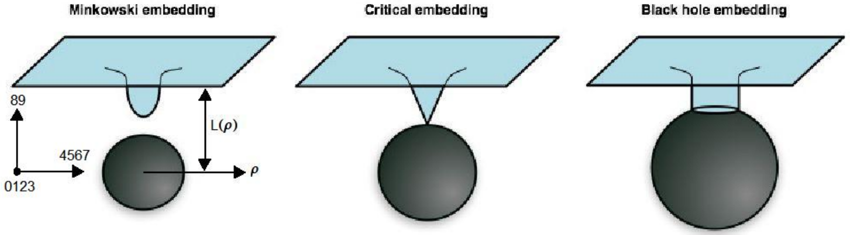
$$\mathcal{L}=\Im\Big[\tau\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\left(\mathrm{tr}(\bar{\Phi}_i e^V \Phi_i e^{-V})+Q_I^\dagger e^V Q^I+\tilde{Q}_I e^V \tilde{Q}^{I\dagger}\right)+$$
$$+\tau\int \mathrm{d}^2\theta\left(\mathrm{tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)+\mathrm{tr}(\epsilon_{ijk}\Phi_i\Phi_j\Phi_k)+\tilde{Q}_I(m+\Phi_3)Q^I\right)\Big],$$
$$I=1,\ldots,N_f$$

1. Field Content:
   - $\mathcal{N}=4$ Glue Vector Multiplet: V, Φ_i , $i=1,2,3$
   - $\mathcal{N}=2$ “Quark” Hypermultiplet: Q , $\tilde{Q}$
2. Global Symmetries ($m=0$):
   $SO(4,2)\times SU(2)_\Phi\times SU(2)_\mathcal{R}\times U(1)_\mathcal{R}\times U(N_f)$
3. Conformal in the $N_c\to\infty$ limit
4. Quenched Approximation!

Flavour at Finite Temperature

Flavour Physics at Finite Temperature

AdS-Schwarzschild $\times$ S^5 (Black brane) , $\quad T = T_{\text{Hawking}} \propto r_s$



Picture: [hep-th/0611099]

1. Embedding: $L(\rho) \overset{\rho\to\infty}{\sim} 2\pi\alpha' m_q + \frac{(2\pi\alpha')^3}{\rho^2}\langle\bar{\psi}\psi\rangle$
2. No Spontaneous CSB: $\langle\bar{\psi}\psi\rangle(m_q = 0) = 0$
3. Fluctuations: Mesons $\rightarrow$ Meson Melting Transition

Electric/Magnetic B-Field [see also 0709.1547, 0709.1554 (hep-th)]

Ansatz for the Kalb-Ramond Field

$$B_{el} = B\mathrm{d}t \wedge \mathrm{d}x \,, \quad B_{mag} = B\mathrm{d}y \wedge \mathrm{d}z$$

- $\mathrm{d}B = 0 \Rightarrow$ No deformation of AdS-Schwarzschild

$$-\frac{T_7}{g_2}\sqrt{-det\left(P[G+B]+2\pi\alpha' F\right)}+\sum_p P[C_p \wedge e^B] \wedge e^{2\pi\alpha' F}$$

- Affects Brane (Flavour) physics: D7 embeddings , thermodynamics, phase transitions , meson spectra ...
- Effect: Background for $U(1)_F$ gauge field, mimics constant $U(1) \in U(N_c)$ field strength

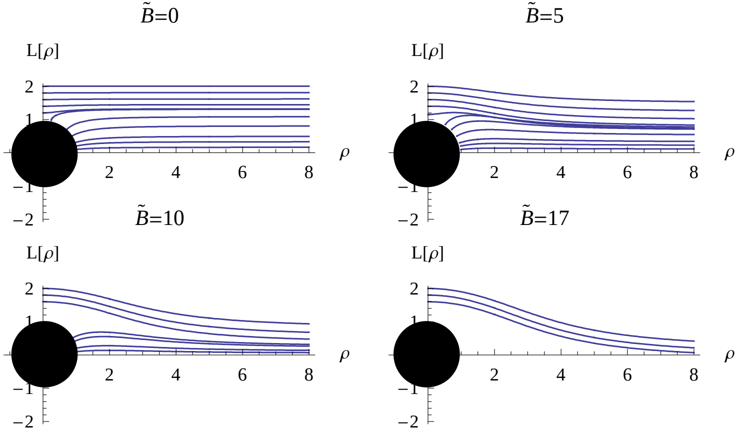
Magnetic $B_{\mu\nu}$

Magnetic Field

$$B_{mag} = B \mathrm{d}y \wedge \mathrm{d}z$$

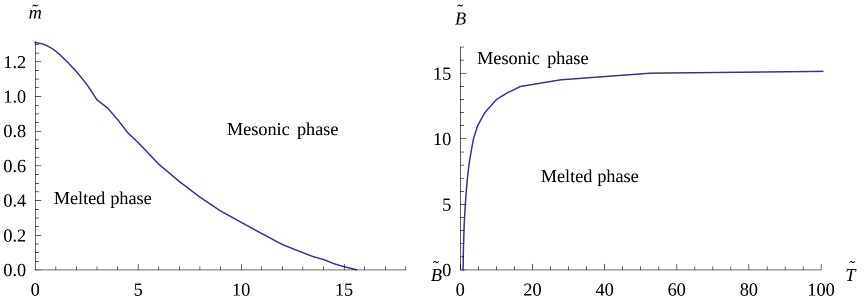
- Zero Temperature: [Filev etal.hep-th/0701001]
  CSB, Goldstone Boson with $M \propto \sqrt{m_q}$ for small m_q, Zeeman splitting
- Finite Temperature:
  - Small B: Meson Melting Transition
  - No molten phase & CSB above a critical magnetic field strength (GMOR)
  - Phase diagram
  - Spectrum of Pseudoscalar Mesons

Magnetic Finite Temperature Embeddings



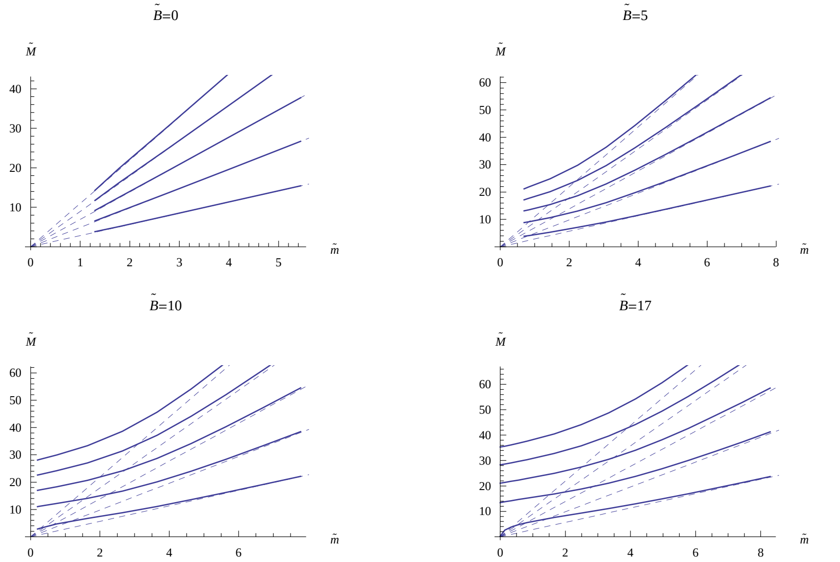
$$\chi SB : \quad \tilde{B}_{crit} \approx 16 \,, \quad \tilde{B} \propto \frac{B}{T^2}$$

Phase Diagram $\tilde{m} = 2m_q/(\sqrt{\lambda}T)$, $\tilde{T} = \tilde{m}^{-1}$

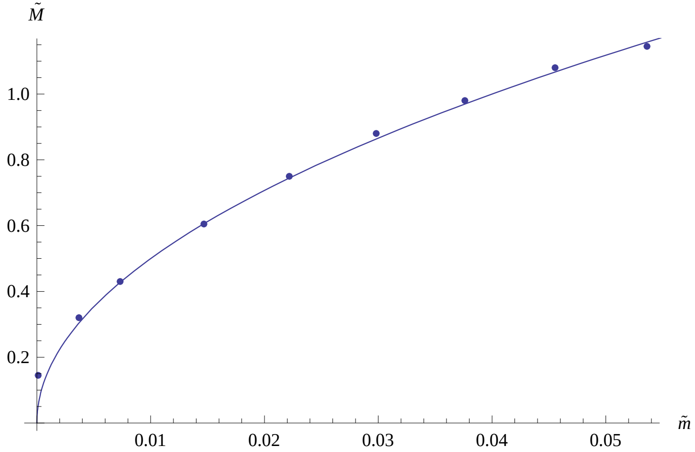


- Meson Melting Transition below $\tilde{B}_{crit}$
- No molten phase and spontaneous CSB above $\tilde{B}_{crit}$
- Magnetic KR-Field acts repells the D7s from the origin

Φ Meson Spectrum (Upper Branch, $\tilde{M} = \frac{M\sqrt{\lambda}}{\sqrt{\pi}\, m_q}$)



Goldstone Boson ($\tilde{B} = 16$)



$$\tilde{M} = 5\sqrt{\tilde{m}}$$

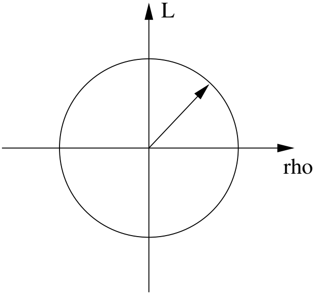
Electric $B_{\mu\nu}$

$$B_{el} = B \mathrm{d}t \wedge \mathrm{d}x$$

- Problem: Zero Locus of DBI action

$$\sqrt{-\det P[G+B]} = 0 \quad \text{for } \rho_{IR}^2 + L(\rho_{IR})^2 = \frac{BR^2}{2} + \frac{1}{2}\sqrt{4r_s^4 + B^2 R^4}$$

Zero Locus



Electric $B_{\mu\nu}$

$$B_{el} = B \mathrm{d}t \wedge \mathrm{d}x$$

- Solution: $U(1)_f$ gauge field $A_x(\rho), A_t(\rho)$ [hep-th:0705.3890]

$$\partial_a \left(\frac{\delta \mathcal{L}_{D7}}{\delta \partial_a A_b} \right) = 0 \;\Rightarrow\; \underline{\text{Two Conserved Quantities}}$$

$$A_t(\rho) \simeq \mu - \frac{\mathcal{D}}{\rho^2} \leftrightarrow \text{finite baryon number density } \langle J_t \rangle = \mathcal{D}$$

$$A_x(\rho) \simeq \frac{\mathcal{B}}{\rho^2} \leftrightarrow \text{baryon number current in } x\text{-direction } \langle J_x \rangle = \mathcal{B}$$

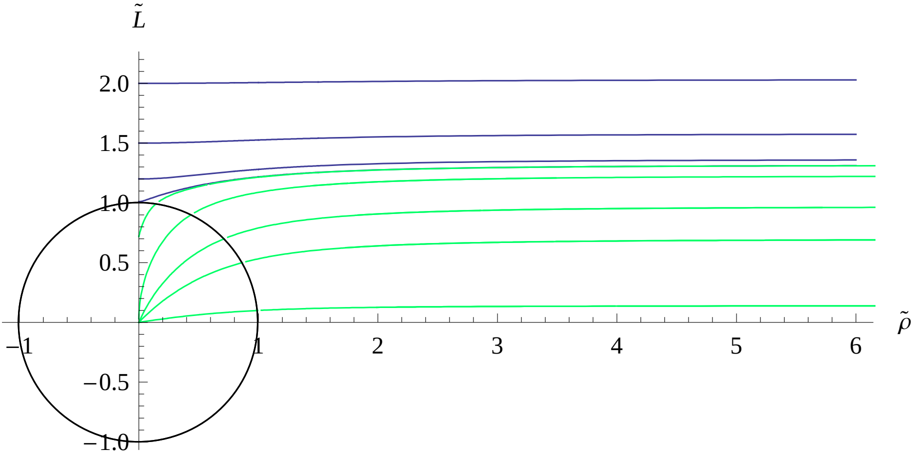
$$\Rightarrow A_x'(\rho) = f'(\mathcal{D}, \mathcal{B}, \rho)\,,\; A_t'(\rho) = g'(\mathcal{D}, \mathcal{B}, \rho) \;\Rightarrow \text{Legendre transform} \Rightarrow$$

Require $\tilde{S}_{D7}[L(\rho), \mathcal{D}, \mathcal{B}]$ to be well-defined :

$$\mathcal{B} = \mathcal{B}(\rho_{IR}, T, B, \mathcal{D})$$

$\Rightarrow$ Well-defined EOMs for $L(\rho)$!

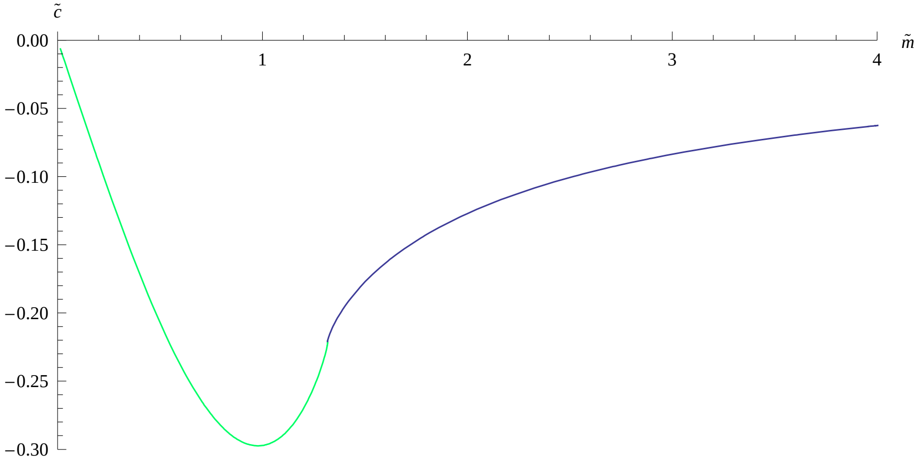
Electric Embeddings at $T = 0$: No CSB, Phase Transition



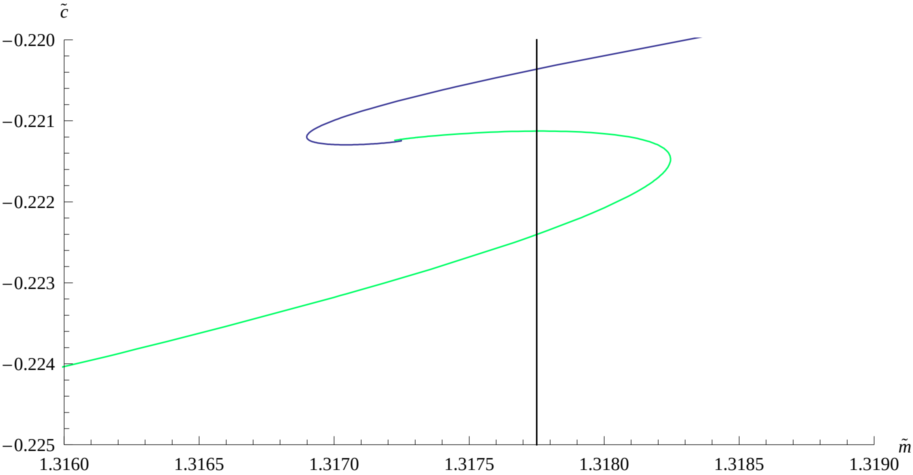
→ Dissociation of Mesons?

→ Conical singularities?

Condensate vs. Mass

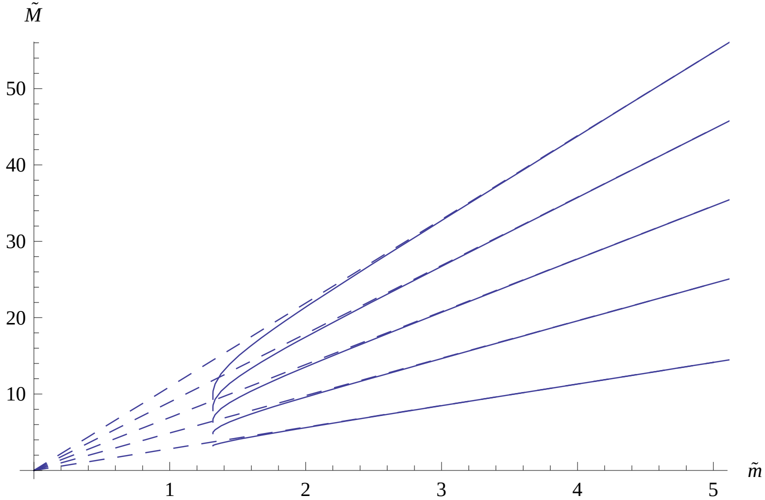


Condensate vs. Mass: Phase Transition



Area$\propto$ F

Φ (l=0) Meson Spectrum at $T = 0$: $\Delta M < 0$



Incoming Wave Boundary Conditions → Dissociation of Mesons!

Electric $B_{\mu\nu}$: Finite Temperature

What to expect at Finite Temperature?

- Meson melting enhanced by dissociation
- Any nonzero electric field will decrease the melting temperature
- No SCSB
- Finite T: One or two transitions?

Open Questions

- Fate of the conically singular solutions? They are either
  - physical $\rightarrow$ What creates the singularity?
  - unphysical $\rightarrow$ What happens in that mass range?
- Energy of the system is time-dependent $\rightarrow$
  - Is this still equilibrium physics?
  - Thermodynamics in the presence of external currents?

Summary: Electric/Magnetic Background Fields

- Magnetic Background
  - Induced SCSB
  - Magnetic Field Stabilizes Mesons
  - Mesons don't melt for large enough B (SCSB)
  - Zeeman splitting
- Electric Background
  - No SCSB
  - Dissociation (& Meson Melting)
  - Stark shift

Backup Slides

Condensate vs. Mass